1

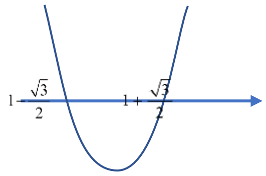
$mx-3=mx^2-(m-1)x-2$

$mx^2-2mx+x+1=0$

$(-2m+1)^2-4m\geq 0$

$4m^2-4m+1-4m\geq 0$

$4m^2-8m+1\geq 0$



$m=\frac{-(-8)\pm\sqrt{8^2-4(4)(1)}}{2(4)}$

$m=\frac{8\pm 4\sqrt{3}}{8}=1\pm\frac{\sqrt{3}}{2}$

$m\leq 1-\frac{\sqrt{3}}{2}$ or $m\geq 1+\frac{\sqrt{3}}{2}$

2

$3e^{2x}>e^x+70$

$3e^{2x}-e^x-70>0$

Let $u=e^x$, then $3u^2-u-70>0$

Consider $3u^2-u-70=0$, then

$u=\frac{1\pm\sqrt{1-4(3)(-70)}}{2(3)}$

$u=5$ or $u=-\frac{28}{6}=-\frac{14}{3}$

For $3u^2-u-70>0$,

$u>5$ or $u<-\frac{28}{6}$

$e^x>5$ or $e^x<-\frac{28}{6}$ (reject as $e^x>0$)

$x>\ln 5$

3(i)

Let \$$x$, \$$y$, \$$z$ be the amount invested in plan I, II and III respectively. $x+y+z=10000$ … (1)

$y=2x\Rightarrow 2x-y=0$ … (2)

$0.014(2)x+0.035y+8(50)(\frac{z}{10000})=3560$

$0.028x+0.035y+0.04z=3560$ … (3)

From GC,

$x=20000$, $y=40000$, $z=40000$

She deposits \$20000 in plan I, \$40000 in plan II and \$40000 in plan III

3(ii)

If she deposits in savings plans II and III, the total interest is

$$0.035(40000)+8(50)\left(\frac{40000}{10000}\right)=3000$$

If she deposits \$80000 in savings plan IV, the interest ranges from
Min 0.037(80000) = \$2960 to Max 0.039(80000) = \$3120. The average interest is 3.8%, 0.038(80 000) = \$3040

Tammy is advised to deposit in savings plan IV as she can earned \$120 more against loss \$40 over savings plans II and III. Further the average interest is \$40 more than the plan II and III.

4

$$\int_1^4 \left(3-\frac{1}{x}\right)^2 \mathrm{d}x$$

$$=\int_1^4 \left(9-\frac{6}{x}+\frac{1}{x^2}\right) \mathrm{d}x$$

$$=\left[9x-6\ln x-\frac{1}{x}\right]_1^4$$

$$=\left(36-6\ln 4-\frac{1}{4}\right)-\left(9-6\ln 1-\frac{1}{1}\right)$$

$$=\frac{111}{4}-6\ln 4$$

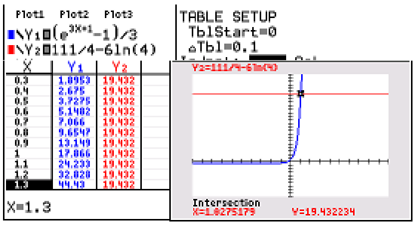
Or 27.75 – 6ln4

$$\int_{-\frac{1}{3}}^{p} \mathrm{e}^{3x+1}\,\mathrm{d}x=\int_1^4 \left(3-\frac{1}{x}\right)^2 \mathrm{d}x$$

$$\Rightarrow \left[\frac{1}{3}\mathrm{e}^{3x+1}\right]_{-\frac{1}{3}}^{p}=\frac{111}{4}-6\ln 4$$

$$\Rightarrow \frac{1}{3}\left(\mathrm{e}^{3p+1}-1\right)=\frac{111}{4}-6\ln 4$$

By GC, p = 1.0275179
= 1. 0275 (4dp)



Method 2

$$\mathrm{e}^{3p+1}-1=\frac{333}{4}-18\ln 4$$

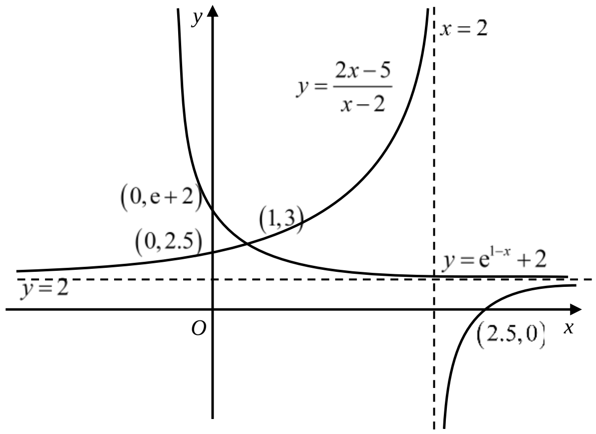
e^{3p+1} = 59.2967015
$3p$ + 1 = ln(59.2967015)
$3p$ = 4.082553681 – 1
p = 1. 027517894 = 1.0275 (4dp)

5(i)

$$y=\frac{2x-5}{x-2}=\frac{2(x-2)-1}{x-2}$$

$$=2-\frac{1}{x-2}$$

5(ii)



5(iii) The exact area of the region bounded by the curves C_1, C_2, and the y-axis

$$=\int_0^1 e^{1-x}+2-\frac{2x-5}{x-2}\,dx$$

$$=\int_0^1 e^{1-x}+2-2+\frac{1}{x-2}\,dx$$

$$=\int_0^1 e^{1-x}+\frac{1}{x-2}\,dx$$

$$=\left[-e^{1-x}+\ln(|x-2|)\right]_0^1$$

$$=-1+e\ -\ln 2$$

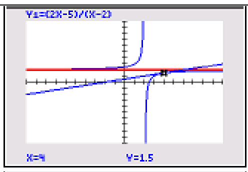
5(iv) $y=\dfrac{2x-5}{x-2}=2-\dfrac{1}{x-2}$

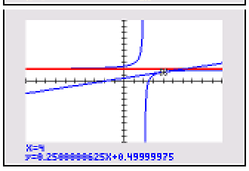
$$\frac{dy}{dx}=\frac{1}{(x-2)^2}$$

When $x=4$, $y=1.5$ and $\dfrac{dy}{dx}=\dfrac{1}{4}$.

$$y-1.5=\frac{1}{4}(x-4)$$

$$y=\frac{x+2}{4}$$

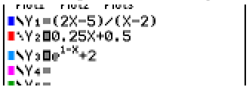


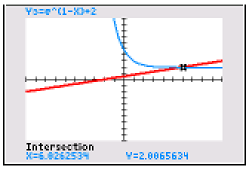


Equation of tangent is $y=\dfrac{x+2}{4}$ or $y=0.25x+0.5$

5(v)

Enter equation of tangent, $y = 0.25x + 0.5$. Find intersection with C_2 giving $x = 6.0262534$





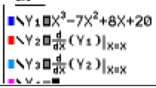
From GC,

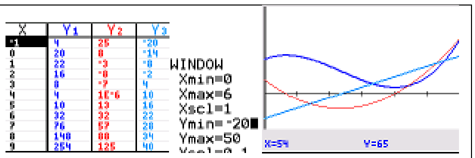
$x < 6.0262534 = 6.03$ (3sf)

6(i)

$C = t^3 - 7t^2 + 8t + 20$

$\frac{dC}{dt} = 3t^2 - 14t + 8$





For stationary points on C, $\frac{dC}{dt} = 0$

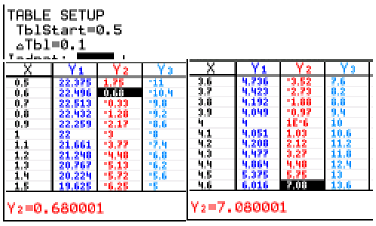
$3t^2 - 14t + 8 = 0$

$(3t-2)(t-4) = 0$

$\Rightarrow t = \frac{2}{3}$ *or* $t = 4$

t	0.6	$\frac{2}{3}$	0.7
$\frac{dC}{dt}$	0.68	0	−0.33
slope	/	—	\

t	3.9	4	4.1
$\frac{dC}{dt}$	-0.97	0	1.03
slope	\	—	/



Or $\frac{d^2C}{dt^2} = 6t - 14$

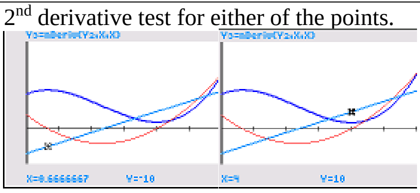
When $t = \frac{2}{3}$,

$\frac{d^2C}{dt^2} = 6(\frac{2}{3}) - 14 = -10 < 0$

When $t = 4$,

$\frac{d^2C}{dt^2} = 6(4) - 14 = 10 > 0$

2nd derivative test for either of the points.



Therefore, C is a maximum when $t=\frac{2}{3}$ and minimum when $t=4$.	

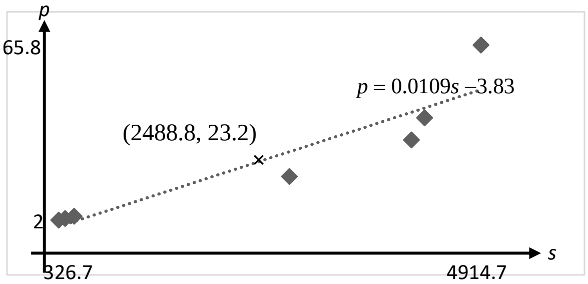
6(ii) Area of the region $\int_0^6 \left(t^3-7t^2+8t+20\right)\ \mathrm{d}t$ $=84$	
The total production cost to manufacture the bottled drinks over a period of 6 months is \$84 000.	
6(iii) $P=100\left(\mathrm{e}^{-0.25C}\right)+3.2$ $\frac{\mathrm{d}P}{\mathrm{d}C}=100\left(-0.25\mathrm{e}^{-0.25C}\right)+0=-25\mathrm{e}^{-0.25C}$ When t = 6, from **(i)** $C=6^3-7(6)^2+8(6)+20=32$ $\frac{\mathrm{d}P}{\mathrm{dC}}=-25\mathrm{e}^{-0.25(32)}$ $=-25\mathrm{e}^{-8}$	
6(iv) $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{\mathrm{d}P}{\mathrm{d}C}\times\frac{\mathrm{d}C}{\mathrm{d}t}$ When t = 6, from **(i),** $\frac{\mathrm{d}C}{\mathrm{d}t}=3(6^2)-14(6)+8=32$ $\frac{\mathrm{d}P}{\mathrm{d}t}=(-25\mathrm{e}^{-8})(32)=-800\mathrm{e}^{-8}$ = 0.2683701023 The rate of decrease in profit when t = 6 is \$268.37 per month.	

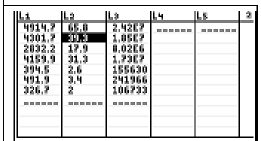
7(i) Mean = 0.5(0.83 + 2.41) = 1.62. Let X be the height of the plants. If X is normally distributed then $X \sim \mathrm{N}(1.62, 1.5^2)$, then $\mathrm{P}(X \le 0) = 0.14007109$. This means that about 14% of the large number of plants will height that is less than 0*. Hence, a normal distribution would not be a good model.
7(ii) $\mathrm{E}(X) = 1.62$, $\mathrm{Var}(X) = 1.5^2$. Let $\overline{X}$ be the sample mean height of the plants. Since the sample size is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(1.62, \frac{1.5^2}{100}\right)$ approximately $\mathrm{P}\left(1.2<\overline{X}<1.6\right)$ = 0.444409753 = 0.444 (to 3 s.f.)

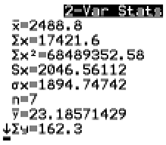
8(i)	P(Accident) $= (0.3\times0.01)+(0.5\times0.03)+(0.2\times0.06)$ = 0.03 P(class H | accident) $=\dfrac{P(\text{accident}\cap\text{class H})}{P(\text{accident})}$ $=\dfrac{0.2\times0.06}{0.03}=0.4$
8(ii)	P(all three drivers are of class H and exactly one has at least an accident) $=(0.2\times0.94)^2\times0.2\times0.06\times3$ = 0.00127 (to 3sf)

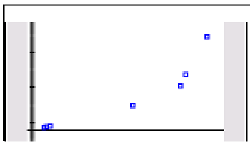
9(ai) Method 1 Number of ways Ann and Alice separated from each other $= 10!\ {}^{11}P_2 = 399168000$
Method 2 Number of ways to bundle Ann and Alice as a unit and arrange the students = 11!2! = 79833600 Require ways = 12! − 11!2! = 399168000
9(aii) Number of ways to bundle 3 boys between Ann and Alice as a unit and arrange them $=\binom{7}{3}3!2!$ Arrange the bundle and 7 others in a row = 8! Number of different sitting arrangements required $=\binom{7}{3}3!2!8! = 16\,934\,400$
(b) Number of ways Ann and Alice are team leaders and Ann's team has 2 girls and Alice's team has Case 1: 1 girl (i.e. only Alice), $\binom{3}{1}\binom{7}{3}\binom{2}{0}\binom{4}{4} = 105$ Case 2: 3 girls (i.e. Alice and 2 other girls), $\binom{3}{1}\binom{7}{3}\binom{2}{2}\binom{4}{2} = 630$ Number of ways Ann and Alice are team leaders and exactly one team has 2 girls = 2(105 + 630) = 1470

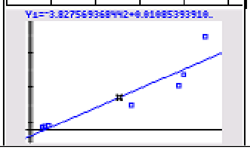
10(i)



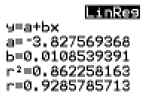






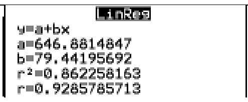


(ii)



The equation of the least-squares regression line of p on s is $p = 0.010853939s - 3.827569368$

$p = 0.0109s - 3.83$ (3sf).



The equation of the least-squares regression line of s on p is $s = 646.8814847 + 79.44195692p$

$s = 647 + 79.4p$ (3sf).

(iii)	The product moment correlation coefficient between s and p, r = 0.928578571 = 0.929 (3 s.f)
	As r is close to 1, there is strong positive linear correlation between s and p.
(vi)	If $p = 70$, $s = 646.8814847 + 79.44195692(70)$ $= 6207.818469 = 6210$ (3sf) The value of the asset is US$ 6210 billion
	Alternative solution As profits depends on assets, use p on s $p = 0.010853939s - 3.827569368$ $70 = 0.010853939s - 3.827569368$ $s = (70 + 3.827569368)/0.010853939$ $= 6801.914896 = 6800$ (3sf)
	The estimate is not reliable as $p = 70$ is outside the data range [2.0, 65.8].

11(i)	The probability of a switch being faulty is a constant and the same for all switches. OR A switch being faulty is independent of any other switches being faulty.
(ii)	$np = 0.54$

	$np(1-p) = 0.5319$ $1-p = 0.5319/0.54$ $= 0.985$ $p = 0.015$ $n = 0.54/0.015$ $= 36$
(iii)	$X \sim B(36, 0.015)$ $P(X \le 5) - P(X \le 1)$ or $P(2 \le X \le 5)$ $= 0.999984928 - 0.898541026$ $= 0.101443902$ $= 0.101$ (3sf)
(iv)	$P(X > 3)$ $= 1 - P(X \le 3)$ $= 0.002032816$ (at least 5sf)
	Let Y be the number of days where more than 3 switches are found to be faulty out of 30 days $Y \sim B(30, 0.002032816)$ $P(Y \le 1)$ or $P(Y = 0) + P(Y = 1)$ $= 0.998269258$ $= 0.998$ (3sf)
(v)	$P(X > 3) = 0.002032816$ The expected number of days $= 0.002032816 \times 30 = 0.060984478$ $= 0.0610$ (3sf)

12(i) Unbiased estimate of the population mean, $\bar{x} = \frac{\sum(x-250)}{60} + 250$ $= \frac{200}{60} + 250 = 253\frac{1}{3}$ or $\frac{760}{3}$ Note: 253.3333333 is wrong as the value is not exact
Unbiased estimate of the population variance, $s^2 = \frac{1}{60-1}\left(\sum(x-250)^2 - \frac{\left(\sum(x-250)\right)^2}{60}\right)$ $= \frac{1}{59}\left(13\,800 - \frac{200^2}{60}\right) = \frac{39400}{177}$ Note: 222.5988701 is wrong as the value is not exact
(ii) Let μ be that the population mean time to repair cars. To test H_0: $\mu = 250$ against H_1: $\mu > 250$ at 5% level of significance. Under H_0, since $n = 60$, is large, by Central Limit Theorem $\bar{X} \sim N\left(250, \frac{39400}{(177)60}\right)$ approximately

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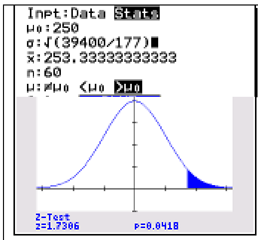		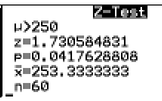

Using a one-tailed test, reject H_0 if p-value ≤ 0.05

Using $\bar{x} = 253\frac{1}{3}$ gives $z_{cal} = 1.730584831$ and

p-value $= 0.04176292 \le 0.05$

We reject H_0 and conclude that there is **sufficient evidence** at the 5% level of significance to conclude that the manager's claim is valid or the population mean time to repair cars is more than 250 minutes.

(iii) Test H_0: $\mu = 250$ against H_1: $\mu > 250$ at 5% level of significance.

Under H_0, since sample size is large, by Central Limit Theorem $\bar{X} \sim \mathrm{N}\left(250, \frac{m}{50}\right)$ approximately.

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

~~Value of test statistic,~~

$$z_{cal} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{254 - 250}{\sqrt{\frac{m}{50}}} = 4\sqrt{\frac{50}{m}}$$

Using a one-tailed test, reject H_0 if $z_{cal} \ge 1.64485$

~~Reject H_0. (manager's claim should be accepted)~~

Since H_0 is rejected (manager's claim should be accepted)~~Using Critical Region,~~

$z_{cal} \ge 1.64485$

$$4\sqrt{\frac{50}{m}} \ge 1.64485$$

$17.195665386 \ge \sqrt{m}$

Squaring, $295.6905116 \ge m$

Thus $0 < m \le 295$

13a(i)	Let W be the random variable the amount of time he has to wait before that particular bus arrives. Then $W \sim \mathrm{N}(16, 5^2)$. Let D be the random variable the amount of time the bus takes to reach his school. Then $D \sim \mathrm{N}(28, 9^2)$ Let T be the total time lapsed between the time Henry starts waiting for the bus and the time he reaches school. As $T \sim \mathrm{N}(16 + 28, 5^2 + 9^2)$. So $T \sim \mathrm{N}(44, 106)$
	Henry will be late if he takes more than 50 min (7.35am – 6.45am)
	Probability that Henry is late for school if he starts waiting for his bus at 6.45am $\mathrm{P}(T > 50)$ $= 0.280023561 = 0.280$ (3sf)

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(ii)	Let t be the time lapsed between the time Henry starts waiting for the bus and 7.35am. If $P(T \le t)$, Henry will not be late. If $P(T > t)$, Henry will be late. For Henry to be late does not exceed (not more than) 5% of the time, $P(T > t) \le 0.05$
	By GC, $t \ge 60.9$ Hence Henry should start waiting for the bus no later than 6.34am. (61 mins or more before 7.35am)

13(bi)
Let K and P denote the masses, in grams, of a randomly chosen Kunning fish and a randomly chosen Pomfret fish G respectively.
Then $K \sim N(50, 3^2)$ and $P \sim N(320, 15^2)$
(i) $E(K_1 + \ldots + K_{20} - 3P)$
$= E(K_1 + \ldots + K_{20}) - 3E(P)$
$= (50 + \ldots + 50 - 3(320)$
$= 20 \times 50 - 3(320) = 40$
$\text{Var}(K_1 + \ldots + K_{20} - 3P) = \text{Var}(K_1 + \ldots + K_{20}) + 3^2\text{Var}(P)$
$= (3^2 + \ldots + 3^2) + 3^2(15^2)$
$= 20 \times 3^2 + 3^2(15^2) = 2205$
Then $K_1 + \ldots + K_{20} - 3P \sim N(40, 2205)$
$P(-50 \le K_1 + \ldots + K_{20} - 3P \le 50)$
$= 0.556677976 = 0.557$ (3sf)

(bii)
Let \$$T$ denote the total cost of 100 randomly chosen Kunning fish and 20 randomly chosen Pomfret fish.
Then $T = \dfrac{0.85}{100}(K_1 + \ldots + K_{100}) + \dfrac{1.2}{100}(P_1 + \ldots + P_{20})$
$E(T) = \dfrac{0.85}{100}E(K_1 + \ldots + K_{100}) + \dfrac{1.2}{100}E(P_1 + \ldots + P_{20})$
$= 0.0085(100)(50) + 0.012(20)(320)$
$= 119.30$
$\text{Var}(T) = \left(\dfrac{0.85}{100}\right)^2 \text{Var}(K_1 + \ldots + K_{100}) + \left(\dfrac{1.2}{100}\right)^2 \text{Var}(P_1 + \ldots + P_{20})$
$= (0.0085)^2(100)(3^2) + (0.012)^2(20)(15^2)$
$= 0.713025$

Then $T \sim N(119.3, 0.713025)$
$P(115 < T < 120)$
$= 0.79644283 = 0.796$ (3sf)