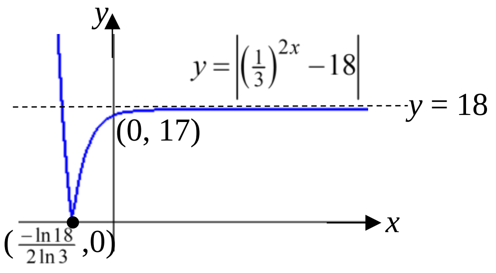
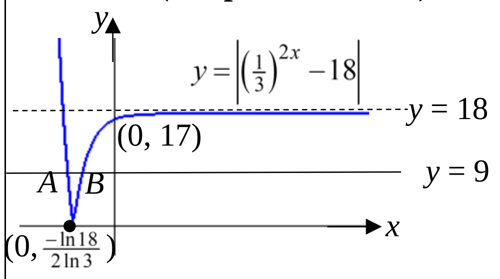
Q1	Suggested Answers
(i)	$\mathrm{f}(x)=ax^3+bx^2+cx+d$ At $(-1, -1)$, $-a+b-c+d=-1$-----------------------------(1) $\mathrm{f}'(x)=3ax^2+2bx+c$ At minimum point $(-1, -1)$, $3a-2b+c=0$ ---------------(2) $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x=\frac{9}{4}$ $\int_0^1 \left(ax^3+bx^2+cx+d\right)\mathrm{d}x=\frac{9}{4}$ $\left[\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+dx\right]_0^1=\frac{9}{4}$ $\frac{a}{4}+\frac{b}{3}+\frac{c}{2}+d=\frac{9}{4}$ ---(3) When f(x) is divided by (x + 2), the remainder is 3 $\mathrm{f}(-2)=a\left(-2\right)^3+b\left(-2\right)^2+c\left(-2\right)+d=3$ $-8a+4b+-2c+d=3$--------------------------------------(4) Solving, $a = -1$, $b = 0$, $c = 3$ and $d = 1$

<table>
<tr><th>Q2</th><th>Suggested Answers</th></tr>
<tr><td>(i)</td><td>
$\left(\frac{1}{3}\right)^{2x}-18=0$
$2x=\frac{\ln 18}{\ln\left(\frac{1}{3}\right)} \Rightarrow x=\frac{\ln 18}{-2\ln 3}$</td></tr>
<tr><td>(ii)</td><td>Method 1 (Graphical method):

To find x-coordinates of point A,
$\left(\frac{1}{3}\right)^{2x}-18=9 \Rightarrow 3^{-2x}=3^3$</td></tr>
</table>

	$x=-\frac{3}{2}$ To find *x*-coordinates of point *B*, $-\left[\left(\frac{1}{3}\right)^{2x}-18\right]=9 \Rightarrow 3^{-2x}=3^{2}$ $x=-1$ $\therefore -\frac{3}{2}\le x\le -1$
(ii)	**Method 2 (Algebraic method):** $\left\|\left(\frac{1}{3}\right)^{2x}-18\right\|\le 9$ $-9\le\left(\frac{1}{3}\right)^{2x}-18\le 9$ $9\le\left(\frac{1}{3}\right)^{2x}\le 27$ $3^{2}\le 3^{-2x}\le 3^{3}\Rightarrow 2\le -2x\le 3$ $\therefore -\frac{3}{2}\le x\le -1$ Alternatively, $\left(\frac{1}{3}\right)^{-2}\le\left(\frac{1}{3}\right)^{2x}\le\left(\frac{1}{3}\right)^{-3}$ $\Rightarrow -2\ge 2x\ge -3$ $\therefore -\frac{3}{2}\le x\le -1$
(ii)	**Method 3 (Algebraic method):** $\left\|\left(\frac{1}{3}\right)^{2x}-18\right\|\le 9$ $-9\le\left(\frac{1}{3}\right)^{2x}-18\le 9$ $9\le\left(\frac{1}{3}\right)^{2x}\le 27$

	$\left(\frac{1}{3}\right)^{2x}=9 \Rightarrow x=\frac{\ln 9}{2\ln\frac{1}{3}}=-1$ $\left(\frac{1}{3}\right)^{2x}=27 \Rightarrow x=\frac{\ln 27}{2\ln\frac{1}{3}}=-\frac{3}{2}$ $\therefore -\frac{3}{2}\le x\le -1$

Q3	**Suggested Answers**
(i)	$x^3+2y^3-3xy-40=0$ Differentiate with respect to *x*, $3x^2+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}-3x\frac{\mathrm{d}y}{\mathrm{d}x}-3y=0\cdots(1)$ At stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, Thus $y=x^2$ Sub $y=x^2$ into $x^3+2y^3-3xy-40=0$, we get $x^3+2x^6-3x^3-40=0$ $2x^6-2x^3-40=0$ $x^6-x^3-20=0$ $\left(x^3-5\right)\left(x^3+4\right)=0$ Thus $x=5^{\frac{1}{3}},-2^{\frac{2}{3}}$. When $x=5^{\frac{1}{3}}, y=5^{\frac{2}{3}}$. When $x=-2^{\frac{2}{3}}, y=2^{\frac{4}{3}}$ The coordinates of the stationary points are $(5^{\frac{1}{3}},5^{\frac{2}{3}})$ and $(-2^{\frac{2}{3}},2^{\frac{4}{3}})$
(ii)	Differentiating equation (1) wrt *x* $6x+12y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+6y^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)-3x\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)-3\frac{\mathrm{d}y}{\mathrm{d}x}-3\frac{\mathrm{d}y}{\mathrm{d}x}=0$ At stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. At $(-2^{\frac{2}{3}},2^{\frac{4}{3}})$, $6\left(-2^{\frac{2}{3}}\right)+6\left(2^{\frac{8}{3}}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+3\left(2^{\frac{2}{3}}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=0$

	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\dfrac{6\left(2^{\frac{2}{3}}\right)}{6\left(2^{\frac{8}{3}}\right)+3\left(2^{\frac{2}{3}}\right)}>0$ Thus $(-2^{\frac{2}{3}},2^{\frac{4}{3}})$ is minimum point.

Q4	Suggested Answers
(i)	**Method 1:** $y=\cos\left(1-e^{-x}\right)$ $\frac{dy}{dx}=-\sin\left(1-e^{-x}\right)\times e^{-x}$ $e^{x}\frac{dy}{dx}=-\sin\left(1-e^{-x}\right)$ $e^{x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)=-\cos\left(1-e^{-x}\right)\times e^{-x}$ $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)=-y$ $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+y=0$ (shown)
(i)	**Method 2:** $y=\cos\left(1-e^{-x}\right)$ $\frac{dy}{dx}=-\sin\left(1-e^{-x}\right)\times e^{-x}$ ------------------------------(1) $\frac{d^2y}{dx^2}=e^{-x}\times\sin\left(1-e^{-x}\right)-e^{-x}\cos\left(1-e^{-x}\right)\times e^{-x}$ ----(2) (1) + (2): $\frac{d^2y}{dx^2}+\frac{dy}{dx}=-e^{-2x}\cos\left(1-e^{-x}\right)$ $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)=e^{2x}\times\left[-e^{-2x}\cos\left(1-e^{-x}\right)\right]=-y$

	$e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+y=0$ (shown)
(ii)	Differentiate $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+y=0$ again, $e^{2x}\left(\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}\right)+2e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+\frac{dy}{dx}=0$ When $x = 0$, $y=1$, $\frac{dy}{dx}=0$, $\frac{d^2y}{dx^2}=-1$, $\frac{d^3y}{dx^3}=3$ $\cos\left(1-e^{-x}\right)=1-\frac{1}{2}x^2+\frac{1}{2}x^3+...$
(iii)	$\int_0^{0.05}\cos\left(1-e^{-x}\right)dx \approx \int_0^{0.05}\left(1-\frac{1}{2}x^2+\frac{1}{2}x^3\right)dx$ = 0.0499799 (to 7 dec. pl.)
(iv)	The answer from part **(iii)** is the same as the given result in the question up to 7 decimal places. This is because the values of x are between 0 and 0.05, which are small, such that x^4 and higher powers of x can be neglected. Hence the estimation in part **(iii)** is accurate.

<table>
<tr><th>Q5</th><th>Suggested Answers</th></tr>
<tr><td>(i)</td><td>Since the equation does not have all real coefficients, therefore Conjugate Root Theorem will not be applicable. Hence $-1-2\mathrm{i}$ may not be a root.</td></tr>
<tr><td>(ii)</td><td>Let $z^3+2(1+\mathrm{i})z^2+(5+4\mathrm{i})z+10\mathrm{i}=(z+1-2\mathrm{i})(z^2+Az+B)$
Comparing constant, $10\mathrm{i}=(1-2\mathrm{i})B$
$$B=\frac{10\mathrm{i}}{(1-2\mathrm{i})}=-4+2\mathrm{i}$$
Comparing z^2 term, $2(1+\mathrm{i})=A+(1-2\mathrm{i})$
$A=1+4\mathrm{i}$

$\therefore\ z^3+2(1+\mathrm{i})z^2+(5+4\mathrm{i})z+10\mathrm{i}=0$
$(z+1-2\mathrm{i})(z^2+(1+4\mathrm{i})z+(-4+2\mathrm{i}))=0$
$(z+1-2\mathrm{i})=0$ or $z^2+(1+4\mathrm{i})z+(-4+2\mathrm{i})=0$
$z=-1+2\mathrm{i}$ or $z=\dfrac{-1-4\mathrm{i}\pm\sqrt{(1+4\mathrm{i})^2-4(-4+2\mathrm{i})}}{2}$
$$z=\frac{-1-4\mathrm{i}\pm\sqrt{1}}{2}$$
$$z=\frac{-1-4\mathrm{i}\pm1}{2}$$
$$z=\frac{-2-4\mathrm{i}}{2}\quad\text{or}\quad z=\frac{-4\mathrm{i}}{2}$$
Therefore, the roots are
$z=-1+2\mathrm{i}$ or $z=-1-2\mathrm{i}$ or $z=-2\mathrm{i}$</td></tr>
<tr><td>(iii)</td><td>From $z^3+2(1+\mathrm{i})z^2+(5+4\mathrm{i})z+10\mathrm{i}=0$, if we replace z with $\mathrm{i}z$,
$(\mathrm{i}z)^3+2(1+\mathrm{i})(\mathrm{i}z)^2+(5+4\mathrm{i})(\mathrm{i}z)+10\mathrm{i}=0$</td></tr>
</table>

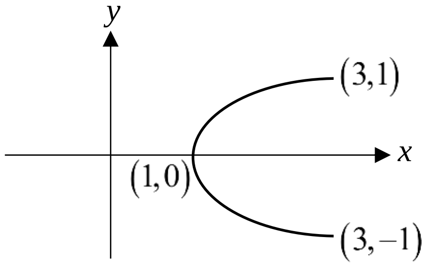
	$-\mathrm{i}z^3 - 2(1+\mathrm{i})z^2 + (5\mathrm{i}-4)z + 10\,\mathrm{i} = 0$ $\mathrm{i}z^3 + 2(1+\mathrm{i})z^2 + (4-5\,\mathrm{i})z - 10\,\mathrm{i} = 0$ So $\quad \mathrm{i}z = -1+2\mathrm{i}$ or $\mathrm{i}z = -1-2\mathrm{i}$ or $\mathrm{i}z = -2\mathrm{i}$ $\quad z = 2+\mathrm{i}$ or $z = \mathrm{i}-2$ or $z = -2$

Q6	Suggested Answers
(a)(i)	$\frac{2}{r^3-r}=\frac{2}{(r-1)r(r+1)}=\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}$ $\sum_{r=2}^{N}\frac{2}{r^3-r}=\sum_{r=2}^{N}\left[\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right]=\frac{1}{1}-\frac{2}{2}+\cancel{\frac{1}{3}}$ $+\frac{1}{2}-\cancel{\frac{2}{3}}+\cancel{\frac{1}{4}}$ $+\cancel{\frac{1}{3}}-\cancel{\frac{2}{4}}+\cancel{\frac{1}{5}}$ $+\ldots$ $+\cancel{\frac{1}{N-3}}-\cancel{\frac{2}{N-2}}+\cancel{\frac{1}{N-1}}$ $+\cancel{\frac{1}{N-2}}-\cancel{\frac{2}{N-1}}+\frac{1}{N}$ $+\cancel{\frac{1}{N-1}}-\frac{2}{N}+\frac{1}{N+1}$ $=\frac{1}{2}-\frac{1}{N}+\frac{1}{N+1}$ $\sum_{r=2}^{N}\left(\frac{4}{5}\right)^{r-1}=\left(\frac{4}{5}\right)+\left(\frac{4}{5}\right)^2+\ldots+\left(\frac{4}{5}\right)^{N-1}=\frac{\frac{4}{5}\left[1-\left(\frac{4}{5}\right)^{N-1}\right]}{1-\frac{4}{5}}=4\left[1-\left(\frac{4}{5}\right)^{N-1}\right]$

	$\sum_{r=2}^{N}\left(\frac{2}{r^3-r}-\left(\frac{4}{5}\right)^{r-1}\right)=\frac{1}{2}-\frac{1}{N}+\frac{1}{N+1}-4\left[1-\left(\frac{4}{5}\right)^{N-1}\right]$ $=-\frac{7}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$
(a)(ii)	$\sum_{r=2}^{N}\left(u_r-u_{r-1}\right)=u_N-u_1$ Hence, $u_N-u_1=-\frac{7}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$ $u_N=3-\frac{7}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$ $=-\frac{1}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$
(b)	$1-\left(\frac{1}{2}\right)^n<b_n<1+\left(\frac{1}{3}\right)^n$ Let $\mathrm{a}_n=1-\left(\frac{1}{2}\right)^n$ and $\lim_{n\to\infty}\mathrm{a}_n=1$ Let $\mathrm{c}_n=1+\left(\frac{1}{3}\right)^n$ and $\lim_{n\to\infty}\mathrm{c}_n=1$ Since $\lim_{n\to\infty}a_n=\lim_{n\to\infty}c_n=1$ By squeeze theorem, b_n is convergent and $\lim_{n\to\infty}b_n=1$.

Q7	**Suggested Answers**
(i)	$x=\dfrac{at}{t+1}$, $y=\dfrac{at^2}{t+1}$ $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{a(t+1)-at}{(t+1)^2}=\dfrac{a}{(t+1)^2}$, $\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{2at(t+1)-at^2}{(t+1)^2}=\dfrac{at^2+2at}{(t+1)^2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=t^2+2t$ Equation of normal at point T: $y-\dfrac{at^2}{t+1}=-\dfrac{1}{t(t+2)}\left[x-\dfrac{at}{t+1}\right]$ $t(t+2)y-\dfrac{at^2}{t+1}\times t(t+2)=-x+\dfrac{at}{t+1}$ $t(t+1)(t+2)y-at^3(t+2)=-(t+1)x+at$ $t(t+1)(t+2)y+(t+1)x=at(t^3+2t^2+1)$ (shown)
(ii)	At $P\left(-a,\dfrac{1}{2}a\right)$, $\dfrac{at}{t+1}=-a\Rightarrow t=-\dfrac{1}{2}$ Equation of normal at P is $-\dfrac{3}{8}y+\dfrac{1}{2}x=-\dfrac{11}{16}a$ $\Rightarrow -6y+8x=-11a$ Alternatively, when $t=-\dfrac{1}{2}$, $x=-a$, $y=\dfrac{1}{2}a$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{3}{4}$ Equation of normal at P is $y-\dfrac{1}{2}a=\dfrac{4}{3}(x+a)$ $\Rightarrow y=\dfrac{4}{3}x+\dfrac{11}{6}a$

	Given that this normal meets the curve again at Q, $-6\left(\dfrac{at^2}{t+1}\right)+8\left(\dfrac{at}{t+1}\right)=-11a$ $\Rightarrow 6t^2-8t=11(t+1)$ $\Rightarrow 6t^2-19t-11=0$ $\Rightarrow (2t+1)(3t-11)=0 \Rightarrow t=-\dfrac{1}{2}$ or $\dfrac{11}{3}$ When $t=\dfrac{11}{3}$, $x=\dfrac{a\left(\frac{11}{3}\right)}{\left(\frac{11}{3}\right)+1}=\dfrac{11}{14}a$, $y=\dfrac{a\left(\frac{11}{3}\right)^2}{\left(\frac{11}{3}\right)+1}=\dfrac{121}{42}a$ The coordinates of Q are $\left(\dfrac{11}{14}a,\dfrac{121}{42}a\right)$
(iii)	Mid-point of PQ, M is $\left(-\dfrac{3}{28}a,\dfrac{71}{42}a\right)$ $x=-\dfrac{3}{28}a, y=\dfrac{71}{42}a \Rightarrow y=-\dfrac{142}{9}x$

<table>
<tr><th>Q8</th><th>Suggested Answers</th></tr>
<tr><td>(i)</td><td>
</td></tr>
<tr><td>(ii)</td><td>Method 1:
$$\begin{aligned}3-\int_1^3 y\,\mathrm{d}x &= 3-\int_{\frac{3\pi}{4}}^{\pi}\cos 2t\cdot 4\cos 2t\,\mathrm{d}t\\ &= 3-4\int_{\frac{3\pi}{4}}^{\pi}\cos^2 2t\,\mathrm{d}t\\ &= 3-4\int_{\frac{3\pi}{4}}^{\pi}\frac{1+\cos 4t}{2}\,\mathrm{d}t\\ &= 3-2\int_{\frac{3\pi}{4}}^{\pi}1+\cos 4t\,\mathrm{d}t\\ &= 3-2\left[t+\frac{1}{4}\sin 4t\right]_{\frac{3\pi}{4}}^{\pi}\\ &= 3-2\left[\pi-\frac{3\pi}{4}\right]\\ &= 3-\frac{\pi}{2}\text{ units}^2\end{aligned}$$</td></tr>
</table>

(ii)	**Method 2:** $\int_0^1 x\,\mathrm{d}y = \int_{\frac{3\pi}{4}}^{\pi} (2\sin 2t+3)\cdot(-2\sin 2t)\,\mathrm{d}t$ $= -2\int_{\frac{3\pi}{4}}^{\pi} 2\sin^2 2t + 3\sin 2t\,\mathrm{d}t$ $= -2\int_{\frac{3\pi}{4}}^{\pi} 1-\cos 4t + 3\sin 2t\,\mathrm{d}t$ $= -2\left[t - \frac{1}{4}\sin 4t - \frac{3}{2}\cos 2t\right]_{\frac{3\pi}{4}}^{\pi}$ $= -2\left[\left(\pi - \frac{3}{2}\right) - \frac{3\pi}{4}\right]$ $= -2\left[\frac{\pi}{4} - \frac{3}{2}\right]$ $= 3 - \frac{\pi}{2}$ units^2
(iii)	$x = 2\sin 2t + 3$ $\qquad$ $y = \cos 2t$ $\sin 2t = \frac{x-3}{2}$ Since $\sin^2 2t + \cos^2 2t = 1$, $\frac{(x-3)^2}{4} + y^2 = 1 \Rightarrow y^2 = 1 - \frac{(x-3)^2}{4}$

(iv)	$\left(y+1\right)^2 = 1-\dfrac{\left(x-3\right)^2}{4}$ $y=-1+\sqrt{1-\dfrac{\left(x-3\right)^2}{4}}$ (Since $y \geq -1$) Volume $=\pi\left(1^2\right)\cdot 1+\pi\int_1^3 y^2\,\mathrm{d}x$ $=\pi\left(1^2\right)\cdot 1+\pi\int_1^3\left(-1+\sqrt{1-\dfrac{\left(x-3\right)^2}{4}}\right)^2\mathrm{d}x$ $\approx 3.7440\text{ units}^3 \approx 3.74\text{ units}^3$

Q9	Suggested Answers
(i)	$a+d=b \quad -(1)$ $a+3d=br^3 \quad -(2)$ $a+5d=br^7 \quad -(3)$ $(3)-(2)=(2)-(1)$ $br^7-br^3=br^3-b$ $r^7-2r^3+1=0$ (since b is non-zero)
(ii)	Using G.C. $r=1,\ r=-1.2578,\ r=0.92057$ Since d is non-zero, $r\neq 1$. The geometric progression has positive terms, so r must be positive. Hence $r=0.921$ is the only answer. Since $\lvert r\rvert<1$, the geometric progression is convergent.
(iii)	$\left\lvert \frac{2n}{2}\left[2k+(2n-1)(3k)\right]-4k\left(0.92057\right)^{n-1}\right\rvert \leq 1000k$ $\left\lvert n(6n-1)-4\left(0.92057\right)^{n-1}\right\rvert-1000\leq 0$ <table><tr><td>n</td><td>$\left\lvert n(6n-1)-4\left(0.92057\right)^{n-1}\right\rvert-1000$</td></tr><tr><td>13</td><td>−0.482</td></tr><tr><td>14</td><td>160.64</td></tr></table>Largest value of n is 13

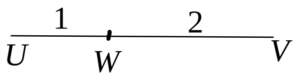
(iv)	Since $\ln\left(\frac{1}{u_n}\right)-\ln\left(\frac{1}{u_{n-1}}\right)=-\ln u_n+\ln u_{n-1}$ $=\ln\frac{u_{n-1}}{u_n}=\ln\frac{1}{r}$ is a constant, the sequence is an arithmetic progression. Alternatively, $\ln\left(\frac{1}{u_n}\right)-\ln\left(\frac{1}{u_{n-1}}\right)=-\ln u_n+\ln u_{n-1}$ $=-\ln br^{n-1}+\ln br^{n-2}$ $=\ln\frac{br^{n-2}}{br^{n-1}}=\ln r^{-1}=-\ln r$ is a constant, the sequence is an arithmetic progression.

Q10	Suggested Answers
(i)	$F_1:\ \mathbf{r}=\begin{pmatrix}3+6p\\1+4p+q\\6+2p-4q\end{pmatrix}=\begin{pmatrix}3\\1\\6\end{pmatrix}+p\begin{pmatrix}6\\4\\2\end{pmatrix}+q\begin{pmatrix}0\\1\\-4\end{pmatrix}$ $\mathbf{n}_1=\begin{pmatrix}6\\4\\2\end{pmatrix}\times\begin{pmatrix}0\\1\\-4\end{pmatrix}=\begin{pmatrix}-18\\24\\6\end{pmatrix}=6\begin{pmatrix}-3\\4\\1\end{pmatrix}$ $F_1:\mathbf{r}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}=\begin{pmatrix}3\\1\\6\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}=1$
(ii)	$F_2:\ \mathbf{r}=\begin{pmatrix}-9+3p\\1+p-2q\\3-p+8q\end{pmatrix}=\begin{pmatrix}-9\\1\\3\end{pmatrix}+p\begin{pmatrix}3\\1\\-1\end{pmatrix}+q\begin{pmatrix}0\\-2\\8\end{pmatrix}$ $\mathbf{n}_2=\begin{pmatrix}3\\1\\-1\end{pmatrix}\times\begin{pmatrix}0\\-2\\8\end{pmatrix}=\begin{pmatrix}6\\-24\\-6\end{pmatrix}=-6\begin{pmatrix}-1\\4\\1\end{pmatrix}$ $\theta=\cos^{-1}\left\lvert\dfrac{\begin{pmatrix}-3\\4\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\4\\1\end{pmatrix}}{\sqrt{9+16+1}\sqrt{1+16+1}}\right\rvert=\cos^{-1}\left\lvert\dfrac{20}{\sqrt{26}\sqrt{18}}\right\rvert\approx 22.4^0$

(iii)

$$\overrightarrow{OU}=\begin{pmatrix}3+6p\\1+4p+q\\6+2p-4q\end{pmatrix} \qquad \overrightarrow{OV}=\begin{pmatrix}-9+3p\\1+p-2q\\3-p+8q\end{pmatrix}$$

Given $2UW=WV$



$$\frac{UW}{WV}=\frac{1}{2}$$

$$\overrightarrow{OW}=\frac{1}{3}\left(\overrightarrow{OV}+2\overrightarrow{OU}\right)$$

$$=\frac{1}{3}\left[\begin{pmatrix}-9+3p\\1+p-2q\\3-p+8q\end{pmatrix}+2\begin{pmatrix}3+6p\\1+4p+q\\6+2p-4q\end{pmatrix}\right]$$

$$=\frac{1}{3}\begin{pmatrix}-3+15p\\3+9p\\15+3p\end{pmatrix}=\begin{pmatrix}-1+5p\\1+3p\\5+1p\end{pmatrix}=\begin{pmatrix}-1\\1\\5\end{pmatrix}+p\begin{pmatrix}5\\3\\1\end{pmatrix}$$

$$l: \quad \mathbf{r}=\begin{pmatrix}-1\\1\\5\end{pmatrix}+p\begin{pmatrix}5\\3\\1\end{pmatrix},\ p\in\mathbb{R}$$

(iv)	<u>Method 1:</u> Since point A is on F_1 and on the line $\mathbf{r} = s\mathbf{k} = s\begin{pmatrix}0\\0\\1\end{pmatrix}$,we have $s\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}=1$. Therefore, $s = 1$ and $\overrightarrow{OA}=\begin{pmatrix}0\\0\\1\end{pmatrix}$
	<u>Method 2:</u> Since point A is on F_1 and on the line $\mathbf{r} = s\mathbf{k} = s\begin{pmatrix}0\\0\\1\end{pmatrix}$,we have $\mathbf{s}\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}3\\1\\6\end{pmatrix}+p\begin{pmatrix}6\\4\\2\end{pmatrix}+q\begin{pmatrix}0\\1\\-4\end{pmatrix}---(*)$ $3+6p=0$ and $1+4p+q=0$ $p=-\frac{1}{2}$ and $1+4(-\frac{1}{2})+q=0$ $q=1$ Sub back (*), $\overrightarrow{OA}=\begin{pmatrix}3\\1\\6\end{pmatrix}-\frac{1}{2}\begin{pmatrix}6\\4\\2\end{pmatrix}+\begin{pmatrix}0\\1\\-4\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}$

	To find perpendicular distance between A and line l: <u>Method 1:</u> Distance of A to l = $\left\lvert \dfrac{\left(\begin{pmatrix}-1\\1\\5\end{pmatrix}-\begin{pmatrix}0\\0\\1\end{pmatrix}\right)\times\begin{pmatrix}5\\3\\1\end{pmatrix}}{\sqrt{25+9+1}} \right\rvert = \left\lvert \dfrac{\begin{pmatrix}-1\\1\\4\end{pmatrix}\times\begin{pmatrix}5\\3\\1\end{pmatrix}}{\sqrt{25+9+1}} \right\rvert$ $= \dfrac{\left\lvert\begin{pmatrix}-11\\21\\-8\end{pmatrix}\right\rvert}{\sqrt{25+9+1}} = \dfrac{\sqrt{626}}{\sqrt{35}} = 4.23 \text{ (to 3 s.f.)}$
(iv)	To find perpendicular distance between A and line l: <u>Method 2:</u> Let P be a point on l $\overrightarrow{OP} = \begin{pmatrix}-1\\1\\5\end{pmatrix} + p\begin{pmatrix}5\\3\\1\end{pmatrix}$ $\overrightarrow{AP} = \begin{pmatrix}-1\\1\\5\end{pmatrix} + p\begin{pmatrix}5\\3\\1\end{pmatrix} - \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\4\end{pmatrix} + p\begin{pmatrix}5\\3\\1\end{pmatrix}$

	Since $\overrightarrow{AP}$ is perpendicular to $\begin{pmatrix}5\\3\\1\end{pmatrix}$, $\left[\begin{pmatrix}-1\\1\\4\end{pmatrix}+p\begin{pmatrix}5\\3\\1\end{pmatrix}\right]\bullet\begin{pmatrix}5\\3\\1\end{pmatrix}=0$ $2+(25+9+1)\,p=0$ $p=-\frac{2}{35}$ $\overrightarrow{AP}=\begin{pmatrix}-1\\1\\4\end{pmatrix}-\frac{2}{35}\begin{pmatrix}5\\3\\1\end{pmatrix}=\frac{1}{35}\begin{pmatrix}-45\\29\\138\end{pmatrix}$ Distance of A to $l=\left\|\overrightarrow{AP}\right\|=\frac{\sqrt{(-45)^2+(29)^2+(138)^2}}{35}\approx 4.23$

Q11	Suggested Answers
(i)	$\dfrac{\mathrm{d}(2x)}{\mathrm{d}t}=m(8-x)(6-x)$ $2\dfrac{\mathrm{d}x}{\mathrm{d}t}=m(8-x)(6-x)$ $\dfrac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$, where $k=\dfrac{m}{2}$ Equivalently, $\dfrac{\mathrm{d}x}{\mathrm{d}t}=k(x-8)(x-6)$
(ii)	**Method 1:** $\dfrac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$ $\displaystyle\int\frac{1}{(8-x)(6-x)}\,\mathrm{d}x=\int k\,\mathrm{d}t$ $\displaystyle\int\frac{1}{(x-7)^2-1}\,\mathrm{d}x=\int k\,\mathrm{d}t$ $\dfrac{1}{2}\ln\left\|\dfrac{(x-7)-1}{(x-7)+1}\right\|=kt+C$ $\dfrac{1}{2}\ln\left\|\dfrac{x-8}{x-6}\right\|=kt+C$ Since $0\le x\le 6$, $\left\|\dfrac{x-8}{x-6}\right\|=\dfrac{x-8}{x-6}$ So, $\dfrac{1}{2}\ln\left(\dfrac{x-8}{x-6}\right)=kt+C$

	$\frac{x-8}{x-6}=\mathrm{e}^{2kt+2C}$ $\frac{x-8}{x-6}=A\mathrm{e}^{2kt}$ where $A=\mathrm{e}^{2C}$ When $t=0$, $x=0$, $A=\frac{4}{3}$ $\frac{x-8}{x-6}=\frac{4}{3}\mathrm{e}^{2kt}$ When $t=5$, $x=5$, $3=\frac{4}{3}\mathrm{e}^{10k}$ $\left(\mathrm{e}^{k}\right)^{10}=\frac{9}{4}$ $\mathrm{e}^{k}=\left(\frac{9}{4}\right)^{\frac{1}{10}}$ $\frac{x-8}{x-6}=\frac{4}{3}\mathrm{e}^{2kt}$ $3x-24=\left(4x-24\right)\mathrm{e}^{2kt}$ $3x-4x\mathrm{e}^{2kt}=24-24\mathrm{e}^{2kt}$ $x\left(3-4\mathrm{e}^{2kt}\right)=24-24\mathrm{e}^{2kt}$ $x=\frac{24-24\mathrm{e}^{2kt}}{3-4\mathrm{e}^{2kt}}$

	$x=\dfrac{24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}}{3-4\left(\frac{9}{4}\right)^{\frac{1}{5}t}}$ $x=\dfrac{24\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-24}{3\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-4}$
(ii)	**Method 2:** $\dfrac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$ $\displaystyle\int\frac{1}{(8-x)(6-x)}\,\mathrm{d}x=\int k\,\mathrm{d}t$ $\displaystyle\int\frac{1}{(x-7)^2-1}\,\mathrm{d}x=\int k\,\mathrm{d}t$ $\dfrac{1}{2}\ln\left\|\dfrac{(x-7)-1}{(x-7)+1}\right\|=kt+C$ $\dfrac{1}{2}\ln\left\|\dfrac{x-8}{x-6}\right\|=kt+C$ $\left\|\dfrac{x-8}{x-6}\right\|=\mathrm{e}^{2kt+C}$ $\dfrac{x-8}{x-6}=\pm\mathrm{e}^{2kt+2C}$ $\dfrac{x-8}{x-6}=A\mathrm{e}^{2kt}$ where $A=\pm\mathrm{e}^{2C}$

	When $t=0$, $x=0$, $$A=\frac{4}{3}$$ $$\frac{x-8}{x-6}=\frac{4}{3}\mathrm{e}^{2kt}$$ When $t=5$, $x=5$, $$3=\frac{4}{3}\mathrm{e}^{10k}$$ $$\mathrm{e}^{10k}=\frac{9}{4}$$ $$10k=\ln\frac{9}{4}$$ $$k=\frac{1}{10}\ln\frac{9}{4}$$ $$\frac{x-8}{x-6}=\frac{4}{3}\mathrm{e}^{\left(\frac{1}{5}\ln\frac{9}{4}\right)t}$$ $$3x-24=(4x-24)\mathrm{e}^{\left(\ln\left(\frac{9}{4}\right)^{\frac{1}{5}}\right)t}$$ $$3x-24=(4x-24)\left(\frac{9}{4}\right)^{\frac{1}{5}t}$$ $$3x-4x\left(\frac{9}{4}\right)^{\frac{1}{5}t}=24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}$$ $$x\left(3-4\left(\frac{9}{4}\right)^{\frac{1}{5}t}\right)=24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}$$

	$$x=\frac{24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}}{3-4\left(\frac{9}{4}\right)^{\frac{1}{5}t}}$$ $$x=\frac{24\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-24}{3\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-4}$$
(ii)	**Alternative method (use of partial fractions):** $$\frac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$$ $$\int\frac{1}{(8-x)(6-x)}\,\mathrm{d}x=\int k\,\mathrm{d}t$$ $$\int\frac{-\frac{1}{2}}{8-x}+\frac{\frac{1}{2}}{6-x}\,\mathrm{d}x=\int k\,\mathrm{d}t$$ $$\frac{1}{2}\ln\lvert 8-x\rvert-\frac{1}{2}\ln\lvert 6-x\rvert=kt+C$$ $$\frac{1}{2}\ln\left\lvert\frac{x-8}{x-6}\right\rvert=kt+C$$ $$\vdots$$

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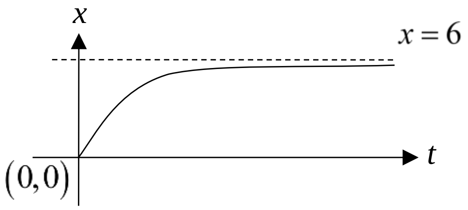
<tr><td>(iii)</td><td>
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<tr><td>(iv)</td><td>
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -8\mathrm{e}^{-2t} - \frac{16}{9}(t+1)^{-\frac{7}{3}}$$
$$\frac{\mathrm{d}x}{\mathrm{d}t} = 4\mathrm{e}^{-2t} + \frac{4}{3}(t+1)^{-\frac{4}{3}} + A$$
$$x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + At + B$$
As $t \to \infty$, $x \to 6$, $A = 0$ (else $At \to \pm\infty$), $B = 6$
$$x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + 6$$
<u>Alternatively</u>,
When $t = 0$, $x = 0$,
$$0 = -2 - 4 + B \quad \Rightarrow \quad B = 6$$
$$x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + At + 6$$
As $t \to \infty$, $x \to 6$, $A = 0$
$$x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + 6$$
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