NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2

Preliminary Examination

NAME

SUBJECT CLASS	2ma1		REGISTRATION NUMBER							

H1 MATHEMATICS

8865/01

12 September 2022

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in the brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	4	
2	4	
3	5	
4	5	
5	10	
6	12	
7	5	
8	6	
9	8	
10	9	
11	10	
12	10	
13(a)	6	
13(b)	6	
Presentation Deduction		**–1 / –2**
TOTAL	**100**	

This document consists of **21** printed pages and **3** blank pages.

[Turn over

Section A: Pure Mathematics [40 marks]

1 Find the exact set of values of m such that the line $y = mx - 3$ intersects the curve $y = mx^2 - (m-1)x - 2$. [4]

2 By using a suitable substitution, solve the inequality $3e^{2x} > e^x + 70$, giving your answer in logarithmic form. [4]

3 Tammy deposits $100\,000$ in three different savings plans for 2 years as shown in the table below:

Savings Plan	Returns
I	Simple interest of 1.4% per annum.
II	One time interest of 3.5% for a 2-year period.
III	Quarterly payout of 50 for every $10\,000$ invested. Investments are made in multiples of $10\,000$.

At the end of 2 years, she received an interest of $3\,560$. The amount of money she deposits in savings plan II is twice the amount she deposits in savings plan I.

(i) Find the amount of money she deposits in each savings plan. [3]

For the amount of money deposited in savings plan II and III, Tammy was given an option to deposit the amount in savings plan IV, which involves a variable return, as shown below.

Savings Plan	Returns
IV	Daily interest rate that fluctuates between 3.7% to 3.9% for a period of 2 years.

(ii) Explain if Tammy should take up savings plan IV, justifying your answer mathematically. [2]

4 Find the exact value of $\int_{1}^{4} \left(3 - \dfrac{1}{x}\right)^2 \, dx$, expressing your answer in the form $a + b\ln c$, where a, b and c are constants to be determined. [3]

Hence find the value of p such that $\int_{-\frac{1}{3}}^{p} e^{3x+1} \, dx = \int_{1}^{4} \left(3 - \dfrac{1}{x}\right)^2 \, dx$, leaving your answer correct to four decimal places. [2]

5 The curves C_1 and C_2 have equations $y = \dfrac{2x-5}{x-2}$ and $y = e^{1-x} + 2$ respectively.

    (i) Express $\dfrac{2x-5}{x-2}$ in the form $p + \dfrac{q}{x-2}$, where p and q are integers to be determined. [1]

    (ii) On the same axes, sketch the graphs of C_1 and C_2, showing clearly the coordinates of any points of intersection between C_1 and C_2, axial intercepts and the equations of any asymptotes. [3]

    (iii) Find the exact area of the region bounded by the curves C_1, C_2 and the y-axis. [3]

    (iv) Without the use of a calculator, find the integers a and b, where $y = \dfrac{x+b}{a}$ is the equation of the tangent to C_1 when $x = 4$. [2]

    (v) Hence, using the values of a and b found in part **(iv)**, find the set of values of x such that $e^{1-x} + 2 > \dfrac{x+b}{a}$. [1]

6 Mark owns a factory that manufactures bottled drinks. He wants to model the production cost, C thousand dollars per month over a period of t months. He believes that

$$C = t^3 - 7t^2 + 8t + 20, \text{ for } 0 \le t \le 6.$$

    (i) Use differentiation to find the exact values of t which give stationary points on the graph of C against t. For each point, justify whether it is minimum or maximum. [5]

    (ii) Use your calculator to find the value of $\int_0^6 \left(t^3 - 7t^2 + 8t + 20 \right) dt$. In the context of the question, what does this value represent? [2]

Mark is also interested in modelling his monthly profit, P thousand dollars, which is related to C by the equation $P = 100 \left(e^{-0.25C} \right) + 3.2$.

    (iii) When $t = 6$, show that $\dfrac{dP}{dC} = ke^m$, where k and m are integer to be found. [3]

    (iv) Hence, find the rate of decrease in profit per month when $t = 6$. [2]

Section B: Probability and Statistics [60 marks]

7 In a forest, 30% of the plants have heights less than 0.83 m and 30% have heights greater than 2.41 m. The standard deviation of the height of the plants is 1.5 m.

    (i) State the mean height of the plants in the forest. Give a reason why a normal distribution, with this standard deviation, would not give a good approximation to the distribution of the height of the plants. [2]

    (ii) A random sample of 100 plants in the forest were taken and their heights measured. Find the probability that the mean height of the sample lies between 1.2 m and 1.6 m. [3]

8 A vehicle insurance company classifies the drivers it insures as class L, M and H according to whether they are of low risk, medium risk or high risk with regard to having an accident respectively. The company estimates that 30% of the drivers who are insured are class L and 50% are class M. The probability that a class L driver will have at least one accident in a year is 0.01, the corresponding probabilities for class M and class H are 0.03 and 0.06 respectively.

    (i) The company insures a driver and within a year, the driver had at least one accident. Show that the probability that the driver is of class H is 0.4. [3]

    (ii) Three drivers insured by the company are chosen randomly. Find the probability that all three drivers are of class H and exactly one of them has at least one accident in a year. [3]

9 A group of 12 students comprises of 5 girls and 7 boys. Among the 5 girls are Ann and Alice.

    (a) The 12 students are seated in a row to take a group photo.

        (i) Find the number of different seating arrangements in which Ann and Alice are separated from each other. [2]

        (ii) Find the number of different seating arrangements in which there are exactly 3 boys between Ann and Alice. [3]

    (b) A teacher wants to form two teams of 5 students for an activity from the group of 5 girls and 7 boys. Ann is selected as the leader of one team and Alice is selected as the leader of the other team.

    Find the number of possible teams that can be formed with exactly one team comprising of 2 girls. [3]

10 A financial magazine publishes an annual ranking of the financial services companies in the world. The ranking is based on sales, profit, assets and market value. A random sample of 7 pairs of assets, US$ s billion and profit, US$ p billion is shown in the table below.

Assets, s	4914.7	4301.7	2832.2	4159.9	394.5	491.9	326.7
Profit, p	65.8	39.3	17.9	31.3	2.6	3.4	2.0

(i) Give a sketch of the scatter diagram for the data, labelling the axes. [2]

(ii) Find the equations of the least-squares regression line of p on s and s on p. Draw the least-squares regression line of p on s and plot ($\overline{s}$, $\overline{p}$) in the scatter diagram in part **(i)**. [3]

(iii) Find the product moment correlation coefficient between s and p and comment on its value in the context of the data. [2]

(iv) Using an appropriate regression line from part **(ii)**, calculate an estimate for the value of the asset when the profit is \$70 billion. Comment on the reliability of your estimate. [2]

11 A factory produces a wide variety of components for use in computers.

The factory makes switches for motherboards. A fixed number of chosen switches are tested each day and the number of faulty switches is denoted by X.

(i) State, in the context of the question, an assumption needed for X to be well modelled by a binomial distribution. [1]

Assume now that X has the distribution B(n, p). X has mean 0.54 and variance 0.5319.

(ii) Show that $p = 0.015$ and find the value of n. [2]

(iii) Find the probability that in a randomly chosen day, more than 1 and at most 5 switches are found to be faulty. [2]

(iv) Find the probability that over a period of 30 days, more than 3 switches are found to be faulty on at most 1 day. [3]

(v) Find the expected number of days in which more than 3 switches are found to be faulty over a period of 30 days. [2]

12 The manager of a service station decided to take a random sample of the time taken, x minutes, to repair 60 cars. The times taken are summarised by

$$\sum(x-250) = 200, \qquad \sum(x-250)^2 = 13\,800.$$

(i) Calculate the exact unbiased estimates of the population mean and variance for time taken to repair cars. [3]

(ii) The manager wishes to test a claim that the population mean time to repair cars is more than 250 minutes. Test at the 5% significance level whether the manager's claim is supported by data, defining any symbols that you use. [4]

(iii) You are given instead the sample mean time to service 50 cars is 254 minutes and the population variance is m, and that the result of a test of 5% significance level is that the manager's claim should be accepted. Find the set of possible values of m. [3]

13 In this question you should state the parameters of any distribution you use.

(a) Henry commutes to school by a particular public bus service. On any one school day, the amount of time he has to wait before that particular bus arrives is normally distributed

with mean 16 minutes and standard deviation 5 minutes. Upon boarding the bus, the amount of time the bus takes to reach his school is normally distributed with mean 28 minutes and standard deviation 9 minutes. Henry is late for school if he reaches school after 7.35am.

(i) Suppose on a randomly chosen school day, Henry starts to wait for the bus at 6.45am. Find the probability that Henry is late on that day. [3]

(ii) After some warnings for being late, Henry decides to leave his house earlier. Find the latest time he needs to start waiting for the bus such that the probability that he is late does not exceed 5%. [3]

(b) There are two types of fishes sold at a fish stall, the Kunning fish and the Pomfret fish. The masses, in grams, of Kunning fish and Pomfret fish are independent and normally distributed with means 50 grams and 320 grams respectively, and standard deviation 3 grams and 15 grams respectively.

(i) Find the probability that the masses of 20 randomly chosen Kunning fish is within 0.05 kg of 3 times the mass of a randomly chosen Pomfret fish. [3]

Kunning fish costs $0.85 per 100g and Pomfret fish costs $1.20 per 100g. A chef purchased 100 randomly chosen Kunning fish and 20 randomly chosen Pomfret fish from the fish stall.

(ii) Find the probability that the chef has to pay between $115 and $120 in total. [3]