Class	Index Number

Name : ____________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2022
Secondary 4

Monday **ADDITIONAL MATHEMATICS** **4049/01**

15 August 2022 **Paper 1** 2 h 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A triangle with a base of $\left(\sqrt{3}+2\right)$ cm has an area of $\left(12+7\sqrt{3}\right)$ cm^2. Find the perpendicular height of the triangle, leaving your answer in the form $\left(a+b\sqrt{3}\right)$ cm, where a and b are integers. [4]

$\frac{1}{2}\left(\sqrt{3}+2\right)\times h=12+7\sqrt{3}$ M1

$\frac{1}{2}h=\frac{12+7\sqrt{3}}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$ M1

$\frac{1}{2}h=\frac{12\sqrt{3}-24+7(3)-14\sqrt{3}}{3-4}$

$\frac{1}{2}h=\frac{-2\sqrt{3}-3}{-1}$ M1

$\frac{1}{2}h=3+2\sqrt{3}$

$h=6+4\sqrt{3}$ A1

2 **(a)** The equation of a curve is $y = px^2 - 4x + 16p$. Find the range of values of p given that the curve lies completely above the x-axis. [3]

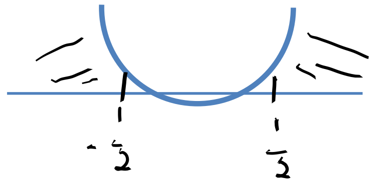
Discriminant = $(-4)^2 - 4(p)(16p)$

$= 16 - 64p^2$

$16 - 64p^2 < 0$ M1

$16 < 64p^2$

$p^2 > \frac{1}{4}$



$p < -\frac{1}{2}$, $p > \frac{1}{2}$ A1

A1 (reject as $p > 0$)

(b) Find the value of h for which the line $y = 2x + h$ is a tangent to the curve $y = 2x^2 - 6x + 5$. [4]

$y = 2x + h$ --- (1)

$y = 2x^2 - 6x + 5$ --- (2)

Sub (1) in (2): $2x + h = 2x^2 - 6x + 5$

$2x^2 - 6x - 2x + 5 - h = 0$

$2x^2 - 8x + 5 - h = 0$

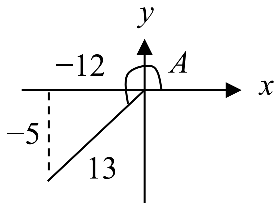
Discriminant = $(-8)^2 - 4(2)(5-h) = 0$

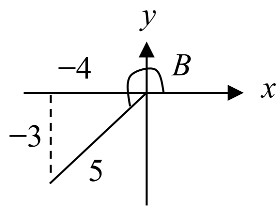
$64 - 40 + 8h = 0$

$h = -3$

3 Given that $\tan A = \frac{5}{12}$ and $\sin B = -\frac{3}{5}$, and that A and B are in the same quadrant, **without using a calculator**, calculate the values of

(a) $\operatorname{cosec}(-A)$, [2]





$$\operatorname{cosec}(-A) = \frac{1}{\sin(-A)}$$ - M1

$$= \frac{1}{-\sin A}$$

$$= \frac{1}{-\left(\frac{-5}{13}\right)}$$

$$= \frac{13}{5}$$ - A1

(b) $\sin 2B$, [1]

$$\sin 2B = 2\sin B \cos B$$

$$\sin 2B = 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{24}{25}$$ - A1

(c) $\tan(A+B)$. [2]

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\left(\frac{5}{12}\right)+\left(\frac{3}{4}\right)}{1-\left(\frac{5}{12}\right)\times\left(\frac{3}{4}\right)}$$ - M1

$$= \frac{56}{33}$$ - A1

4 **(a)** Solve the equation $2^{2x+1}-17\left(2^{x}\right)+21=0$. [4]

$2^{2x+1}-17\left(2^{x}\right)+21=0$

$\left(2^{x}\right)^{2}\times 2^{1}-17\left(2^{x}\right)+21=0$

Let $u=2^{x}$,

$2u^{2}-17u+21=0$ $\boxed{\text{M1}}$

$(2u-3)(u-7)=0$

$u=\frac{3}{2}$, $u=7$

$2^{x}=\frac{3}{2}$, $2^{x}=7$ $\boxed{\text{M1}}$

$x\lg 2=\lg\left(\frac{3}{2}\right)$, $x\lg 2=\lg 7$ $\boxed{\text{M1}}$

$x=\dfrac{\lg\left(\frac{3}{2}\right)}{\lg 2}$, $x=\dfrac{\lg 7}{\lg 2}$

$x=0.585$, $x=2.81$ $\boxed{\text{A1}}$

(b) Explain why the equation $2^{2x+1} - 17\left(2^x\right) + p = 0$ has no solution if $p > 36\frac{1}{8}$. [2]

Discriminant $= (-17)^2 - 4(2)(p)$

$= 289 - 8p$ M1

Since $p > 36\frac{1}{8}$, $-8p < -289$

$289 - 8p < 0$

$\therefore$ Discriminant < 0 A1

Hence, the equation has no solution if $p > 36\frac{1}{8}$.

Let $u = 2^x$, $2u^2 - 17u + p = 0$

D < 0, $\left(-17\right)^2 - 4(2)(p) < 0$ M1

$289 - 8p < 0$

$p > 36\frac{1}{8}$

Hence, the equation has no solution if $p > 36\frac{1}{8}$. A1

5 The mass, m grams, of a radioactive substance remaining, t days after being measured is given by $m = 10e^{-0.01t} + 0.2$.

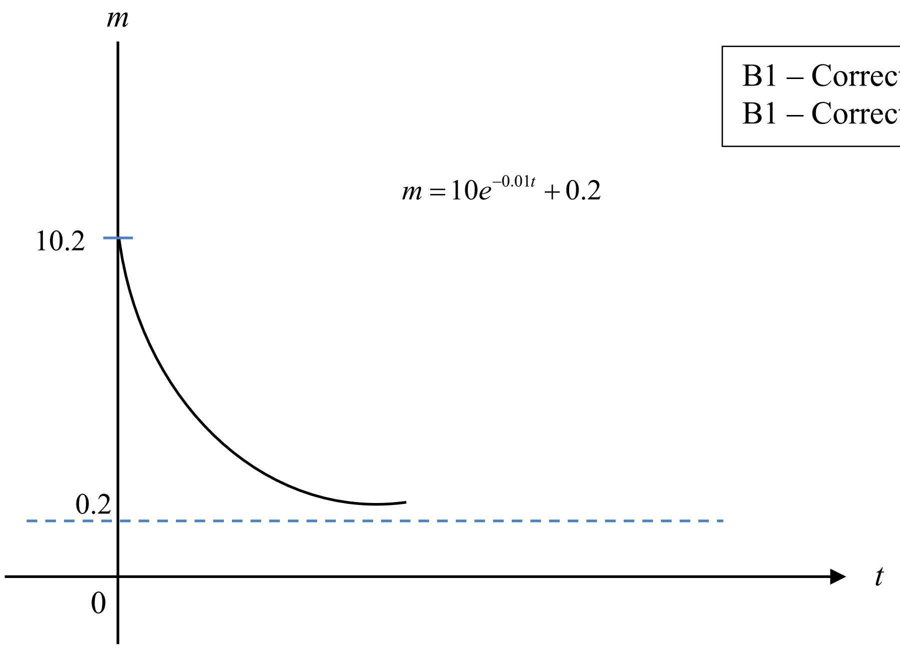
(a) Find the initial mass. [1]

Initial mass $= 10e^{-0.01(0)} + 0.2$

$= 10.2$ g

A1

(b) Sketch the graph $m = 10e^{-0.01t} + 0.2$ for $t \geq 0$. [2]



B1 – Correct shape
B1 – Correct intercept

(c) Find the least number of days it takes before the amount of substance is reduced to 5% of its initial mass. [3]

$10e^{-0.01t} + 0.2 < 0.05(10.2)$ M1

$10e^{-0.01t} + 0.2 < 0.51$

$10e^{-0.01t} < 0.31$

$e^{-0.01t} < 0.031$

$-0.01t < \ln(0.031)$ M1

$t > 347.37$

Least number of days = 348 A1

(d) Explain why the mass of the radioactive substance can never be less than 0.2 g. [2]

Since $e^{-0.01t} > 0$ A1

$10e^{-0.01t} > 0$

$10e^{-0.01t} + 0.2 > 0.2$ A1

Therefore, the mass of the radioactive substance can never be less than 0.2g.

6 The polynomial $2x^3 - 3ax^2 - 2ax + b$ has a factor $2x-1$ and leaves a remainder of -8 when divided by $x-1$.

(a) Find the values of a and of b. [4]

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 3a\left(\frac{1}{2}\right)^2 - 2a\left(\frac{1}{2}\right) + b = 0$$

$$\frac{1}{4} - \frac{3a}{4} - a + b = 0$$

$$7a - 4b = 1 \quad \text{-- (1)} \qquad \boxed{\text{M1}}$$

$$f(1) = 2 - 3a - 2a + b = -8$$

$$5a - b = 10$$

$$b = 5a - 10 \quad \text{-- (2)} \qquad \boxed{\text{M1}}$$

Sub (2) in (1):

$$7a - 4(5a - 10) = 1 \qquad \boxed{\text{M1}}$$

$$7a - 20a + 40 = 1$$

$$-13a + 39 = 0$$

$$a = 3$$

$$\boxed{\text{A1}}$$

$$b = 5(3) - 10$$

$$= 5$$

(b) Using the values of a and b in part **(a)**, factorise the polynomial completely. [3]

$$\begin{array}{rrrrr}
 & x^2 & -4x & -5 & \\ \cline{2-5}
2x-1 & \big)\,2x^3 & -9x^2 & -6x & +5 \\
- & & & & \\
 & 2x^3 & -x^2 & & \\ \cline{2-5}
 & & -8x^2 & -6x & +5 \\
 & - & & & \\
 & & -8x^2 & +4x & \\ \cline{2-5}
 & & & -10x & +5 \\
 & & - & & \\
 & & & -10x & +5 \\ \cline{2-5}
 & & & & 0
\end{array}$$

M1

$$2x^3 - 3ax^2 - 6x + 5 = (2x-1)(x^2-4x-5)$$ M1

$$= (2x-1)(x-5)(x+1)$$ A1

7 A circle, C_1 has a diameter AB where A is the point (6, –2) and B is the point (12, 6).

(a) Find the equation of C_1. [2]

Centre = $\left(\frac{6+12}{2}, \frac{-2+6}{2}\right) = (9,2)$

Radius = $\sqrt{(9-6)^2+(2+2)^2} = 5$

$\boxed{\text{M1 for either 1}}$

Eqn of C1: $(x-9)^2+(y-2)^2 = 25$ or $x^2+y^2-18x-4y+60=0$ $\boxed{\text{A1}}$

(b) Show that the equation of the tangent to the circle at A is $4y+3x=10$. [2]

Gradient of AB = $\frac{6+2}{12-6} = \frac{4}{3}$

Gradient of tangent = $-\frac{3}{4}$ $\boxed{\text{M1}}$

At (6, –2), $y+2=-\frac{3}{4}(x-6)$

$4y+3x=10$ $\boxed{\text{A1}}$

(c) Another circle, C_2, has its centre at A. Given that the area of C_2 is $\frac{1}{9}$ that of C_1, find the equation of C_2. [2]

Area of $C_2 = \frac{1}{9} \times$ area of C_1

Radius of $C_2 = \frac{1}{3} \times$ radius of C_1

$= \frac{5}{3}$ $\boxed{\text{M1}}$

Hence, Eqn of C_2: $(x-6)^2 + (y+2)^2 = \frac{25}{9}$ $\boxed{\text{A1}}$

8 The function $y = 1 - 0.5\cos(bx)$ is defined for $0 \le x \le \pi$ and b is a positive integer.

(a) State the amplitude. [1]

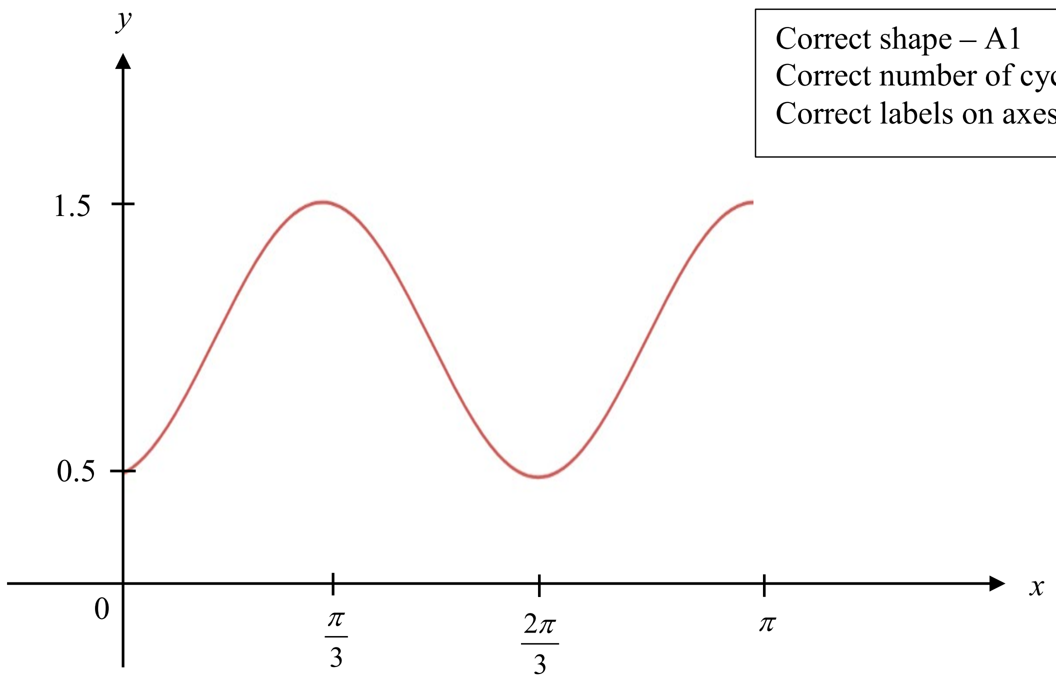
Amplitude = 0.5 $\boxed{\text{B1}}$

(b) It is given that the period of y is $\frac{2\pi}{3}$. Find the value of b. [1]

$$\frac{2\pi}{b} = \frac{2\pi}{3}$$

$b = 3$ $\boxed{\text{A1}}$

(c) Using your answer in part **(b)**, sketch the graph of $y = 1 - 0.5\cos(bx)$, for $0 \le x \le \pi$. [3]



Correct shape – A1
Correct number of cycles – A1
Correct labels on axes – A1

(d) Hence, explain how the solutions of the equation $\pi\cos(bx)+3x=0$ can be obtained from the graph sketched in part **(c)**. [3]

$$\pi\cos(3x)+3x=0$$
$$3x=-\pi\cos(3x)$$
$$\frac{3x}{2\pi}=-\frac{1}{2}\cos(3x) \qquad \boxed{\text{M1}}$$
$$\frac{3x}{2\pi}+1=1-\frac{1}{2}\cos(3x)$$

Draw the line $y=\dfrac{3x}{2\pi}+1$ $\boxed{\text{A1}}$

The solutions to $\pi\cos(3x)+3x=0$ are the <u>x-coordinates</u> of the <u>points of intersections</u> between the line and the curve. $\boxed{\text{A1}}$

9 Solve $3\cos 2x = 2\sin^2 2x$ for $0^\circ \le x \le 360^\circ$. [5]

$3\cos 2x = 2(1-\cos^2 2x)$ $\boxed{\text{M1}}$

$3\cos 2x = 2-2\cos^2 2x$

$2\cos^2 2x + 3\cos 2x - 2 = 0$

$(2\cos 2x - 1)(\cos 2x + 2) = 0$

$2\cos 2x - 1 = 0$ $\boxed{\text{M1}}$ or $\cos 2x + 2 = 0$ $\boxed{\text{M1}}$

$\cos 2x = \dfrac{1}{2}$ $\cos 2x = -2$

$\alpha = 60^\circ$ $\therefore$ There are no solutions. $\boxed{\text{A1}}$

$2x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$ $\boxed{\text{A1}}$

10 **(a)** Given that $y = \dfrac{x}{3x-4}$, find $\dfrac{dy}{dx}$. [2]

$$\frac{dy}{dx} = \frac{(3x-4)(1) - 3x}{(3x-4)^2} \quad \boxed{\text{M1}}$$

$$= -\frac{4}{(3x-4)^2} \quad \boxed{\text{A1}}$$

(b) Hence, show that y is a decreasing function for all real values of x. [2]

For all real values of x, $(3x-4)^2 > 0$

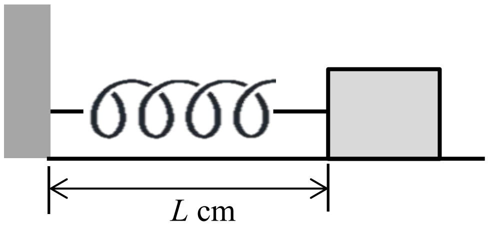
$$\frac{1}{(3x-4)^2} > 0 \quad \boxed{\text{M1}}$$

$$\frac{-4}{(3x-4)^2} < 0$$

$$\therefore \frac{dy}{dx} < 0 \quad \boxed{\text{A1}}$$

Since $\dfrac{dy}{dx} < 0$, y is a decreasing function for all real values of x.

11 In the diagram below, the length, L cm, of a spring at time, t seconds is given by the equation $L = 20 + 3\sin\left(4t - \frac{\pi}{2}\right)$.



(a) Show that the shortest length of the spring is 17 cm. [2]

Shortest length occurs when $\sin\left(4t - \frac{\pi}{2}\right) = -1$ $\boxed{\text{M1}}$

$\therefore L_{\min} = 20 + 3(-1) = 17cm$ $\boxed{\text{A1}}$

$-1 \le \sin\left(4t - \frac{\pi}{2}\right) \le 1$ $\boxed{\text{M1}}$

$-3 \le 3\sin\left(4t - \frac{\pi}{2}\right) \le 3$

$17 \le 20 + 3\sin\left(4t - \frac{\pi}{2}\right) \le 23$

$\therefore L_{\min} = 17cm$ $\boxed{\text{A1}}$

(b) Find the time when the spring first reaches 22 cm. [3]

$$22 = 20 + 3\sin\left(4t - \frac{\pi}{2}\right)$$ M1

$$2 = 3\sin\left(4t - \frac{\pi}{2}\right)$$

$$\frac{2}{3} = \sin\left(4t - \frac{\pi}{2}\right)$$

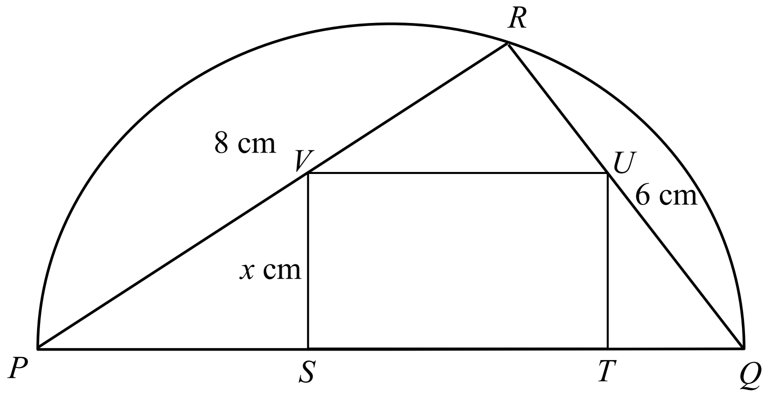
$$\alpha = 0.72972$$ M1

$$4t - \frac{\pi}{2} = 0.72972\,,\ \ldots$$

$$t = 0.5751,\ \ldots$$

$$\therefore t = 0.575\ \text{sec}$$ A1

12 The diagram below shows a semicircle with diameter *PQ*, with the point *R* on the circumference of the semicircle. A rectangle *STUV* is drawn within the triangle *PQR* where *ST* lies on *PQ*. *U* and *V* are points on *QR* and *PR* respectively. It is given that *PR* = 8 cm and *QR* = 6 cm.



(a) Find the perpendicular distance from *R* to *PQ*. [2]

Since $\angle PRQ = 90^\circ$ ($\angle$ in a semicircle), $PQ = \sqrt{8^2 + 6^2} = 10cm$ M1

Area $\Delta PRQ = \frac{1}{2} \times 6 \times 8 = 24cm^2$

Perpendicular Distance = $\frac{24}{0.5 \times 10} = 4.8cm$ A1

(b) It is given that *SV* is *x* cm. Show that the area of rectangle *STUV, A* m^2 is given by $A = 10x - \frac{25}{12}x^2$. [3]

Since ΔRVU is similar to ΔRPQ, $\frac{VU}{PQ} = \frac{h_1}{h_2}$

$$\frac{VU}{10} = \frac{4.8 - x}{4.8} \qquad \boxed{\text{M1}}$$

$$VU = 10 - \frac{25}{12}x \qquad \boxed{\text{M1}}$$

Area = $x\left(10 - \frac{25}{12}x\right)$

$= = 10x - \frac{25}{12}x^2$ (shown) $\boxed{\text{A1}}$

(c) Calculate the value of *x* for which *A* has a stationary value. [2]

$$\frac{\text{dA}}{\text{d}x} = 10 - \frac{50}{12}x \qquad \boxed{\text{M1}}$$

Since $\frac{\text{dA}}{\text{d}x} = 0$, $10 - \frac{50}{12}x = 0$

$$x = \frac{12}{5} \qquad \boxed{\text{A1}}$$

13 A function $f(x)$ is defined for all real values of x such that $f''(x) = 4e^{-2x}$. It is given that the gradient of the curve $y = f(x)$ is 3 when $x = 0$ and the curve passes through the point $\left(2, e^{-4}\right)$.

(a) Find an expression for $f(x)$. [5]

$$f''(x) = 4e^{-2x}$$

$$f'(x) = \frac{4e^{-2x}}{-2} + c \qquad \boxed{\text{M1}}$$

$$3 = \frac{4e^{0}}{-2} + c$$

$$c = 5 \qquad \boxed{\text{M1}}$$

$$f'(x) = -2e^{-2x} + 5$$

$$f(x) = e^{-2x} + 5x + c_1 \qquad \boxed{\text{M1}}$$

$$e^{-4} = e^{-2(2)} + 5(2) + c_1$$

$$c_1 = -10$$

$$\therefore f(x) = e^{-2x} + 5x - 10 \qquad \boxed{\text{A1}}$$

(b) Find the coordinates of the stationary point of the curve and determine the nature of this stationary point. [4]

$$f'(x) = -2e^{-2x} + 5 = 0 \qquad \boxed{\text{M1}}$$

$$-2x = \ln\left(\frac{5}{2}\right)$$

$$x = -0.45814$$

$$f''(x) = 4e^{-2(0.45814)} \quad (> 0) \qquad \boxed{\text{M1}}$$

$\therefore (-0.458, -9.79)$ is a <u>minimum point</u>. $\boxed{\text{A1}}$

$\boxed{\text{A1}}$

14 A particle moves in a straight line such that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = t^2 - 6t + 5$. The particle comes to instantaneous rest, firstly at A and then at B.

(a) Find an expression, in terms of t, for the distance of the particle from O at time t. [2]

$v = t^2 - 6t + 5$

$s = \int (t^2 - 6t + 5)dt$

$= \frac{t^3}{3} - 3t^2 + 5t + c$ $\boxed{\text{M1}}$

$s = 0$, $t = 0$, $c = 0$

$\therefore s = \frac{t^3}{3} - 3t^2 + 5t$ $\boxed{\text{A1}}$

(b) Find the total distance travelled by the particle in the first 5 seconds after passing O. [4]

$v = 0$

$t^2 - 6t + 5 = 0$ $\boxed{\text{M1}}$

$(t-5)(t-1) = 0$

$t = 1$, $t = 5$

$\boxed{\text{M1}}$ $s = 2\frac{1}{3}$, $s = -8\frac{1}{3}$ $\boxed{\text{M1}}$

Distance travelled = $\left(2\frac{1}{3} \times 2\right) + 8\frac{1}{3}$

= 13m $\boxed{\text{A1}}$

(c) Given that C is a point at which the particle has zero acceleration, determine, with working, whether C is nearer to O or to B. [3]

Max speed → $a = 0$

$a = \frac{\text{dv}}{\text{dt}}$

$2t - 6 = 0$ $\boxed{\text{M1}}$

$t = 3$

$s = 9 - 27 + 15 = -3$ $\boxed{\text{M1}}$

At $t = 3$, distance from B = $\frac{16}{3}$

Hence, C is nearer O. $\boxed{\text{A1}}$

Blank Page