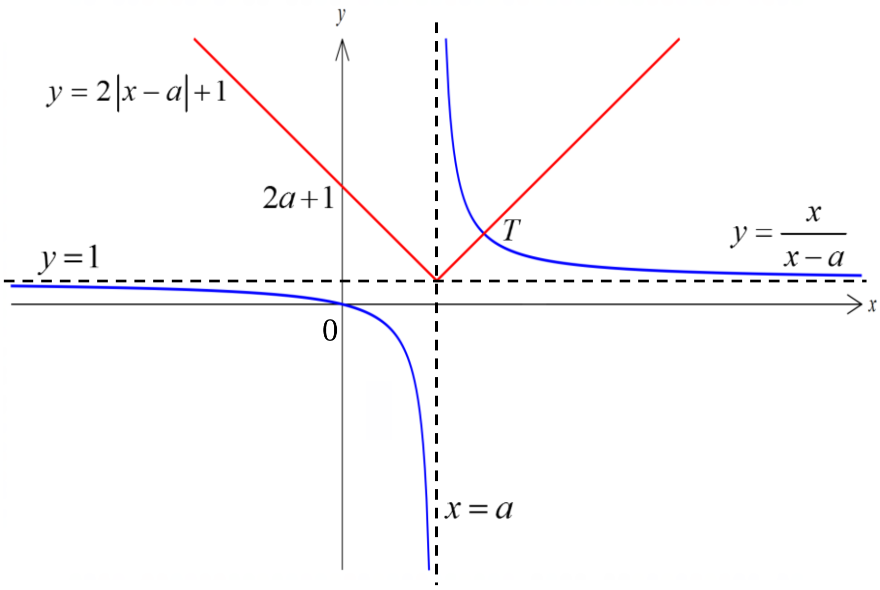
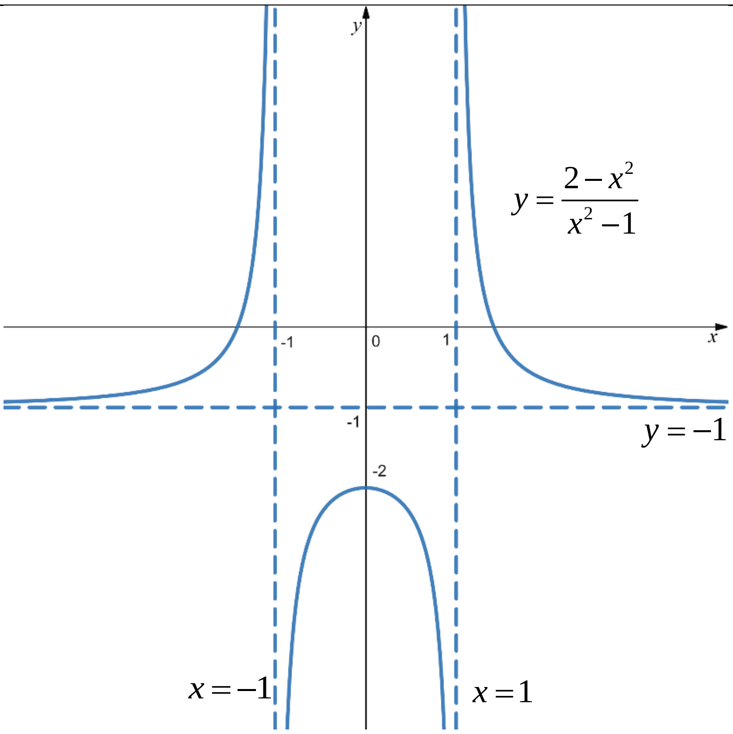
RAFFLES INSTITUTION
2022 Year 6 H2 Mathematics Preliminary Examination Paper 1 Solutions

1(i)	
(ii)	Let the 2 graphs intersect at the point T. To solve for T, consider $\frac{x}{x-a}=2(x-a)+1$ $x=2(x-a)^2+x-a$ $(x-a)^2=\frac{a}{2}$ $x=a\pm\sqrt{\frac{a}{2}}$ Since $x>a$ at the point T, $x=a+\frac{\sqrt{2a}}{2}$. Thus the solution is $a<x<a+\frac{\sqrt{2a}}{2}$.

2(i)	$\dfrac{1+2x-x^2}{2(x^2-2x)}$ $=\dfrac{1}{2}\left[\dfrac{2-(x^2-2x+1)}{(x^2-2x+1)-1}\right]$ $=\dfrac{1}{2}\left[\dfrac{2-(x-1)^2}{(x-1)^2-1}\right]$ Thus $p=-1$ and $q=\dfrac{1}{2}$.
(ii)	The 2 transformations (in either order) are 1. A translation of 1 unit in the positive x-direction 2. A scaling parallel to the y-axis by a factor of $\dfrac{1}{2}$.
(iii)	 The equation $\dfrac{2-x^2}{x^2-1}=k$ has no real roots for values of k in the set $(-2,\ -1]$.
	Alternative Solution: $\dfrac{2-x^2}{x^2-1}=k$ $2-x^2=kx^2-k$ $(k+1)x^2-(k+2)=0$ If $k=-1$, $(k+1)x^2-(k+2)=0$ becomes $-1=0$, so there is no solution. If $k\neq-1$, the quadratic equation will have no real roots if $0-4\left[-(k+2)(k+1)\right]<0$ $(k+2)(k+1)<0$ $-2<k<-1$ So $-2<k\leq-1$ and the required solution set is $(-2,\ -1]$.

3(i)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=-\dfrac{a}{t^2}$; $\quad \dfrac{\mathrm{d}y}{\mathrm{d}t}=a\left(1+\dfrac{2}{t^3}\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{a\left(1+\dfrac{2}{t^3}\right)}{-\dfrac{a}{t^2}}=-\dfrac{t^3+2}{t}$ When $t=-\dfrac{1}{2}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{15}{4}$, $x=-a$, $y=-\dfrac{9}{2}a$ Equation of tangent at P: $y+\dfrac{9}{2}a=\dfrac{15}{4}(x+a)$ $y=\dfrac{15}{4}x-\dfrac{3}{4}a$ Gradient of normal at $P=-\dfrac{4}{15}$ Equation of normal at P: $y+\dfrac{9}{2}a=-\dfrac{4}{15}(x+a)$ $y=-\dfrac{4}{15}x-\dfrac{143}{30}a$
(ii)	At point Q, $x=0$, $y=-\dfrac{3}{4}a$ At point R, $x=0$, $y=-\dfrac{143}{30}a$ Area of triangle PQR $=\dfrac{1}{2}\lvert a\rvert\left(\dfrac{143}{30}\lvert a\rvert-\dfrac{3}{4}\lvert a\rvert\right)$ $=\dfrac{1}{2}\left(\dfrac{241}{60}\right)\lvert a\rvert^2$ $=\dfrac{241}{120}a^2$ (shown)

4(a)

$$\mathbf{r}=\begin{pmatrix}a\\2\\3\end{pmatrix}\times\begin{pmatrix}1\\4\\5\end{pmatrix}=\begin{pmatrix}-2\\-5a+3\\4a-2\end{pmatrix}=\begin{pmatrix}-2\\3\\-2\end{pmatrix}+a\begin{pmatrix}0\\-5\\4\end{pmatrix},\ a\in\mathbb{R}$$

R lies on the line passing through the point with coordinates $(-2,3,-2)$, and the line is parallel to the vector $-5\mathbf{j}+4\mathbf{k}$.

(b)

Since F lies on the line PQ, then

$$\overrightarrow{OF}=\mathbf{p}+\lambda(\mathbf{q}-\mathbf{p})\ \text{, for some } \lambda\in\mathbb{R}\ .$$

Since $\overrightarrow{OF}$ is perpendicular to the line PQ, then

$$\left(\mathbf{p}+\lambda(\mathbf{q}-\mathbf{p})\right)\cdot(\mathbf{q}-\mathbf{p})=0$$

$$\mathbf{p}\cdot(\mathbf{q}-\mathbf{p})+\lambda\left|\mathbf{q}-\mathbf{p}\right|^2=0$$

$$\mathbf{p}\cdot\mathbf{q}-\left|\mathbf{p}\right|^2+\lambda\left|\mathbf{q}-\mathbf{p}\right|^2=0$$

$$\lambda=\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}$$

Substitute value of λ into (1) :

$$\overrightarrow{OF}=\mathbf{p}+\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}(\mathbf{q}-\mathbf{p})$$

$$=\left(1-\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}\right)\mathbf{p}+\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}\mathbf{q}$$

$$=(1-\lambda)\mathbf{p}+\lambda\mathbf{q}$$

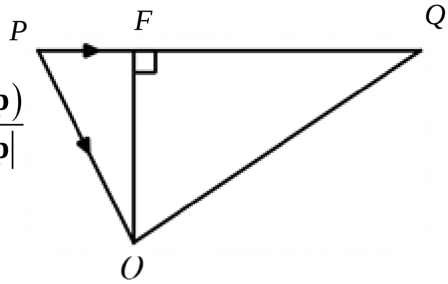
Alternative Method:

Consider $\overrightarrow{PF}$ as the projection vector of $\overrightarrow{PO}$ onto $\overrightarrow{PQ}$,

$$\overrightarrow{PF}=\left(\overrightarrow{PO}\cdot\overrightarrow{PQ}\right)\overrightarrow{PQ}$$

$$=\frac{-\mathbf{p}\cdot(\mathbf{q}-\mathbf{p})}{\left|\mathbf{q}-\mathbf{p}\right|}\times\frac{(\mathbf{q}-\mathbf{p})}{\left|\mathbf{q}-\mathbf{p}\right|}$$

$$=\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}(\mathbf{q}-\mathbf{p})$$



Hence we have

$$\overrightarrow{OF}=\overrightarrow{OP}+\overrightarrow{PF}$$

$$=\mathbf{p}+\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}(\mathbf{q}-\mathbf{p})$$

$$=\left(1-\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}\right)\mathbf{p}+\left(\frac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}\right)\mathbf{q}=(1-\lambda)\mathbf{p}+\lambda\mathbf{q}$$

where $\lambda=\dfrac{\left|\mathbf{p}\right|^2-\mathbf{p}\cdot\mathbf{q}}{\left|\mathbf{q}-\mathbf{p}\right|^2}$ (shown)

(b)(ii)

For F to lie within the line segment PQ,

$$0\le\lambda\le1$$

5(i)	$z=\sin\left(\frac{\pi}{6}\right)+\mathrm{i}\cos\left(\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{2}-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$ $=\cos\left(\frac{\pi}{3}\right)+\mathrm{i}\sin\left(\frac{\pi}{3}\right)=\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}.$ $\therefore \lvert z\rvert=1$ and $\arg(z)=\frac{\pi}{3}.$ Hence $z^3=\left(\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}\right)^3=\mathrm{e}^{\mathrm{i}\pi}=-1.$ **Alternatively**: $z=\sin\left(\frac{\pi}{6}\right)+\mathrm{i}\cos\left(\frac{\pi}{6}\right)=\frac{1}{2}+\mathrm{i}\frac{\sqrt{3}}{2}.$ $\lvert z\rvert=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=1,\ \arg(z)=\tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right)=\frac{\pi}{3}$
(ii)	$w=\sqrt{2}\left[\sin\left(\frac{\pi}{6}\right)+\mathrm{i}\cos\left(\frac{\pi}{3}\right)\right]=\sqrt{2}\left(\frac{1}{2}+\mathrm{i}\frac{1}{2}\right)=\frac{\sqrt{2}}{2}+\mathrm{i}\frac{\sqrt{2}}{2}.$ Thus $w^4=\left(\frac{\sqrt{2}}{2}+\mathrm{i}\frac{\sqrt{2}}{2}\right)^4=\left(\frac{1}{2}+\mathrm{i}-\frac{1}{2}\right)^2=\mathrm{i}^2=-1$ **Alternatively:** $\lvert w\rvert=\sqrt{\left(\sqrt{2}\sin\frac{\pi}{6}\right)^2+\left(\sqrt{2}\cos\frac{\pi}{3}\right)^2}=\sqrt{2\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)^2}=1.$ $\arg(w)=\tan^{-1}\left(\frac{\sqrt{2}\cos\frac{\pi}{3}}{\sqrt{2}\sin\frac{\pi}{6}}\right)=\tan^{-1}(1)=\frac{\pi}{4}.$ Thus $w^4=\left(\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\right)^4=\mathrm{e}^{\mathrm{i}\pi}=-1.$
(iii)	Hence $z^{2022}-w^{2020}=\left(z^3\right)^{674}-\left(w^4\right)^{505}=(-1)^{674}-(-1)^{505}=1-(-1)=2$ OR $\mathrm{e}^{674\pi\mathrm{i}}-\mathrm{e}^{505\pi\mathrm{i}}=\mathrm{e}^{0\pi\mathrm{i}}-\mathrm{e}^{\pi\mathrm{i}}=1-(-1)=2$

6(a)(i)	$u_1 = 2$ $u_2 = \frac{4}{u_1} = \frac{4}{2} = 2$ $u_3 = \frac{4}{u_2} = \frac{4}{2} = 2$ The sequence is a constant sequence of 2.
(a)(ii)	$u_1 = 3$ $u_2 = \frac{4}{u_1} = \frac{4}{3}$ $u_3 = \frac{4}{u_2} = \frac{4}{\frac{4}{3}} = 3$ The sequence alternates between 3 and $\frac{4}{3}$.
(b)(i)	$v_2 = v_1 + 2$ $v_3 = v_2 + 3$ $\quad = v_1 + 2 + 3$ $v_4 = v_3 + 4$ $\quad = v_1 + 2 + 3 + 4$ $v_n = v_1 + 2 + 3 + 4 + ... + n$ $\quad = v_1 + \frac{n-1}{2}(2+n)$ $\quad = v_1 + \frac{(n-1)(n+2)}{2}$ $\therefore v_n = A + \frac{(n-1)(n+2)}{2}$
(b)(ii)	$\sum_{r=1}^{n} v_r = \sum_{r=1}^{n} \left(A + \frac{(r-1)(r+2)}{2} \right)$ $= \sum_{r=1}^{n} \left(A + \frac{r^2 + r - 2}{2} \right)$ $= \sum_{r=1}^{n} A + \frac{1}{2}\sum_{r=1}^{n} r^2 + \frac{1}{2}\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$ $= \sum_{r=1}^{n} (A-1) + \frac{1}{2}\sum_{r=1}^{n} r^2 + \frac{1}{2}\sum_{r=1}^{n} r$ $= n(A-1) + \frac{1}{12}n(n+1)(2n+1) + \frac{1}{4}(n)(n+1)$

7(i)	$y=\sqrt{2+\cos^2 x} \quad \Rightarrow \quad y^2=2+\cos^2 x$ Differentiating w.r.t x, $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=2\cos x(-\sin x)=-\sin 2x$ (shown) ------ (1)
(ii)	Differentiating (1) w.r.t x, $2y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=-2\cos 2x$ $y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-\cos 2x$ (shown) ------ (2)
	Differentiating (2) w.r.t x, $y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\sin 2x$ $y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+3\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=2\sin 2x$ ------- (3) Differentiating (3) w.r.t x, $y\dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}+\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)+3\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)+3\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=4\cos 2x$ $y\dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}+4\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)+3\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2=4\cos 2x$ ------ (4) When $x=0,\ y=\sqrt{3},\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=0,\ \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{\sqrt{3}}{3},\ \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=0,\ \dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}=\sqrt{3}$ By Maclaurin's Theorem, $y\approx\sqrt{3}-\dfrac{\sqrt{3}}{6}x^2+\dfrac{\sqrt{3}}{24}x^4$
	$\sqrt{2+\cos^2\left(\dfrac{\pi}{6}\right)}\approx\sqrt{3}-\dfrac{\sqrt{3}}{6}\left(\dfrac{\pi}{6}\right)^2+\dfrac{\sqrt{3}}{24}\left(\dfrac{\pi}{6}\right)^4$ $\sqrt{2+\left(\dfrac{\sqrt{3}}{2}\right)^2}\approx\sqrt{3}-\dfrac{\sqrt{3}}{216}\pi^2+\dfrac{\sqrt{3}}{31104}\pi^4$ $\dfrac{\sqrt{11}}{2}\approx\sqrt{3}\left(1-\dfrac{\pi^2}{216}+\dfrac{\pi^4}{31104}\right)$ $\sqrt{11}\approx 2\sqrt{3}\left(1-\dfrac{\pi^2}{216}+\dfrac{\pi^4}{31104}\right)$ (shown)

8(i)

Required volume

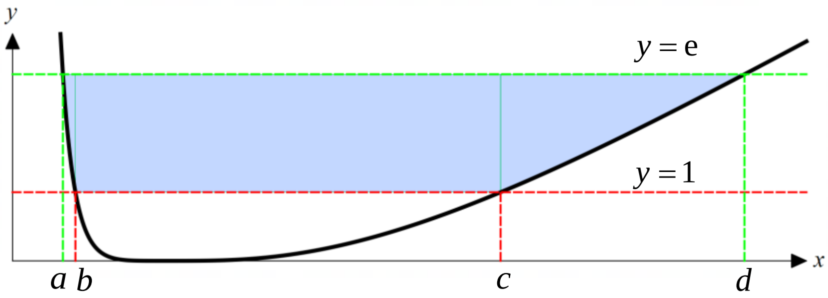
$$= \pi \int_1^{\mathrm{e}} \left(\frac{(\ln x)^4}{\sqrt{x}} \right)^2 \mathrm{d}x$$

$$= \pi \int_1^{\mathrm{e}} \frac{1}{x} (\ln x)^8 \, \mathrm{d}x$$

$$= \pi \left[\frac{(\ln x)^9}{9} \right]_1^{\mathrm{e}}$$

$$= \frac{\pi}{9}$$

(ii)



$$\frac{(\ln x)^4}{\sqrt{x}} = \mathrm{e} \quad \Rightarrow \quad x = 0.32735,\ 4.76172$$

$$\frac{(\ln x)^4}{\sqrt{x}} = 1 \quad \Rightarrow \quad x = 0.40892, 3.17520$$

Let a = 0.32735, b = 0.40892, c = 3.1752 and d = 4.7617 (5 sf)

<u>Method 1</u>
Required area

$$= (d - a)\mathrm{e} - \int_a^b \frac{(\ln x)^4}{\sqrt{x}} \mathrm{d}x - (c - b) - \int_c^d \frac{(\ln x)^4}{\sqrt{x}} \mathrm{d}x$$

= 6.2482 (5 sf) = 6.25 (3 sf).

<u>Method 2</u>
Required area

$$= \int_a^b \left(\mathrm{e} - \frac{(\ln x)^4}{\sqrt{x}} \right) \mathrm{d}x + (c - b)(\mathrm{e} - 1) + \int_c^d \left(\mathrm{e} - \frac{(\ln x)^4}{\sqrt{x}} \right) \mathrm{d}x.$$

= 6.2482 (5 sf) = 6.25 (3 sf).

<u>Method 3</u>

Required area $= \int_a^d \left(\mathrm{e} - \frac{(\ln x)^4}{\sqrt{x}} \right) \mathrm{d}x - \int_b^c \left(1 - \frac{(\ln x)^4}{\sqrt{x}} \right) \mathrm{d}x.$

= 6.2482 (5 sf) = 6.25 (3 sf).

9(i)

Let $y = \mathrm{e}^{-3x}$. Then $-3x = \ln y \Rightarrow x = -\frac{1}{3}\ln y$.

So $\mathrm{g}^{-1} : x \mapsto -\frac{1}{3}\ln x, \quad x \in \mathbf{R}, \quad x > 1.$

Let $y = \frac{x+1}{x-1}$.

Then $yx - y = x + 1 \Rightarrow yx - x = y + 1$

$$\Rightarrow x(y-1) = y+1$$

$$\Rightarrow x = \frac{y+1}{y-1}.$$

So $\mathrm{h}^{-1} : x \mapsto \frac{x+1}{x-1}, \quad x \in \mathbf{R}, x \neq 1.$

(ii)

$\mathrm{ff}(x) = (x^3 - 1)^3 - 1$.

(iii)

Given that $\mathrm{ff}(\alpha) = 0$ means

$$\left(\alpha^3 - 1\right)^3 - 1 = 0 \Rightarrow \left(\alpha^3 - 1\right)^3 = 1$$

$$\Rightarrow \alpha^3 - 1 = 1$$

$$\Rightarrow \alpha^3 = 2$$

$$\Rightarrow \alpha = \sqrt[3]{2}.$$

So $\mathrm{g}^{-1}(\alpha) = -\frac{1}{3}\ln\alpha = -\frac{1}{3}\ln 2^{\frac{1}{3}} = -\frac{1}{9}\ln 2$ (Shown).

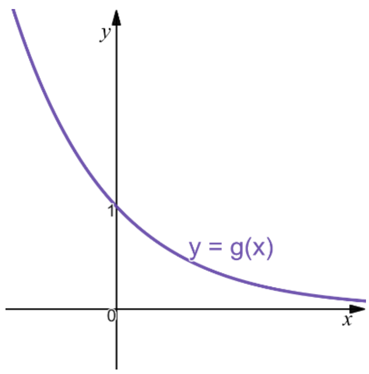
Thus $k = -\frac{1}{9}$.

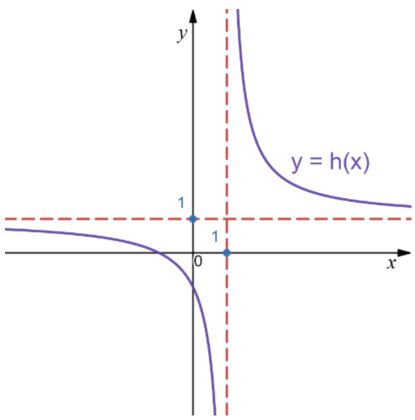
(iv)

Since $\mathrm{R_g} = (1, \infty) \subseteq \mathrm{D_h} = (-\infty, 1) \cup (1, \infty)$, hg exists (Shown).

$$(-\infty, 0) \xrightarrow{g} (1, \infty) \xrightarrow{h} (1, \infty)$$

The range of the function is $(1, \infty)$.



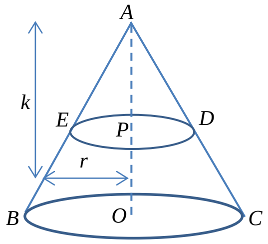


(v)

Since $\mathrm{h}^2(x) = x$ (i.e. h is a self-inverse function), and $\mathrm{h}^m(x) = x$ if m is an even integer.

Thus $\mathrm{h}^m(x) = \mathrm{h}^{-1}(-1)$ becomes $x = \mathrm{h}^{-1}(-1) = 0$.

10(i)	$\frac{\mathrm{d}G}{\mathrm{d}t} = -\lambda G$ $\int \frac{1}{G}\,\mathrm{d}G = \int -\lambda\ \mathrm{d}t$ $\ln G = -\lambda t + C$, where $C \in \mathbb{R}$ $G = A\mathrm{e}^{-\lambda t}$, where $A = \mathrm{e}^{C}$.
(ii)	When $t = 0$, $G = 80 \Rightarrow A = 80$ $G = 80\mathrm{e}^{-0.005t}$ For $G \leq 70$, $\mathrm{e}^{-0.005t} \leq 0.875$ $-0.005t \leq \ln 0.875$ $t \geq 26.706$ The approximate time is 26.7 minutes.
(iii)	$\frac{\mathrm{d}G}{\mathrm{d}t} = \mu - 0.005G = -0.005\left(G - 200\mu\right)$ $\int \frac{1}{G-200\mu}\,\mathrm{d}G = \int -0.005\ \mathrm{d}t$ $\ln\lvert G - 200\mu\rvert = -0.005t + D$, where $D \in \mathbb{R}$ $G - 200\mu = B\mathrm{e}^{-0.005t}$, where $B = \pm\mathrm{e}^{D}$. When $t = 0$, $G = 80 \Rightarrow B = 80 - 200\mu$ $\therefore G = 200\mu + (80 - 200\mu)\mathrm{e}^{-0.005t}$
(iv)	When $\mu = 0.7$, $G = 140 - 60\mathrm{e}^{-0.005t}$ Observe that when $t \to \infty$, $G \to 140$. This means it is **<u>not recommended</u>** as after a long period of time, the glucose level in the blood stream will exceed 100 mg/dL. <u>Alternatively:</u> For $G \geq 100$, we have $140 - 60\mathrm{e}^{-0.005t} \geq 100$ $\Rightarrow 60\mathrm{e}^{-0.005t} \leq 40$ $\Rightarrow t \geq 81.093$ This means it is **<u>not recommended</u>** as after approximately 81 minutes the glucose level in the blood stream will exceed 100 mg/dL.
(v)	$70 < 200\mu < 100 \Rightarrow 0.35 < \mu < 0.5$

11(i)	Let C be the material cost of a can $\pi r^2 h = 100\pi$ $h = \frac{100}{r^2}$ ------ (1) $C = 2\pi rh(0.9\times10^{-4}) + 2\pi r^2(1.2\times10^{-4})$ ------ (2) Sub (1) into (2), $C = 2\pi r\frac{100}{r^2}(0.9\times10^{-4}) + 2\pi r^2(1.2\times10^{-4})$ $= \frac{0.018\pi}{r} + 0.00024\pi r^2$ $\frac{\mathrm{d}C}{\mathrm{d}r} = -\frac{0.018\pi}{r^2} + 0.00048\pi r$ when $\frac{\mathrm{d}C}{\mathrm{d}r} = 0,\ r^3 = \frac{0.018}{0.00048}$ $r = 3.3472 = 3.35$ (3sf) (shown) $h = 8.9258$ or 8.9107 (if use r = 3.35) $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} = \frac{0.036\pi}{r^3} + 0.00048\pi,$ for $r > 0,$ $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$, so C is minimum Thus, the most economical can has a radius of 3.35 cm (3sf) and height 8.93 cm (3sf)
(ii)	The cost of the can, $C = \frac{0.018\pi}{3.35} + 0.00024\pi(3.35)^2 = 0.0253$ (3sf) The most economical can costs 2.5 cents each. (1dp)
(iii)	 Refer to the diagram above. Let BC be the diameter of the top of the cylindrical part of can with O as the centre and let ED be the diameter of the lid with P as the centre. The lines BE and CD are extended to meet at A as shown in the diagram. The points A, B, C, D and E lie in the same plane and ABC and AED form two right cones. Let $OB = 3$ cm, $PE = 1.5$ cm, then $BE = EA = 2.5$ cm. Hence $BA = 5$ cm, $OA = 4$ cm. Let r be the radius of the liquid surface, and k be the vertical distance from the liquid surface to A. Using similar triangles, $\frac{r}{k} = \frac{3}{4}$

	Volume of liquid above level BC, $V=\frac{1}{3}\pi(3^2)(4)-\frac{1}{3}\pi r^2 k$ $=\frac{1}{3}\pi(3^2)(4)-\frac{3}{16}\pi k^3$ $\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}k}=-\frac{9}{16}\pi k^2$ When the liquid level is 1cm from the lid of the can, $k=3$ $\frac{\mathrm{d}k}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}k}{\mathrm{d}V}$ $=90\pi\times\left(-\frac{16}{9\pi(3)^2}\right)$ $=-\frac{160}{9}$ Since k is decreasing at $\frac{160}{9}$ cm/s, it follows that the liquid level in the can is increasing at $\frac{160}{9}$ cm/s when it is 1cm from the top of the can.