# CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

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| CANDIDATE NAME | |

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## ADDITIONAL MATHEMATICS — 4049/01
Paper 1

**30 August 2023**

**2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

| For Examiner's Use |
|---|
| 90 |

This document consists of **21** printed pages and **1** blank page.

**[Turn over**

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 Express $\dfrac{2x^3-4x^2+x-18}{x^3-2x^2+4x-8}$ in partial fractions. [6]

**2** Two vertices of a rhombus *ABCD* are $A(-2, -5)$ and $C(4, 7)$.

**(a)** Find the equation of the diagonal *BD*. [3]

If the gradient of the side *BC* is 3, find

**(b)** the coordinates of *B* and of *D*. [4]

**3** The equation of a curve is $y = x^3 + hx^2 + kx + 9$, where $h$ and $k$ are constants.

**(a)** Show that if $y$ increases as $x$ increases, then $3k - h^2 > 0$. [3]

**(b)** In the case when $h = -5$ and $k = 3$, find the $x$-coordinate of each of the points at which the curve meets the $x$-axis. [3]

**4** **(a)** Given that the constant term in the binomial expansion of $\left(x+\frac{k}{x}\right)^6$ is $-160$, find the value of the constant $k$. [3]

**(b)** Using the value of $k$ found in part **(a)**, show that there is no constant term in the expansion of $\left(x+\frac{k}{x}\right)^6\left(2x^2+3\right)$. [3]

**5** **(a)** The equation of a quadratic curve is $y = 2x^2 + px + 16$. Given that $y < 0$ only when $2 < x < k$, find the value of *p* and of *k*. [3]

**(b)** In the case where $p = -14$, find the value of *m* for which the line $y = 2x + m$ is a tangent to the quadratic curve, $y = 2x^2 + px + 16$. [3]

**6** Mary and Sally took part in a shot put competition. The heights, in metres, of Mary's and Sally's shot put throws can be modelled by the quadratic functions
$\mathrm{f}(x) = -\frac{7}{180}(x-6)^2 + 3$ and $\mathrm{g}(x) = -\frac{1}{35}x^2 + \frac{2}{5}x + \frac{8}{5}$ respectively, where $x$ m is the horizontal distance of the shot put from the starting line.

**(a)** Express g($x$) in the form $\mathrm{g}(x) = a(x+b)^2 + c$ where $a$, $b$ and $c$ are constants. [2]

**(b)** Evaluate f(0) and g(0) and hence interpret the meaning of your answers. [2]

**(c)** The winner of the competition is the one whose shot put has the further horizontal distance from the starting line. Explain mathematically who is the winner of the competition. [3]

**7** The table shows experimental values of two variables $x$ and $y$.

| $x$ | 0.5 | 1.3 | 2.1 | 3.5 | 4.3 | 5.5 |
|---|---|---|---|---|---|---|
| $y$ | 3.3 | 2.5 | 2 | 1.5 | 1.3 | 1.1 |

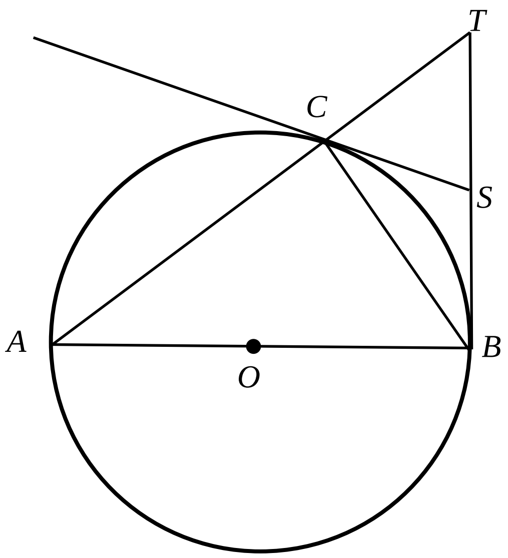
It is known that $x$ and $y$ are related by the equation $y = \dfrac{a}{x+b}$, where $a$ and $b$ are constants.

**(a)** On the grid on page 11, plot $xy$ against $y$ and obtain a straight line graph. [2]

**(b)** Use your graph on page 11 to estimate the value of $a$ and of $b$. [4]

**(c)** Obtain the value of the gradient of the straight line obtained when $\dfrac{1}{y}$ is plotted against $x$. [2]

**8** In the diagram, $AB$ is a diameter of the circle with centre $O$. $CS$ and $BT$ are the tangents to the circle at $C$ and $B$ respectively. $ACT$ and $BST$ are straight lines.




**(a)** Prove that triangle $TCS$ is an isosceles triangle. [4]

**(b)** Show that $AB^2 = AC \times AT$ . [4]

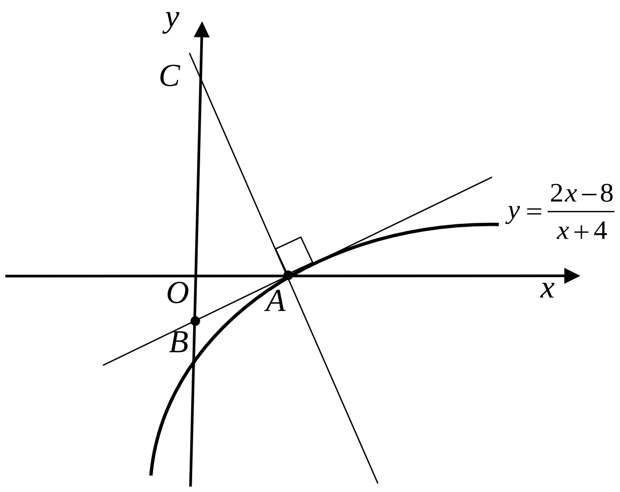
**9** It is given that $x$ is a function of $t$, $\frac{dx}{dt} = 1 - e^{2t}$ and $x = 2$ when $t = 0$.

**(a)** Express $x$ in terms of $t$. [3]

It is also given that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 5x + \sqrt{x+5}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 60$ when $x = 4$.

**(b)** Find the value of $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $t = 1$. [5]

**10** The diagram shows part of the curve $y = \dfrac{2x-8}{x+4}$, $x > -4$.




**(a)** Explain why the curve $y = \dfrac{2x-8}{x+4}$ does not have a stationary point. [2]

**(b)** The curve cuts the *x*-axis at *A*. The tangent and the normal to the curve at *A* intersect the *y*-axis at *B* and *C* respectively.

**(i)** Find the equation of the normal *AC*. [3]

**(ii)** Find the area of triangle *ABC*. [4]

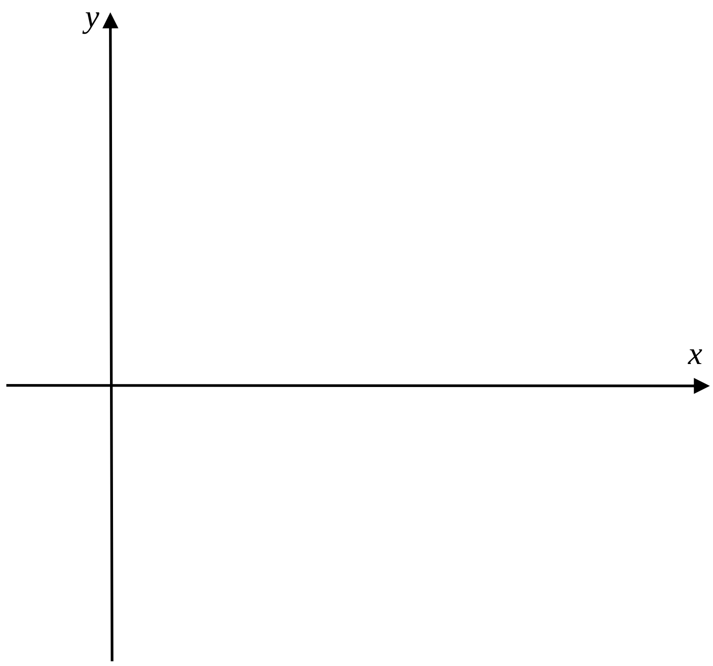
**(c)** By expressing $\dfrac{2x-8}{x+4} = D + \dfrac{E}{x+4}$, explain why the line $y = 2$ does not intersect the curve. [2]

**11** The curve $y = a\cos bx + c$, where $a$, $b$ and $c$ are positive integers, is defined for $0 \le x \le \pi$.

The curve has an amplitude of 3 and a period of $\frac{\pi}{3}$ radians. The minimum value of $y$ is 4.

**(a)** State the value of $a$, $b$ and $c$. [3]

**(b)** Sketch the graph of $y = a\cos bx + c$ for $0 \le x \le \pi$. [3]



**(c)** On the same axes in part **(b)**, sketch the graph of $y = -\frac{3}{\pi}x + 10$ for $0 \le x \le \pi$. [1]

**(d)** Hence, for $0 \le x \le \pi$, state the number of solutions of the equation

$$-3x + 10\pi = \pi\left(a\cos bx + c\right).$$ [2]

**12** A particle moves in a straight line and passes a fixed point $O$. The velocity, $v$ m/s, of the particle, $t$ seconds after passing $O$, is given by $v = 6t^2 + mt + 9$, where $m$ is a constant. The particle travels with a deceleration of $9\text{ m/s}^2$ when $t = 1$.

**(a)** Show that the value of $m$ is $-21$. [1]

**(b)** Find the value(s) of $t$ when the particle is at instantaneous rest. [2]

**(c)** Explain clearly why the total distance travelled by the particle in the interval from $t = 0$ to $t = 4$ is not obtained by finding the value of the displacement of the particle at $t = 4$. [2]

**(d)** Find the total distance travelled by the particle in the interval $t = 0$ to $t = 4$. [3]

End of Paper

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