2022 Sec 4OLP Mathematics Preliminary Examinations Paper 2 Solution

Qn	Answer
	Total Marks: [100 Marks]
1ai	$\frac{3x+1}{2} < 1-\frac{2x}{5} \le 5$ $\frac{3x+1}{2} < 1-\frac{2x}{5}$, $1-\frac{2x}{5} \le 5$ $5(3x+1) < 10-4x$, $5-2x \le 25$ $19x < 5$, $-2x \le 20$ $x < \frac{5}{19}$, $x \ge -10$ $\therefore -10 \le x < \frac{5}{19}$
1aii	Largest integer = 0
1b	$\frac{(4x-10)^2}{10+21x-10x^2} = \frac{4(2x-5)^2}{(5x+2)(-2x+5)}$ $= \frac{-4(2x-5)^2}{(5x+2)(2x-5)}$ $= \frac{-4(2x-5)}{5x+2}$
1c	$\frac{6}{x+2} - \frac{x-5}{3x^2-12} = 1$ $\frac{6}{x+2} - \frac{x-5}{3(x-2)(x+2)} = 1$ $18(x-2)-(x-5) = 3x^2-12$ $18x-36-x+5 = 3x^2-12$ $3x^2-17x+19 = 0$ $x = \frac{-(-17) \pm \sqrt{(-17)^2-4(3)(19)}}{2(3)}$ $x = \frac{17 \pm \sqrt{61}}{6}$ $x \approx 4.13504$ or $x \approx 1.53162$ $\therefore x = 4.14$ or $x = 1.53$ (3 s.f.)
2a	No. of distinctions = 0.35 × 20 = 7 students $\therefore n = 36$
2b	Let the smallest score be x and the greatest score be y. $y - x = 44 \cdots\cdots\cdots (1)$ $x + y = 56 \cdots\cdots\cdots (2)$ $(2)-(1)$: $2x = 12$ $x = 6$ $y = 50$ $\therefore a = 6$, $b = 0$

2c	Mean (EL) = 33.3 , Standard Deviation (EL) = 11.4 Mean (Sci) = 38 , Standard Deviation (Sci) = 6 Since the mean score for the Science test is higher than that of the English test, the students generally performed better for the Science test. Since the standard deviation for the Science test is smaller than that of the English test, the scores for the Science test is less spread out and more consistent than the English test.
3a	$\mathbf{N} = \begin{pmatrix} 45 & 35 & 40 \end{pmatrix}$
3b	$\mathbf{C} = 0.9\begin{pmatrix} 45 & 35 & 40 \end{pmatrix}\begin{pmatrix} 15 & 18 \\ 20 & 15 \\ 12 & 16 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 40.5 & 31.5 & 36 \end{pmatrix}\begin{pmatrix} 15 & 18 \\ 20 & 15 \\ 12 & 16 \end{pmatrix}$ $\therefore \mathbf{C} = \begin{pmatrix} 1669.5 & 1777.5 \end{pmatrix}$
3c	\$1669.50 and \$1777.50 represents the **<u>total cost</u>** that **<u>Jon and Lim paid respectively (or each person paid)</u>** for their purchases after a 10% discount.
3di	$\mathbf{Q} = \begin{pmatrix} 32 & 0 & 0 \\ 0 & 72 & 0 \\ 0 & 0 & 45 \end{pmatrix}$
3dii	$\mathbf{R} = \begin{pmatrix} 480 & 576 \\ 1440 & 1080 \\ 540 & 720 \end{pmatrix}$
3e	Total amt. collected $= 1.2(1669.5)$ $= \$2003.40$ Amt. collected from oranges $= 2003.4 - 6\left(\frac{480}{5}\right) - 540$ $= \$887.40$ No. of pears that can be sold $= 0.85(1440)$ $= 1224$ $\therefore$ Price of each pear $= 887.4 \div 1224$ $= 0.725$ $= \$0.73$ (nearest cent)
4ai	**<u>Method 1</u>** $\angle DAB = 180° - 70°$ (angles on a str. line) $= 110°$ $\therefore \angle BCD = 180° - 110°$ (angles in opp. segment) $= 70°$
4ai	**<u>Method 2</u>** $\therefore \angle BCD = \angle EAD = 70°$ (ext. angle of cyclic quad.)

2022 Sec 4OLP Mathematics Preliminary Examinations Paper 2 Solution

4aii	**<u>Method 1</u>** $\angle ACD = 70° - 36°$ $= 34°$ $\therefore \angle ABD = \angle ACD$ (angles in same segment) $= 34°$
4aii	**<u>Method 2</u>** $\angle ADB = \angle ACB$ (angles in same segment) $= 36°$ $\therefore \angle ABD = 180° - 36° - 110°$ (sum of angles in ΔABD) $= 34°$
4aiii	$\angle BCT = \dfrac{180° - 72°}{2}$ ($BT = CT$, tangents from ext. pt.) $= 54°$ $\therefore \angle ACT = 54° + 36°$ $= 90°$
4b	Since **<u>$\angle ACT = 90°$</u>** , **<u>by tangent perpendicular to radius</u>**, **<u>AC is the diameter of the circle</u>**.
4c	Since $\angle AQB = 72° = 2(36°) = 2\angle ADB$, **<u>by angle at the centre, twice angle at the circumference</u>**, **<u>O is the centre of the circle</u>**.
5a	-0.4
5b	Correct plotting of points. Smooth curve
5c	Exact k: Either 1.14 or 2.86 (3s.f.) Accept: 1.0, 1.05, 1.1, 1.15 Accept: 2.85, 2.9, 2.95
5d	Drawing of tangent line. Exact gradient: -1.4 Accept: $-1.5 \le m \le -1.3$
5ei	$2 + x - \dfrac{x^3}{5} = -\dfrac{x}{4} + \dfrac{5}{4}$ $40 + 20x - 4x^3 = -5x + 25$ $4x^3 - 25x - 15 = 0$ $\therefore A = 4,\ B = 25$
5eii	Drawing of line $y = -\dfrac{x}{4} + \dfrac{5}{4}$. Exact x: -2.12, -0.642, 2.76 Accept: -2.05, -2.1, -2.15 Accept: -0.7, -0.65, -0.6, -0.55 Accept: 2.7, 2.75, 2.8, 2.85
6a	$\angle ADF = \angle GCF$ (alt. angles, AD parallel to CG) $\angle AFD = \angle GFC$ (vert. opp. angles) $DF = CF$ (F is the midpoint of DC) $\therefore$ By ASA, ΔADF is congruent to ΔGCF.

6b	$\angle CFG = \angle BAG$ (corrs. angles, AB parallel to FC) $\angle FCG = \angle ABG$ (corrs. angles, AB parallel to FC) $\angle CGF = \angle BGA$ (common angle) $\therefore \Delta FCG$ is similar to ΔABG.
6c	ΔAET is similar to ΔGBT ; ΔAET is similar to ΔBAT ΔAET is similar to ΔGFC ; ΔFDA is similar to ΔABG
6di	1 : 16
6dii	1 : 3
6diii	1 : 11
7a	**Method 1** $\angle N_Q QS = \angle N_S SR = 145°$ (corrs. angles) $\angle N_Q QP = 360° - 145° - 57° = 158°$ $\therefore$ Bearing of Q from $P = 180° - 158° = 022°$
7a	**Method 2** $\angle SRS_R = \angle N_S SR = 145°$ (alt. angles) $\angle N_P PR = \angle PRS_R$ (alt. angles) $= 145° - 68°$ $= 77°$ $\angle QPR = 180° - 68° - 57° = 55°$ $\therefore$ Bearing of Q from $P = 77° - 55° = 022°$
7b	$\angle QPR = 180° - 68° - 57° = 55°$ $\frac{\sin 57°}{90} = \frac{\sin 55°}{QR}$ $QR = \frac{90 \sin 55°}{\sin 57°}$ ≈ 87.9054 $\therefore SR = 87.9054 - 40$ ≈ 47.9054 $= 47.9$ m (3 s.f.) (shown)
7c	$PS = \sqrt{90^2 + 47.9054^2 - 2(90)(47.9054)\cos 68°}$ $PS \approx 84.6445$ $\therefore PS = 84.6$ m (3 s.f.)
7d	Let the new position of the surveyor be S'. $\cos 68° = \frac{RS'}{90}$ $RS' = 90 \cos 68°$ ≈ 33.71459 $\therefore$ Dist. to walk $= 47.9054 - 33.71459$ ≈ 14.19081 $= 14.2$ m (3 s.f.) The surveyor should walk towards point ***R*** for a distance of **14.2** m given that the distance between him and point P is a minimum.

2022 Sec 4OLP Mathematics Preliminary Examinations Paper 2 Solution

8a	$\left\|\overrightarrow{AB}\right\| = \sqrt{4.5^2 + 6^2} = 7.5$ units
8b	$\overrightarrow{BC} = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix}$
8c	$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$ $= \begin{pmatrix} -4.5 \\ -6 \end{pmatrix} + \begin{pmatrix} 7.5 \\ 0 \end{pmatrix}$ $\therefore \overrightarrow{BD} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$
8d	Gradient of line $= -\frac{6}{3} = -2$ Equation of line: $y - 7 = -2(x - 3)$ $\therefore y = -2x + 13$
8ei	**<u>Method 1</u>** $\overrightarrow{PD} = \frac{1}{2}\overrightarrow{BD}$ $\overrightarrow{OD} - \overrightarrow{OP} = \frac{1}{2}\begin{pmatrix} 3 \\ -6 \end{pmatrix}$ $\overrightarrow{OP} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 1.5 \\ -3 \end{pmatrix}$ $\therefore \overrightarrow{OP} = \begin{pmatrix} 3.5 \\ 1 \end{pmatrix}$
8ei	**<u>Method 2</u>** $\overrightarrow{PD} = \frac{1}{2}\overrightarrow{BD} = \begin{pmatrix} 1.5 \\ -3 \end{pmatrix}$ Coordinates of P : $(5 - 1.5, -2 + 3) = (3.5, 1)$ $\therefore \overrightarrow{OP} = \begin{pmatrix} 3.5 \\ 1 \end{pmatrix}$
8eii	Since the **<u>diagonals of a rhombus intersect perpendicularly</u>**, ***<u>$\angle CPD = 90°$</u>***. Hence, by **<u>right-angle in a semi-circle</u>** property, a circle passing through C, P and D **<u>can be drawn</u>** with CD as the diameter of the circle.
9a	$\cos \angle CMQ = \frac{3}{5} = \frac{6}{10} = \frac{MQ}{CM}$ $\therefore PQ = 6 \times 2 = 12$ m (shown)
9b	$MR = \sqrt{6^2 + 14.4^2} = 15.6$ m $\therefore$ Angle of elevation $= \tan^{-1}\left(\frac{10}{15.6}\right)$ ≈ 32.6609 $= 32.7°$ (1 d.p.)

9c

$$\angle AMC = 180° - 2\left[\cos^{-1}\left(\frac{3}{5}\right)\right] \approx 73.7397°$$

$$AP = \sqrt{10^2 - 6^2} = 8 \text{ m}$$

$$\text{Perimeter } ABCQP = \frac{73.7397°}{360°} \times 2\pi(10) + 8 + 12 + 8$$
$$\approx 40.87000 \text{ m}$$

$$\therefore \text{Total surface area of barn} = 40.87000 \times 14.4$$
$$\approx 588.528$$
$$= 589 \text{ m}^2 \quad (3 \text{ s.f.})$$

9d

Method 1

Let l_1 and l_2 be the lengths of the small and large barns respectively.

$$\frac{l_1}{l_2} = \sqrt[3]{\frac{1}{3}}$$

$$\frac{A_1}{A_2} = \left(\sqrt[3]{\frac{1}{3}}\right)^2$$

Since the cost of painting the roof is directly proportional to its area, then

$$\text{Cost of painting smaller barn} = \left(\sqrt[3]{\frac{1}{3}}\right)^2 \times 1000$$
$$\approx 480.7498$$
$$= \$480.75 \quad \text{(nearest cent)}$$

9d

Method 2

$$\text{Area of roof of larger barn} = \frac{73.7397°}{360°} \times 2\pi(10) \times 14.4$$
$$\approx 185.328 \text{ m}^2$$

$$\text{Area of roof of smaller barn} = \left(\sqrt[3]{\frac{1}{3}}\right)^2 \times 185.328$$
$$\approx 89.0964 \text{ m}^2$$

$$\therefore \text{Cost of painting smaller barn} = \frac{1000}{185.328} \times 89.0964$$
$$\approx 480.7498$$
$$= \$480.75 \quad \text{(nearest cent)}$$

10a

$$\text{Price of laptop for Smith} = \frac{5249}{107} \times 100$$
$$= \text{S}\$4905.6074$$

$$\text{Price of laptop on hire purchase} = 1000 + 18(230)$$
$$= \text{S}\$5140$$

$$\therefore \text{Additional amt. paid} = 5140 - 4905.6074$$
$$= \text{S}\$234.39 \quad \text{(nearest cent)}$$

10b

Method 1

Let the amount of euros that Smith has be €x.

Euros → SGD

S$1 = €0.6854

Amt. of Euros in SGD = S$$\left(\frac{x}{0.6854}\right)$

Euros → USD→ SGD

€1 = US$1.05

Amt. of USD = US1.05x$

S$1 = US$0.7193

Amt. of Euros in SGD = S$$\left(\frac{1.05x}{0.7193}\right)$

$$\frac{x}{0.6854}+8=\frac{1.05x}{0.7193}$$

$$0.7193x+3.94406576=0.71967$$

$$x=\frac{3.94406576}{0.71967-0.7193}$$

$$x\approx 10659.6371$$

Amount of Euros that Smith has is €10 659.64.

Plan A

Interest earned at end of 3 years in Euros

$$=\frac{(10659.6371-5000)(2.5)(3)}{100}$$

$$\approx €424.4727$$

Since S$$\left(\frac{1.05x}{0.7193}\right)$ gives more Singapore dollars, it will be used for all conversion from Euros to Singapore dollars.

Interest in S$ for Plan A = $\frac{1.05(424.4727)}{0.7193}$

= S$619.63 (nearest cent)

Plan B

Interest earned at end of 3 years in SGD

$$=\frac{\frac{1.05(10659.6371)}{0.7193}\times 1.8\times 3}{100}$$

$$=\text{S\$840.26} \quad \text{(nearest cent)}$$

Since **S$619.63 < S$840.26**, Plan A will not give higher interest than Plan B.

Hence, I **disagree with Smith**. In addition, Smith is a tourist and thus, does not qualify for Plan C.

10b

Method 2

Let the amount of euros that Smith has be €x.

Euros → SGD

S$1 = €0.6854

Amt. of Euros in SGD = S$$\left(\frac{x}{0.6854}\right)$

Euros → USD→ SGD

€1 = US$1.05

Amt. of USD = US1.05x$

S$1 = US$0.7193

Amt. of Euros in SGD = S$$\left(\frac{1.05x}{0.7193}\right)$

$$\frac{x}{0.6854}+8=\frac{1.05x}{0.7193}$$

$$0.7193x+3.94406576=0.71967$$

$$x=\frac{3.94406576}{0.71967-0.7193}$$

$$x\approx 10659.6371$$

Amount of Euros that Smith has is €10 659.64.

Plan A

Interest earned at end of 3 years in Euros

$$=\frac{(10659.6371-5000)(2.5)(3)}{100}$$

$$\approx €424.4727$$

Since S$$\left(\frac{x}{0.6854}\right)$ gives less Singapore dollars, it will be used for all conversion from Euros to Singapore dollars so that the amount of interest he will get is a minimum.

Interest in S$ for Plan A = $\frac{424.4727}{0.6854}$

= S$619.31 (nearest cent)

Plan B

Interest earned at end of 3 years in SGD

$$=\frac{\frac{10659.6371}{0.6854}\times 1.8\times 3}{100}$$

= S$839.83 (nearest cent)

Since **S$619.31 < S$839.83**, Plan A will not give higher interest than Plan B.

Hence, I **disagree with Smith**. In addition, Smith is a tourist and thus, does not qualify for Plan C.