

Name : \_\_\_\_\_

Class      Index Number

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# METHODIST GIRLS' SCHOOL

Founded in 1887



## PRELIMINARY EXAMINATION 2022 Secondary 4

Thursday

**ADDITIONAL MATHEMATICS**

4049/02

18 August 2022

**PAPER 2**

2 hours 15 minutes

Candidates answer on the Question Paper.  
No Additional Materials are required.

### READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.

Write in dark blue or black pen

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

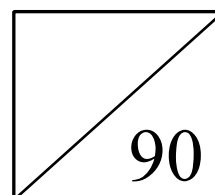
The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.



**1. ALGEBRA*****Quadratic Equation***

For the quadratic equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

***Binomial Expansion***

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$ .

**2. TRIGONOMETRY*****Identities***

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

***Formulae for  $\triangle ABC$*** 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** Given that  $y = 4x \sin 2x$ .

**(i)** Show that  $\frac{dy}{dx} = 4(2x \cos 2x + \sin 2x)$  [2]

**(ii)** Given that  $x$  is increasing at  $\frac{2}{5}$  radians per second, find the rate of change of  $y$  when  $x = \frac{\pi}{2}$ . [2]

**2**      **(i)**      Show that  $\frac{d}{dx}\left(\frac{\ln x^2}{x}\right) = \frac{2-2\ln x}{x^2}$ . [2]

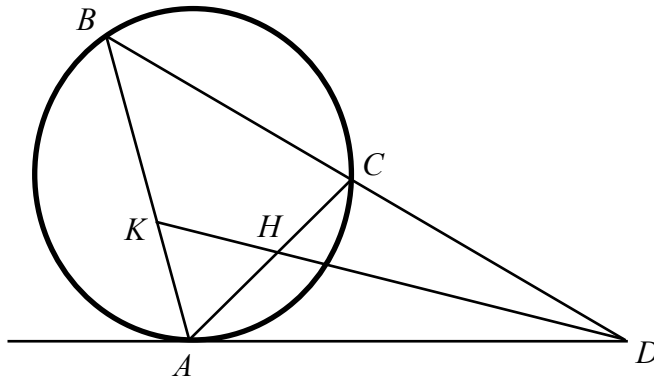
**(ii)**      Hence evaluate  $\int_1^e \left(\frac{2\ln x}{x^2}\right) dx$ . [4]

**3**      **(a)**      Find the coefficient of  $x^3$  in the expansion of  $(2-3x)^6$ . [2]

**(b)**      **(i)**      Write down the first three terms, in ascending powers of  $x$ , in the binomial expansion of  $(1+ax)^8$ , where  $a$  is a constant. [2]

**(ii)**      Given that the expansion of  $(1-3x)^2(1+ax)^8$  in ascending powers of  $x$  is  $1+10x+bx^2+\dots$ , calculate the value of  $a$ , and of  $b$ . [3]

- 4 In the diagram,  $AD$  is a tangent to the circle at  $A$ .  $BCD$  is a straight line that intersects the tangent at  $D$ .  $KD$  bisects  $\angle ADC$  and it cuts  $AC$  at  $H$  and meets  $AB$  at  $K$ .



Prove that

- (i)  $\triangle AHD$  is similar to  $\triangle BKD$ . [3]

- (ii)  $HD \times BD = KD \times AD$ , [2]

(iii)  $\triangle AHK$  is isosceles.

[3]

5      (a)      Show that  $\operatorname{cosec} x \sec x \equiv \tan x + \cot x$ . [4]

(b)      Hence solve the equation  $\operatorname{cosec} x \sec x = 2$  for  $0 < x < 3\pi$ . Leave your answers in terms of  $\pi$ . [3]



- 6 Express  $\frac{7x+4}{(x-2)(x^2+5)}$  in partial fractions. [5]

- 7      **(a)**      Express  $2x^2 - 6x + 7$  in the form  $a(x+b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants.  
Without solving, explain why the curve  $y = 2x^2 - 6x + 7$  will not intersect the line  $y = 2$ . [4]

- (b)**      By using a suitable substitution, solve the equation  
 $\frac{1}{2} \log_3 x = 3 + 2 \log_x 81$ . [4]

(c) Solve the simultaneous equations

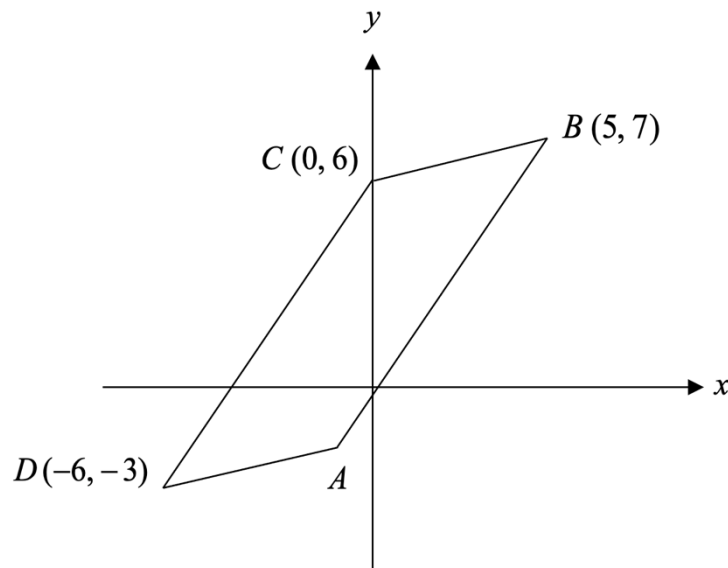
[4]

$$4^x \div 8^y = 32$$

$$\log_2 x - \log_2 (y+1) = 1.$$

**8 Solutions to this question by accurate drawing will not be accepted.**

$ABCD$  is a parallelogram where  $B$  is  $(5, 7)$ ,  $C$  is  $(0, 6)$  and  $D$  is  $(-6, -3)$ . The perpendicular from  $C$  to  $AB$  meets  $DA$  extended at a point  $E$ .



**(a)** Show that the coordinates of  $A$  is  $(-1, -2)$ . [1]

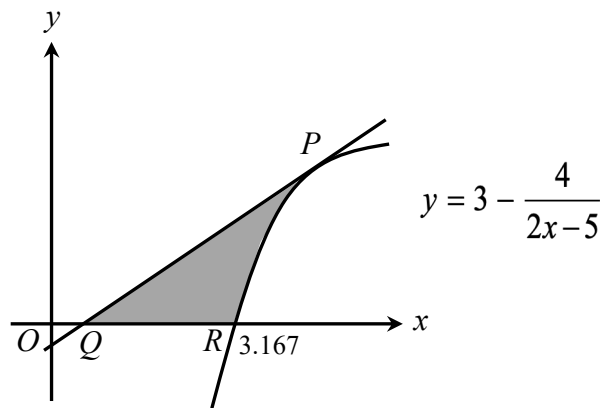
**(b) (i)** Find the equations of the line  $CE$  and line  $DA$  respectively. [3]

**(ii)** Hence, find the coordinates of  $E$ , and deduce the ratio  $DA : AE$  . [3]

**(c)** Name the special quadrilateral  $BCDE$  and explain clearly your reasons for it. [2]

**(d)** Find the area of quadrilateral  $BCDE$  . [2]

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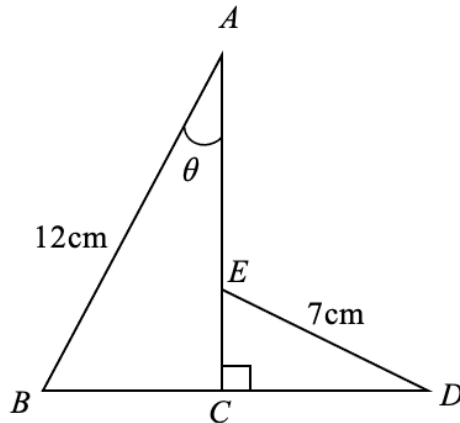
The diagram shows part of the curve  $y = 3 - \frac{4}{2x-5}$ , cutting the  $x$ -axis at  $R(3.167, 0)$ .

The tangent to the curve at  $P$  cuts the  $x$ -axis at  $Q$ . Given that the normal to the curve at  $P$  is parallel to the line  $y = -2x + 6$ , find

- (i) the equation of the tangent to the curve at  $P$ , [6]

- (ii) the area of the shaded region  $RPQ$ . [5]

- 10** The diagram shows a quadrilateral  $ABCDE$  in which  $\angle DCE = 90^\circ$ ,  $AB = 12$  cm,  $DE = 7$  cm and  $\angle BAC = \theta$  where  $0^\circ < \theta < 90^\circ$ .  $\triangle ABC$  is similar to  $\triangle DEC$ .



- (i) Show that the perimeter,  $P$  cm, of the quadrilateral is given by  $P = 19 + 19 \cos \theta + 5 \sin \theta$ . [3]

- (ii) Express  $P$  in the form  $k + R \cos(\theta - \alpha)$ , where  $k$  is a constant. [3]



- (iii) Find the maximum value of the perimeter,  $P$ , and the corresponding value of  $\theta$ . [2]

- (iv) Find the value of  $\theta$  for which  $P = 32$ . [2]

- 11** An experiment was conducted to estimate the population of insects,  $y$ , present in a colony  $x$  weeks after the start of the experiment.

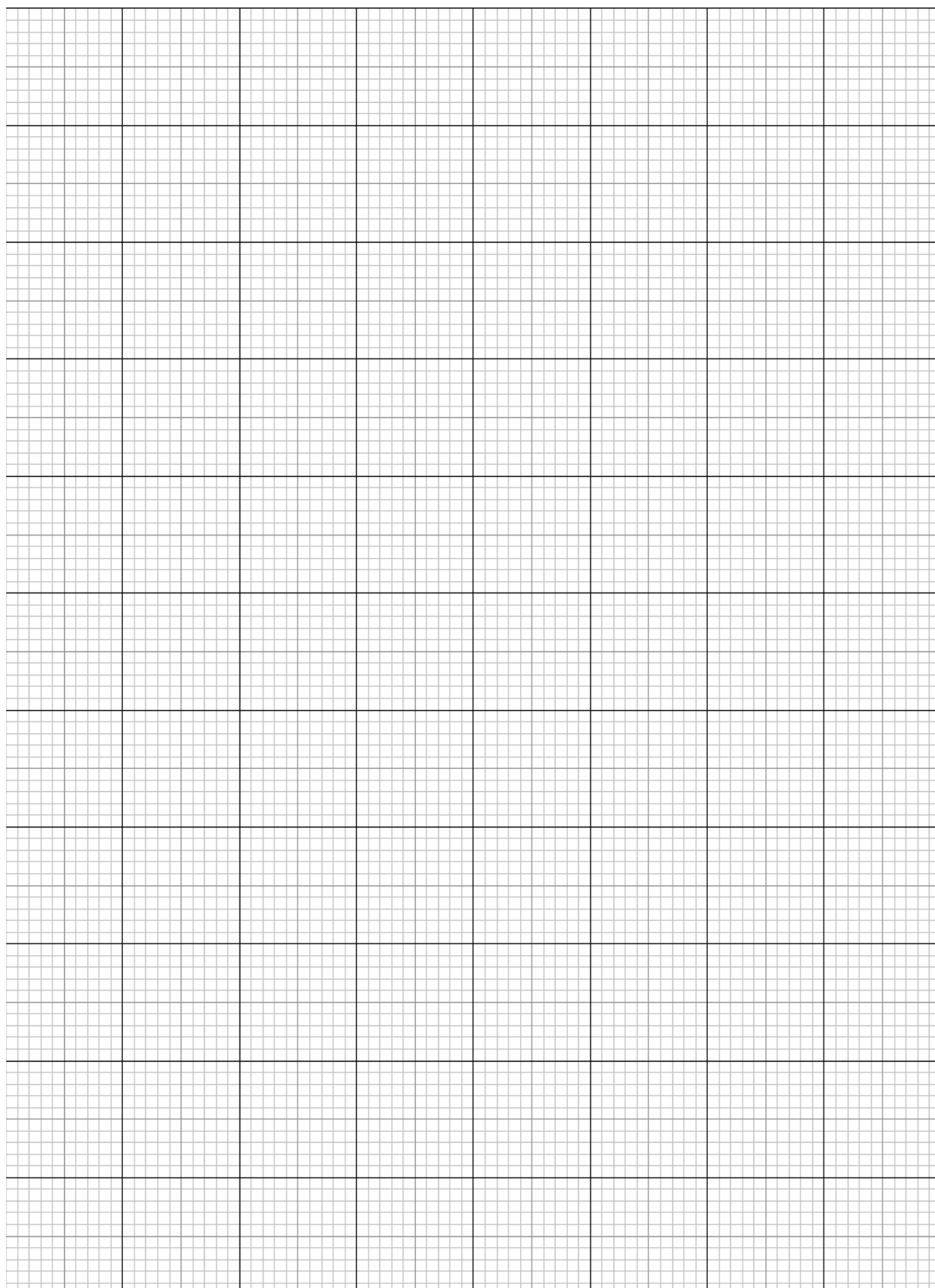
It is known that the variables  $x$  and  $y$  can be modelled by the equation  $y = kp^{\frac{x}{3}}$ , where  $k$  and  $p$  are constants. The table below shows the values of  $x$  and  $y$ .

$x$	2	4	6	8	10
$y$	1027	1341	1763	2342	3103

- (i) Plot  $\ln y$  against  $x$  on the grid provided and draw a straight line graph. The vertical  $\ln y$ -axis should start from 6.2 and have a scale of 2cm to 0.2. [3]
- (ii) Use your graph to estimate the value of  $k$  and of  $p$ . [3]
- (iii) Use your graph to estimate the number of weeks needed for the population of the insect to be doubled from the start of the experiment. [2]

- (iv) Explain why this model cannot be used five years from the start of the experiment.

[1]



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