

TEMASEK JUNIOR COLLEGE
2022 JC2 Preliminary Examination
Higher 1



CANDIDATE
NAME

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INDEX
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MATHEMATICS

8865/01

Paper 1

26 August 2022

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Marks	

Section A: Pure Mathematics [40 marks]

- 1 Find algebraically the set of values of k for which

$$2kx^2 - 4x + (k+1) > 0$$

for all real values of x .

[4]

- 2 (a) Differentiate $\ln\left(9x^{\frac{1}{3}} - 2x^2\right)^3$ with respect to x .

[3]

(b) Find $\int \frac{(2x-3)^2}{\sqrt{x}} dx$.

[3]

- 3 (i) Find the exact root of the equation $2e^{2x} - 1 = 3e^{-2x}$. [3]
 (ii) Sketch on the same diagram, the graphs of $y = 2e^{2x} - 1$ and $y = 3e^{-2x}$, showing clearly the equations of any asymptotes and the coordinates of the y -intercepts. [2]
 (iii) Use (i) and (ii) to deduce in exact form, the range of values of x that satisfy the inequality $2e^{2x} - 3e^{-2x} - 1 > 0$. [1]
 (iv) Deduce the range of values of p for which $2e^{-p} - 3e^p - 1 > 0$. [1]

- 4 A curve C satisfies the equation $\frac{dy}{dx} = \frac{a}{(x-5)^2}$, where a is a constant. The equation of the tangent to C at the point P where the x -coordinate is 2 is $3y + 2x = 16$.

(i) Show that the equation of C is $y = 6\left(\frac{1}{x-5} + 1\right)$. [4]

- (ii) Sketch C , stating the equation(s) of the asymptotes and coordinates of any points of intersection with the axial axes. [2]

- (iii) Write down the equation of the line which passes through the origin and P . [1]

- (iv) Hence, find the exact area of the finite region bounded by C , the line in (iii) and the x -axis. [4]

- 5 A small high-end collectible manufacturer decides to produce ninth, sixth and quarter scale diecast figurings of superhero War-Robot. The total cost of making 20 ninth scale War-Robots is \$30 more than the cost of making 4 sixth scale War-Robots. The total production cost of making 30 ninth scale, 16 sixth scale and 6 quarter scale War-Robots is \$8517. Based on experience, if the manufacturer decides that the profit earned from selling a ninth, sixth and quarter scale War-Robot are 120%, 100% and 80% of their production cost respectively, the total profit earned from selling a set of 3 different scale War-Robots will be \$661.50.

- (i) By writing down three linear equations, find the cost price of making each scale of War-Robot, correct to the nearest cent. [4]

Due to high demand for the sixth scale War-Robot, the manufacturer decides to use a new manufacturing process with new molds. It is given that the manufacturer must produce at least 21 sixth scale War-Robots to be sold so that it will be operationally viable. The manufacturer is interested in modelling his manufacturing costs, C thousand dollars, for number of sixth scale War-Robot, w . The model the manufacturer uses is

$$C = \frac{w}{20} - \ln(3w - 60) + 11, \text{ for } w > 20.$$

- (ii) Use differentiation to find the value of w which gives a stationary point on the graph of C against w . Justify whether C is a minimum or maximum. [4]
- (iii) Sketch the graph of C against w for $w > 20$. Estimate the minimum cost C . [2]

The manufacturer decides to sell each sixth scale War-Robot for \$420.

- (iv) Write down an equation relating the profit earned, P thousand dollars, from selling a sixth scale War-Robot. What is the minimum order you would advise the manufacturer to accept? [2]

Section B: Probability and Statistics [60 marks]

- 6 The elected college student councillors consist of 9 boys and 5 girls. An executive committee consisting of 5 members is to be formed from these student councillors.

(i) How many different committees can be formed if there must be at least one girl and at least one boy? [2]

The 5 chosen members consist of 3 boys and 2 girls. They are to stand in a line to take a photoshoot.

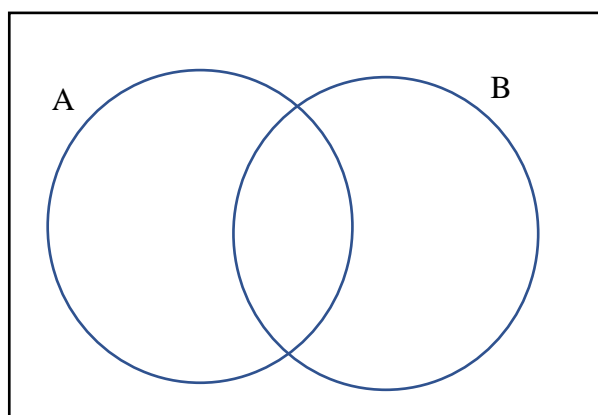
(ii) In how many ways can they stand in a line where Linda, one of the girls, must stand in between 2 boys? [2]

- 7 Events A and B are such that $P(A \cup B) = 0.8$, $P(B) = 0.4$ and $P(A|B) = 0.3$.

(i) Explain what is meant by $P(A' \cap B)$ and find its value. [3]

(ii) Find $P(A)$. [2]

(iii) Complete the Venn diagram to show the probability in each of the four regions. [1]



(iv) Explain whether A and B are independent. [1]

- 8 A fruit grower produces a large number of peaches every day. A small portion p of these peaches is infected. A check is carried out each day by taking a random sample of 60 peaches and examining them for infection. The number of infected peaches, X , in the sample may be well modelled by a binomial distribution.

(i) It is given that for a particular sample, the probability that at most one peach is infected is 0.61. Form an equation in p and find the value of p numerically. [3]

Using a special cultural method, p is now controlled at 0.02.

(ii) Find the probability that, for a particular day, at least 2 peaches are infected in a random sample of 60 peaches. [1]

The day's production is accepted as satisfactory if at most one infected peach is found from the random sample of 60 peaches.

Find the probability that in, a month of 30 days, the number of days with day's production accepted as satisfactory is within one standard deviation from its mean. [4]

- 9 A company engaged an advertising company to help to promote a new product to the market. The advertising costs, w dollars per week, together with the number of items sold, v , per week, are given in the following table.

w	55	60	65	70	75	80	85	90
v	400	420	410	430	460	490	550	520

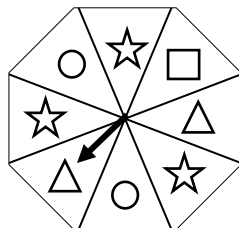
- (i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [1]
 (ii) Find the value of the product moment correlation coefficient and comment on its value in the context of the data. [2]
 (iii) Calculate the equations of the regression line of w on v and that of v on w . [2]
 (iv) With justification, choose a suitable regression line from (iii) and use it to calculate an estimate of the advertising cost given the number of items sold in the week was 500. Comment on the reliability of your estimate. [3]

The weekly advertising costs are all increased by a fixed percentage due to inflation.

- (v) Without any further calculations, state what will happen to the value of the product moment correlation coefficient found in part (ii). [1]

10

At a funfair carnival, a game is designed such that each player has to spin a fair spinner 3 times on a regular octagon turntable as shown in the diagram below. The spinner has 8 sectors and an arrow which has an equal chance of coming to rest over any of the 8 sectors. The spinner has 1 sector labelled \square , 2 sectors labelled \triangle , 2 sectors labelled \circ and 3 sectors labelled \star (see diagram).



- (i) Given that a player spins exactly two stars for his game, find the probability that the first spin did not show a star. [3]

For one game, a participant wins a \$6 voucher if he spins 3 identical shapes. He wins a \$2 voucher if he spins the star once or twice only. Otherwise, he receives nothing.

Find the probability that, for one game, a participant wins

- (ii) \$6, [1]
 (iii) \$2. [2]

Anand and Charlie each plays the game three times. Find the probability that Anand wins \$10 worth of vouchers in total or Charlie does not win anything, but not both. [3]

- 11 It is known that under standard growing conditions, the mean height of a certain type of plant is 31.4 cm in an island located on earth northern hemisphere. 100 of these plants were genetically modified and the height of each plant, x cm, was recorded. The data obtained is summarised by

$$\sum (x - 30) = 260 \quad \text{and} \quad \sum (x - 30)^2 = 7844.$$

- (i) Find unbiased estimates of the population mean and variance. [2]
 (ii) Test at the 5% significance level whether the height of the plants has increased under the genetic engineering. [4]
 (iii) State, with a reason, whether it is necessary to assume that the heights of these plants are distributed normally for the test to be valid. [1]

Another test is carried out with another 100 of the same type of plants in an island located on earth southern hemisphere. The mean height of the plants in this sample is h cm and the sample variance is 55 cm^2 . It was found at the 10% significance level that the mean height of the plants is larger than the mean height of the plant under standard growing condition.

- (iv) Find the range of possible values of h , correct to 3 decimal places. [4]

- 12 The duration of a patient's consultation, in minutes, with a general practitioner (GP) and a specialist in a polyclinic are modelled as having independent normal distributions with mean and standard deviation as shown in the following table.

	Mean	Standard Deviation
Consultation with GP	6.5	1.7
Consultation with specialist	μ	σ

- (i) The duration of a patient's consultation with a specialist is equally likely to be less than 11.8 minutes and more than 12.8 minutes, with a probability of 0.406. Find μ and σ , correct to 1 decimal place. [3]

For the rest of the question, use $\mu = 10.5$ and $\sigma = 2.9$.

- (ii) Find the probability the total duration of 3 randomly chosen patients' consultation with the GP is shorter than twice the duration of a randomly chosen patient's consultation with the specialist. [3]
- (iii) Find the duration of consultation with a GP such that 10% of the patients in the polyclinic will exceed. [1]

The consultation fee charged by the GP is made up of 2 components: a fixed component of \$10 and a variable component of \$1 per minute. Similarly, the consultation fee charged by the specialist has a fixed component of \$25 and a variable component of \$2 per minute.

- (iv) Find the probability that the consultation fees of 2 randomly chosen patients visiting the specialist is within 5 dollars of the consultation fees of 5 randomly chosen patients visiting the GP. [3]

It is also known that the mean and standard deviation of duration of a patient's consultation with a dentist is 15.2 minutes and 3.1 minutes respectively.

- (v) In a particular week, 80 patients visited the dentist. Estimate the probability that the average consultation times of these 80 patients is between 16 minutes and 18 minutes. [2]