Section A: Pure Mathematics [40 marks]

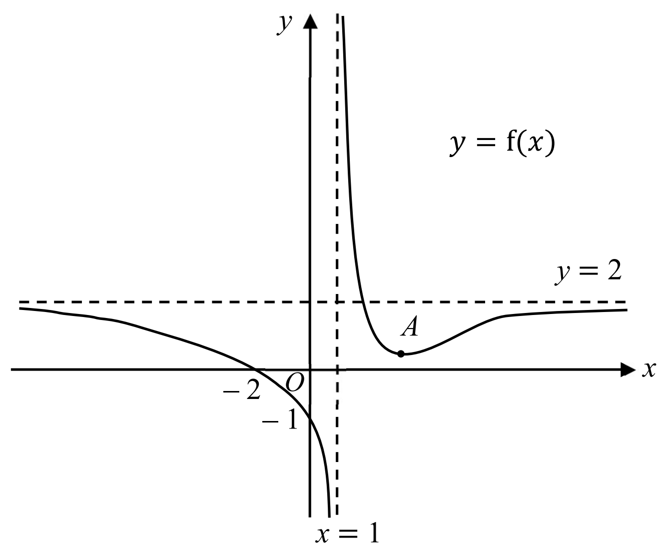
1 It is given that $I = \int \frac{x}{\sqrt{4-2x}}\,\mathrm{d}x$.

(a) Use integration by parts to find an expression for I. [2]

(b) Use the substitution $u = 4-2x$ to find another expression for I. [2]

(c) Show algebraically that the answers to parts **(a)** and **(b)** differ by a constant. [2]

2



The diagram shows the curve $y = \mathrm{f}(x)$ with a turning point $A\left(3, \frac{1}{2}\right)$. The curve crosses the axes at $x=-2$ and $y=-1$ and the lines $x=1$ and $y=2$ are the asymptotes of the curve.

Sketch the following curves on separate diagrams, stating, if it is possible to do so, the equations of any asymptotes and the coordinates of any points where each curve crosses the axes and of any turning points.

(a) $y = \mathrm{f}'(x)$, [2]

(b) $y = \frac{1}{\mathrm{f}(x)}$. [3]

3 Functions f and g are defined by

$$\mathrm{f}: x \mapsto x^2 + 3x - 1,\ x \in \mathbb{R},\ x \le k,$$

$$\mathrm{g}: x \mapsto \sqrt{x+5},\ x \in \mathbb{R},\ \ x \ge -5.$$

(a) Given that f^{-1} exists, state the largest possible value of k. Using this value of k, find $\mathrm{f}^{-1}(x)$. [3]

For the rest of this question, let $k = -2$.

(b) Find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [2]

(c) Determine whether the composite functions fg and gf exist. If the composite function exists, give a definition (including the domain) of the function. [3]

(d) Hence find the exact range of the composite function that exists. [1]

4 **(a)** State a sequence of transformations that will transform the curve with equation $y = \ln x$ onto the curve with equation $y = \ln\,(2x+3)^3$. [3]

A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \begin{cases} \ln 64 & \text{for } x > \dfrac{1}{2}, \\ \ln\,(2x+3)^3 & \text{for } -\dfrac{1}{2} \le x \le \dfrac{1}{2}, \\ \ln 8 & \text{otherwise.} \end{cases}$$

(b) Sketch the curve for $-1 \le x \le 1$. [3]

(c) Find the numerical value of the volume generated when the region bounded by the curve $y = \mathrm{f}(x)$, the line $x = 1$ and the line $y = \ln 27$ is rotated completely about the y-axis. Give your answer correct to 3 decimal places. [3]

5 The plane p has equation $\mathbf{r}=\begin{pmatrix}6\\2\\-5\end{pmatrix}+\lambda\begin{pmatrix}0\\3\\-1\end{pmatrix}+\mu\begin{pmatrix}2\\-2\\1\end{pmatrix}$, where λ and μ are parameters.

The line l passes through the points A and B with position vectors $4\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $4\mathbf{j}+2\mathbf{k}$ respectively.

(a) Find the coordinates of the point of intersection between p and l. [5]

(b) Find the cartesian equations of the planes such that the perpendicular distance from each plane to p is $\sqrt{41}$. [3]

Another line m has equation $\dfrac{1-x}{3}=y+2=\dfrac{z-3}{a}$.

(c) Find the value of a such that p and m do not meet in a unique point. [3]

Section B: Probability and Statistics [60 marks]

6 A group of 12 people consists of 5 men and 7 women. One of the women is the wife of one of the men.

(a) How many committees of 5 can be formed which include at least 3 women? [2]

(b) The 12 people sit at random at a round table. Find the probability that the husband and his wife are seated together and no two men are next to each other. [3]

7 Bag A contains 4 balls numbered 3, 5, 6 and 9. Bag B contains 5 balls numbered 1, 2, 7, 9 and 9. Bag C contains 8 balls numbered 3, 4, 4, 8, 8, 9, 9 and 9. All the balls are indistinguishable apart from the number on the balls. One ball is selected at random from each bag.

- X is the event that exactly two of the selected balls have the same number.
- Y is the event that the ball selected from bag A is numbered 5.

(a) Show that $\mathrm{P}(X)=\dfrac{21}{80}$. [2]

(b) Find $\mathrm{P}(X\cap Y)$ and hence determine whether X and Y are independent. [3]

(c) Find the probability that one ball is numbered 7, given that exactly two of the selected balls have the same number. [2]

8 A fair cubical die has faces each labelled with one of the four distinct numbers: a, $2a$, b and $3b$. The die is thrown once and the number on the uppermost face is the score T.

It is given that the mode of T is a and that $\mathrm{P}(T = 2a) = \mathrm{P}(T = 3b) = \frac{1}{6}$.

(a) Draw a table to show the probability distribution of T. [2]

(b) Given that the mean score is $\frac{25}{6}$, find the variance of the score in terms of a. [5]

9 A flower shop makes 75 bouquets of flowers daily. On average, p% of the bouquets have LED lights. Assume that X, the number of bouquets of flowers with LED lights made daily, follows a binomial distribution.

(a) Given that there is a probability of 0.0288 that fewer than 2 bouquets made in a day have LED lights, write down an equation in terms of p and hence find p correct to 4 decimal places. [3]

It is now given that $p = 7.5$.

(b) Find the most likely number of bouquets with LED lights made in a day. [2]

(c) 30 days are randomly selected. Find the probability that the mean number of bouquets with LED lights made per day is at least 5. [3]

10 In an experiment, a chemist applied different quantities, x ml, of a chemical to 7 samples of a type of metal, and the times, t hours, for the metal to discolour were measured. The results are given in the table.

x	1.2	2.0	2.7	3.8	4.8	5.6	7.0
t	2.2	4.5	5.8	7.3	8.0	9.0	10.5

(a) Draw a scatter diagram for these values, labelling the axes. [1]

(b) Find, correct to 4 decimal places, the product moment correlation coefficient between

(i) $\ln x$ and t,

(ii) e^{-x} and t. [2]

(c) Explain which of the two cases in part **(b)** is more appropriate and find the equation of a suitable regression line for this case. [3]

(d) Use the equation of your regression line to estimate the value of the quantity of chemical applied to the metal when the time taken for the metal to discolour is 8.5 hours. Explain whether your estimate is reliable. [2]

11 Farm A claims that the duck eggs from their farm have a mean mass of 70 grams. A random sample of 50 duck eggs is selected. The masses, x grams, are summarised as follows.

$$\sum(x-70)=186.35, \quad \sum(x-70)^2=10494.$$

(a) Calculate unbiased estimates of the population mean and variance. [2]

(b) Test, at the 5% level of significance, whether Farm A's claim is valid. [4]

(c) State, with a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

(d) Explain the meaning of 'at the 5% level of significance' in the context of the question. [1]

Farm B claims that their duck eggs have a mean mass of more than 70 grams. A random sample of 40 duck eggs is taken, and it is found that their mean mass and variance are k grams and 146 grams^2 respectively. Given that a test at the 3% significance level indicates that Farm B's claim is valid, find the set of values of k. [4]

12 **In this question you should state the parameters of any distributions that you use.**

A supermarket sells two types of sugar. White sugar is sold in packets with the labelled mass of 1 kg. The mass of a packet of white sugar may be regarded as a normally distributed random variable with mean 1.05 kg and standard deviation 0.03 kg.

(a) What mass is exceeded by 80% of the packets of white sugar? Give your answer correct to 3 decimal places. [1]

(b) A packet of white sugar that weighs less than 98% of the labelled mass is considered underweight. Find the probability that at most 1 out of 10 randomly chosen packets of white sugar is underweight. [3]

The masses of packets of brown sugar are normally distributed with mean m kg and standard deviation 0.05 kg and the masses of packets of white sugar and brown sugar have independent normal distributions.

(c) Given that the probability that the total mass of 4 randomly chosen packets of white sugar exceeds twice the mass of a randomly chosen packet of brown sugar is 0.15, find m. [5]

It is now given that $m = 2.03$.

(d) Find the probability that the average mass of 4 randomly chosen packets of white sugar and 5 randomly chosen packets of brown sugar exceeds 1.6 kg. [4]