

2022 JC2 H2MA Preliminary Examination Paper 2

<p>1(a)</p>	<p>Let $u = x \Rightarrow \frac{du}{dx} = 1$</p> $\frac{dv}{dx} = (4-2x)^{-\frac{1}{2}} \Rightarrow v = -(4-2x)^{\frac{1}{2}}$ $I = \int \frac{x}{\sqrt{4-2x}} dx$ $= -x(4-2x)^{\frac{1}{2}} + \int (4-2x)^{\frac{1}{2}} dx$ $= -x(4-2x)^{\frac{1}{2}} + \frac{(4-2x)^{\frac{3}{2}}}{\frac{3}{2}(-2)} + C$ $= -x(4-2x)^{\frac{1}{2}} - \frac{(4-2x)^{\frac{3}{2}}}{3} + C$
<p>(b)</p>	<p>$u = 4-2x \Rightarrow \frac{du}{dx} = -2$</p> $I = \int \frac{x}{\sqrt{4-2x}} dx$ $= \int \frac{4-u}{2\sqrt{u}} \left(-\frac{1}{2}\right) du$ $= \int \frac{u-4}{4\sqrt{u}} du$ $= \int \frac{1}{4} u^{\frac{1}{2}} - u^{-\frac{1}{2}} du$ $= \frac{1}{6} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + D$ $= \frac{1}{6} (4-2x)^{\frac{3}{2}} - 2(4-2x)^{\frac{1}{2}} + D$

(c)

$$\left[-x(4-2x)^{\frac{1}{2}} - \frac{(4-2x)^{\frac{3}{2}}}{3} + C \right] - \left[\frac{1}{6}(4-2x)^{\frac{3}{2}} - 2(4-2x)^{\frac{1}{2}} + D \right]$$

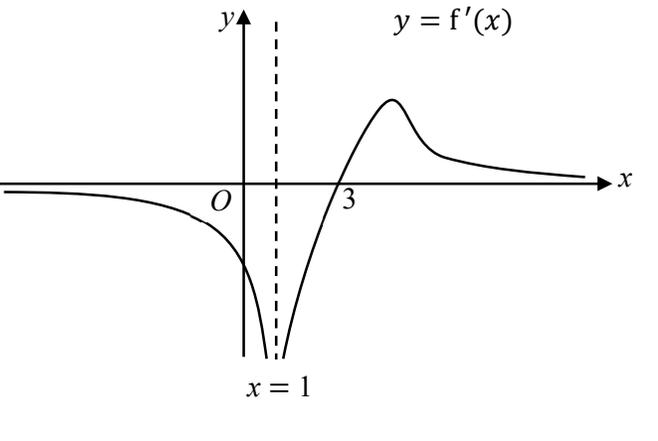
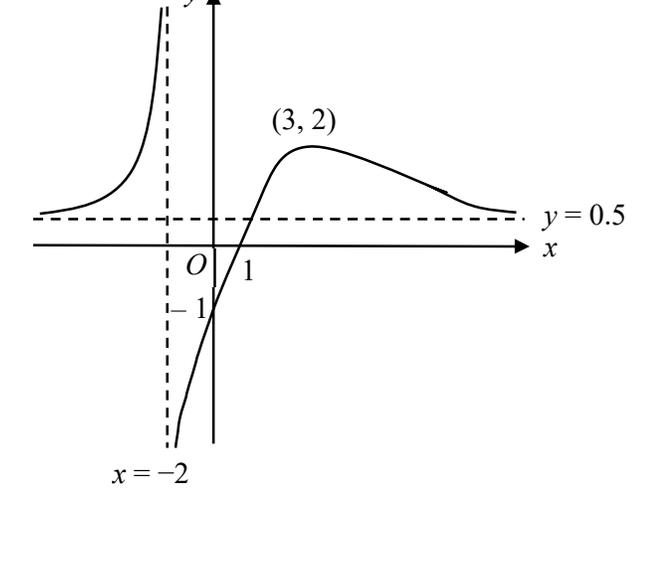
$$= -x(4-2x)^{\frac{1}{2}} - \frac{(4-2x)^{\frac{3}{2}}}{3} - \frac{1}{6}(4-2x)^{\frac{3}{2}} + 2(4-2x)^{\frac{1}{2}} + C - D$$

$$= (4-2x)^{\frac{1}{2}} \left[-x - \frac{1}{3}(4-2x) - \frac{1}{6}(4-2x) + 2 \right] + C - D$$

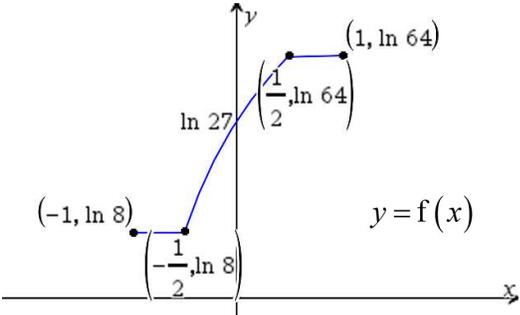
$$= (4-2x)^{\frac{1}{2}} \left(-x + \frac{2}{3}x + \frac{1}{3}x - \frac{4}{3} - \frac{2}{3} + 2 \right) + C - D$$

$$= (4-2x)^{\frac{1}{2}} (0) + C - D$$

$$= C - D \text{ (shown)}$$

Qn	Solution
2(a)	 <p>The graph shows the derivative function $y = f'(x)$ plotted against x. The origin is labeled O. A vertical dashed line represents a vertical asymptote at $x = 1$. The curve approaches $-\infty$ as x approaches 1 from the left and $+\infty$ as x approaches 1 from the right. The curve crosses the x-axis at $x = 3$. For $x > 3$, the curve is positive and asymptotically approaches the x-axis from above.</p>
2(b)	 <p>The graph shows a function plotted against x and y. The origin is labeled O. A vertical dashed line represents a vertical asymptote at $x = -2$. A horizontal dashed line represents a horizontal asymptote at $y = 0.5$. The curve passes through the point $(3, 2)$ and the y-axis at $(0, -1)$. The curve is increasing and concave down for $x > -2$.</p>

3(a)	<p>Largest possible $k = -1.5$</p> $y = x^2 + 3x - 1$ $= (x + 1.5)^2 - 2.25 - 1$ $(x + 1.5)^2 = y + 3.25$ $x = -1.5 \pm \sqrt{y + 3.25}$ <p>since $x \leq -1.5$</p> $x = -1.5 - \sqrt{y + 3.25}$ $f^{-1}(x) = -1.5 - \sqrt{x + 3.25}$
(b)	<p>$f(x) = f^{-1}(x)$</p> <p>Since the graphs of f and f^{-1} intersect at $y = x$,</p> $f(x) = x$ $x^2 + 3x - 1 = x$ $x^2 + 2x - 1 = 0$ $x = \frac{-2 \pm \sqrt{2^2 - 4(-1)}}{2}$ $= \frac{-2 \pm \sqrt{8}}{2}$ $x = -1 - \sqrt{2} \quad \text{or} \quad x = -1 + \sqrt{2}$ <p>Since $x \leq -2$, $\therefore x = -1 - \sqrt{2}$</p>
(c)	<p>fg does not exist as $R_g = [0, \infty) \not\subseteq D_f = (-\infty, -2]$</p> <p>$gf$ exists as $R_f = [-3, \infty) \subseteq D_g = [-5, \infty)$</p> $gf(x) = g(x^2 + 3x - 1)$ $= \sqrt{x^2 + 3x - 1 + 5}$ $= \sqrt{x^2 + 3x + 4}, x \leq -2$
(d)	<p>$R_f = [-3, \infty)$</p> <p>$R_{gf} = [\sqrt{2}, \infty)$</p>

	Solution
4(a)	<p>$\ln (2x+3)^3 = 3\ln(2x+3) = 3g(2x+3)$ where $g(x) = \ln x$.</p> <p>This is the sequence of transformations:</p> <ol style="list-style-type: none"> (1) Translate the graph $y = \ln x$ 3 units in the negative x-direction (2) Scale parallel to x-axis by a scale factor of $\frac{1}{2}$ (3) Scale parallel to y-axis by a scale factor of 3
(b)	
(c)	$y = \ln (2x+3)^3$ $e^y = (2x+3)^3$ $x = \frac{1}{2} \left((e^y)^{\frac{1}{3}} - 3 \right)$ $x^2 = \frac{1}{4} \left(e^{\frac{y}{3}} - 3 \right)^2$ <p>Required volume = volume of cylinder, with radius 1, height $(\ln 64 - \ln 27)$</p> $-\pi \int_{\ln 27}^{\ln 64} x^2 dy$ $= \pi(1^2)(\ln 64 - \ln 27) - \pi \int_{\ln 27}^{\ln 64} \frac{1}{4} \left(e^{\frac{y}{3}} - 3 \right)^2 dy$ $= 2.501 \text{ (to 3 d.p.)}$

5(a)

Method 1

$$\overline{AB} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}$$

$$l: \mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}, t \in \mathbb{R} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}, t \in \mathbb{R}$$

$$\text{Plane } p: \mathbf{r} = \begin{pmatrix} 6 \\ 2 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

Normal of plane p :

$$\begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3(1) - (-2)(-1) \\ -(0(1) - 2(-1)) \\ 0(-2) - 2(3) \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix}$$

$$\text{Plane } p: \mathbf{r} \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = 32$$

Let X be the point of intersection between plane p and line l .

$$\overline{OX} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}, \text{ for some } t \in \mathbb{R}$$

$$\text{Or } \overline{OX} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}, \text{ for some } t \in \mathbb{R}$$

Since X lies on p ,

$$\left[\begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = 32 \quad \text{or} \quad \left[\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = 32$$

$$(-8 - 12) + t(-4 - 4 - 18) = 32$$

$$-26t = 52$$

$$t = -2$$

$$\overline{OX} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - 2 \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ -4 \end{pmatrix} \quad \text{or} \quad \overline{OX} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$$

The coordinates of the point of intersection is

$$(8, 0, -4)$$

Method 2

Equating $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}, t \in \mathbb{R}$ and

$$\mathbf{r} = \begin{pmatrix} 6 \\ 2 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

$$-4t = 6 + 0\lambda + 2\mu \Rightarrow 0\lambda + 2\mu + 4t = -6 \text{ ---- (1)}$$

$$4 + 2t = 2 + 3\lambda - 2\mu \Rightarrow 3\lambda - 2\mu - 2t = 2 \text{ ---- (2)}$$

$$2 + 3t = -5 - \lambda + \mu \Rightarrow -\lambda + \mu - 3t = 7 \text{ ---- (3)}$$

Using GC,

$$\lambda = 0, \mu = 1, t = -2$$

When $t = -2$,

$$\overrightarrow{OX} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} - 2 \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ -4 \end{pmatrix}$$

The coordinates of the point of intersection is $(8, 0, -4)$.

(b) Let the equation of either plane be $\mathbf{r} \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = D$.

Distance between either plane and plane $p = \sqrt{41}$

$$\Rightarrow \left| \frac{D - 32}{\sqrt{1^2 + 2^2 + 6^2}} \right| = \sqrt{41}$$

$$\Rightarrow \left| \frac{D - 32}{\sqrt{41}} \right| = \sqrt{41}$$

$$\Rightarrow D - 32 = 41 \text{ or } D - 32 = -41$$

$$\Rightarrow D = 73 \text{ or } D = -9$$

\therefore the equations of the planes are

$$\mathbf{r} \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = 73 \quad \text{or} \quad \mathbf{r} \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = -9$$

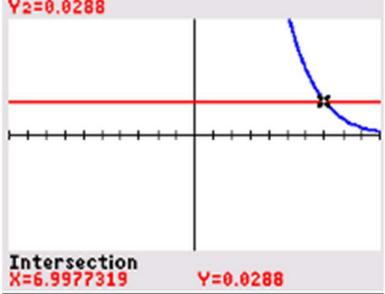
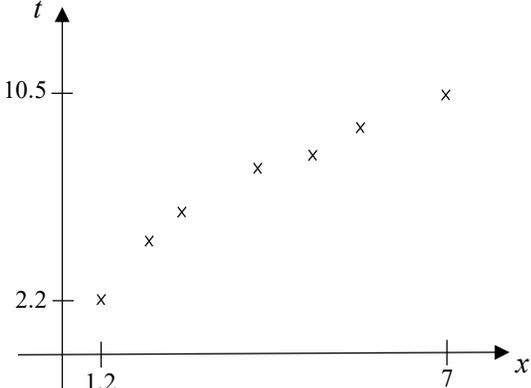
$$x - 2y - 6z = 73 \quad \text{or} \quad x - 2y - 6z = -9$$

(c)	$m : \mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 1 \\ a \end{pmatrix}, t \in \mathbb{R}$ <p>Since p and m do not meet in a unique point, p and m are parallel.</p> $\begin{pmatrix} -3 \\ 1 \\ a \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -6 \end{pmatrix} = 0$ $-3 - 2 - 6a = 0$ $a = -\frac{5}{6}$
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6(a)	<p>Method 1:</p> <p>Case 1: 3W2M – No. of ways = ${}^7C_3 {}^5C_2 = 350$</p> <p>Case 2: 4W1M – No. of ways = ${}^7C_4 {}^5C_1 = 175$</p> <p>Case 3: 5W0M – No. of ways = ${}^7C_5 {}^5C_0 = 21$</p> <p>Total number of ways = $350 + 175 + 21 = 546$</p> <p>Method 2: (Using complementary method)</p> <p>Case 1: 0W5M – No. of ways = ${}^7C_0 {}^5C_5 = 1$</p> <p>Case 2: 1W4M – No. of ways = ${}^7C_1 {}^5C_4 = 35$</p> <p>Case 2: 2W3M – No. of ways = ${}^7C_2 {}^5C_3 = 210$</p> <p>Total number of ways = ${}^{12}C_5 - 1 - 35 - 210 = 546$</p>
6(b)	<p>Required probability</p> $= \frac{(7-1)! \times 2! \times {}^6C_4 \times 4!}{(12-1)!}$ $= \frac{1}{77}$

7(a)	$P(X)$ $= P(\text{Two 3s or two 9s and one other})$ $= P(A3, B \text{ any}, C3) + [P(A9, B9, C \text{ not } 9)$ $\quad + P(A9, B \text{ not } 9, C9) + P(A \text{ not } 9, B9, C9)]$ $= \left(\frac{1}{4}\right)\left(\frac{5}{5}\right)\left(\frac{1}{8}\right) + \left[\left(\frac{1}{4}\right)\left(\frac{2}{5}\right)\left(\frac{5}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{5}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{5}\right)\left(\frac{3}{8}\right)\right]$ $= \frac{21}{80}$
7(b)	$P(X \cap Y) = P(A5, B9, C9)$ $= \left(\frac{1}{4}\right)\left(\frac{2}{5}\right)\left(\frac{3}{8}\right)$ $= \frac{3}{80}$ $P(Y) = P(A5, B \text{ any}, C \text{ any}) = \frac{1}{4}$ $P(X) \times P(Y) = \frac{21}{320} \neq P(X \cap Y)$ <p>Therefore, events X and Y are not independent.</p>
7(c)	$P(\text{One ball is numbered } 7 X)$ $= \frac{P(A3, B7, C3) + P(A9, B7, C9)}{P(X)}$ $= \frac{\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{5}\right)\left(\frac{3}{8}\right)}{\frac{21}{80}}$ $= \frac{2}{21}$
8(a)	<p>Possible values of T are $\{a, 2a, b, 3b\}$.</p> <p>Since $P(T = 2a) = P(T = 3b) = \frac{1}{6}$, then</p> $P(T = a) + P(T = b) = \frac{4}{6}.$ <p>If $P(T = a) = P(T = b) = \frac{2}{6}$, then the modes of T are a and b, which contradicts the question.</p> <p>Hence, $P(T = a) = \frac{3}{6} = \frac{1}{2}$ and $P(T = b) = \frac{1}{6}$.</p>

	t	a	$2a$	b	$3b$
	$P(T=t)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
8(b)	$E(T) = a\left(\frac{1}{2}\right) + 2a\left(\frac{1}{6}\right) + b\left(\frac{1}{6}\right) + 3b\left(\frac{1}{6}\right)$ $= \frac{5}{6}a + \frac{2}{3}b$ <p>Given that $E(T) = \frac{25}{6}$,</p> $\frac{5}{6}a + \frac{2}{3}b = \frac{25}{6} \Rightarrow b = \frac{25}{4} - \frac{5}{4}a$ $= \frac{5}{4}(5 - a)$ $E(T^2) = a^2\left(\frac{1}{2}\right) + (2a)^2\left(\frac{1}{6}\right) + b^2\left(\frac{1}{6}\right) + (3b)^2\left(\frac{1}{6}\right)$ $= \frac{1}{2}a^2 + \frac{2}{3}a^2 + \frac{1}{6}b^2 + \frac{3}{2}b^2$ $= \frac{7}{6}a^2 + \frac{5}{3}b^2$ $\text{Var}(T) = E(T^2) - [E(T)]^2$ $= \frac{7}{6}a^2 + \frac{5}{3}b^2 - \left(\frac{25}{6}\right)^2$ $= \frac{7}{6}a^2 + \frac{5}{3}\left[\frac{5}{4}(5 - a)\right]^2 - \frac{625}{36}$ $= \frac{7}{6}a^2 + \frac{125}{48}(5 - a)^2 - \frac{625}{36}$ $= \frac{181}{48}a^2 - \frac{625}{24}a + \frac{6875}{144}$				
9(a)	$X \sim B\left(75, \frac{p}{100}\right)$ $P(X \leq 1) = 0.0288$ $\binom{75}{0}\left(\frac{p}{100}\right)^0\left(1 - \frac{p}{100}\right)^{75} + \binom{75}{1}\left(\frac{p}{100}\right)\left(1 - \frac{p}{100}\right)^{74} = 0.0288$ $\left(1 - \frac{p}{100}\right)^{75} + \frac{3}{4}p\left(1 - \frac{p}{100}\right)^{74} = 0.0288$				

	 <p>$p = 6.9977$ (to 4 d.p.)</p>
9(b)	<p>$X \sim B(75, 0.075)$</p> <p>From GC,</p> <p>$P(X = 4) = 0.1517$</p> <p>$P(X = 5) = 0.1747$</p> <p>$P(X = 6) = 0.1652$</p> <p>The most likely number of bouquets with LED lights made in a day is 5.</p>
9(c)	<p>$X \sim B(75, 0.075)$</p> <p>$E(X) = 75 \times 0.075 = 5.625,$</p> <p>$\text{Var}(X) = 75(0.075)(1 - 0.075) = 5.203125$</p> <p>Since $n = 30$ is large, by Central Limit Theorem,</p> <p>$\bar{X} \sim N\left(5.625, \frac{5.203125}{30}\right)$ approximately</p> <p>Required probability</p> <p>$P(\bar{X} \geq 5)$</p> <p>$= 0.933$ (3 sf)</p>
10 (a)	
10 (b)	<p>The product moment correlation coefficient for case (i) is 0.9965 (to 4 d.p.) while that of the case (ii) is -0.9182 (to 4 d.p.).</p>

<p>10 (c)</p>	<p>Considering the values of r, since the value for case (i) is closer to 1 as compared to that of case (ii), there is a stronger linear correlation between $\ln x$ and t as compared to e^{-x} and t. Hence case (i) is more appropriate.</p> <p>A suitable regression line of t on $\ln x$ will be $t = 1.309671 + 4.517503 \ln x$ $t = 1.31 + 4.52 \ln x$ (to 3 s.f.)</p>
<p>10 (d)</p>	<p>As x is the controlled variable, regression line of t on $\ln x$ should be used even though we are asked to estimate x. $t = 1.309671 + 4.517503 \ln x$ When $t = 8.5$, $x \approx 4.911896 \approx 4.91$</p> <p>Since $r = 0.9965$ is close to +1 and that $t = 8.5$ is within the data range of t, the estimate of the value of quantity of chemical applied to the metal is reliable.</p>
<p>11 (a)</p>	<p>Unbiased estimate of the population mean $\bar{x} = \frac{186.35}{50} + 70$ $= 73.727$</p> <p>Unbiased estimate of the population variance $s^2 = \frac{1}{49} \left(10494 - \frac{186.35^2}{50} \right)$ ≈ 199.989 ≈ 200 (3 s.f.)</p>
<p>11 (b)</p>	<p>$H_0 : \mu = 70$ $H_1 : \mu \neq 70$ Test at 5% significance level. Under H_0, since $n = 50$ is large, by Central Limit Theorem, $\bar{X} \sim N \left(70, \frac{199.989}{50} \right)$ approximately, where $s^2 \approx 199.989$ is a good estimate of σ^2.</p> <p>Using GC, p-value = 0.0624 > 0.05 Do not reject H_0 and conclude that there is insufficient evidence, at 5% significance level, that the population mean mass of duck eggs is not equal to 70 grams.</p>

	Hence Farm A 's claim is valid.
11 (c)	No, it is not necessary to assume that X is normally distributed as the sample size of 50 is large. Central Limit Theorem could be applied and thus \bar{X} is approximately normally distributed and a Z test can be carried out.
11 (d)	There is a probability of 0.05 that the test will indicate that the population mean mass of duck eggs is not 70 grams when in fact it is 70 grams.
	<p>Unbiased estimate of the population variance</p> $s^2 = \frac{40}{39}(146)$ $= 149.744$ <p>Let Y be the mass, in grams, of a randomly chosen duck egg from Farm B.</p> <p>$H_0 : \mu = 70$ $H_1 : \mu > 70$</p> <p>Test at 3% significance level. Under H_0, since $n = 40$ is large, by Central Limit Theorem,</p> $\bar{Y} \sim N\left(70, \frac{149.744}{40}\right) \quad \text{approximately, where}$ <p>$s^2 \approx 149.744$ is a good estimate of σ^2. OR</p> <p>or $\bar{Y} \sim N\left(70, \frac{146}{39}\right)$ approximately, where</p> <p>$s^2 = 40 \times \frac{146}{39}$ is a good estimate of σ^2.</p> <p>Since Farm B's claim is valid, we reject H_0.</p> $\frac{k - 70}{\sqrt{\frac{149.744}{40}}} \geq 1.8808 \quad \text{OR} \quad \frac{k - 70}{\sqrt{\frac{146}{39}}} \geq 1.8808$ $k - 70 \geq 3.6390$ $k \geq 73.639$ $k \geq 73.6 \text{ (3 s.f.)}$ $\{k \in \mathbb{R} : k \geq 73.6\}$
12 (a)	Let W be the random variable denoting the mass of a randomly chosen packet of white sugar.

	$W \sim N(1.05, 0.03^2)$ Let k kg be the mass exceeded by 80% of the packets of white sugar. $P(W > k) = 0.8$ $k = 1.025$ (3 d.p.)
12 (b)	98% of the labelled mass is 0.98 kg $P(W < 0.98) = 0.0098153$ Let X be the number of packets of white sugar, out of 10, that are underweight. $X \sim B(10, 0.0098153)$ $P(X \leq 1) = 0.996$ (3 s.f.)
12 (c)	Let B be the random variable denoting the mass of a randomly chosen packet of brown sugar. $B \sim N(m, 0.05^2)$ $P(W_1 + W_2 + W_3 + W_4 > 2B) = 0.15$ Let $S = W_1 + W_2 + W_3 + W_4 - 2B$ $E(S) = 4 \times 1.05 - 2m$ $= 4.2 - 2m$ $\text{Var}(S) = 4 \times 0.03^2 + 2^2 \times 0.05^2$ $= 0.0136$ $S \sim N(4.2 - 2m, 0.0136)$ $P(S > 0) = 0.15$ $P\left(Z > \frac{0 - 4.2 + 2m}{\sqrt{0.0136}}\right) = 0.15$ $\frac{2m - 4.2}{\sqrt{0.0136}} = 1.03643338$ $m = \frac{1}{2}(4.2 + 1.03643338\sqrt{0.0136})$ $m \approx 2.16$ (3 s.f.)
12 (d)	$B \sim N(2.03, 0.05^2)$ Let $A = \frac{1}{9}(W_1 + W_2 + W_3 + W_4 + B_1 + B_2 + B_3 + B_4 + B_5)$ $E(A) = \frac{1}{9}[4 \times 1.05 + 5 \times 2.03] = 1.59444$ $\text{Var}(A) = \frac{1}{81}[4 \times 0.03^2 + 5 \times 0.05^2]$ $= \frac{1}{81}(0.0161)$ or 0.0001987654321

$A \sim N\left(1.59444, \frac{0.0161}{81}\right)$ $P(A > 1.6) = 0.347 \text{ (3 s.f.)}$
