TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME**: _____**

CIVICS GROUP: **_____**

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H2 MATHEMATICS   **9758/01**
Paper 1   13 SEPTEMBER 2022
                                                   3 hours

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

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READ THESE INSTRUCTIONS FIRST

Write your name and Civics Group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or
1 decimal place in the case of angles in degrees, unless a different
level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless
a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not
allowed in a question, you are required to present the mathematical
steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your
answers.

For Examiners' Use	
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Total	

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

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This document consists of <u>6</u> printed pages and <u>0</u> blank pages.

[Turn Over

1 **(i)** Express $\dfrac{3}{r}+\dfrac{2}{r+1}-\dfrac{5}{r+2}$ as a single fraction. [1]

    (ii) Hence find $\displaystyle\sum_{r=1}^{n}\dfrac{4r+3}{r(r+1)(r+2)}$. [3]

    (iii) Use your answer to part **(ii)** to find $\displaystyle\sum_{r=3}^{n}\dfrac{4r-5}{r(r-1)(r-2)}$. [2]

2 **(a)** Find $\displaystyle\int\dfrac{1}{\sqrt{\left(1-x^2\right)}\sin^{-1}x}\,\mathrm{d}x$. [2]

    (b) Find $\displaystyle\int\dfrac{x-3}{x^2-2x+4}\,\mathrm{d}x$. [3]

3 The curve C is defined by the parametric equations

$$x=a^2t^2,\quad y=\mathrm{e}^{at},\qquad \text{for } t\geq 0,$$

where a is a positive constant.

Find the exact area enclosed by C, the axes and the line $x=8$. [5]

4 On the same axes, sketch the curves with equation $y=-x^2+5x-3$ and $y=\left|3-x\right|$, labelling the axial intercepts. [2]

Hence, without using a calculator, solve the inequality $-x^2+5x-3<\left|3-x\right|$. [4]

**5** Given that $\ln y=\sin kx$ where k is a non-zero constant, show that

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+k^2y\ln y-\dfrac{1}{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=0$. Hence obtain the expansion of y in ascending powers of

x, up to and including the term in x^2. [5]

Using the standard series given in MF26, verify that the same result is obtained and determine the coefficient of x^3. [4]

6 **(a)** Disposable cups in the shape of an inverted right cone of radius 4 cm and height 8 cm are being produced by a factory. For quality check, the factory supervisor took a cup and tested it by filling it completely with water. Water was found leaking at the vertex of the cup at a constant rate of 1.5 cm^3 per second. The diagram below shows the cup.
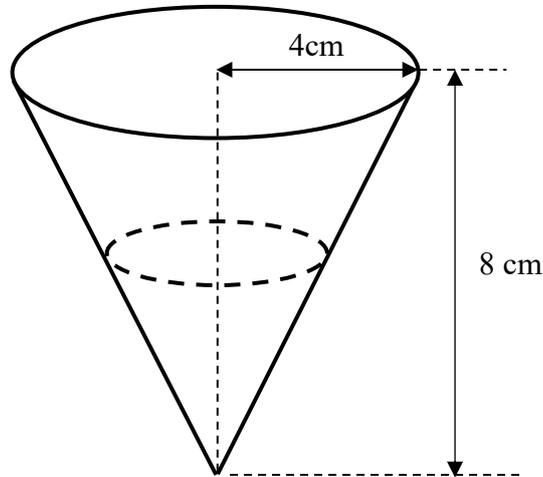
Find the exact rate at which the water level is decreasing when the depth of the water is 2 cm. [4]

[The volume of a cone of base radius r and height h is given by $\dfrac{1}{3}\pi r^2 h.$]

(b) The diagram below shows a trapezium $ABCD$ with height 5 cm such that $DA + AB + BC = 20$ cm. Points M and N are the foot of the perpendiculars from points A and B to line DC respectively, and $\angle DAM = \angle CBN = \theta$, where $0 < \theta < \dfrac{\pi}{2}$.

(i) Show that the area, A cm^2, of trapezium $ABCD$ is $100 - 50\sec\theta + 25\tan\theta$. [3]

(ii) Hence, by differentiation, find the maximum area of trapezium $ABCD$, giving your answer in exact form. [You do not need to verify that it is a maximum area.] [4]

[Turn Over

7 **(a)** The function f is defined as

$$f : x \mapsto ax + \frac{a}{x-1}, \quad x \in \mathbb{R}, \ x \neq 1,$$

where a is a constant greater than 1.

   (i) Sketch the graph of $y = f(x)$, indicating clearly the coordinates of the turning points and the equations of the asymptotes. [3]

   (ii) Explain why f does not have an inverse. [1]

   The function g is defined as

$$g : x \mapsto \frac{1}{x^2 - 1}, \quad x \in \mathbb{R}, \ x \neq -1, \ x \neq 1.$$

   (iii) Determine, with a reason, whether gf exists. [2]

(b) A curve C has equation $y = e^{\frac{x^2 - 4}{12}}$, where $x > 0$.

The curve C undergoes the following sequence of transformations:

   A: Stretch by factor $\frac{1}{2}$ parallel to the x-axis.

   B: Translate by 1 unit in the negative y-direction.

   C: Reflection in the line $y = x$.

Find the equation of the new curve in the form $y = q(x)$ and state the domain of q.

[5]

8 The plane p_1 has cartesian equation $x + z = 3$. The plane p_2 is perpendicular to p_1 and contains the line l_1 with equation $\dfrac{x+2}{5} = \dfrac{y+1}{2} = \dfrac{3-z}{3}$.

   (i) Show that the cartesian equation of the plane p_2 is $-x + 4y + z = 1$. [2]

   (ii) Find a vector equation of the line l_2 given that p_1 and p_2 intersect at l_2. [2]

   (iii) It is given that the point B with coordinates $(0, 4, 3)$ is on p_1 and the perpendicular distance from B to p_2 is k. Find the position vector of the foot of perpendicular from B to p_2 and deduce the value of k. [4]

   (iv) Hence, find the vector equations of the lines in p_1 such that the perpendicular distance from each line to p_2 is k. [3]

9   **(a)**   The complex number z is given by $z = x + y\mathrm{i}$, where x and y are non-zero real numbers. Given that $|z| = 1$, find the possible values of z for which $\dfrac{\left(z^2\right)^*}{z}$ is real.

[6]

(b)   Without the use of a calculator, find the roots of the equation $z^2 = 33 + 56\mathrm{i}$, expressing your answer in cartesian form $x + \mathrm{i}y$ where x and y are real.   [4]

Hence, find in cartesian form, the roots of the equation $w^2 = -33 + 56\mathrm{i}$.   [2]

10   A squirrel falls vertically from a tall tree. The distance, x metres, that the squirrel has fallen from the tree after t seconds is observed. It is given that $x = 0$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0$ when $t = 0$.

The motion of the squirrel is modelled by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 0.1\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = 10.$$

(i)   By substituting $y = \dfrac{\mathrm{d}x}{\mathrm{d}t}$, show that the differential equation can be written as

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 10 - 0.1y^2.$$

[1]

(ii)   Find y in terms of t and hence find x in terms of t.   [8]

(iii)   How far has the squirrel fallen after 2 seconds?   [1]

(iv)   For a falling object, the terminal velocity is the value approached by the velocity after a long time. Find the terminal velocity of the falling squirrel.   [2]

11 Mrs Toh intends to invest $1000 per year in a savings plan, starting from 1 January 2023. Savings plan A allows her to invest a fixed amount of $1000 into account A on the first day of every year. The amount in account A earns an interest of 3.5% per annum at the end of each year of investment.

  (i) Show that the total amount in account A at the end of n years is in the form $p(q^n - 1)$, where p and q are exact constants to be determined. [3]

  (ii) On what date will the total amount in account A first exceed 36000? [3]

Savings plan B allows Mrs Toh to invest $1000 into account B on 1 January 2023. The amount to be invested on the first day of the subsequent years will increase by $\$k$ per annum. A fixed annual bonus of $40 is added to account B at the end of each year of investment.

  (iii) If $k = 36$, find the year in which the total amount in account A first exceeds the total amount in account B. [4]

  (iv) Find the least value of k, giving your answer to the nearest whole number, such that the total amount in account B is more than the total amount in account A at the end of 31 December 2032. [2]

End of Paper