SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME	Solutions		
CLASS	4	REGISTER NUMBER	
CENTRE NUMBER	S	INDEX NUMBER	

MATHEMATICS **4048/01**

PAPER 1

Monday **22 August 2022** **2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 Expressed as the product of its prime factors, $1224 = 2^3 \times 3^2 \times 17$.

(a) The number $1224 \div \dfrac{m}{n}$, where m and n are prime numbers, is a perfect cube.
Write down the value of m and of n.

$$1224 \div \frac{m}{n} = \frac{2^3 \times 3^2 \times 17 \times n}{m}$$ [1]

$m = 17$ and $n = 3$

(b) The highest common factor and lowest common multiple of three numbers are 6 and 1224 respectively.
Two of these numbers are 36 and 102, and the third number lies between 50 and 500.
Find two possible values of the third number.

HCF = 6 = 2×3 , LCM = $2^3 \times 3^2 \times 17$ [2]

$36 = 2^2 \times 3^2$, $102 = 2 \times 3 \times 17$

Third number
$= 2^3 \times 3 \times 3 = 72$ or $2^3 \times 3 \times 17 = 408$

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2 A town is represented by an area of 20 cm^2 on a map of scale 1 : 400 000.
On a second map, the town is represented by an area of 51.2 cm^2.
Express the scale of the second map in the form 1 : n.

Map 1, 1cm^2 represents 16 km^2 [2]
Actual area = 20(16) = 320 km^2

Map 2, 51.2cm^2 represents 320 km^2
1cm^2 represents 6.25 km^2
Distance represented by 1 cm = 2.5 km

Therefore, scale is 1 : 250 000

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3 The estimated number of rhinoceroses in the wild decreased from 70 000 in 1970 to 27 000 in 2021.
The number decreased by r % every year.
Find the value of r.

$$27000 = 70000\left(1 - \frac{r}{100}\right)^{51}$$ [2]

$r = 1.85$ (3sf)

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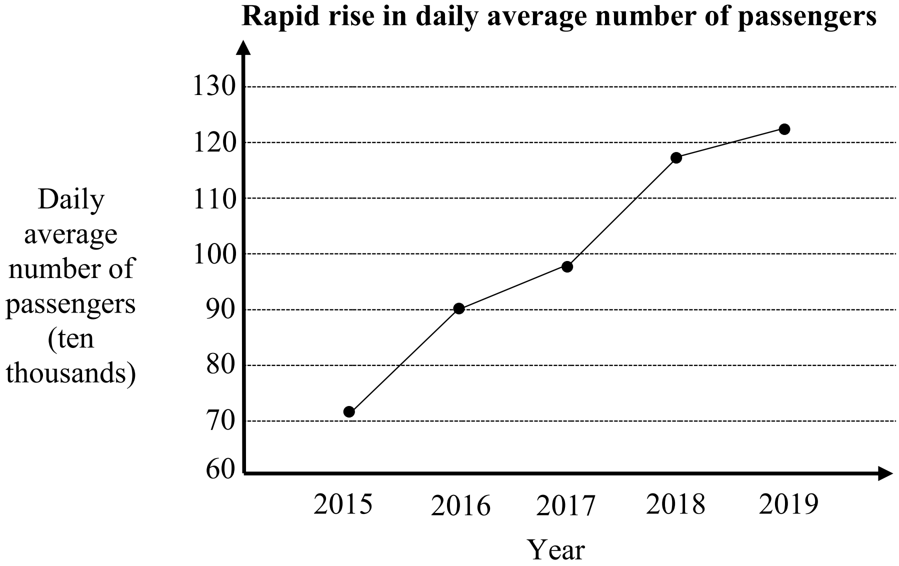
4 The graph shows the daily average number of passengers using the MRT system in Singapore from 2015 to 2019.



(a) State one misleading feature of the graph.

The vertical axis does not start from zero. [1]

(b) Explain how this feature affects the reader's interpretation of the graph.

As the vertical axis does not start from zero, this could **exaggerate the increase** in the daily average number of passengers over the years. For example, due to the reduced scale, the reader might think that the number of passengers in 2016 is approximately 2.5 times that in 2015. [1]

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5 Arnold purchased a bicycle from a shop at a discount of 20 % off the marked price.
After a week, he sold it at $2205 and made a profit of 125 %.
Calculate the marked price of the bicycle at the shop.

[2]

$$\text{Amount he paid} = \frac{100}{225}(2205)$$
$$= \$\,980$$

$$\text{Marked price} = \frac{100}{80}(980)$$
$$= \$1225$$

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6 **(a)** Express $20-9x+x^2$ in the form $(x+p)^2+q$.

$$\left(x-\frac{9}{2}\right)^2-\frac{1}{4}$$ [2]

(b) Hence, solve $x^2+20=9x$.

$$\left(x-\frac{9}{2}\right)^2=\frac{1}{4}$$ [2]

$$x-\frac{9}{2}=\frac{1}{2} \quad ; \quad x-\frac{9}{2}=-\frac{1}{2}$$

$$x=5 \qquad\qquad x=4$$

---

7 When a number is first decreased by p % and then increased by p %, where p is a positive integer, the overall percentage decrease is 4 %. Find p.

$$\left(\frac{100+p}{100}\right)\left(\frac{100-p}{100}\right)=\frac{96}{100} \quad \text{or} \quad 1-\left(\frac{100+p}{100}\right)\left(\frac{100-p}{100}\right)=\frac{4}{100}$$ [2]

$$100^2-\left(100^2-p^2\right)=400$$

Since $p>0$, $p=20$

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8 The area of triangle LMN is 796.6 m^2.
LM = 62 m and MN = 52 m.

Find the two possible sizes of the angle LMN.

$$\frac{1}{2}(62)(52)\sin\angle LMN=796.6$$ [2]

$$\angle LMN = 29.6^\circ \quad \text{or} \quad 150.4^\circ$$

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9 **(a)** The first four terms of a sequence are 1, 5, 12 and 22.
Find an expression, in terms of n, for the nth term of the sequence.

$T_n = \frac{3}{2}n^2 - \frac{1}{2}n$ [1]

(b) A series of rectangles was drawn.
The table shows the width and the area of the first four rectangles drawn.

Width (cm)	Area (cm^2)
2	10
4	36
6	78
8	136

(i) Find an expression, in terms of w, for the length of the rectangle with width w cm.

Length	Width (cm)	Area (cm^2)
5	2	10
9	4	36
13	6	78
17	8	136

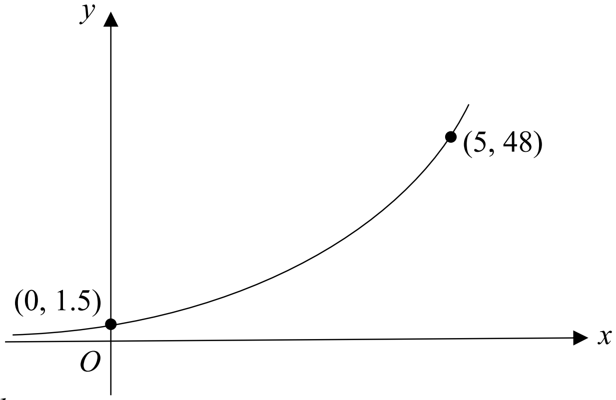
[1]

Length = $2w+1$

(ii) Would it be possible for the perimeter of a rectangle in this sequence to be 596 cm?
Explain your answer clearly.

Perimeter = $2w+2(2w+1)$ [2]
$= 6w + 2$
Let $6w + 2 = 596$
$w = 99$
Since w must be a **positive even number**, it is not possible for the perimeter of a rectangle in this sequence to be 596 cm.

10 The sketch shows the graph of $y = ka^x$.
The points (0, 1.5) and (5, 48) lie on the graph.



Find the value of a and of k.

At (0, 1.5), $1.5 = k(a)^0$ At (5, 48), $48 = 1.5(a)^5$ [2]

$k = 1.5$ $a = 2$

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11 **(a)** Simplify $\left(\dfrac{e^4}{343f^3}\right)^{-\frac{1}{3}}$.

[1]

$$\left(\frac{e^4}{343f^3}\right)^{-\frac{1}{3}} = \left(\frac{343f^3}{e^4}\right)^{\frac{1}{3}}$$
$$= \frac{7f}{e^{\frac{4}{3}}}$$

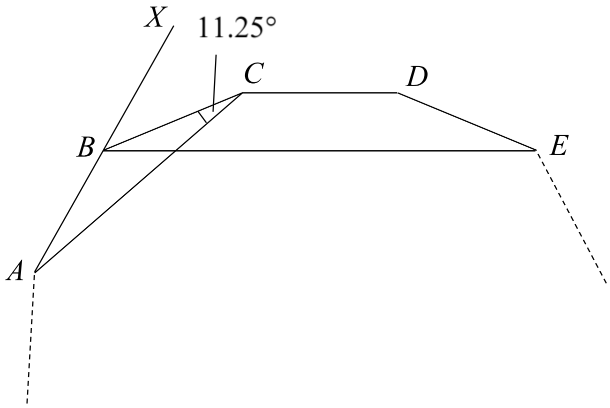
(b) Solve $\dfrac{15^{\frac{1}{2}x}}{27^x\left(5^{3x}\right)} = 225$.

[3]

$$\frac{15^{\frac{1}{2}x}}{27^x\left(5^{3x}\right)} = 225$$
$$\frac{15^{\frac{1}{2}x}}{15^{3x}} = 15^2$$
$$15^{\frac{1}{2}x-3x} = 15^2$$
$$\frac{1}{2}x - 3x = 2 \quad \Rightarrow \quad x = -\frac{4}{5}$$

[Turn Over

12 A, B, C, D, E, … are some of the vertices of a regular polygon.
ABX is a straight line and angle $BCA = 11.25°$.



(a) Calculate

(i) angle CBX,

Angle $CBX = 2\ (11.25^{o})$ (exterior angle of triangle) [1]
$= 22.5^{o}$

(ii) angle BED.

Angle $BED = 22.5^{o}$ (alternate angles) [1]

Or, Angle $BCD = 180^{o} - 22.5^{o}$ (adjacent angles on a straight line)
$= 157.5^{o}$
Angle $BED = 180^{o} - 157.5^{o}$ (interior angles)
$= 22.5^{o}$

Or, Angle $BCD = 180^{o} - 22.5^{o}$ (adjacent angles on a straight line)
$= 157.5^{o}$
Angle $BED = \frac{1}{2}\left[360^{o} - 2\left(157.5^{o}\right)\right]$ (angle sum of quadrilateral)
$= 22.5^{o}$

(b) A, C and E are consecutive vertices of a second regular polygon.
Find the number of sides of this polygon.

Angle $ACE = 157.5^{o} - 2(11.5^{o})$ [2]
$= 135^{o}$

Number of sides $= \dfrac{360^{o}}{180^{o} - 135^{o}}$
$= 8$

Or, Let n be number of sides.
$180\left(n-2\right) = 135n$
$n = 8$

13 Given that $a = c + \sqrt{\dfrac{2\left(b^2+1\right)}{a}}$, express b in terms of a and c.

[3]

$$a = c + \sqrt{\frac{2\left(b^2+1\right)}{a}}$$

$$a\left(a-c\right)^2 = 2b^2 + 2$$

$$b^2 = \frac{a\left(a-c\right)^2 - 2}{2}$$

$$b = \pm\sqrt{\frac{a\left(a-c\right)^2 - 2}{2}}$$

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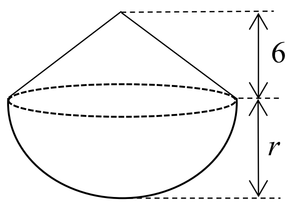
14 The diagram shows a solid made from a cone and a hemisphere of radius r cm.
The cone has a height of 6 cm.
The volume of the hemisphere is 2.5 times the volume of the cone.
Find r.

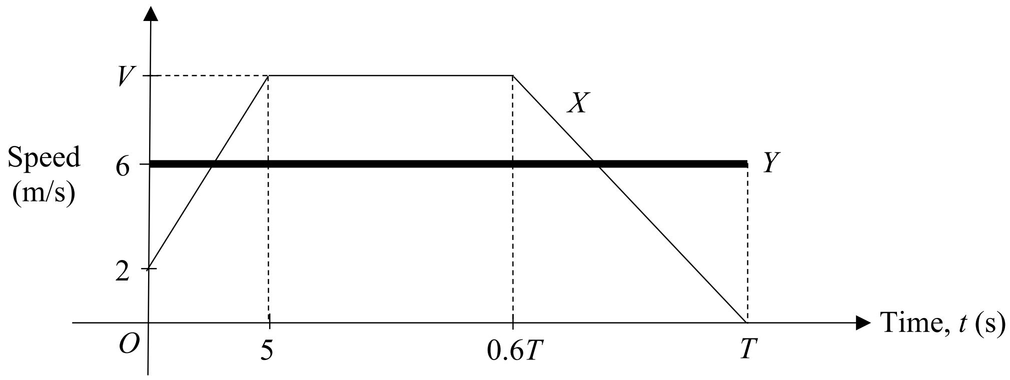


[2]

$$2.5\left(\frac{1}{3}\pi r^2(6)\right) = \frac{2}{3}\pi r^3$$

$$r = 7.5$$

15 The diagram shows the speed-time graphs for two cyclists, X and Y.



X accelerated uniformly for 5 seconds and then travelled at a constant speed of V m/s before decelerating uniformly to a stop at $t = T$.
Y travelled at a constant speed of 6 m/s.

(a) The acceleration of X in the first 5 seconds is 1.4 m/s^2. Show that $V = 9$. [1]

$$\frac{V-2}{5} = 1.4$$
$$V - 2 = 7$$
$$V = 9$$

(b) Find the speed of X when $t = 3$. [1]

Speed = $2 + 1.4(3)$
= 6.2 m/s

(c) At $t = T$, X travelled 12.5 m more than Y.
Find T. [3]

$$6T + 12.5 = \frac{1}{2}(5)(2+9) + \frac{1}{2}(9)(T - 5 + 0.6T - 5)$$
$$12T + 25 = 55 + 14.4T - 90$$
$$T = 25$$

(d) Find the average speed of X. [2]

Average speed = $\dfrac{6(25)+12.5}{25}$
= 6.5 m/s

Or, Average speed = $\dfrac{\frac{1}{2}(5)(2+9)+\frac{1}{2}(9)(10+20)}{25}$
= 6.5 m/s

16 **(a)** Simplify $\dfrac{m^2-6mn+9n^2}{10}\times\dfrac{5m}{9n-3m}$.

[2]

$$\begin{aligned}&\frac{m^2-6mn+9n^2}{10}\times\frac{5m}{9n-3m}\\&=\frac{(m-3n)^2}{10}\times\frac{5m}{-3(m-3n)}\\&=\frac{m-3n}{2}\times\frac{m}{-3}\\&=\frac{3mn-m^2}{6}\end{aligned}$$

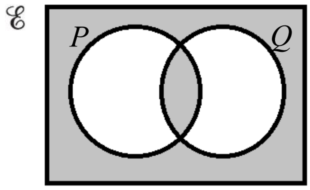
(b) Factorise completely $8p-6q^2+2q-24pq$.

[2]

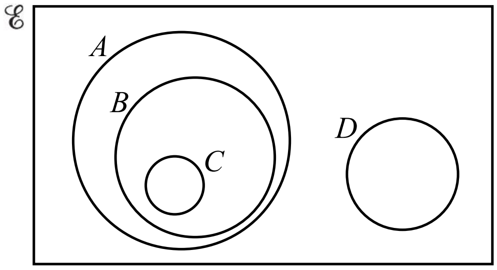
$$\begin{aligned}&8p-6q^2+2q-24pq\\&=2\left[4p(1-3q)+q(1-3q)\right]\\&=2(1-3q)(4p+q)\end{aligned}$$

17 (a) On the Venn diagram, shade the region(s) which represent $(P' \cap Q') \cup (P \cap Q)$.

[1]



(b)



(i) Use one of the symbols below to complete each statement.

$\subset \quad \phi \quad \not\subset \quad \in \quad \notin$

(a) $C \subset A$ [1]

(b) $B \cap D = \phi$ [1]

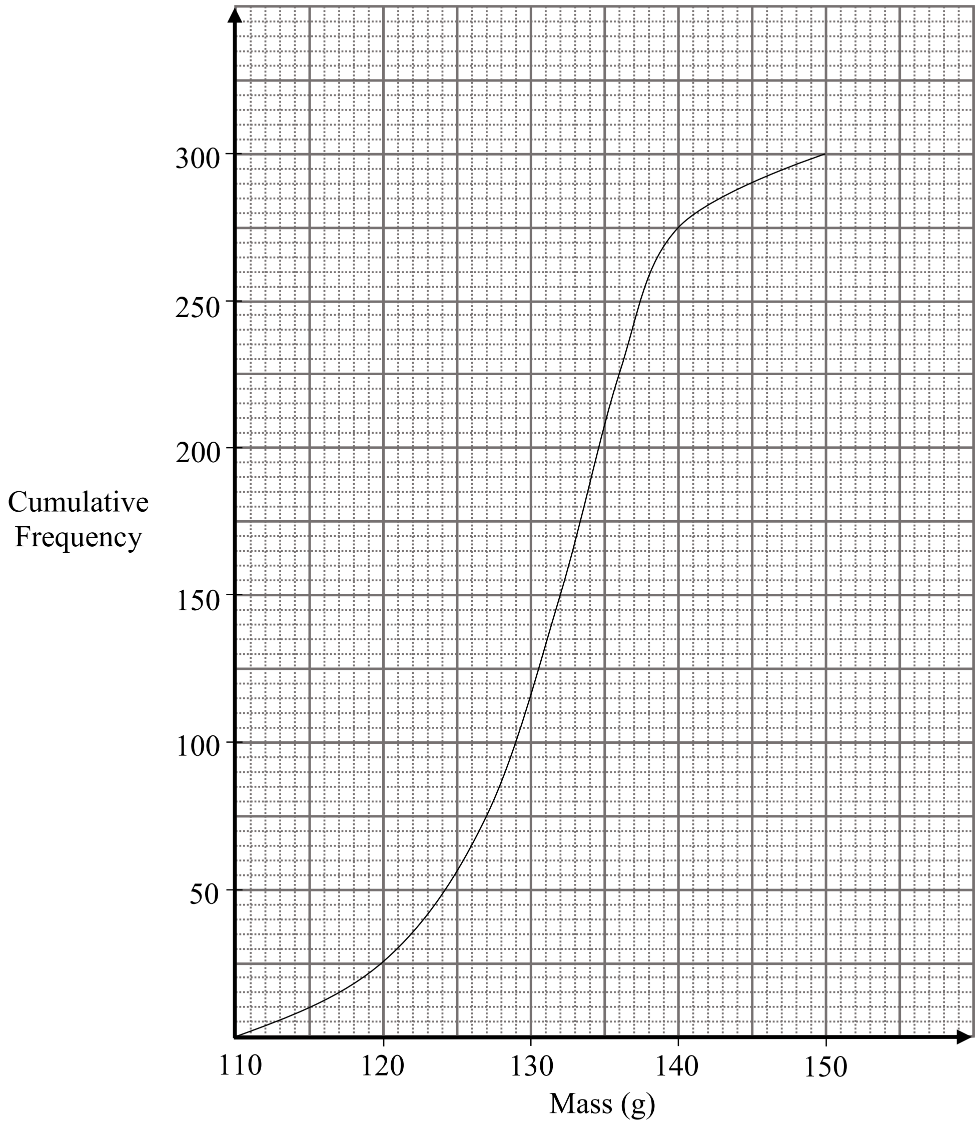
(ii) Given $\mathscr{E}$ = {quadrilaterals}
A = {parallelograms}
B = {rectangles}

name the quadrilaterals in the set C and the set D.

C = { squares } [1]

D = { trapeziums } or { kites } [1]

18 The cumulative frequency curve shows the distribution of the masses, in grams, of 300 oranges.



(a) Use the curve to estimate

(i) the median,

Median = 132 g [1]

(ii) the interquartile range.

Interquartile range = 136 – 127 [2]
= 9 g

(b) Oranges of mass at least 140 g are considered Grade A oranges.
Estimate the number of Grade A oranges.

Number of oranges = 25 [1]

[Turn Over

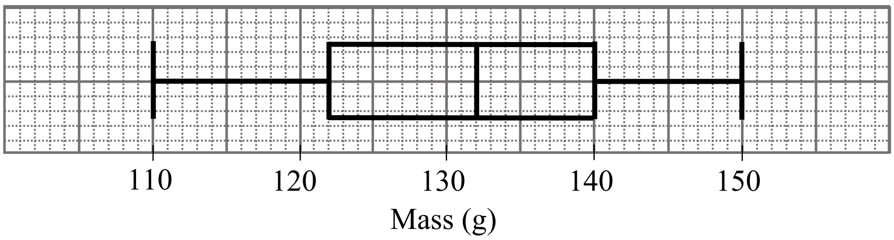
(c) Two oranges are chosen at random.
Find the probability that one is a Grade A orange and the other is not a Grade A orange.

P(one Grade A and the other not Grade A)

$= \frac{25}{300}\left(\frac{275}{299}\right)(2)$ [2]

$= \frac{275}{1794}$

(d) The box-and-whisker plot shows the distribution of the masses, in grams, of 300 apples.



If the distribution of the masses of the apples is represented using a cumulative frequency curve instead, explain how the cumulative frequency curve for the apples will differ from the cumulative frequency curve for the oranges. Give a reason for your answer.

The cumulative frequency curve for the apples will be **less steep about the median** than the cumulative frequency curve for the oranges. The bottom half of the curve will shift to the left while the top half of the curve will shift to the right. [2]

This is because the **interquartile range for the apples** is **larger** than that for the oranges, suggesting that the masses of the apples are more widely spread.

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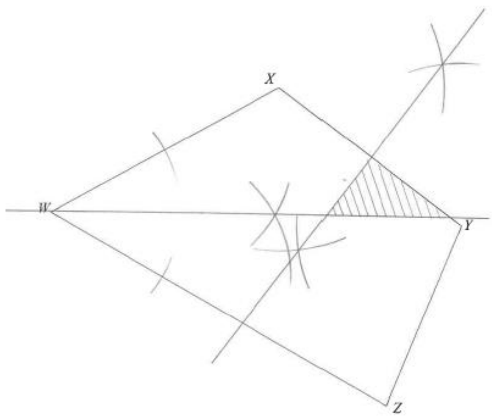
19 The diagram shows a field *WXYZ* on horizontal ground. [3]



20 A game consists of two separate and independent segments, Segment A and Segment B.
A player must complete both Segments A and B in order to complete the game.
If a player fails to complete any one segment, he is allowed to re-attempt that particular segment.

The probability of Carl completing Segment A in any one attempt is 0.75 .
The probability of Carl completing Segment B in any one attempt is 0.8 .

(a) Find the probability that Carl completes Segment A in the second attempt.

P(complete Segment A in second attempt) [1]
$= (0.25)(0.75)$
$= \dfrac{3}{16}$ or 0.1875

(b) Find, in terms of n, the probability that Carl completes Segment A in the n^{th} attempt.

P(complete Segment A in the nth attempt)
$= (0.25)^{n-1}(0.75)$

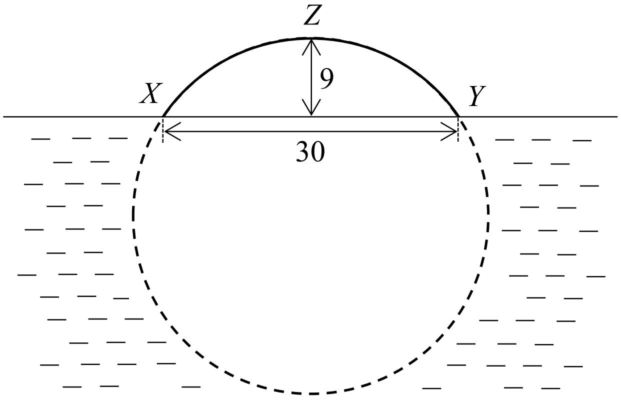
(c) Find the probability that Carl completes Segment B in not more than two attempts.

P(complete Segment B in no more than two attempts) [2]
$= 0.8 + 0.2(0.8)$
$= \dfrac{24}{25}$ or 0.96

(d) Find the probability that Carl completes the game with no more than two attempts in each segment.

P(complete Segments A and B in no more than two attempts) [2]
$= \dfrac{24}{25}[0.75 + 0.25(0.75)]$
$= \dfrac{9}{10}$ or 0.9

21 The diagram shows the cross-section of a cylindrical log of length 1.9 m partially floating in water.



X and Y are points on the log that are on the water surface.
$XY = 30$ cm.
The highest point of the log, Z, is 9 cm above the water surface.

Calculate the volume of the log, in cubic centimetres, which is below the water surface.

Let O be the centre of the circle, M be the midpoint of XY, W be the lowest point on the circle and r cm be the radius of the circle. [5]

$$r^2 = (r-9)^2 + 15^2$$
$$r = 17$$

$$\sin \angle MOY = \frac{15}{17}$$
or $\quad 30^2 = 17^2 + 17^2 - 2(17)(17)\cos \angle MOY$

$$\angle MOY = 1.0808$$

$$\angle XOY = 2.1616$$

Area of major segment XWY
$= \frac{1}{2}(17)^2\left[(2\pi - 2.1616) + \sin 2.1616\right]$

Volume below the water surface
$= 190\left(\frac{1}{2}\right)(17)^2\left[(2\pi - 2.1616) + \sin 2.1616\right]$
$= 136\ 000$ cm^3 (3sf)

Alternatively,

Let O be the centre of the circle, M be the midpoint of XY, W be the lowest point on the circle and r cm be the radius of the circle. [5]

$$r^2 = (r-9)^2 + 15^2$$
$$r = 17$$

$$\sin \angle MOY = \frac{15}{17}$$
$$\angle MOY = 61.927^o$$
or $\quad 30^2 = 17^2 + 17^2 - 2(17)(17)\cos \angle MOY$
$$\angle XOY = 123.855^o$$

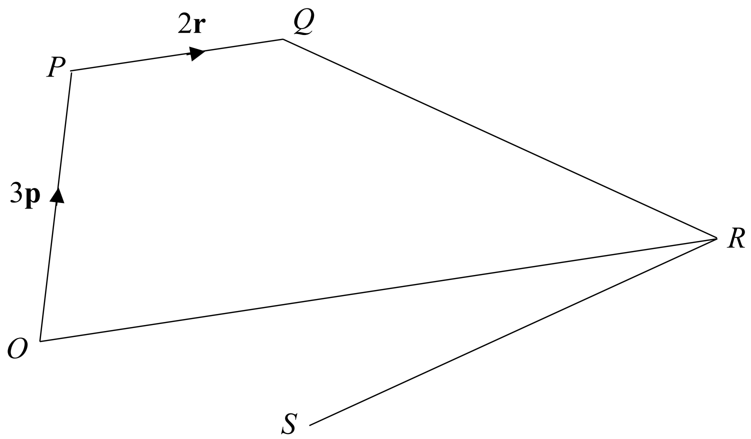
Area of major segment XWY
$= \dfrac{360-123.855}{360}(\pi)(17)^2 + \dfrac{1}{2}(17)^2 \sin 123.855^o$

Volume below the water surface
$= 190\left[\dfrac{360-123.855}{360}(\pi)(17)^2 + \dfrac{1}{2}(17)^2 \sin 123.855^o\right]$
$= 136\ 000$ cm^3 (3sf)

22 $OPQR$ is a trapezium where OR is parallel to PQ.

$\overrightarrow{OP} = 3\mathbf{p}$, $\overrightarrow{PQ} = 2\mathbf{r}$ and $OR : PQ = 3 : 1$.

S is a point such that $\overrightarrow{SR} = 2\mathbf{p} + \frac{10}{3}\mathbf{r}$.



(a) Express $\overrightarrow{QR}$, as simply as possibly, in terms of $\mathbf{p}$ and $\mathbf{r}$.

[1]

$$\overrightarrow{OR} = 6\mathbf{r}$$

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OP} - \overrightarrow{PQ}$$
$$= 4\mathbf{r} - 3\mathbf{p}$$

(b) Explain why $\overrightarrow{OS}$ is parallel to $\overrightarrow{QR}$.

[2]

$$\overrightarrow{OS} = \overrightarrow{OR} - \overrightarrow{SR}$$
$$= 6\mathbf{r} - 2\mathbf{p} - \frac{10}{3}\mathbf{r}$$
$$= \frac{8}{3}\mathbf{r} - 2\mathbf{p}$$
$$= \frac{2}{3}(4\mathbf{r} - 3\mathbf{p})$$

Since $\overrightarrow{OS}$ is $\frac{2}{3}\overrightarrow{QR}$, $\overrightarrow{OS}$ is parallel to $\overrightarrow{QR}$.

(c) Find $\dfrac{\text{area of triangle } OPQ}{\text{area of pentagon } OPQRS}$. [1]

Area triangle *OPQ* : area triangle *OQR* : area triangle *OSR*
= 1 : 3 : 2

$$\frac{\text{area of triangle } OPQ}{\text{area of pentagon } OPQRS} = \frac{1}{6}$$
