VICTORIA JUNIOR COLLEGE

JC 1 PROMOTIONAL EXAMINATION 2022

CANDIDATE NAME	

CLASS		INDEX NUMBER	

H2 MATHEMATICS **9758/01**
Paper 1 **3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)
Writing paper

READ THESE INSTRUCTIONS FIRST

Write your class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Marks	

1 When the polynomial $\mathrm{P}(x) = ax^3 + bx^2 + cx - 1$ is divided by $(x+1)$ and $(x-3)$, the remainders are -12 and 140 respectively. It is given that $(2x-1)$ is a factor of $\mathrm{P}(x)$.
Find the values of a, b and c. [4]

2 **(i)** Show that x^2+2x+2 is always positive for all real values of x. [1]

(ii) Without using a calculator, solve the inequality $\dfrac{x-12}{x^2+3x-10}>1$. [3]

3 The curve C has equation $\dfrac{9(y+3)^2}{4} - \dfrac{(x-2)^2}{4} = 1$.

(i) Sketch C, stating the equations of any asymptotes and the coordinates of any turning points. [4]

(ii) Hence find the set of values of m such that the straight line with gradient m that passes through the point $(2,-3)$ intersects C at least once. [2]

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4 The curve C_1 has equation $y = x^3 + 1$ and the curve C_2 has equation $y = b(-x+a)^3 + b$, where a and b are positive constants.

(i) Describe a sequence of transformations that transforms C_1 onto C_2. [3]

(ii) The stationary point on C_1 corresponds to the point (k, l) on C_2 after the sequence of transformations in part **(i)**. Find k and l in terms of a and b. [2]

5 Find

(a) $\displaystyle\int \frac{5x}{\left(x^2-1\right)^{10}}\,\mathrm{d}x,$ [2]

(b) $\displaystyle\int x\sin x\,\mathrm{d}x,$ [2]

(c) $\displaystyle\int \frac{1}{\sec x - 2\tan x}\,\mathrm{d}x.$ [3]

6 The complex number u is given by $u = \frac{1}{2}(1 + \cos\theta + \mathrm{i}\sin\theta)$, where $0 < \theta < \pi$.

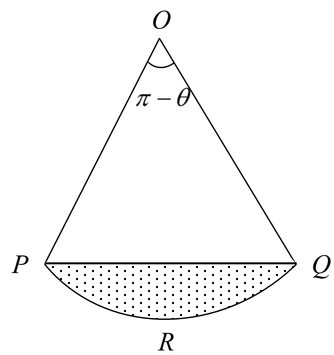
(i) Show that $|u| = \cos\frac{\theta}{2}$ and $\arg(u) = \frac{\theta}{2}$. [3]

(ii) Write down $\left|\frac{1}{u}\right|$ and $\arg\left(\frac{1}{u}\right)$ in terms of θ. [2]

(iii) Hence show that $\frac{1}{u} = a - \mathrm{i}\tan\left(\frac{\theta}{2}\right)$, where a is a real constant to be determined. [2]

7 **(a)** Expand $\dfrac{1}{(3+x)^2}$ in ascending powers of x, up to and including the term in x^3. State the values of x for which this expansion is valid. [4]

(b)



An arc PQ of a circle subtends an angle $(\pi - \theta)$ radians at the centre O. The length of chord PQ is L units and the area of the shaded segment PRQ is A square units.

Given that θ is small and $A = kL^2$, show that $2k\theta^2 - 2\theta + \pi - 8k \approx 0$. [5]

8 A curve C has equation $y = \dfrac{x^2 - 2x + 7}{x - 3}$.

(i) Using an algebraic method, find the range of values that y cannot take. Leave your answer in exact form. [4]

(ii) Sketch the curve C, stating the equations of any asymptotes and the exact coordinates of any points where C crosses the axes and of any turning points. [4]

[continued]

(iii) By considering a suitable graph, find the range of values of k such that the equation $x^4 - 2x^3 + 7x^2 = kx - 3k$ has exactly one positive root and one negative root. [2]

9 The complex number z satisfies the equation

$$z^2 + (2 - 4\mathrm{i})z = c,$$

where c is a complex constant.

(i) Given that z_1 and z_2 are two distinct complex roots of the above equation, show that $\dfrac{z_1 + z_2}{2} = -1 + 2\mathrm{i}$. [2]

It is given that $c = 4\mathrm{i}$ and $\operatorname{Im}(z_1) < \operatorname{Im}(z_2)$.

(ii) Find z_1 and z_2 in the form $x + \mathrm{i}y$, where x and y are real numbers. Leave the values of x and y in exact form. [3]

[continued]

(iii) On an Argand diagram, indicate clearly the points P_1 and P_2 which represent z_1 and z_2 respectively. [2]

(iv) Point O and Point Q represent complex number 0 and w respectively. It is given that OP_1P_2Q is a parallelogram. Using diagram in part **(iii)**, find w. [2]

10 **(a)** **(i)** Show that $9x$ can be expressed as $A(12-18x)+B$, where A and B are constants to be determined. [1]

(ii) Without the use of a calculator, find $\int_{\frac{1}{3}}^{1} \frac{9x}{\sqrt{12x-9x^2}}\,\mathrm{d}x$. [4]

(b) Use the substitution $x+1=\sqrt{2}\sin\theta$ to show that, for $0<\alpha<1$,

$$\int_{-2}^{\alpha-1} \frac{1}{\left(1-2x-x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x = \frac{1}{2}\left(\frac{\alpha}{\sqrt{2-\alpha^2}}+1\right).$$

[5]

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(c) A function g satisfies the equation

$$7 + \mathrm{g}\left(x\right) = 3x^2 \int_{-1}^{1} \mathrm{g}(x)\,\mathrm{d}x$$

for all real values of x. Find $\int_{-1}^{1} \mathrm{g}(x)\,\mathrm{d}x$. [3]

11 In a laboratory experiment, a tank is transported to and fro between station A and station B.

On the 1st visit to station A, 200 ml of water is added into the tank and on subsequent visits, 200 ml more water than the previous visit to station A will be added. That is, 400 ml and 600 ml of water will be added into the tank on the 2nd and 3rd visit to station A respectively. On each visit to station B, b ml of water is removed from the tank.

It is given that the tank is empty at the start of the experiment and it starts from station A.

(i) On the m^{th} visit to station A, 3200 ml of water is added into the tank. Find m. [2]

(ii) It is given that immediately after the 16th visit to station B, there is more than 25 litres of water in the tank. Find the set of values that b can take. [3]

In another laboratory experiment, a tank is transported to and fro between station A and station C. 10% of the water in the tank is removed on each visit to station C. It is given that the tank is empty at the start of the experiment and it starts from station A.

(iii) Show that the amount of water in the tank immediately after the 3rd visit to station C is 1009.8 ml. [2]

(iv) Show that the amount of water in the tank immediately after the n^{th} visit to station C is $1800n - 16200\left(1 - 0.9^{n}\right)$. [4]

[continued]

(v) The experiment stops when the amount of water in the tank immediately after it leaves station C exceeds 10 litres. After the k^{th} visit to station C, the experiment stopped. Find k. [1]

12 A curve C has parametric equations

$$x = \mathrm{e}^{u}, \qquad y = u^2, \qquad \text{for } u \in \mathbb{R}.$$

(i) Sketch C, stating the equation of the asymptote and the coordinates of any points where C cuts the axes. [3]

(ii) Find the equation of the tangent to C at the point where $u = -1$. Leave your answer in the form of $y = mx + c$ where m and c are exact values to be determined. [3]

For the rest of the question, all lengths are measured in kilometres (km).

The region bounded by C and the line $y = 5$ represents a man-made lake. A circle with radius 0.5 and centre $P(1, 2)$ represents a man-made island on the lake. A bridge is built from a point Q on C, where $u \leq 0$, to a point R on the edge of the circular island such that QR produced passes through point P. It is given that the distance between Q and P is denoted by l.

(iii) Show that $l^2 = \mathrm{e}^{2u} - 2\mathrm{e}^{u} + u^4 - 4u^2 + 5$. [2]

(iv) Use differentiation to find the values of u which give stationary values of l^2. [2]

(v) Show that only one of the values of u found in part **(iv)** gives a minimum value of l^2. Hence find the shortest length of the bridge QR. [4]

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