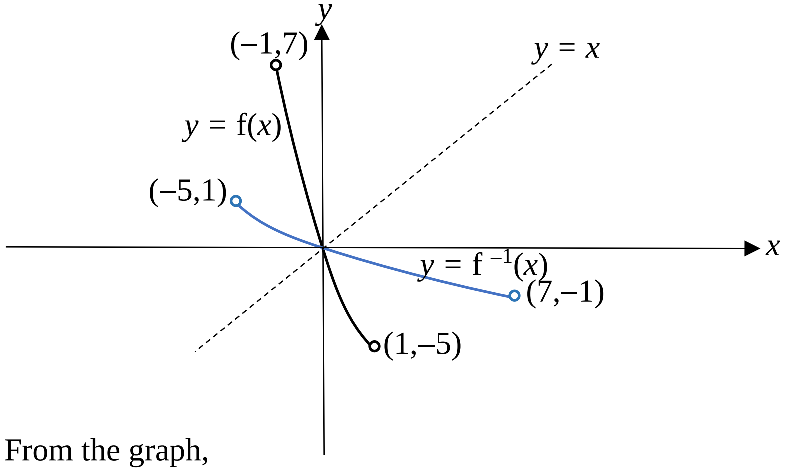
2022 NYJC J2 H2 Mathematics Preliminary Exam 9758/2 Marking Guide

Q1	Suggested Answers
(i)	A plane containing the vectors **a** and **b**, and the origin *O*. OR A plane containing *O*, *A* and *B*.
(ii)	It is the length of projection of $\overrightarrow{OP}$ onto the plane containing the vectors **a** and **b**, and the origin *O*. It is the length of projection of $\overrightarrow{OP}$ onto the plane containing *O*, *A* and *B*.
(iii)	$\mathbf{c}=\dfrac{\mathbf{b}+9\mathbf{a}}{10}=\dfrac{1}{10}\mathbf{b}+\dfrac{9}{10}\mathbf{a}$ $\overrightarrow{OC}\cdot\overrightarrow{AB}=0$ $(\dfrac{1}{10}\mathbf{b}+\dfrac{9}{10}\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})=0$ $\dfrac{1}{10}\mathbf{b}\cdot\mathbf{b}-\dfrac{1}{10}\mathbf{b}\cdot\mathbf{a}+\dfrac{9}{10}\mathbf{a}\cdot\mathbf{b}-\dfrac{9}{10}\mathbf{a}\cdot\mathbf{a}=0$ Since **a** is perpendicular to **b**, $\mathbf{a}\cdot\mathbf{b}=0$ $\dfrac{1}{10}\lvert\mathbf{b}\rvert^2-\dfrac{9}{10}\lvert\mathbf{a}\rvert^2=0$ $\lvert\mathbf{b}\rvert^2=9\lvert\mathbf{a}\rvert^2\Rightarrow\dfrac{\lvert\mathbf{b}\rvert^2}{\lvert\mathbf{a}\rvert^2}=9$ $\therefore OB:OA=3:1$

Q2	**Suggested Answers**
	$w-z=1-\sqrt{3}$ ----(1) $\mathrm{i}z+w=(\sqrt{3}+1)\mathrm{i}$ ----(2) From (1), $z=w-1+\sqrt{3}$ sub into (2) $\mathrm{i}(w-1+\sqrt{3})+w=(\sqrt{3}+1)\mathrm{i}$ $\mathrm{i}w-\mathrm{i}+\sqrt{3}\mathrm{i}+w=\sqrt{3}\mathrm{i}+\mathrm{i}$ $(1+\mathrm{i})w=\sqrt{3}\mathrm{i}+\mathrm{i}+\mathrm{i}-\sqrt{3}\mathrm{i}$ $(1+\mathrm{i})w=2\mathrm{i}$ $w=\dfrac{2\mathrm{i}}{1+\mathrm{i}}=1+\mathrm{i}$ $w=1+\mathrm{i}=\sqrt{2}\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$ $\mathrm{i}w=\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}\times\sqrt{2}\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}=\sqrt{2}\mathrm{e}^{\frac{3\pi}{4}\mathrm{i}}$ $(\mathrm{i}w)^n=(\sqrt{2})^n\mathrm{e}^{\frac{3n\pi}{4}\mathrm{i}}$ For $(\mathrm{i}w)^n$ to be real, $\dfrac{3n\pi}{4}=k\pi,\quad k\in\mathbb{Z}$ $n=\dfrac{4k}{3},\quad k\in\mathbb{Z}$ $\therefore$ 3 smallest positive whole number values n = 4, 8, 12

Q3	**Suggested Answers**
	Volume $= k = \pi(15x)^2 w + \frac{1}{3}\pi(15x)^2(8x)$ ---(1) $A = 2\pi(15x)w + \pi(15x)^2 + \pi(15x)\sqrt{(8x)^2 + (15x)^2}$ $= 2\pi(15x)w + \pi(15x)^2 + \pi(15x)(17x)$ $= 30\pi xw + 480\pi x^2$ ----(2) From (1), $w = \frac{k - 600\pi x^3}{225\pi x^2}$ Subst. into (2) $A = 30\pi x\left[\frac{k - 600\pi x^3}{225\pi x^2}\right] + 480\pi x^2$ $= \frac{2k}{15x} + 400\pi x^2$ $\frac{\mathrm{d}A}{\mathrm{d}x} = -\frac{2k}{15x^2} + 800\pi x$ Let $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$, thus $x^3 = \frac{k}{6000\pi}$, or $x = \frac{1}{10}\left(\frac{k}{6\pi}\right)^{\frac{1}{3}}$ $w = \frac{1}{225\pi\frac{k^{\frac{2}{3}}}{100(6^{\frac{2}{3}})(\pi^{\frac{2}{3}})}}\left[k - 600\pi \times \frac{k}{6000\pi}\right]$ $= \frac{100(6^{\frac{2}{3}})}{225\pi^{\frac{1}{3}}k^{\frac{2}{3}}}\left[\frac{9}{10}k\right] = \frac{12}{5}\left(\frac{k}{6\pi}\right)^{\frac{1}{3}}$

Q4	Suggested Answers
(i)	Let $y=x^2-6x$ $y=x^2-6x=(x-3)^2-9$ $(x-3)^2=y+9$ $x=3\pm\sqrt{y+9}$ Alternatively, $x^2-6x-y=0$ $x=\dfrac{-(-6)\pm\sqrt{36-4(-y)}}{2}=3\pm\sqrt{9+y}$ Since $-1<x<1, x=3-\sqrt{y+9}$ $\mathrm{f}^{-1}(x)=3-\sqrt{x+9}$ $\mathrm{D}_{\mathrm{f}^{-1}}=(-5,7)$
(ii)	 From the graph, $\mathrm{f}(x)=\mathrm{f}^{-1}(x)\Rightarrow x=0$

(iii)	$\mathrm{h}(x)=\mathrm{gf}(x)=1+\dfrac{1}{x^2-6x+5}$ $\mathrm{h}'(x)=\dfrac{-(2x-6)}{\left(x^2-6x+5\right)^2}$ $-1<x<1 \Rightarrow -8<2x-6<-4$ $4<-(2x-6)<8$ Since $-(2x-6)>0$ and $\left(x^2-6x+5\right)^2>0$, $\therefore \mathrm{h}'(x)>0$ for $-1<x<1$. Hence h(x) increases as x increases.

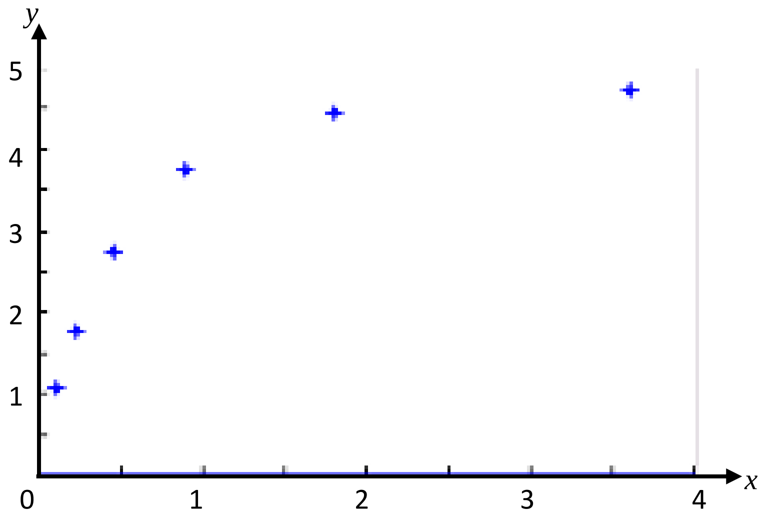
Q5	Suggested Answers
(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x^2+4}\right)=\frac{2x}{2\sqrt{x^2+4}}=\frac{x}{\sqrt{x^2+4}}$
(a)(ii)	$u=x^2 \qquad \frac{\mathrm{d}v}{\mathrm{d}x}=\frac{x}{\sqrt{x^2+4}}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=2x \qquad v=\sqrt{x^2+4}$ $\int\frac{x^3}{\sqrt{x^2+4}}\,\mathrm{d}x=x^2\sqrt{x^2+4}-\int 2x\sqrt{x^2+4}\,\mathrm{d}x$ $=x^2\sqrt{x^2+4}-\frac{2\left(x^2+4\right)^{\frac{3}{2}}}{3}+C$
(b)(i)	$\int\sin mx\cos mx\,\mathrm{d}x=\frac{1}{2}\int\sin 2mx\,\mathrm{d}x$ $=-\frac{1}{4m}\cos 2mx+C$ OR $\int\sin mx\cos mx\,\mathrm{d}x=-\frac{1}{m}\int(-m\sin mx)\cos mx\,\mathrm{d}x$ $=-\frac{1}{2m}\cos^2 mx+C$ OR $\int\sin mx\cos mx\,\mathrm{d}x=\frac{1}{m}\int(m\cos mx)\sin mx\,\mathrm{d}x$ $=\frac{1}{2m}\sin^2 mx+C$

(ii)	Method 1: $\int(\cos 3x\cos x)\sin 2x\,\mathrm{d}x=\int\frac{1}{2}(\cos 4x+\cos 2x)\sin 2x\,\mathrm{d}x$ $=\frac{1}{2}\int\cos 4x\sin 2x+\cos 2x\sin 2x\,\mathrm{d}x$ $=\frac{1}{4}\int(\sin 6x-\sin 2x)\,\mathrm{d}x+\frac{1}{2}\int\cos 2x\sin 2x\,\mathrm{d}x$ $=-\frac{1}{24}\cos 6x+\frac{1}{8}\cos 2x-\frac{1}{16}\cos 4x+C$ Equivalent answers: $-\frac{1}{24}\cos 6x+\frac{1}{8}\cos 2x-\frac{1}{8}\cos^2 2x+C$ Or $-\frac{1}{24}\cos 6x+\frac{1}{8}\cos 2x+\frac{1}{8}\sin^2 2x+C$
(ii)	Method 2: $\int(\cos 3x\sin 2x)\cos x\,\mathrm{d}x=\int\frac{1}{2}(\sin 5x-\sin x)\cos x\,\mathrm{d}x$ $=\frac{1}{2}\int\sin 5x\cos x-\sin x\cos x\,\mathrm{d}x$ $=\frac{1}{2}\int\frac{1}{2}(\sin 6x+\sin 4x)-\sin x\cos x\,\mathrm{d}x$ $=\frac{1}{4}\int\sin 6x+\sin 4x-\sin 2x\,\mathrm{d}x$ $=\frac{1}{4}\left(-\frac{1}{6}\cos 6x-\frac{1}{4}\cos 4x+\frac{1}{2}\cos 2x\right)+C$

(ii)	Method 3: $\int(\sin 2x\cos x)\cos 3x \,\mathrm{d}x = \int \frac{1}{2}(\sin 3x+\sin x)\cos 3x \,\mathrm{d}x$ $= \frac{1}{2}\int \sin 3x\cos 3x+\sin x\cos 3x \,\mathrm{d}x$ $= \frac{1}{2}\int \frac{1}{2}\sin 6x+\frac{1}{2}(\sin 4x+\sin(-2x))\,\mathrm{d}x$ $= \frac{1}{4}\int \sin 6x+\sin 4x-\sin 2x \,\mathrm{d}x$ $= \frac{1}{4}\left(-\frac{1}{6}\cos 6x-\frac{1}{4}\cos 4x+\frac{1}{2}\cos 2x\right)+C$

Q6	Suggested Answers
(i)	Largest number of unsuccessful login attempts $= {}^{26}C_4 \times {}^{10}C_2 \times 6! - 1 = 484379999$
(ii)	Number of passwords $= {}^{26}C_4 \times {}^{10}C_2 \times 2!$ $= 1345500$
(iii)	Method 1: Number of passwords with exactly one vowel in the first four distinct letters and last two digits are distinct $= {}^{21}C_3 \times {}^{5}C_1 \times 4! \times {}^{10}C_2 \times 2!$ Number of passwords such that the first four letters are distinct and the last two digits are distinct $= {}^{26}C_4 \times 4! \times {}^{10}C_2 \times 2!$ Required probability $= \dfrac{{}^{21}C_3 \times {}^{5}C_1 \times 4! \times {}^{10}C_2 \times 2!}{{}^{26}C_4 \times 4! \times {}^{10}C_2 \times 2!} = \dfrac{133}{299}$ Alternative 1: Required probability $= \dfrac{{}^{21}C_3 \times {}^{5}C_1 \times 4! \times {}^{10}C_2 \times 2!}{{}^{26}C_4 \times 4! \times {}^{10}C_2 \times 2!}$ $= \dfrac{{}^{21}C_3 \times {}^{5}C_1}{{}^{26}C_4} = \dfrac{133}{299}$ Alternative 2:

	Required probability $= \dfrac{{}^{21}C_3\,{}^{5}C_1 4!\,{}^{10}C_2\,2! \big/ {}^{26}C_4\,{}^{10}C_2\,6!}{{}^{26}C_4 4!\,{}^{10}C_2\,2! \big/ {}^{26}C_4\,{}^{10}C_2\,6!}$ $= \dfrac{{}^{21}C_3\,{}^{5}C_1}{{}^{26}C_4} = \dfrac{133}{299}$

7	Suggested Answers
(i)	
(ii)	Using GC, product moment correlation coefficient, r = 0.84162, i.e. $r = 0.842$, suggesting a positive linear correlation between the concentration of a reactant, x, and rate of enzymatic reactions, y. However, the scatter diagram in **(i)** shows that this linear relation is not strong since as x increases, y increases at a decreasing rate. **Or** The value of r is not close to 1, indicating the linear correlation between x and y is not strong as indicated by / agreeing with the scatter diagram in **(i)**, showing that as x increases, y increases at a decreasing rate.

(iii)	**Method 1** $y=\dfrac{ax}{b+x}$ $\dfrac{1}{y}=\dfrac{b+x}{ax}=\dfrac{b}{a}\left(\dfrac{1}{x}\right)+\dfrac{1}{a}$ Using GC, $r_{new}=0.99956\approx 0.9996$ $\dfrac{b}{a}=0.084164$ and $\dfrac{1}{a}=0.18190$ Hence a = 5.4974 = 5.50 and b = 0.462695 = 0.463 (3 sig fig) **Method 2** $\dfrac{1}{y}=A\left(\dfrac{1}{x}\right)+B$ Using GC, $r_{new}=0.99956\approx 0.9996$ $A=0.084164$ and $B=0.18190$ $\dfrac{1}{y}=0.084164\left(\dfrac{1}{x}\right)+0.18190$ $=\dfrac{0.084164+0.18190x}{x}$ $y=\dfrac{x}{0.18190(0.46269+x)}$ $=\dfrac{5.4975x}{0.46269+x}$ Hence a = 5.4975 = 5.50 and b = 0.46269 = 0.463 (3 sig fig)

(iv)	When $y=5$, $5=\dfrac{5.4974x}{0.462695+x}$ Hence $x=\dfrac{5\times 0.462695}{0.4974}=4.6511=4.65$ Since $y=5$ is outside the data range [1.081, 4.717], estimate is unreliable.

8	Suggested Answers
(i)	Whether the shooter achieves / hits bullseye in an attempt is independent of whether the shooter achieves / hits bullseye in any of the other attempts. The shooter has the same probability of achieving / hitting bullseye for each / every attempt made.
(ii)	The shooter achieving / hitting bullseye in any attempt may <u>not</u> be <u>independent:</u> Or The shooter may <u>not</u> have <u>the same probability</u> of achieving / hitting bullseye for each / every attempt made: • as the shooter may be tired from the previous attempts made • his accuracy may improve with more attempts • his accuracy may be affected by the changing weather conditions
(iii)	Let X be the random variable denoting the number of bullseyes achieved out of 35 attempts. $X \sim \mathrm{B}(35, 0.7)$ Using GC, <table><tr><td>x</td><td>$\mathrm{P}(X = x)$</td></tr><tr><td>24</td><td>0.14160</td></tr><tr><td>25</td><td>0.14537</td></tr><tr><td>26</td><td>0.13046</td></tr></table>Hence most probable number of bullseye m = 25 $\mathrm{P}(X = 25) = 0.14537 \approx 0.145$
(iv)	$\mathrm{P}(X \geq 25) = 1 - \mathrm{P}(X \leq 24)$ $= 0.50996 \approx 0.510$
(v)	<u>Method 1:</u> $X \sim \mathrm{B}(35, 0.7)$

	$\mathrm{E}(X)=24.5$ $\mathrm{Var}(X)=35(0.7)(0.3)=7.35$ Since number of trainings (= 40) is large, by Central Limit Theorem, $\overline{X}\sim \mathrm{N}\left(24.5,\dfrac{7.35}{40}\right)$ approximately $\mathrm{P}\left(\overline{X}\geq 25\right)=0.12172\approx 0.122$
(v)	<u>Method 2:</u> $X\sim \mathrm{B}(35,0.7)$ $\mathrm{E}(X)=24.5$ $\mathrm{Var}(X)=35(0.7)(0.3)=7.35$ Let $T = X_1 + X_2 +\ldots + X_{40}$ Since number of trainings (= 40) is large, by Central Limit Theorem, $T\sim \mathrm{N}(24.5\times 40, 7.35\times 40)$ ie $T\sim \mathrm{N}(980, 294)$ approximately $\mathrm{P}(T\geq 25\times 40)=\mathrm{P}(T\geq 1000)=0.12172\approx 0.122$

<table>
<tr><th>Q9</th><th>Suggested Answers</th></tr>
<tr><td>(i)</td><td>

$$\mathrm{P}(S=-1)=\mathrm{P}(\{\mathrm{Y,B}\})=\frac{2(3\times 1)}{(x+4)(x+3)}=\frac{6}{(x+4)(x+3)}$$

$$\mathrm{P}(S=0)=\mathrm{P}(\{\mathrm{B,R}\})=\frac{2x}{(x+4)(x+3)}$$

$$\mathrm{P}(S=2)=\mathrm{P}(\{\mathrm{Y,Y}\})=\frac{3\times 2}{(x+4)(x+3)}=\frac{6}{(x+4)(x+3)}$$

$$\mathrm{P}(S=3)=\mathrm{P}(\{\mathrm{Y,R}\})=\frac{2(3x)}{(x+4)(x+3)}=\frac{6x}{(x+4)(x+3)}$$

$$\mathrm{P}(S=4)=\mathrm{P}(\{\mathrm{R,R}\})=\frac{x(x-1)}{(x+4)(x+3)}$$

s	-1	0	2	3	4
$\mathrm{P}(S=s)$	$\frac{6}{(x+4)(x+3)}$	$\frac{2x}{(x+4)(x+3)}$	$\frac{6}{(x+4)(x+3)}$	$\frac{6x}{(x+4)(x+3)}$	$\frac{x(x-1)}{(x+4)(x+3)}$

</td></tr>
<tr><td>(ii)</td><td></td></tr>
</table>

	$\mathrm{E}(S)$ $= -1 \cdot \frac{6}{(x+4)(x+3)} + 0 \cdot \frac{2x}{(x+4)(x+3)} + 2 \cdot \frac{6}{(x+4)(x+3)} + 3 \cdot \frac{6x}{(x+4)(x+3)} + 4 \cdot \frac{x(x-1)}{(x+4)(x+3)}$ $= \frac{-6+12+18x+4x(x-1)}{(x+4)(x+3)}$ $= \frac{4x^2+14x+6}{(x+4)(x+3)}$ $= \frac{2(2x+1)(x+3)}{(x+4)(x+3)}$ $= \frac{2(2x+1)}{x+4}$ $\mathrm{Var}(S)$ $= (-1)^2 \cdot \frac{6}{(x+4)(x+3)} + 0 \cdot \frac{2x}{(x+4)(x+3)} + 2^2 \cdot \frac{6}{(x+4)(x+3)} + 3^2 \cdot \frac{6x}{(x+4)(x+3)} + 4^2 \cdot \frac{x(x-1)}{(x+4)(x+3)} - \left(\frac{2(2x+1)}{x+4}\right)^2$ $= \frac{30+38x+16x^2}{(x+4)(x+3)} - \frac{4(2x+1)^2}{(x+4)^2}$ $= \frac{(30+38x+16x^2)(x+4)-4(2x+1)^2(x+3)}{(x+4)^2(x+3)}$

	$=\dfrac{2\left(19x^2+65x+54\right)}{(x+4)^2(x+3)}$ $=\dfrac{2(19x+27)(x+2)}{(x+4)^2(x+3)}$
(iii)	Method 1: $\mathrm{E}(S)=\dfrac{2(2x+1)}{x+4}>2$ Since $x>1$, $2(2x+1)>2x+8$ $x>3$ Least $x=4$ Method 2: $\mathrm{E}(S)=\dfrac{2(2x+1)}{x+4}>2$ Using GC, <table><tr><td>x</td><td>$\mathrm{E}(S)$</td></tr><tr><td>3</td><td>2</td></tr><tr><td>4</td><td>2.25</td></tr></table> Least $x=4$

Q10	**Suggested Answers**
(i)	Unbiased estimates of the population mean and variance are: $\bar{x}=\dfrac{650}{40}=16.25$ $s^2=\dfrac{\Sigma(x-\bar{x})^2}{n-1}$ $=\dfrac{944}{39}\approx 24.205\approx 24.2$
(ii)	Let X be the waiting time of a randomly chosen customer and μ be the population mean waiting time of a randomly chosen customer. To test $H_0: \mu=15$ $H_1: \mu>15$ at 5% level of significance. Under H_0, since $n=40$ is large, by Central Limit Theorem, $\bar{X}\sim N\left(15, \dfrac{944/39}{40}\right)$ approximately and $Z=\dfrac{\bar{X}-15}{\sqrt{\dfrac{944/39}{40}}}\sim \mathrm{N}(0,1)$ approximately. Reject H_0 if p-value ≤ 0.05 or $z_{\text{cal}}\ge 1.64485$ $z_{\text{cal}}=\dfrac{16.25-15}{\sqrt{\dfrac{944/39}{40}}}=1.60689$ Calculations: $p\text{-value}=0.05404$

	Since p-value > 0.05 (or z_{cal} < 1.64485), we do not reject H_0 and conclude that there is insufficient evidence at the 5% level of significance to conclude that the population mean waiting time is longer than 15 minutes.
(iii)	There is no need to know the distribution since the sample size $n = 40$ is large, by Central Limit Theorem, the sample mean waiting time, $\overline{X}$, is approximately normally distributed, ie $\overline{X} \sim N\left(\mu, \frac{s^2}{n}\right)$ approximately and z-test can be used.
(iv)	Let Y be the waiting time of a randomly chosen customer. To test $H_0 : \mu = 15$ $H_1 : \mu > 15$ at 5% level of significance. Under H_0, since $n = 32$ is large, by Central Limit Theorem, $\overline{Y} \sim N\left(15, \frac{30.25}{32}\right)$ approximately and $Z = \frac{\overline{Y} - 15}{\sqrt{30.25/32}} \sim \mathrm{N}(0,1)$ approximately. Reject H_0 if $z_{\text{cal}} \geq 1.64485$ $z_{\text{cal}} = \frac{\overline{y} - 15}{\sqrt{30.25/32}} \geq 1.64485$ $\overline{y} \geq 16.599$ Thus if bank director's suspicion is confirmed, we need $\overline{y} \geq 16.6$

11	Suggested Answers
(i)	Let B, H and L be random variables denoting the time taken to do a repetition of bench press, a hack squat and a leg press respectively. $L \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $\mu = \dfrac{2.9+5.1}{2} = 4$
	<u>Method 1:</u> $\mathrm{P}\left(L < 2.9\right) = 0.15$ $\mathrm{P}\left(Z < \dfrac{2.9-4}{\sigma}\right) = 0.15$ Using GC, $\dfrac{2.9-4}{\sigma} = -1.0364$ $\sigma = 1.0613 \approx 1.06$
	<u>Method 2:</u> $\mathrm{P}\left(L > 5.1\right) = 0.15$ $\mathrm{P}\left(Z > \dfrac{5.1-4}{\sigma}\right) = 0.15$ Using GC, $\dfrac{5.1-4}{\sigma} = 1.0364$ $\sigma = 1.0613 \approx 1.06$
	<u>Method 3:</u> $\mathrm{P}\left(2.9 \le L \le 5.1\right) = 0.7$

	$\mathrm{P}\left(\frac{2.9-4}{\sigma} \le Z \le \frac{5.1-4}{\sigma}\right)=0.7$ Using GC, $\mathrm{P}(-1.0364 \le Z \le 1.0364)=0.7$ $\frac{2.9-4}{\sigma}=-1.0364$ or $\frac{5.1-4}{\sigma}=1.0364$ $\sigma=1.0613 \approx 1.06$
(ii)	Let C be random variable denoting the time taken for a circuit. $C=B_1+\cdots+B_{10}+H_1+\cdots+H_{10}+L_1+\cdots+L_{10}+120$ Expected time taken for a circuit, $\mathrm{E}(C)=10(5)+10(3)+10(4)+120=240$ (shown)
(iii)	$\mathrm{Var}(C)=10(0.5^2)+10(0.1^2)+10(1.0613^2)=13.8635769$ $C \sim \mathrm{N}(240, 13.8635769)$ $C_1-C_2 \sim \mathrm{N}(0, 27.7271538)$ $\mathrm{P}(C_1>C_2+5)=\mathrm{P}(C_1-C_2>5)=0.1711 \approx 0.171$
(iv)	Let R be the random variable denoting the time taken for two circuits. $R=C_1+C_2+120$ $\mathrm{E}(R)=2(240)+120=600$ $\mathrm{Var}(R)=2(13.8635769)=27.7271538$ $R-2C \sim \mathrm{N}(600-2(240), 27.7271538+4(13.8635769))$ i.e. $R-2C \sim \mathrm{N}(120, 83.1814614)$ $\mathrm{P}(R-2C>150)=0.00050218 \approx 0.000502$
(v)	Let $D=B_1+\cdots+B_{10}+H_1+\cdots+H_{10}+L_1+\cdots+L_{10}+2k$ $D \sim \mathrm{N}(120+2k, 13.8635769)$ $\mathrm{P}(D>180)<0.01$

Method 1

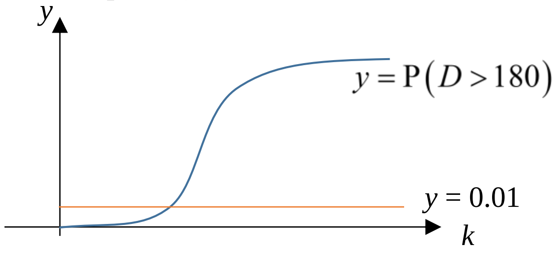
$\mathrm{P}\left(Z > \frac{180-120-2k}{\sqrt{13.8635769}}\right) < 0.01$

$\frac{60-2k}{\sqrt{13.8635769}} > 2.3263$

$k < 25.669$

Hence largest integer k is 25 seconds

Method 2 (Graphical)



Using GC, $k = 25.669$

Hence $0 < k < 25.669$

Largest integer k is 25 seconds

Method 3

$\mathrm{P}(D > 180) - 0.01 < 0$

k	$\mathrm{P}(D > 180) - 0.01$
24	$-0.0093654 < 0$
25	$-0.0063814 < 0$
26	$0.0058336 > 0$

Largest integer k is 25 seconds