CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	Solutions		
CLASS	4	INDEX NUMBER	
CENTRE/ INDEX NO	/		

ADDITIONAL MATHEMATICS

Paper 1

4049/01

30 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **21** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 Express $\dfrac{2x^3-4x^2+x-18}{x^3-2x^2+4x-8}$ in partial fractions. [6]

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad\quad\quad 2 \\ x^3-2x^2+4x-8 & 2x^3-4x^2+x-18 \\ & -\left(2x^3-4x^2+8x-16\right) \\ \hline & \quad\quad\quad\quad -7x-2 \end{array}$$

$$\frac{2x^3-4x^2+x-18}{(x-2)\left(x^2+4\right)}=2+\frac{-7x-2}{(x-2)\left(x^2+4\right)}$$

$-7x-2=A\left(x^2+4\right)+(Bx+C)(x-2)$

When $x=2$, $-14-2=8A \Rightarrow A=-2$

Comparing coefficients of x^2, $0=A+B \Rightarrow B=2$

Comparing constants, $-2=4A-2C \Rightarrow 2C=-8+2=-6$
$C=-3$

$$\frac{2x^3-4x^2+x-18}{x^3-2x^2+4x-8}=2-\frac{2}{x-2}+\frac{2x-3}{x^2+4}.$$

2 Two vertices of a rhombus *ABCD* are $A(-2,-5)$ and $C(4,7)$.

(a) Find the equation of the diagonal *BD*. [3]

Gradient of *AC* = $\dfrac{-5-7}{-2-4}=2$

Gradient of *BD* = $-\dfrac{1}{2}$

Midpoint of *AC* = $\left(\dfrac{-2+4}{2},\quad \dfrac{-5+7}{2}\right)=(1,1)$

Equation of *BD*: $y-1=-\frac{1}{2}(x-1)$

$y=-\dfrac{1}{2}x+\dfrac{3}{2}$

If the gradient of the side *BC* is 3, find

(b) the coordinates of *B* and of *D*. [4]

Equation of *AD*: $y+5=3(x+2)$ or Equation of *BC* : $y-7=3(x-4)$

$y=3x+1$ $\qquad y=3x-5$

At *D*, $-\dfrac{1}{2}x+\dfrac{3}{2}=3x+1$

$x=\dfrac{1}{7}$

$y=3\left(\dfrac{1}{7}\right)+1=\dfrac{10}{7}$

Coordinates of *D* = $\left(\dfrac{1}{7},\dfrac{10}{7}\right)$.

At *B*, $-\dfrac{1}{2}x+\dfrac{3}{2}=3x-5$

$x=\dfrac{13}{7}$

$y=3\left(\dfrac{13}{7}\right)-5=\dfrac{4}{7}$

Coordinates of *B* = $\left(\dfrac{13}{7},\dfrac{4}{7}\right)$.

Making use of mid-point formula, let *B* = (x,y) or *D* = (x,y)

$\dfrac{x+\frac{1}{7}}{2}=1$ and $\dfrac{y+\frac{10}{7}}{2}=1$

$x=\dfrac{13}{7}$ and $y=\dfrac{4}{7}$

Coordinates of *B* = $\left(\dfrac{13}{7},\dfrac{4}{7}\right)$.

$\dfrac{x+\frac{13}{7}}{2}=1$ and $\dfrac{y+\frac{4}{7}}{2}=1$

$x=\dfrac{1}{7}$ and $y=\dfrac{10}{7}$

Coordinates of *D* = $\left(\dfrac{1}{7},\dfrac{10}{7}\right)$.

3 The equation of a curve is $y = x^3 + hx^2 + kx + 9$, where h and k are constants.

(a) Show that if y increases as x increases, then $3k - h^2 > 0$. [3]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2hx + k$$

If y increases as x increases, then $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2hx + k > 0$,

As $3 > 0$, then $b^2 - 4ac < 0$

$(2h)^2 - 4(3)(k) < 0$

$4h^2 - 12k < 0$

$3k - h^2 > 0.$

(b) In the case when $h = -5$ and $k = 3$, find the x-coordinate of each of the points at which the curve meets the x-axis. [3]

Since curve meets x-axis, $x^3 - 5x^2 + 3x + 9 = 0$

Let $\mathrm{f}(x) = x^3 - 5x^2 + 3x + 9$

Since $\mathrm{f}(-1) = (-1)^3 - 5(-1)^2 - 3 + 9 = 0$

$(x+1)$ is a factor of f(x).

$$\begin{array}{r|l} & x^2 - 6x + 9 \\ \hline x+1 & x^3 - 5x^2 + 3x + 9 \end{array}$$

Therefore, $(x^2 - 6x + 9)(x+1) = 0$

$(x-3)^2(x+1) = 0$

$x = 3$ or $x = -1$

4 **(a)** Given that the constant term in the binomial expansion of $\left(x+\frac{k}{x}\right)^6$ is -160, find the value of the constant k. [3]

General term = $\binom{6}{r}(x)^{6-r}\left(\frac{k}{x}\right)^r$

$6-2r=0$

$r=3$

Since constant term is -160,

$\binom{6}{3}k^3=-160$

$k=\sqrt[3]{\frac{-160}{20}}=-2$

(b) Using the value of k found in part **(a)**, show that there is no constant term in the expansion of $\left(x+\frac{k}{x}\right)^6\left(2x^2+3\right)$. [3]

$\left(x-\frac{2}{x}\right)^6\left(2x^2+3\right)=\left(2x^2+3\right)\left(\text{...Term in } x^{-2}+\text{Constant term }+\text{...}\right)$

For term in x^{-2}, $6-2r=-2$

$r=4$

Term in x^{-2} = $\binom{6}{4}(x)^2\left(\frac{-2}{x}\right)^4=\frac{240}{x^2}$

Constant term in expansion = $2\left(240\right)+3(-160)=0$

Hence there is no constant term in the expansion.

5 **(a)** The equation of a quadratic curve is $y = 2x^2 + px + 16$. Given that $y < 0$ only when $2 < x < k$, find the value of p and of k. [3]

Since $2x^2 + px + 16 < 0$ when $2 < x < k$,
$2(x-2)(x-k) < 0$
$2x^2 - (2k+4)x + 4k < 0$
By comparing, $4k = 16 \Rightarrow k = 4$.
By comparing, $p = -(2k+4) = -(8+4) = -12$

(b) In the case where $p = -14$, find the value of m for which the line $y = 2x + m$ is a tangent to the quadratic curve, $y = 2x^2 + px + 16$. [3]

Since line cuts curve, $2x + m = 2x^2 - 14x + 16$.
$2x^2 - 16x + 16 - m = 0$
Since line is a tangent to curve, $b^2 - 4ac = 0$
$(-16)^2 - 4(2)(16-m) = 0$
$(16-m) = 256 \div 8$
$m = -16$

6 Mary and Sally took part in a shot put competition. The heights, in metres, of Mary's and Sally's shot put throws can be modelled by the quadratic functions $f(x) = -\frac{7}{180}(x-6)^2 + 3$ and $g(x) = -\frac{1}{35}x^2 + \frac{2}{5}x + \frac{8}{5}$ respectively, where x m is the horizontal distance of the shot put from the starting line.

(a) Express g(x) in the form $g(x) = a(x+b)^2 + c$ where a, b and c are constants. [2]

$$g(x) = -\frac{1}{35}\left(x^2 - 14x + 7^2 - 7^2\right) + \frac{8}{5}$$
$$g(x) = -\frac{1}{35}\left((x-7)^2 - 49\right) + \frac{8}{5}$$
$$g(x) = -\frac{1}{35}(x-7)^2 + 3$$

(b) Evaluate f(0) and g(0) and hence interpret the meaning of your answers. [2]

$$f(0) = -\frac{7}{180}(0-6)^2 + 3 = 1.6$$
$$g(0) = 1.6$$

Both Mary and Sally threw the shot put from a height of 1.6 m.

(c) The winner of the competition is the one whose shot put has the further horizontal distance from the starting line. Explain mathematically who is the winner of the competition. [3]

As the shot put touches the ground, $\mathrm{f}(x)=0$ and $\mathrm{g}(x)=0$

$-\frac{7}{180}(x-6)^2+3=0$ and $-\frac{1}{35}(x-7)^2+3=0$

$x=\sqrt{\frac{3\times180}{7}}+6=14.8$ $x=\sqrt{3\times105}+7=17.2$

As Mary threw a distance of 14.8 m and Sally a distance of 17.2 m, Sally is the winner.

7 The table shows experimental values of two variables x and y.

x	0.5	1.3	2.1	3.5	4.3	5.5
y	3.3	2.5	2	1.5	1.3	1.1

It is known that x and y are related by the equation $y = \dfrac{a}{x+b}$, where a and b are constants.

(a) On the grid on page 11, plot xy against y and obtain a straight line graph. [2]

y	3.3	2.5	2	1.5	1.3	1.1
xy	1.65	3.25	4.2	5.25	5.59	6.05

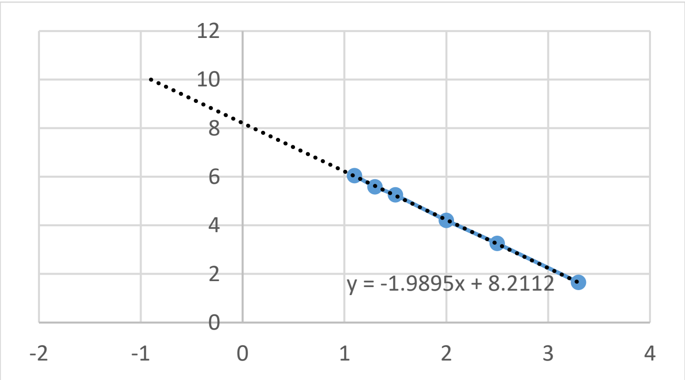
$y = \dfrac{a}{x+b} \Rightarrow xy = -by + a$ where $Y = xy$ and $X = y$, gradient = $-b$ and

Y-intercept = a

(b) Use your graph to estimate the value of a and of b. [4]

$a = 8.0 - 8.4$

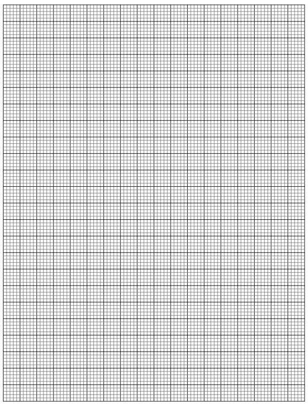
$b = 1.8$ to 2.2



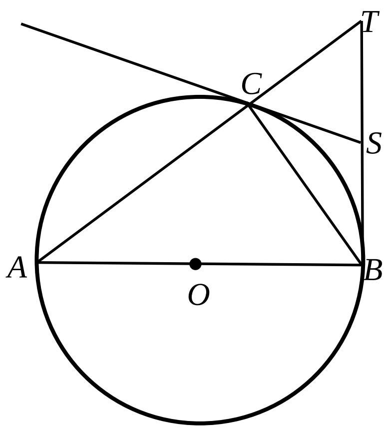
(c) Obtain the value of the gradient of the straight line obtained when $\dfrac{1}{y}$ is plotted against x. [2]

$y = \dfrac{a}{x+b} \Rightarrow \dfrac{1}{y} = \dfrac{x}{a} + \dfrac{b}{a}$ where $Y = \dfrac{1}{y}$ and $X = x$, gradient = $\dfrac{1}{a}$

Gradient is $\dfrac{1}{8.2} = 0.122$



8 In the diagram, AB is a diameter of the circle with centre O. CS and BT are the tangents to the circle at C and B respectively. ACT and BST are straight lines.



(a) Prove that triangle TCS is an isosceles triangle. [4]

Let $\angle CAB = x°$

$\therefore \angle SCB = x°$ (Angle in alternate segment or Tangent Chord Theorem)

$\angle ACB = 90°$ (Angle in a semi-circle)

$\therefore \angle TCS = 90° - x°$

$\angle TBA = 90°$ (Rad $\perp$ Tan)

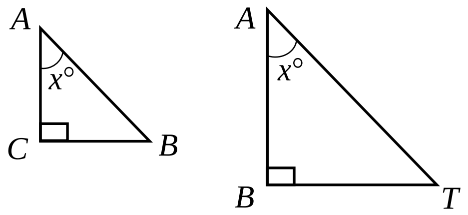
$\therefore \angle CTS = 180° - 90° - x°$ (Angle sum of Triangle)

$= 90° - x°$

Since $\angle CTS = \angle TCS$, TCS is an isosceles triangle.

(b) Show that $AB^2 = AC \times AT$. [4]

(1) $\angle CAB = \angle TAB$(common angle)
(2) $\angle ACB = \angle TBA = 90°$
Therefore, Triangles CAB and BAT are similar. (AA Similarity)



$\frac{AB}{AT} = \frac{AC}{AB}$ (all corr sides are proportional)
$AB^2 = AC \times AT$

9 It is given that x is a function of t, $\frac{\mathrm{d}x}{\mathrm{d}t} = 1 - e^{2t}$ and $x = 2$ when $t = 0$.

(a) Express x in terms of t. [3]

$$x = \int \left(1 - e^{2t}\right) \mathrm{d}t$$
$$= t - \frac{e^{2t}}{2} + c_1$$

Since $x = 2$ when $t = 0$, $2 = 0 - \frac{1}{2} + c_1 \Rightarrow c_1 = \frac{5}{2}$

$$x = t - \frac{e^{2t}}{2} + \frac{5}{2}$$

It is also given that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 5x + \sqrt{x+5}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 60$ when $x = 4$.

(b) Find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ when $t = 1$. [5]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5x^2}{2} + \frac{2(x+5)^{\frac{3}{2}}}{3} + c_2$$

When $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 60$, $x = 4$, $60 = \dfrac{5(4)^2}{2} + \dfrac{2(4+5)^{\frac{3}{2}}}{3} + c_2$

$c_2 = 60 - 40 - 18 = 2$

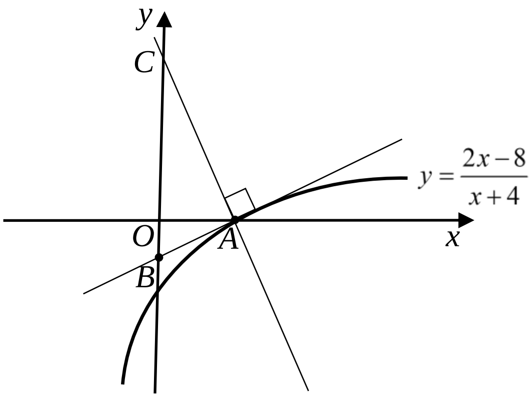
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5x^2}{2} + \frac{2(x+5)^{\frac{3}{2}}}{3} + 2$$

When $t = 1$, $x = 1 - \dfrac{e^2}{2} + \dfrac{5}{2} = \dfrac{7}{2} - \dfrac{e^2}{2} = -0.19453$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$$

$$= \left(\frac{5(-0.19453)^2}{2} + \frac{2(-0.19453+5)^{\frac{3}{2}}}{3} + 2 \right) \times (1 - e^2) = -58.3$$

10 The diagram shows part of the curve $y=\dfrac{2x-8}{x+4}$, $x>-4$.



(a) Explain why the curve $y=\dfrac{2x-8}{x+4}$ does not have a stationary point. [2]

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+4)2-(2x-8)}{(x+4)^2}=\frac{16}{(x+4)^2}$$

Since $\dfrac{\mathrm{d}y}{\mathrm{d}x}\neq 0$, y does not have a stationary point.

(b) The curve cuts the x-axis at A. The tangent and the normal to the curve at A intersect the y-axis at B and C respectively.

(i) Find the equation of the normal AC. [3]

When $y=0, x=4$.

Coordinates of A = $(4,0)$

Gradient of tangent at A = $\dfrac{16}{8^2}=\dfrac{1}{4}$

Gradient of normal at A = -4

Equation of normal AC: $y-0=-4(x-4)$

$y=-4x+16$

(ii) Find the area of triangle *ABC*. [4]

Equation of tangent *AB*: $y-0=\frac{1}{4}(x-4)$

$$y=\frac{1}{4}x-1$$

Therefore, coordinates of *B* = $(0,-1)$

Area of Triangle *ABC* = Area of Triangle *OAB* + Area of Triangle *OAC*

$=\frac{1}{2}\times 1\times 4+\frac{1}{2}\times 16\times 4=34$ sq units

Or Area of Triangle *ABC*

$=\frac{1}{2}\begin{vmatrix} 4 & 0 & 0 & 4 \\ 0 & 16 & -1 & 0 \end{vmatrix}=\frac{1}{2}[64-(-4)]=34$ sq units

Or Area of Triangle *ABC* = $\frac{1}{2}\times(16+1)\times 4=34$ sq units

(c) By expressing $\frac{2x-8}{x+4}=D+\frac{E}{x+4}$, explain why the line $y=2$ does not intersect the curve. [2]

$$\begin{array}{r|l} & \quad 2 \\ x+4 & \overline{\;2x-8} \\ & \text{-)}\;\underline{2x+8} \\ & \quad\;\; \underline{-16} \end{array}$$

$\frac{2x-8}{x+4}=2-\frac{16}{x+4}$

As $x>-4$, $-\frac{16}{x+4}<0$

Since $2-\frac{16}{x+4}<2$, the line $y=2$ does not intersect the curve.

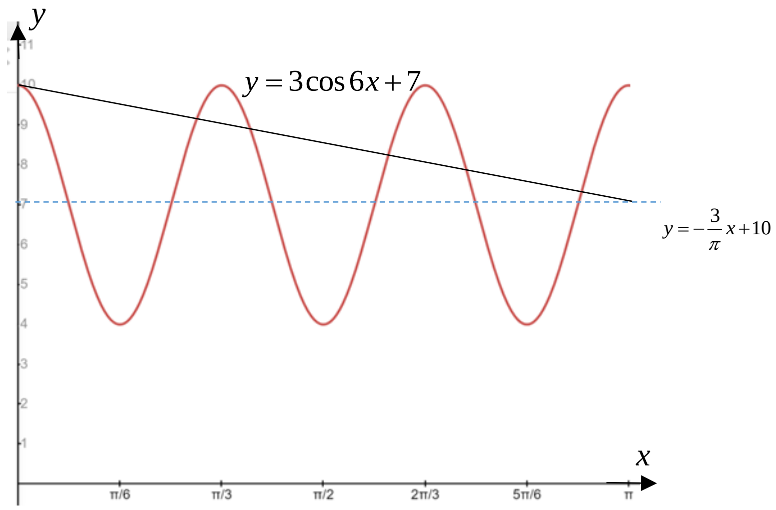
11 The curve $y = a\cos bx + c$, where a, b and c are positive integers, is defined for $0 \le x \le \pi$.

The curve has an amplitude of 3 and a period of $\frac{\pi}{3}$ radians. The minimum value of y is 4.

(a) State the value of a, b and c. [3]

$$a = 3, b = \frac{2\pi}{\frac{\pi}{3}} = 6 \quad \text{and} \quad c = 4 + 3 = 7$$

(b) Sketch the graph of $y = a\cos bx + c$ for $0 \le x \le \pi$. [3]



(c) On the same axes in part (b), sketch the graph of $y=-\frac{3}{\pi}x+10$ for $0\le x\le \pi$. [1]

Drawing of line

(d) Hence, for $0\le x\le \pi$, state the number of solutions of the equation

$$-3x+10\pi=\pi\left(a\cos bx+c\right).$$ [2]

$-\frac{3}{\pi}x+10=a\cos bx+c$

$-3x+10\pi=\pi\left(a\cos bx+c\right)$

There are 6 solutions.

12 A particle moves in a straight line and passes a fixed point O. The velocity, v m/s, of the particle, t seconds after passing O, is given by $v = 6t^2 + mt + 9$, where m is a constant. The particle travels with a deceleration of 9 m/s^2 when $t = 1$.

(a) Show that the value of m is -21. [1]

$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = 12t + m$

When $a = -9$ and $t = 1$, $12 + m = -9$

$m = -21$

(b) Find the value(s) of t when the particle is at instantaneous rest. [2]

When particle is at instantaneous rest, $v = 6t^2 - 21t + 9 = 0$

$3(2t-1)(t-3) = 0$

$t = 0.5$ or $t = 3$

(c) Explain clearly why the total distance travelled by the particle in the interval from $t=0$ to $t=4$ is not obtained by finding the value of the displacement of the particle at $t=4$. [2]

The value of the displacement of the particle at $t=4$ will only give the distance of the particle from O when $t=4$. It does not take into account the distances travelled by the particle when it changes its direction of motion when $t=0.5$ or $t=3$.

Concept of displacement as distance from O.
Changing in direction of motion when $t=0.5$ or $t=3$.

(d) Find the total distance travelled by the particle in the interval $t=0$ to $t=4$. [3]

$s=\int\left(6t^2-21t+9\right)\mathrm{d}t$

$=2t^3-\dfrac{21t^2}{2}+9t+c$

When $t=0$, $s=0$, therefore $c=0$

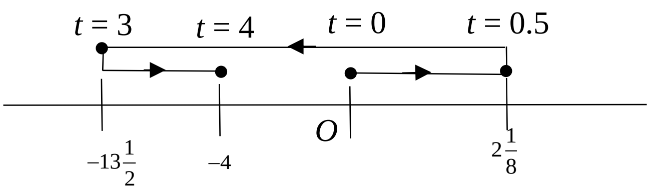
$s=2t^3-\dfrac{21t^2}{2}+9t$

When $t=0.5$, $s=2(0.5)^3-\dfrac{21(0.5)^2}{2}+9(0.5)=2\dfrac{1}{8}$

When $t=3$, $s=2(3)^3-\dfrac{21(3)^2}{2}+9(3)=-13\dfrac{1}{2}$

When $t=4$, $s=2(4)^3-\dfrac{21(4)^2}{2}+9(4)=-4$

Total distance travelled = $2\dfrac{1}{8}\times2+13\dfrac{1}{2}\times2-4=27\dfrac{1}{4}$ = 27.25 m (exact)



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