1 Find

(a) $\int \sin^{-1} 2x \, \mathrm{d}x,$ [3]

(b) $\int \frac{1}{x^2 - nx + n^2} \, \mathrm{d}x,$ where n is a constant. [2]

2 **Do not use a calculator in answering this question.**

The equation $3z^2 - (5 + \mathrm{i})z = k$, where k is a real constant, has a root $1 + \mathrm{i}$.

(i) Find the value of k. [2]

(ii) Find the other complex root. [3]

3 **(i)** On the same axes, sketch the graphs of $y = |x + a|$ and $y = |ax + 1|$ where $0 < a < 1$, and hence solve the inequality $|x + a| < |ax + 1|$. [4]

(ii) Deduce the solution to the inequality $\left|\frac{1}{x} + a\right| < \left|\frac{a}{x} + 1\right|$. [2]

4 It is given that $\ln y = 1 + \tan^{-1}(2x)$.

(i) Show that $(1 + 4x^2)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (8x - 2)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. [2]

(ii) Find the Maclaurin series for y, up to and including the term in x^3, giving exact coefficients for each term. [4]

(iii) Hence find the Maclaurin series for $\mathrm{e}^{\tan^{-1}(2x)}$, up to and including the term in x^3. [2]

5 With reference to the origin O, the points A and B have position vectors **a** and **b** respectively, where **a** and **b** are non-zero and non-parallel.

(i) The point C has position vector **c** given by **c** = 9**a** – 6**b**. Find, in terms of **a** and **b**, the position vector of the point where the lines OC and AB meet. [4]

(ii) The point D has position vector **d** given by **d** = t**a** + $(1 - t)$**b** where t is a constant. Given that **a** = **i** + **j** and **b** = 4**i** – 2**j** +6**k**, and the angle between OA and OD is 60°, form a quadratic equation in t and hence solve for the exact values of t. [4]

6 The sum of the first r terms of a sequence of real numbers $u_1, u_2, u_3, ...$ is denoted by S_r.

It is given that $S_{r+1} = \dfrac{S_r}{1+(2r-1)S_r}$, for $r \geq 1$, and $u_1 = 1$.

(i) Show that $\dfrac{1}{S_{r+1}} - \dfrac{1}{S_r} = 2r - 1$. [1]

(ii) By considering $\displaystyle\sum_{r=1}^{n-1}\left(\frac{1}{S_{r+1}} - \frac{1}{S_r}\right)$ and using the result in part **(i)**, show that

$$S_n = \frac{1}{(n-1)^2+1}, \text{ for } n \geq 2.$$ [5]

(iii) Explain whether the sequence $S_1, S_2, S_3, ...$ is converging. [1]

(iv) Show that u_n is negative for all $n \geq 2$. [2]

7 It is given that

$$f(x) = \begin{cases} \sqrt{4x - x^2} & \text{for } 2 \leq x \leq 4, \\ x - 4 & \text{for } 4 < x \leq 6, \end{cases}$$

and that $f(x) = f(x+4)$ for all real values of x.

(i) Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 7$. [3]

(ii) If the domain of f is restricted to $k \leq x < 4$, state the smallest value of k for which the function f^{-1} exists. For this value of k, find $f^{-1}(x)$ and state its domain. [4]

Another function g is defined by $g(x) = \ln x$, $x \in \mathbb{R}$, $e^3 < x < e^5$.

(iii) Taking the domain of f to be [2, 6], show that fg exists. Find also the exact range of fg. [3]

8 The curve C_1 has equation $\dfrac{x^2}{4} - y^2 = 1$. The curve C_2 has equation with $x^2 = a^2(1 - y^2)$ where $a > 0$, $a \neq 1$.

(a) State a sequence of transformations which transforms the graph with equation $(x+1)^2 + y^2 = 1$ onto the graph of C_2. [2]

(b) **(i)** Sketch C_1, labelling clearly the coordinates of the points of intersection with the axes and the equations of any asymptotes. [2]

(ii) State the value of a such that C_1 and C_2 intersect at exactly 2 points. [1]

It is now given that a is the value found in **(b)(ii)**.

(iii) Sketch C_2 on the same diagram as C_1. [1]

(iv) By using the substitution $x = a\cos\theta$, find the exact area bounded by C_2 and the lines $x = 2$ and $y = 1$. [6]

9 An oil company wants to drill holes to reach an oil deposit 8000 metres below ground level. The company has 2 drilling teams, A and B. The teams have been tasked to drill a hole each on separate sites above the large oil deposit.

Team A decides to drill 190 metres on Day 1. On subsequent days, the team will drill $r\%$ of the depth drilled on the previous day, where $0 < r < 100$.

(a) Find the range of values of r that will result in team A never reaching the oil deposit. [3]

For the rest of the question, let $r = 99$.

(b) On which day will team A reach the oil deposit? [3]

Team B has a different plan. It decides to drill 180 metres on Day 1. On subsequent days, the team will drill 1 metre less than the depth drilled on the previous day. The 2 teams start drilling on the same day.

(c) Find the first day that the depth drilled by team A on that day is less than the depth drilled by team B. [3]

(d) Determine which team will reach the oil deposit first. [3]

10 **(a)** Show that $\dfrac{v}{(16+v)(9-v)} = \dfrac{a}{(16+v)} + \dfrac{b}{(9-v)}$ where a and b are constants to be determined. [2]

(b) A cyclist is riding in one direction along a straight horizontal road. She starts with zero speed, and t seconds later, her speed v metres per second satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{(16+v)(9-v)}{320v}.$$

(i) Find t in terms of v. [4]

(ii) Find the cyclist's theoretical maximum speed. Hence find the time she takes to reach a speed equal to half her theoretical maximum speed. [3]

It is now given that when v is small, an approximate solution to the above differential equation is

$$v = \sqrt{\frac{9t}{10}}.$$

It is known that $v = \dfrac{\mathrm{d}x}{\mathrm{d}t}$ where x metres is the distance travelled by the cyclist after t seconds.

(iii) Find x in terms of t, and hence find the time the cyclist takes to travel 10 metres. [3]

11 A parabola G has equation $3y^2 = 2x - 1$. The point A on G has y-coordinate p, where $p > 0$. The tangent to G at A intersects the y-axis at the point B. The point C is a point on the y-axis such that AC is parallel to the x-axis.

(i) Show that the equation of the tangent to G at A can be expressed as

$$6py = 2x + 3p^2 - 1.$$ [3]

(ii) Show that the area of ΔABC is given by $\frac{1}{24}\left(9p^3 + 6p + \frac{1}{p}\right)$. [2]

(iii) Without the use of a calculator, find the minimum value of the area of ΔABC, proving that it is a minimum. [6]

(iv) Determine the type of triangle ΔABC is when its area is a minimum. [2]