RAFFLES INSTITUTION
2022 YEAR 6 PRELIMINARY EXAMINATION

CANDIDATE NAME

CLASS 22

MATHEMATICS **9758/02**

Paper 2 **3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper. **You may use the blank pages on page 2, 23 and 24 if necessary and you are reminded to indicate the question number(s) clearly.**
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics	Q1	Q2	Section B: Probability & Statistics	Q5	Q6	Q7	Q8
	/ 7	/ 8		/ 7	/ 8	/ 10	/ 10
	Q3	Q4		Q9	Q10	TOTAL	
	/ 12	/ 13		/ 12	/ 13	/ 100	

Section A: Pure Mathematics [40 marks]

1 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\cos^2 x}\right) = \frac{k\sin x}{\cos^n x}$, where the values of the constants k and n are to be determined. [2]

(ii) Hence use integration by parts to evaluate $\int_0^{\frac{\pi}{4}} \sin^2 x \sec^3 x \,\mathrm{d}x$, leaving your answer in the form $a + b\ln c$, where a, b and c are exact constants. [5]

2 **Do not use a calculator in answering this question.**

(a) Let $\mathrm{f}(z)$ be a polynomial in z of degree 4 with real coefficients. The equation $\mathrm{f}(z) = 0$ has four roots, namely α, β, γ and δ such that they satisfy the following two conditions:

$$\alpha\beta\gamma\delta < 0 \text{ and } \alpha^2 + \beta^2 + \gamma^2 + \delta^2 < 0.$$

Based on the two conditions, a student concludes that the equation $\mathrm{f}(z) = 0$ has one positive real root, one negative real root and a pair of complex conjugate roots.

State, with reasons, whether the student's claim is true. [3]

(b) It is given that $\mathrm{g}(z) = z^4 + z^3 - 2z^2 + 4z - 24$.

Verify that $z = 2\mathrm{i}$ is a root of the equation $\mathrm{g}(z) = 0$. Hence find the other roots of the equation. [5]

3 The line L_1 has equation

$$1-y=\frac{z-1}{2},\ x=2,$$

and meets the xy-plane at point P. The point A has position vector $\begin{pmatrix}3\\-1\\2\end{pmatrix}$ with reference to the origin O.

(i) Find a vector equation of the line L_2 which passes through O and P. [3]

(ii) Find an equation of the plane π containing both L_1 and L_2, in the scalar product form. [2]

(iii) The points A and C are on different sides of π such that AC is perpendicular to π. The distance of C from π is t times the distance of A from π. Find, in terms of t, the position vector of C. [5]

(iv) Find the value of t such that the line OC is parallel to the plane with equation $\mathbf{r}\cdot\begin{pmatrix}2\\0\\1\end{pmatrix}=2$. [2]

4 **(a)** The first 3 terms of a geometric progression are a, b, 2 and the first 3 terms of an arithmetic progression are 2, a, b, with non-zero common difference. Find the values of a and b. [4]

(b) $u_1,\ u_2,\ u_3,\ u_4,\ldots,\ u_{2n}$ is a sequence of $2n$ positive terms, with $n>1$. The odd-numbered terms form an arithmetic progression with common difference p and the even-numbered terms form a geometric progression with common ratio $\frac{6}{5}$. Given that $u_1=u_2=p$ and the sum of the odd-numbered terms is less than the sum of the even-numbered terms, find the least value of n. [4]

(c) A geometric progression of n terms has first term q and common ratio r, where q is non-zero and $r\neq 1$. For $k\leq n$, find the difference between the sum of the last k terms and the sum of the first k terms, simplifying your answer. [5]

Section B: Probability and Statistics [60 marks]

5 A club in a school has 5 members from Class P, 4 members from Class Q and 3 members from Class R. Five members are to be chosen for an upcoming competition.

(i) Find the number of ways the team of five can be chosen so that it has exactly two members from each of Class P and Class Q. [1]

(ii) Find the number of ways the team of five can be chosen so that it has at least two members from Class R. [2]

(iii) All the 12 members of the club go to a cinema. Find the number of ways they can sit in a row so that no more than two members from Class P are next to each other. [4]

6 A car insurance company collected the following data about the percentage occurrence of accident-involved vehicles, p% for vehicles of different weight, w tons.

w (tons)	2.2	1.9	1.7	1.5	1.3	1.1	1.0	0.9
p (%)	2.6	3.2	3.8	4.3	5.4	5.3	7.4	8.6

(i) Calculate the value of the product moment correlation coefficient between w and p, and explain whether your answer suggests that a linear model is appropriate. [2]

(ii) Draw a scatter diagram of the data. [1]

One of the values of p appears to be incorrect.

(iii) Indicate the corresponding point on your diagram by labelling it R, and explain why the scatter diagram for the remaining points may be consistent with a model of the form $\ln p = a + bw$. [2]

(iv) Omitting R, calculate least squares estimates of a and b for the model $\ln p = a + bw$. [2]

(v) Assume that the value of w at R is correct. Estimate the value of p for this value of w. [1]

7 On average, 11% of the students in school A have been infected before by a contagious disease. Every class has 20 students. The number of students in a class who has been infected by the disease before is denoted by X.

(i) State, in the context of this question, two assumptions needed for X to be well modelled by a binomial distribution. Explain why your assumptions may not be met. [4]

Assume now that the context above is well-modelled by a binomial distribution.

(ii) A class is chosen at random. Find the probability that at least 1 but fewer than 10 students in the class has been infected by the disease before. [2]

Fatihah is a student reporter tasked to ask students in a randomly chosen class, one by one, if they have been infected by the disease before.

(iii) Find the probability that the 20^{th} student she asks is the fifth student who has been infected by the disease before. [2]

(iv) Without doing any further calculation, is the probability found in **(iii)** higher or lower than the probability that there are exactly 5 students in the class who have been infected by the disease before? Explain your answer. [2]

8 A game is played by throwing a fair coin and two fair four-sided dice. The dice are coloured red or blue, and have faces numbered from 1 to 4. If the coin shows a head, then the score is the number shown on the red die. Otherwise, the score is the sum of the numbers shown on the two dice.

(i) Show that the probability that a game results in a score of 4 is $\frac{7}{32}$. [2]

(ii) Find the expectation and variance of the score. [5]

(iii) The game is played 35 times. Estimate the probability that the average score of the 35 games is at least 4, given that the first and second games result in a score of 3 and 4 respectively. [3]

9 A large company claims that its employees work an average of 41 hours a week. After receiving feedback from some employees that they work longer than claimed, a survey involving 200 randomly chosen employees is conducted. The amount of time they spend at work per week, x thousand hours, are summarised below.

$$\sum x = 8.71 \qquad \sum x^2 = 0.505$$

(i) Calculate unbiased estimates of the population mean and variance of the amount of time spent by an employee at work per week. [2]

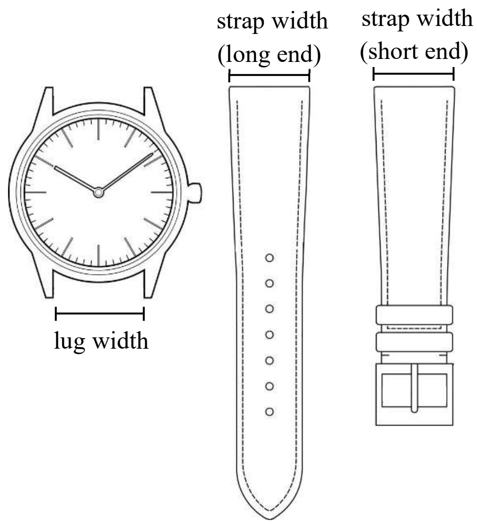
(ii) Test, at the 5% significance level, whether the average working hours of an employee per week is more than 41. State hypotheses for the test, defining any symbols you use. [5]

(iii) After the company restructures its operations, it claims that the average working hours a week is now 40. The human resource manager takes a random sample of 12 employees and finds that they spend an average of 40.1 hours per week at work. Suppose now that the population variance is k hours^2.

(a) Stating a necessary assumption, find the range of possible values of k if the manager concludes that there is insufficient evidence to reject the company's claim that the average working hours is 40, at the 5% significance level. [4]

(b) Explain why the Central Limit Theorem does not apply in this context. [1]

10 In this question you should state the parameters of any normal distributions you use.



A leather craftsman company handcrafts customised leather watch straps according to the lug width of their customers' watches. Over a period of time it is found that the lug widths of their customers' watches are normally distributed; 85% of the watches have lug widths less than 21 mm, and 15% of the watches have lug widths less than 19 mm.

(a) Find the mean and the standard deviation of the lug width of their customers' watches. [3]

The widths of the straps made by the company follow the normal distribution with mean 19.6 mm and standard deviation 1.1 mm.

(b) Find the expected number of straps of width more than 20.2 mm in a randomly chosen batch of 40 straps. [3]

The straps are handcrafted in pairs. Each pair consists of a long end strap and a short end strap. The strap widths of both the long end straps and the short end straps follow the same normal distribution, and are independent of each other. In order for the strap to fit into the lug of the watch, the strap width needs to be shorter than the lug width of the watch. If the strap width is less than 0.2 mm shorter than the lug width of the watch, it is considered a good fit. Otherwise, the strap is a bad fit. A pair of strap is only usable for the watch if the long end strap and the short end strap are both good fits.

(c) A customer walks in with a watch with lug width of 20 mm. He randomly chooses a pair of straps. Show that the probability the pair of straps is usable for his watch is 0.00487, correct to 3 significant figures. [3]

(d) Another customer walks in with two watches of lug widths 18.5 mm and 20 mm respectively. He randomly chooses 2 pairs of straps of different designs. Find the probability that at least 1 pair of straps are usable for any of his watches, giving your answer correct to 5 decimal places. [4]