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CANDIDATE NAME

CLASS

ADMISSION NUMBER

# 2022 Preliminary Examination
## Pre-University 3

**MATHEMATICS** **9758/02**

Paper 2 **19 September 2022**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

**READ THESE INSTRUCTIONS FIRST**

Write your admission number, name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

| **Qn No.** | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | * | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Score** | | | | | | | | | | | | | |
| **Max Score** | **5** | **6** | **9** | **10** | **10** | **7** | **8** | **9** | **11** | **12** | **13** | | **100** |

## Section A: Pure Mathematics [40 marks]

**1** A curve $C$ has equation

$$y = \frac{a}{x^3} + bx^2 + c$$

where $a$, $b$ and $c$ are constants. It is given that $C$ passes through the point with coordinates $(-1.2, 5.4)$, and has its minimum point at $(0.9, 5.9)$.

**(i)** Find the values of $a$, $b$ and $c$, giving you answer correct to 3 decimal places. [4]

**(ii)** State the equation of the line that is an asymptote of $C$. [1]

**2** The amount of glucose, $m$ milligrams at time $t$ minutes in a simulated dialysis machine is modelled by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{1}{120}(120 - m).$$

The machine contains 50 milligrams of glucose initially.

**(i)** Find $m$ in terms of $t$. [4]

**(ii)** Explain what happens to the amount of glucose in the machine for large values of $t$. [2]

**3** **(i)** Using standard series from the List of Formulae (MF26), find the first three non-zero terms of the Maclaurin's series for $y = \ln\left(1 + \mathrm{e}^{-x}\right)$. [4]

**(ii)** Use your answer in part **(i)** to find the approximate value of $\ln\left(\frac{\mathrm{e}^2 + 1}{\mathrm{e}^2}\right)$. [2]

**(iii)** Hence, or otherwise, find an expansion for $\frac{1}{1 + \mathrm{e}^x}$, up to and including the term in $x$. [3]

**4** The equations of the plane $\pi_1$ and the line $l$ are $\mathbf{r}\bullet\begin{pmatrix}1\\0\\1\end{pmatrix}=1$ and $\frac{x}{2}=y+1,\ z=2$ respectively.

The plane $\pi_2$ contains the line $l$ and the point $A$ with position vector **(i)**

the cartesian equation of the plane $\pi_2$, [3]

**(ii)** the vector equation of the line of intersection between the planes $\pi_1$ and $\pi_2$, [1]

**(iii)** the position vector of the foot of the perpendicular from the point $B(-2,\ 1,\ 0)$ to the plane $\pi_1$, [4]

**(iv)** the point of reflection of $B$ in $\pi_1$. [2]

**5** A function f is defined by $\mathrm{f}(x)=\mathrm{e}^{\cos 2x}\sin 4x$, for $0\le x\le\frac{\pi}{2}$.

**(i)** Differentiate $\mathrm{e}^{\cos 2x}$ with respect to $x$. [1]

**(ii)** Use part **(i)** and integration by parts to show that

$$\int \mathrm{f}(x)\,\mathrm{d}x=\mathrm{e}^{\cos 2x}(1-\cos 2x)+c\,.$$ [4]

**(iii)** Find the exact value of the area enclosed by the graph of $y=\mathrm{f}(x)$ and the $x$-axis. [3]

**(iv)** The graph of $y=\mathrm{f}(x)$ is rotated through $2\pi$ radians about the $x$-axis. Find the volume of the solid formed. [2]

## Section B: Probability and Statistics [60 marks]

**6** **(a)** A group of three adults and five children dines in a restaurant. Find the number of ways to seat the group of eight people at a round table for three people and a round table for five people if

**(i)** there are no restrictions, [2]

**(ii)** there is at least one child and at least one adult at each table. [3]

**(b)** The restaurant offers a salad bar for every main course ordered. The salad bar consists of ten different dishes and customers are free to take from any of the ten dishes at the salad bar. Find the number of ways a customer can choose at least one dish from the salad bar. [2]

**7** A bag contains three red balls, $n-1$ blue balls and $n$ white ball, where $n \geq 3$. The balls are identical except for their colour. Two balls are drawn at random, without replacement. Two points is given for each red ball drawn, one point is given for each blue ball drawn and one point is deducted for each white ball drawn. Let $X$ be the score obtained from adding the points of the two balls.

**(i)** Show that $\mathrm{P}(X=0)=\dfrac{n(n-1)}{(n+1)(2n+1)}$. [2]

**(ii)** Find the probability distribution of $X$. [3]

**(iii)** Given that the average score of picking the two balls is at least 0.16, find the largest possible number of white balls in the bag. [3]

**8** The table gives the world record time, in seconds, for the 400-metre race for the various years after 1950. The number of years after 1950 is denoted by $T$, where $T = x - 1950$.

| Year, $x$ | 1955 | 1956 | 1960 | 1967 | 1968 | 1968 | 1988 | 1999 | 2016 |
|---|---|---|---|---|---|---|---|---|---|
| World record time, $y$ | 45.4 | 45.2 | 44.9 | 44.5 | 44.1 | 43.86 | 43.29 | 43.18 | 43.03 |

**(i)** Draw the scatter diagram for the values of $y$ and $T$, labelling the axes clearly. [2]

A statistician believes that the world record time $y$ can be modelled by one of the following formulae

$$y = a + bT \quad \text{or} \quad y = c + d\ln T,$$

where $a$, $b$, $c$ and $d$ are constants.

**(ii)** Using the product moment correlation coefficient, explain which of the formulae $y = a + bT$ or $y = c + d\ln T$ is the better model. [3]

**(iii)** Using the better model found in part **(ii)**, find the equation of a suitable regression line. [2]

**(iv)** Hence estimate the world record time in the year 2024, giving your answer correct to 2 decimal places. Comment on the reliability of this estimate. [2]

**9** A factory produces a large number of chocolates every day. On average, $100p\%$ of the chocolates are milk chocolates and the rest are almond chocolates. The chocolates are packed into bags of fifteen.

**(i)** Given that the most likely number of milk chocolates in a bag is 5, find the range of values of $p$. [4]

Use $p = 0.35$ for the rest of the question.

Find the probability that

**(ii)** there are three milk chocolates in a bag of fifteen chocolates. [1]

**(iii)** the fifteenth chocolate is the third milk chocolate in a bag of fifteen chocolates. [2]

**(iv)** Explain why the answer in part **(ii)** is greater than the answer in part **(iii)**. [1]

Bags of fifteen chocolates each are now packed into cartons. Each carton contains 25 bags of chocolates. Find

**(v)** the probability that a carton contains fewer than 7 bags with three milk chocolates each, [2]

**(vi)** the average number of bags with three milk chocolates each in a carton. [1]

**10** A marine biologist claims that the mean mass of an adult sockeye salmon is at least 4.2 kg. The mass, $x$ kg, of each adult sockeye salmon in a random sample of 100 sockeye salmon is weighed. The results are summarised as follows.

$$\sum(x-4.2)=-7.84 \qquad \sum(x-4.2)^2=32.186$$

**(i)** Find unbiased estimates of the population mean and variance of the mass of adult sockeye salmons. [3]

**(ii)** Test, at the 5% level of significance, whether the marine biologist's claim is valid. [4]

**(iii)** Explain, in the context of the question, the meaning of 'at the 5% level of significance'. [1]

**(iv)** Explain, without further calculation, whether the conclusion of the test in part **(ii)** will remain the same if the test were conducted with the same data but with only the following changes.

**(a)** The marine biologist claims that the mean mass of an adult sockeye salmon is 4.2 kg. [2]

**(b)** The population variance is known and is smaller than the unbiased estimate of the population variance found in part **(i)**. [2]

**11** **In this question you should, where applicable, state clearly all the distributions that you use, together with the values of the appropriate parameters.**

The time spent by each customer at S Supermarket is denoted by the random variable $X$. $X$ follows a normal distribution with mean $a$ minutes and standard deviation $b$ minutes.

**(i)** Given that $\mathrm{P}(X>c)=0.3$, where $c$ is a constant. Find

**(a)** $\mathrm{P}(a<X<c)$, [1]

**(b)** $\mathrm{P}(c<X<a+b)$. [2]

Use $a=35$ and $b=10$ for the rest of this question.

Alice, Barry and Christine are three customers at S Supermarket who do not know each other. Find the probability that

**(ii)** the time spent by Alice differs from the time spent by Barry by less than a minute, [3]

**(iii)** the total time spent by Barry and Christine exceeds twice the time spent by Alice by at least 15 minutes, [3]

**(iv)** the total time spent by Alice, Barry and Christine at S Supermarket is at most 110 minutes. [3]

**(v)** If Barry and Christine were members of the same family, what can you say about the validity of your answers in parts **(ii)**, **(iii)** and **(iv)**? [1]

**End of Paper**