PU3 MATHEMATICS

Paper 9758/02

Section A: Pure Mathematics

Qn	Solution
1(i) **[4]**	$y=\frac{a}{x^3}+bx^2+c$ Given that C passes through $(-1.2, 5.4)$, $5.4=\frac{a}{(-1.2)^3}+b(-1.2)^2+c$ --- (1) Given that C also passes through $(0.9, 5.9)$, $5.9=\frac{a}{(0.9)^3}+b(0.9)^2+c$ --- (2) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3a}{x^4}+2bx$ Given that C has a minimum point at $(0.9, 5.9)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=0.9$ $0=\frac{-3a}{(0.9)^4}+2(0.9)b$ --- (3) Solving the three equations simultaneously using GC, $a=1.428$, $b=3.628$, $c=1.002$ (3d.p.) Hence $y=\frac{1.428}{x^3}+3.628x^2+1.002$
1(ii) **[1]**	Vertical asymptote is $x=0$

Qn	Solution
2(i) **[4]**	$\frac{\mathrm{d}m}{\mathrm{d}t}=\frac{1}{120}(120-m)$ $\frac{1}{120-m}\frac{\mathrm{d}m}{\mathrm{d}t}=\frac{1}{120}$ $\int\frac{1}{120-m}\frac{\mathrm{d}m}{\mathrm{d}t}\,\mathrm{d}t=\int\frac{1}{120}\,\mathrm{d}t$ $\int\frac{1}{120-m}\,\mathrm{d}t=\int\frac{1}{120}\,\mathrm{d}t$ $-\ln\lvert 120-m\rvert=\frac{1}{120}t+c$ $\ln\lvert 120-m\rvert=-\frac{1}{120}t+d$, where $d=-c$ $\lvert 120-m\rvert=\mathrm{e}^{-\frac{1}{120}t+d}=\mathrm{e}^{d}\mathrm{e}^{-\frac{1}{120}t}$ $120-m=A\mathrm{e}^{-\frac{1}{120}t}$, where $A=\mathrm{e}^{d}$ or $-\mathrm{e}^{d}$ $m=120-A\mathrm{e}^{-\frac{1}{120}t}$ When $t=0$, $m=50$, $50=120-A$ $A=70$ $\therefore m=120-70\mathrm{e}^{-\frac{1}{120}t}$
2(ii) **[2]**	As $t\to\infty$, $\mathrm{e}^{-\frac{1}{120}t}\to 0$, $m=120-70\mathrm{e}^{-\frac{1}{120}t}\to 120$. For large values of t, the amount of glucose approaches 120 milligrams.

Qn	Solution
3(i) [4]	$\ln\left(1+e^{-x}\right)\approx\ln\left[1+\left(1-x+\frac{x^2}{2}\right)\right]$ $=\ln\left(2-x+\frac{x^2}{2}\right)$ $=\ln\left[2\left(1-\frac{x}{2}+\frac{x^2}{4}\right)\right]$ $=\ln 2+\ln\left[1+\left(-\frac{x}{2}+\frac{x^2}{4}\right)\right]$ $=\ln 2+\left(-\frac{x}{2}+\frac{x^2}{4}\right)-\frac{\left(-\frac{x}{2}+\frac{x^2}{4}\right)^2}{2}+\dots$ $=\ln 2-\frac{x}{2}+\frac{x^2}{4}-\left(\frac{x^2}{8}\right)+\dots$ $=\ln 2-\frac{x}{2}+\frac{1}{8}x^2+\dots$
3(ii) [2]	$\ln\left(\frac{e^2+1}{e^2}\right)=\ln\left(1+e^{-2}\right)$ $\approx\ln 2-\frac{2}{2}+\frac{1}{8}(2)^2$ $\approx 0.193\ (3\text{ s.f.})$
3(iii) [3]	**<u>'Hence' Method:</u>** Observe that $\frac{d}{dx}\left[\ln\left(1+e^{-x}\right)\right]=\frac{-e^{-x}}{1+e^{-x}}$ $\frac{d}{dx}\left[\ln\left(1+e^{-x}\right)\right]=\frac{d}{dx}\left[\ln 2-\frac{x}{2}+\frac{1}{8}x^2+\dots\right]$ ---(*) $\frac{-e^{-x}}{1+e^{-x}}=-\frac{1}{2}+\frac{1}{4}x+\dots$ $\frac{-e^{-x}\left(e^{x}\right)}{\left(1+e^{-x}\right)\left(e^{x}\right)}=-\frac{1}{2}+\frac{1}{4}x+\dots$ $\frac{1}{1+e^{x}}=\frac{1}{2}-\frac{1}{4}x+\dots$ ---

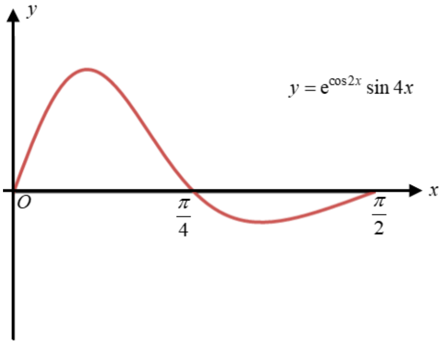
Qn	Solution
	<u>'Otherwise' Method 1:</u>
	$\dfrac{1}{1+e^x}=\left(1+e^x\right)^{-1}$
	$\approx\left(1+1+x\right)^{-1}$
	$=\left(2+x\right)^{-1}$
	$=2^{-1}\left(1+\dfrac{x}{2}\right)^{-1}$
	$=\dfrac{1}{2}\left[1+\left(-1\right)\left(\dfrac{x}{2}\right)+\ldots\right]$
	$=\dfrac{1}{2}-\dfrac{1}{4}x+\ldots$
	<u>'Otherwise' Method 2 (Repeated Differentiation):</u>
	Let $y=\dfrac{1}{1+e^x}$
	Diff. w.r.t x:
	$\dfrac{dy}{dx}=-\left(1+e^x\right)^{-2}\times e^x$
	$=\dfrac{-e^x}{\left(1+e^x\right)^2}$
	At $x=0$:
	$y=\dfrac{1}{2}$,
	$\dfrac{dy}{dx}=\dfrac{-1}{\left(1+1\right)^2}\Rightarrow\dfrac{dy}{dx}=-\dfrac{1}{4}$
	$\therefore\dfrac{1}{1+e^x}\approx\dfrac{1}{2}-\dfrac{1}{4}x$

Qn	Solution
4(i) **[3]**	$l:\mathbf{r}=\begin{pmatrix}0\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix},\ \lambda\in\mathbb{R}$ Let $\overrightarrow{OA}=\begin{pmatrix}-1\\0\\1\end{pmatrix},\ \overrightarrow{OC}=\begin{pmatrix}0\\-1\\2\end{pmatrix}$ Direction vector on plane, $\overrightarrow{AC}$ $=\begin{pmatrix}0\\-1\\2\end{pmatrix}-\begin{pmatrix}-1\\0\\1\end{pmatrix}$ $=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ Normal vector to plane, $\mathbf{n}_2$ $=\begin{pmatrix}1\\-1\\1\end{pmatrix}\times\begin{pmatrix}2\\1\\0\end{pmatrix}$ $=\begin{pmatrix}-1\\2\\3\end{pmatrix}$ Vector equation of plane in scalar product form: $\mathbf{r}.\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}.\begin{pmatrix}-1\\2\\3\end{pmatrix}$ $\mathbf{r}.\begin{pmatrix}-1\\2\\3\end{pmatrix}=4$ Cartesian equation of the plane π_2 is $-x+2y+3z=4$
4(ii) **[1]**	Using GC: Vector equation of line of intersection between the two planes: $\mathbf{r}=\begin{pmatrix}1\\\frac{5}{2}\\0\end{pmatrix}+\mu\begin{pmatrix}-1\\-2\\1\end{pmatrix},\ \mu\in\mathbb{R}$
4(iii) **[4]**	Let F be the foot of the perpendicular from B to π_1.

Qn	Solution
	Equation of line l_{BF} : $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \ \alpha \in \mathbb{R}$, Since F lies on the line BF, thus $\overrightarrow{OF} = \begin{pmatrix} -2+\alpha \\ 1 \\ \alpha \end{pmatrix}$, for some $\alpha \in \mathbb{R}$. Since F also lies in the plane π_1, $\begin{pmatrix} -2+\alpha \\ 1 \\ \alpha \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 1$ $-2+\alpha+\alpha = 1$ $2\alpha = 3$ $\alpha = \frac{3}{2}$ $\overrightarrow{OF} = \begin{pmatrix} -2+\frac{3}{2} \\ 1 \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ \frac{3}{2} \end{pmatrix}$
4(iv) **[2]**	Using Ratio Theorem $\overrightarrow{OF} = \frac{\overrightarrow{OB} + \overrightarrow{OB'}}{2}$ $\overrightarrow{OB'} = 2\overrightarrow{OF} - \overrightarrow{OB} = 2\begin{pmatrix} -\frac{1}{2} \\ 1 \\ \frac{3}{2} \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>5(i)
[1]</td><td>$\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{e}^{\cos 2x}\right]=(-2\sin 2x)\mathrm{e}^{\cos 2x}$
Note: This implies that $\int -2\sin 2x\mathrm{e}^{\cos 2x}\ \mathrm{d}x=\mathrm{e}^{\cos 2x}+c$</td></tr>
<tr><td>5(ii)
[4]</td><td>$\int \mathrm{f}(x)\,\mathrm{d}x$
$=\int \mathrm{e}^{\cos 2x}\sin 4x\,\mathrm{d}x$
$=\int \mathrm{e}^{\cos 2x}\sin 2(2x)\,\mathrm{d}x$
$=\int \mathrm{e}^{\cos 2x}2\sin 2x\cos 2x\,\mathrm{d}x$
$=-\int -\mathrm{e}^{\cos 2x}2\sin 2x\cos 2x\,\mathrm{d}x$
$=-\int \left(-2\sin 2x\mathrm{e}^{\cos 2x}\right)(\cos 2x)\,\mathrm{d}x$

Using integration by parts:
<table><tr><td>$u=\cos 2x$</td><td>$\frac{\mathrm{d}v}{\mathrm{d}x}=-2\sin 2x\mathrm{e}^{\cos 2x}$</td></tr><tr><td>$\frac{\mathrm{d}u}{\mathrm{d}x}=-2\sin 2x$</td><td>$v=\int -2\sin 2x\mathrm{e}^{\cos 2x}\,\mathrm{d}x$
$=\mathrm{e}^{\cos 2x}$</td></tr></table>
$-\int \left(-2\sin 2x\mathrm{e}^{\cos 2x}\right)(\cos 2x)\,\mathrm{d}x$
$=-\left[(\cos 2x)\left(\mathrm{e}^{\cos 2x}\right)-\int\left(\mathrm{e}^{\cos 2x}\right)(-2\sin 2x)\,\mathrm{d}x\right]$
$=-\mathrm{e}^{\cos 2x}\cos 2x+\int \mathrm{e}^{\cos 2x}(-2\sin 2x)\,\mathrm{d}x$
$=-\mathrm{e}^{\cos 2x}\cos 2x+\mathrm{e}^{\cos 2x}+c$
$=\mathrm{e}^{\cos 2x}-\mathrm{e}^{\cos 2x}\cos 2x+c$
$=\mathrm{e}^{\cos 2x}(1-\cos 2x)+c$</td></tr>
</table>

Qn	Solution
5(iii) **[3]**	 Let $y = e^{\cos 2x} \sin 4x = 0$. $e^{\cos 2x} = 0$ (rejected as $e^{\cos 2x} > 0$ for all real values of x) or $\sin 4x = 0$. Solve $\sin 4x = 0$: $\therefore x = 0, \frac{\pi}{4}, \frac{\pi}{2}$. Required Area $= \int_0^{\frac{\pi}{4}} e^{\cos 2x} \sin 4x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} -e^{\cos 2x} \sin 4x \, dx$ $= \int_0^{\frac{\pi}{4}} e^{\cos 2x} \sin 4x \, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\cos 2x} \sin 4x \, dx$ $= \left[e^{\cos 2x} (1 - \cos 2x) \right]_0^{\frac{\pi}{4}} - \left[e^{\cos 2x} (1 - \cos 2x) \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \left[e^{\cos 2\left(\frac{\pi}{4}\right)} \left(1 - \cos 2\left(\frac{\pi}{4}\right)\right) - \left(e^{\cos 2(0)} (1 - \cos 2(0))\right) \right] -$ $\left[e^{\cos 2\left(\frac{\pi}{2}\right)} \left(1 - \cos 2\left(\frac{\pi}{2}\right)\right) - \left(e^{\cos 2\left(\frac{\pi}{4}\right)} \left(1 - \cos 2\left(\frac{\pi}{4}\right)\right) \right) \right]$ $= \left[e^{\cos \frac{\pi}{2}} \left(1 - \cos \frac{\pi}{2}\right) - \left(e^{\cos 0} (1 - \cos 0)\right) \right] -$ $\left[e^{\cos \pi} (1 - \cos \pi) - \left(e^{\cos \frac{\pi}{2}} \left(1 - \cos \frac{\pi}{2}\right) \right) \right]$ $= \left[e^0 (1 - 0) - \left(e^1 (1 - 1)\right) \right] - \left[e^{-1} (1 - (-1)) - \left(e^0 (1 - 0)\right) \right]$ $= [1 - 0] - \left[2e^{-1} - (1) \right]$ $= 1 - 2e^{-1} + 1$ $= 2 - 2e^{-1}$ units^2

Qn	Solution
5(iv) **[2]**	Required Volume $=\pi\int_0^{\frac{\pi}{2}} y^2 \ \mathrm{d}x$ $=\pi\int_0^{\frac{\pi}{2}} \left(\mathrm{e}^{\cos 2x}\sin 4x\right)^2 \ \mathrm{d}x$ $=5.50 \text{ units}^3$ (to 3 s.f.)

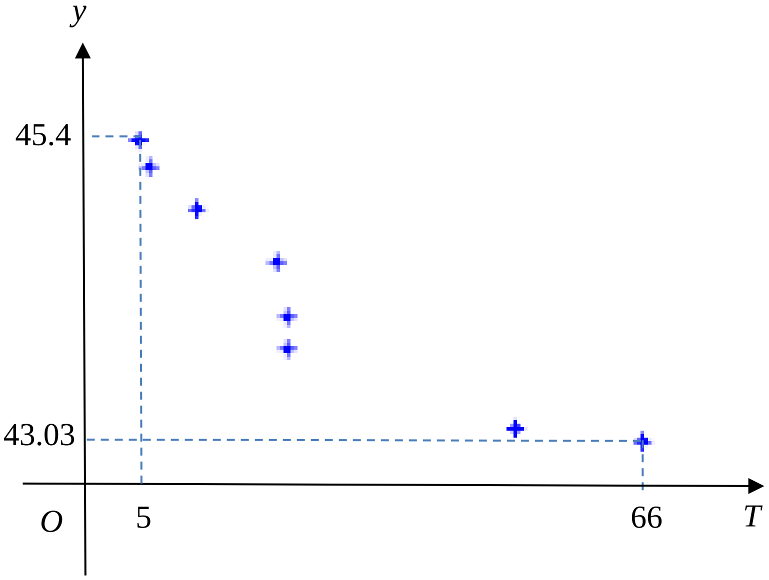
Section B: Probability and Statistics

Qn	Solution
6(a)(i) **[2]**	<u>Method 1:</u> No. of ways = ${}^8C_3\times(3-1)!\times{}^5C_5\times(5-1)!=2688$ <u>Method 2:</u> ${}^8C_5\times(5-1)!\times{}^3C_3\times(3-1)!=2688$
6(a)(ii) **[3]**	<u>Method 1:</u> <u>Case 1: one child at table of 3</u> No. of ways = ${}^5C_1\times{}^3C_2\times(3-1)!\times{}^4C_4\times{}^2C_2\times(5-1)!=720$ <u>Case 2: two children at table of 3</u> No. of ways = ${}^5C_2\times{}^3C_1\times(3-1)!\times{}^3C_3\times{}^2C_2\times(5-1)!=1440$ Total no. of ways = $720+1440=2160$ <u>Method 2:</u> No. of ways = no restriction – (3 children at table of 3) – (3 children at table of 5) = 2688 $-{}^3C_3\times(3-1)!\times{}^5C_5\times(5-1)!$ $-{}^5C_3\times(3-1)!\times{}^3C_3\times{}^2C_2\times(5-1)!$ = 2160
6(b) **[2]**	No. of ways = $2^{10}-1=1023$

Qn	Solution
7(i) **[2]**	Let R = Red ball, B = Blue Ball, W = White ball

Qn	Solution
	$\mathrm{P}(X=0)=\mathrm{P}(B,W)+\mathrm{P}(W,B)$ $=\left(\frac{n-1}{2n+2}\right)\left(\frac{n}{2n+1}\right)\times 2$ $=\frac{n(n-1)}{2(n+1)(2n+1)}\times 2$ $=\frac{n(n-1)}{(n+1)(2n+1)}$
7(ii) **[3]**	$\mathrm{P}(X=-2)=\mathrm{P}(W,W)$ $=\left(\frac{n}{2n+2}\right)\left(\frac{n-1}{2n+1}\right)$ $=\frac{n(n-1)}{2(n+1)(2n+1)}$ $\mathrm{P}(X=1)=\mathrm{P}(R,W)+\mathrm{P}(W,R)$ $=\left(\frac{3}{2n+2}\right)\left(\frac{n}{2n+1}\right)\times 2$ $=\frac{3n}{(n+1)(2n+1)}$ $\mathrm{P}(X=2)=\mathrm{P}(B,B)$ $=\left(\frac{n-1}{2n+2}\right)\left(\frac{n-2}{2n+1}\right)$ $=\frac{(n-1)(n-2)}{2(n+1)(2n+1)}$ $\mathrm{P}(X=3)=\mathrm{P}(R,B)+\mathrm{P}(B,R)$ $=\left(\frac{3}{2n+2}\right)\left(\frac{n-1}{2n+1}\right)\times 2$ $=\frac{3(n-1)}{(n+1)(2n+1)}$ $\mathrm{P}(X=4)=\mathrm{P}(R,R)$ $=\left(\frac{3}{2n+2}\right)\left(\frac{2}{2n+1}\right)$ $=\frac{3}{(n+1)(2n+1)}$

Qn	Solution
7(iii) **[3]**	$\mathrm{E}(X)\geq 0.6 \Rightarrow \sum x\mathrm{P}(X=x)\geq 0.16$ $\Rightarrow -2\left(\frac{n(n-1)}{2(n+1)(2n+1)}\right)+1\left(\frac{3n}{(n+1)(2n+1)}\right)+2\left(\frac{(n-1)(n-2)}{2(n+1)(2n+1)}\right)$ $+3\left(\frac{3(n-1)}{(n+1)(2n+1)}\right)+4\left(\frac{3}{(n+1)(2n+1)}\right)\geq 0.16$ $\Rightarrow \frac{-(n^2-n)+(3n)+(n^2-3n+2)+(9)(n-1)+(12)}{(n+1)(2n+1)}\geq 0.16$ $\Rightarrow \frac{-n^2+n+3n+n^2-3n+2+9n-9+12}{(n+1)(2n+1)}\geq 0.16$ $\Rightarrow \frac{10n+5}{(n+1)(2n+1)}\geq 0.16$ Using GC: <table><tr><td>n</td><td>$\frac{10n+5}{(n+1)(2n+1)}$</td></tr><tr><td>29</td><td>0.1667</td></tr><tr><td>30</td><td>0.1613</td></tr><tr><td>31</td><td>0.1563</td></tr></table> Max number of white balls is 30.

Qn	Solution
8(i) **[2]**	
8(ii) **[3]**	The product moment correlation coefficient between y and $T = 0.90818$ The product moment correlation coefficient between y and $\ln T = 0.98083$ Since the product moment correlation coefficient between y and $\ln T$ is closer to 1, $y = c + d\ln T$ is the better model.
8(iii) **[2]**	The regression line of y on $\ln T$: $y = -0.97506\ln T + 46.983$ $y = -0.975\ln T + 47.0$ (3 s.f)
8(iv) **[2]**	For the year 2024, $T = 2024 - 1950 = 74$. When $T = 74$, $y = -0.97506\ln 74 + 46.983$ $= 42.786$ The required estimate is 42.79 seconds (2 d.p). This estimate **<u>may not be reliable</u>** as $T = 74$ is **<u>outside the range of the given data</u>** (extrapolation). The model $y = -0.975\ln T + 47.0$ may not hold outside the given range of data.

Qn	Solution
9(i) **[4]**	Let X be the number of milk chocolates, out of 15. $X \sim \mathrm{B}(15, p)$

Qn	Solution
	Since the most likely number of milk chocolates is 5, $\mathrm{P}(X=5)>\mathrm{P}(X=4)$ ${}^{15}C_5 p^5(1-p)^{10} > {}^{15}C_4 p^4 (1-p)^{11}$ $2.2p>1-p$ $p>\frac{5}{16}$ (i.e. $p>0.3125$) And $\mathrm{P}(X=5)>\mathrm{P}(X=6)$ ${}^{15}C_5 p^5(1-p)^{10} > {}^{15}C_6 p^6 (1-p)^{9}$ $1-p>\frac{5}{3}p$ $p<\frac{3}{8}$ (i.e. $p<0.375$) Thus $\frac{5}{16}<p<\frac{3}{8}$ (i.e. $0.3125<p<0.375$)
9(ii) **[1]**	Let X be the number of milk chocolates, out of 15. $X \sim \mathrm{B}(15, 0.35)$ Probability $=\mathrm{P}(X=3)\approx 0.11096 = 0.111$ (3 s.f.)
9(iii) **[2]**	**<u>Method 1:</u>** Let Y be the number of milk chocolates, out of 14. $Y \sim \mathrm{B}(14, 0.35)$ Probability $=\mathrm{P}(Y=2)\times 0.35 \approx 0.022192 = 0.0222$ (3 s.f.) -- **<u>Method 2:</u>** Probability $=(0.35)^2(0.65)^{12}\times {}^{14}C_2\left(\text{ or } \frac{14!}{2!12!}\right)\times 0.35$ $\approx 0.022192 = 0.0222$ (3 s.f.)
9(iv) **[1]**	Answer in part **(ii)** is greater than part **(iii)** because the event in part **(iii)** is a **<u>subset</u>** of the event in part **(ii)**. For example, the case that the first three chocolates in a bag are milk chocolates is included in part **(ii)** but not in part **(iii)**.
9(v) **[2]**	Let A be the number of bags with three milk chocolates in a carton, out of 25. $Y \sim \mathrm{B}(25, 0.11096)$

Qn	Solution
	$\text{P}(Y<7)=\text{P}(Y\le 6)$ $\approx 0.98367 = 0.984 \text{ (to 3 s.f.)}$
9(vi) **[1]**	$\text{E}(X)=25(0.11096)=2.774\approx 2.77 \text{ (3 s.f.)}$

Qn	Solution
10(i) **[3]**	The unbiased estimate of population mean, $\bar{x}$ $=\dfrac{-7.84}{100}+4.2$ $=4.1216$ The unbiased estimate of population variance, s^2 $=\dfrac{1}{100-1}\left[32.186-\dfrac{(-7.84)^2}{100}\right]$ $=0.31890$ $=0.319$ (3 s.f)
10(ii) **[4]**	Let μ be the **population mean mass** of adult sockeye salmons. $\text{H}_0: \mu=4.2$ $\text{H}_1: \mu<4.2$ **Under H_0**, since n = 100 is large, by **Central Limit Theorem**, $\bar{X}\sim \text{N}\left(4.2, \dfrac{0.31890}{100}\right)$ approximately. Use a z-test at $\alpha=0.05$. Using GC, p-value $=0.082520=0.0825$ (3 s.f) . Since p-value = 0.0825 > 0.05, we **do not reject H_0**. There is **insufficient evidence at the 5% level of significance** that the **mean mass of adults sockeye salmons is less than 4.2 kg** (i.e. **insufficient evidence that the claim is not valid**).
10(iii)	**0.05** is the probability of **concluding that the mean mass of sockeye salmon is less than 4.2 kg**, when the **mean mass is in fact 4.2 kg**.
10(iv) **(a)** **[2]**	The test becomes a two-tailed test and the hypotheses are: $\text{H}_0: \mu=4.2$ $\text{H}_1: \mu\ne 4.2$ The new p-value $=2(0.08252)=0.16504>0.05$ H_0 will still not be rejected. The conclusion will still be that there is insufficient evidence that the claim is not valid.

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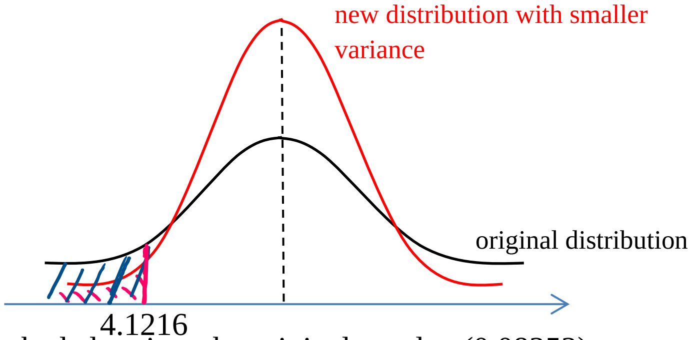
<tr><td>10(iv)
(b)
[2]</td><td>

<u>Method 1 (comparing p-values)</u>

When the population variance is known and is smaller than the unbiased estimate of the population variance found in part (i), the variance of the distribution of $\overline{X}$ $\left(\text{i.e. } \frac{\sigma^2}{100}\right)$ becomes smaller (see diagram).

Thus, with the same value of $\overline{x}$ (4.1216), the new p-value < 0.08252 (see diagram). However, the new p-value may or may not be lower than 0.05.

H_0 may or may not be rejected. The conclusion may or may not be that there is insufficient evidence that the claim is not valid.



Blue shaded region: the original p-value (0.08252)
Red shaded region: the new p-value ($<$ 0.08252)

<u>Method 2a (comparing test statistic with critical value)</u>

The population variance is now used and is smaller than the value of the unbiased estimate used originally. Thus, the critical value is larger than the critical value in the test in part (ii). The test statistic $(\text{i.e. } 4.1216)$ may or may not be smaller than the new critical value.
H_0 may or may not be rejected. The conclusion may or may not be that there is insufficient evidence that the claim is not valid.

<u>Method 2b (comparing test statistic with critical value)</u>

If the test statistic is standardised, the critical value remains the same $(\text{i.e. } -1.6449)$. The population variance is now used and is smaller than the value of the unbiased estimate used originally. Thus, the standardised test statistic i.e. $z = \frac{\overline{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}}$ becomes larger than the original test statistic i.e. more negative in this case. The new test statistic value may or may not be smaller than -1.6449.
H_0 may or may not be rejected. The conclusion may or may not be that there is insufficient evidence that the claim is not valid.

Note:
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<tr><td></td><td>Recall that only the value of variance used is changed in this case.</td></tr>
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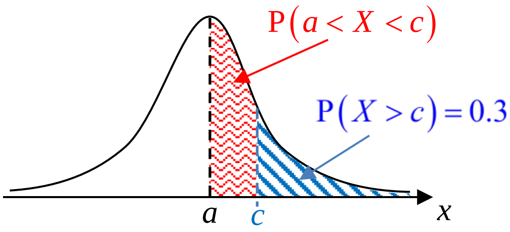
<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>11(i)
(a)
[1]</td><td>

$X \sim \mathrm{N}(a,b^2)$



<u>**Method 1**</u>

$$\begin{aligned}\mathrm{P}(a<X<c)&=\mathrm{P}(X>a)-\mathrm{P}(X>c)\\&=0.5-0.3\\&=0.2\end{aligned}$$

<u>**Method 2**</u>

$$\begin{aligned}\mathrm{P}(a<X<c)&=\mathrm{P}(X<c)-\mathrm{P}(X<a)\\&=0.7-0.5\\&=0.2\end{aligned}$$

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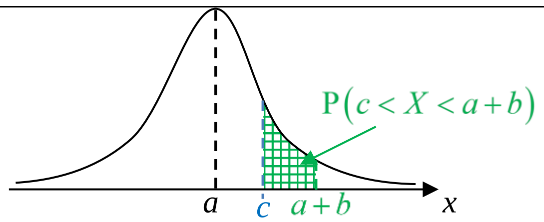
<tr><td>11(i)
(b)
[2]</td><td>

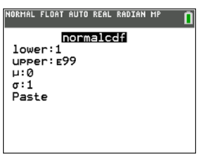


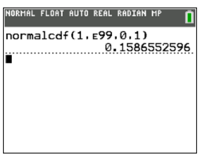
<u>**Method 1**</u>

$$\begin{aligned}\mathrm{P}(c<X<a+b)&=\mathrm{P}(X>c)-\mathrm{P}(X>a+b)\\&=\mathrm{P}(X>c)-\mathrm{P}\left(\frac{X-a}{b}>\frac{a+b-a}{b}\right)\\&=\mathrm{P}(X>c)-\mathrm{P}(Z>1)\\&=0.3-0.15866\\&=0.141\ (3\text{s.f.})\end{aligned}$$

By standardisation,

$$\frac{X-a}{b}=Z\sim \mathrm{N}(0,1)$$

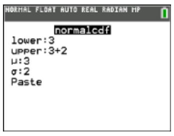
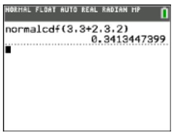
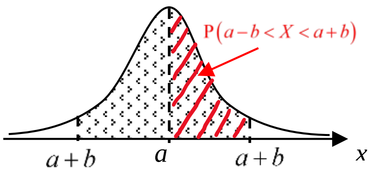




<u>**Method 2**</u>

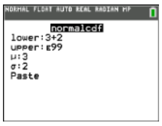
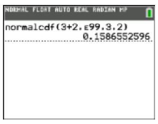
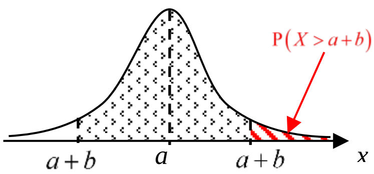
</td></tr>
</table>

Qn	Solution

Using GC, choosing any 2 positive values for *a* and *b*, i.e. $a = 3,\ b = 2$, we can observe that  $\mathrm{P}(a < X < a+b) = 0.34134$	By the '68-95-99.7' rule of normal distribution, $\mathrm{P}(\mu-\sigma < X < \mu+\sigma) = 0.68$, i.e. 68% of the distribution lies within one standard deviation of the mean. For $X \sim \mathrm{N}(a, b^2)$ $\mathrm{P}(a-b < X < a+b) = 0.68$,  which gives us $\mathrm{P}(a < X < a+b) = \dfrac{0.68}{2} = 0.34$

$$\mathrm{P}(c < X < a+b) = \mathrm{P}(a < X < a+b) - \mathrm{P}(a < X < c)$$
$$= 0.34134 - 0.2 \quad \text{OR} \quad 0.34\text{-}0.2$$
$$= 0.141 \text{ (3 s.f)} \quad \text{OR} \quad 0.14$$

<u>Method 3</u>

Using GC, choosing any 2 positive values for *a* and *b*, i.e. $a = 3,\ b = 2$, we can observe that  $\mathrm{P}(X > a+b) = 0.15866$	By the '68-95-99.7' rule of normal distribution, $\mathrm{P}(\mu-\sigma < X < \mu+\sigma) = 0.68$, i.e. 68% of the distribution lies within one standard deviation of the mean. For $X \sim \mathrm{N}(a, b^2)$ $\mathrm{P}(a-b < X < a+b) = 0.68$,  which gives us $\mathrm{P}(X > a+b) = \dfrac{1-0.68}{2} = 0.16$

Qn	Solution
	$\mathrm{P}\left(c < X < a+b\right) = \mathrm{P}\left(X > c\right) - \mathrm{P}\left(X > a+b\right)$ $= 0.3 - 0.15866$ OR $0.3 - 0.16$ $= 0.141$ (3 s.f) OR 0.14
11(ii) **[3]**	$X \sim \mathrm{N}(35,\ 10^2)$ Let X_a and X_b be the time Alicia and Barry spend at S Supermarket respectively. The required probability $= \mathrm{P}\left(\left\|X_a - X_b\right\| < 1\right)$ $= \mathrm{P}\left(-1 < X_a - X_b < 1\right)$ $\mathrm{E}\left(X_a - X_b\right) = 35 - 35 = 0$ $\mathrm{Var}\left(X_a - X_b\right) = 10^2 + 10^2 = 200$ $X_a - X_b \sim \mathrm{N}(0,\ 200)$ The required probability $= 0.056372 = 0.0564$ (3 s.f)
11(iii) **[3]**	$X \sim \mathrm{N}(35,\ 10^2)$ Let X_c be the time Christine spends at S Supermarket. The required probability $= \mathrm{P}\left(X_b + X_c - 2X_a > 15\right)$ $\mathrm{E}\left(X_b + X_c - 2X_a\right) = 35 + 35 - 2(35) = 0$ $\mathrm{Var}\left(X_b + X_c - 2X_a\right) = 10^2 + 10^2 + 2^2(10^2) = 600$ $X_b + X_c - 2X_a \sim \mathrm{N}(0,\ 600)$ The required probability $= 0.27015 = 0.270$ (3 s.f)
11(iv) **[3]**	The required probability $= \mathrm{P}\left(X_a + X_b + X_c \le 110\right)$ $\mathrm{E}\left(X_a + X_b + X_c\right) = 35 + 35 + 35 = 3(35) = 105$ $\mathrm{Var}\left(X_a + X_b + X_c\right) = 10^2 + 10^2 + 10^2 = 3\left(10^2\right) = 300$ $X_a + X_b + X_c \sim \mathrm{N}(105,\ 300)$ The required probability $= 0.61358 = 0.614$ (3 s.f)
11(v) **[1]**	The calculations will not valid as the **time Barry and Christine each spends** at S Supermarket **will not be independent**.