NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME	

CT CLASS	2	1			Centre Number/ Index Number					/					

MATHEMATICS 9758/02

Paper 2

16 September 2022

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For examiner's use only	
Question number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

Section A: Pure Mathematics [40 marks]

1 The origin O and the points A and B lie in the same plane, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\mathbf{a}$ and $\mathbf{b}$ are non-parallel constant vectors.

(i) Interpret geometrically the vector equation $\mathbf{r} = \lambda\mathbf{a} + \mu\mathbf{b}$, where λ and μ are parameters. [1]

(ii) The point P, with position vector $\mathbf{p}$, does not lie in the same plane as O, A and B. Interpret geometrically $|\mathbf{p} \times \mathbf{u}|$, where $\mathbf{u}$ is a unit vector parallel to $\mathbf{a} \times \mathbf{b}$. [1]

(iii) The point C with position vector $\mathbf{c}$ lies on AB, between A and B, such that $10AC = AB$. OC is perpendicular to AB and angle AOB is 90°. Find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and the ratio $OB : OA$. [4]

2 **Do not use a calculator in answering this question.**

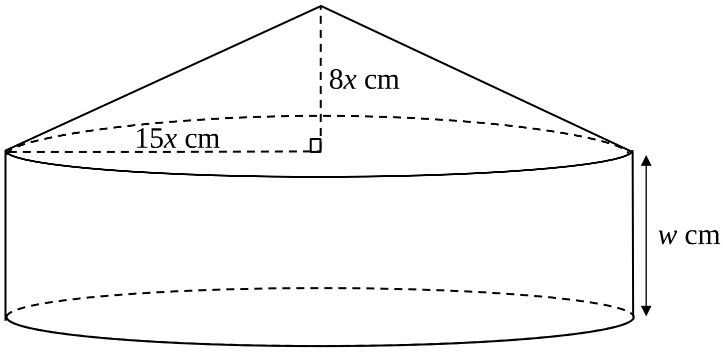
The complex numbers z and w satisfy the following equations

$$w - z = 1 - \sqrt{3},$$

$$\mathrm{i}z + w = (\sqrt{3} + 1)\mathrm{i}.$$

Find w in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. Give r and θ in exact form. Hence find the three smallest positive whole number values of n for which $(\mathrm{i}w)^n$ is a real number. [7]

3 [It is given that a right circular cone with base radius r and height h has volume $\frac{1}{3}\pi r^2 h$ and curved surface area πrl, where l is the slanted height of the cone.]



The model of a house is made up of three parts.

- The roof is modelled by the curved surface of a right circular cone with base radius $15x$ cm and height $8x$ cm.
- The walls are modelled by the curved surface of a cylinder of radius $15x$ cm and height w cm.
- The floor is modelled by a circular disc of radius $15x$ cm.

The three parts are joined together as shown in the diagram. The model is made of material of negligible thickness. It is given that the volume of the model is a fixed value k cm^3, and the external surface area is a minimum. Use differentiation to find the exact values of x and w in terms of k. Simplify your answers.

[8]

4 The function f is defined by $\mathrm{f} : x \mapsto x^2 - 6x$ for $x \in \mathbb{R}, -1 < x < 1$.

(i) Find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(ii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. Hence, determine the value of x for which $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

The functions g and h are defined by

$$\mathrm{g} : x \mapsto 1 + \frac{1}{x+5} \text{ for } x \in \mathbb{R}, x \neq -5,$$

$$\mathrm{h} : x \mapsto \mathrm{gf}(x) \text{ for } x \in \mathbb{R}, -1 < x < 1.$$

(iii) Using differentiation, show that h(x) increases as x increases. [3]

5 **(a)** **(i)** Differentiate $\sqrt{x^2 + 4}$ with respect to x. [1]

(ii) Hence find $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,\mathrm{d}x$. [3]

(b) Find

(i) $\int \sin mx \cos mx \,\mathrm{d}x$, where $m > 0$, [2]

(ii) $\int \cos 3x \cos x \sin 2x \,\mathrm{d}x$. [4]

Section B: Probability and Statistics [60 marks]

6 Jean has forgotten the six-character login password for her laptop. She remembers that the password consists of four distinct letters from the twenty-six letters of the alphabet A − Z and two distinct digits from the ten digits 0 − 9.

(i) Assuming that Jean keys in a six-character password for all her login attempts and she never repeats the same incorrect password, find the largest number of unsuccessful login attempts. [2]

(ii) Find the number of possible six-character passwords if the first four characters are distinct letters in alphabetical order. [2]

(iii) Given that the first four characters are distinct letters, and the last two characters are distinct digits, find the probability that exactly one of the four letters is a vowel. [3]

7 An experiment was conducted to study the relationship between the rate of enzymatic reactions, y, and the concentration of a reactant, x, in a chemical reaction. Six readings, in appropriate units, were obtained.

x	0.1125	0.225	0.45	0.9	1.8	3.6
y	1.081	1.756	2.733	3.735	4.462	4.717

Source: Adapted from https://techvidvan.com/tutorials/nonlinear-regression

(i) Sketch a scatter diagram of the data. [1]

(ii) Find the product moment correlation coefficient between x and y and comment on its value based on the scatter diagram in part **(i)**. [2]

(iii) It is suspected that the data is modelled by the regression equation $y = \frac{ax}{b+x}$. To find the values of a and b, x and y are transformed to $\frac{1}{x}$ and $\frac{1}{y}$ respectively. State the product moment correlation coefficient between $\frac{1}{x}$ and $\frac{1}{y}$, giving your answer correct to 4 decimal places. Find the values of a and b. [3]

(iv) Use the model in part **(iii)** to estimate the value of x when $y = 5$. Comment on the reliability of the estimate found. [2]

8 A shooter can hit the bullseye of his target 7 out of 10 attempts on average. In a particular training, he made 35 attempts on the target.

(i) State two assumptions for the number of bullseyes achieved in a training to be well modelled by a binomial distribution. [2]

(ii) Explain why one of these assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that the number of bullseyes the shooter achieved in a training follows a binomial distribution.

(iii) Given that m is the most probable number of bullseyes the shooter can achieve in a training, find the value of m and state its corresponding probability. [2]

(iv) Find the probability that the shooter achieves at least 25 bullseyes in a training. [2]

(v) The shooter attended 40 trainings. Using a suitable approximation, find the probability that he achieved an average of at least 25 bullseyes per training. [3]

9 A bag contains three yellow marbles, one blue marble and x red marbles, where $x > 1$. In a game, Lily takes 2 marbles at random from the bag, without replacement. She scores 2 points for each red marble taken and 1 point for each yellow marble taken. If a blue marble is taken, she loses 2 points. The random variable S is the sum of the points of the two marbles taken.

(i) Find $\mathrm{P}(S = s)$ for all possible values of s. [4]

(ii) Show that $\mathrm{E}(S) = \frac{2(2x+1)}{x+4}$ and find an expression for $\mathrm{Var}(S)$. [5]

(iii) Find the least value of x if $\mathrm{E}(S) > 2$. [2]

10 The manager of a bank claims that the mean waiting time for customers to be served by a bank consultant is 15 minutes. The bank director suspects that the waiting time is longer than 15 minutes and decides to carry out a hypothesis test on a sample of these customers.

The waiting times, x minutes, of a random sample of 40 customers are summarised below.

$$\Sigma x = 650 \qquad \Sigma(x - \bar{x})^2 = 944$$

(i) Calculate the unbiased estimates of the population mean and variance of the waiting times of the customers. [2]

(ii) Carry out the test, at the 5% level of significance, for the bank director. You should state your hypotheses and define any symbols that you use. [5]

(iii) Explain why there is no need for the bank director to know anything about the population distribution of the waiting times of the customers. [1]

It is given instead that the population variance of the waiting times is 30.25 minutes^2. Another random sample of 32 customers is taken and the bank director now carries out the same test at the 5% level of significance.

(iv) Find the range of possible values of the mean waiting time of this random sample of 32 customers if the bank director's suspicion is confirmed. [4]

11 In this question you should state the parameters of any normal distributions you use.

In strength training, repetitions are the number of times a person completes a single exercise before taking a rest.

Randy goes to a gym for his workout. At the gym, he does three different exercises, namely bench press, hack squat and leg press. The time he takes, in seconds, to do **one repetition** of bench press, hack squat and leg press are independent normal distributions with means and standard deviations given in the table below.

Exercise	Mean	Standard deviation
Bench press	5	0.5
Hack squat	3	0.1
Leg press	μ	σ

The probability that he takes more than 5.1 seconds to do one repetition of leg press is 0.15. The probability that he takes less than 2.9 seconds to do one repetition of leg press is also 0.15.

(i) State the mean time, μ, to do one repetition of leg press and show, with clear working, that the standard deviation, σ, is 1.06 seconds. [3]

A circuit is completed when Randy completes the three different exercises, each exercise consists of ten repetitions. After each exercise, he takes a 60-second break such that in total, a complete circuit comprises two breaks.

(ii) Show that the expected time taken to complete one circuit is 240 seconds. [1]

(iii) Find the probability that the first circuit takes more than 5 seconds longer than the second circuit. [2]

Between any two circuits, Randy takes a 120-second break.

(iv) Find the probability that the total time taken for any two consecutive circuits is longer than twice the time taken for any one circuit by more than 150 seconds. [3]

Randy decides to modify his training plan for a circuit such that each 60-second break is k-second instead.

(v) Find the largest integer k such that the probability that the time taken for one circuit is longer than 3 minutes is less than 0.01. [3]

BLANK PAGE