

Name : \_\_\_\_\_

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# METHODIST GIRLS' SCHOOL

Founded in 1887

## PRELIMINARY EXAMINATION 2022 Secondary 4



Monday

**ADDITIONAL MATHEMATICS****4049/01**

15 August 2022

**Paper 1**

2 h 15 min

Candidates answer on the Question Paper.  
No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your class, index number and name in the spaces at the top of this page.

Write in dark blue or black pen

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

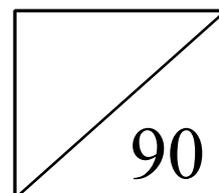
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.



*Mathematical Formulae***1. ALGEBRA*****Quadratic Equation***

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

***Binomial Expansion***

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$$

**2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for  $\Delta ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 A triangle with a base of  $(\sqrt{3} + 2)$  cm has an area of  $(12 + 7\sqrt{3})$  cm<sup>2</sup>. Find the perpendicular height of the triangle, leaving your answer in the form  $(a + b\sqrt{3})$  cm, where  $a$  and  $b$  are integers. [4]

- 2 (a) The equation of a curve is  $y = px^2 - 4x + 16p$ . Find the range of values of  $p$  given that the curve lies completely above the  $x$ -axis. [3]

- (b) Find the value of  $h$  for which the line  $y = 2x + h$  is a tangent to the curve  $y = 2x^2 - 6x + 5$ . [4]

- 3 Given that  $\tan A = \frac{5}{12}$  and  $\sin B = -\frac{3}{5}$ , and that  $A$  and  $B$  are in the same quadrant, **without using a calculator**, calculate the values of

(a)  $\operatorname{cosec}(-A)$ , [2]

(b)  $\sin 2B$ , [1]

(c)  $\tan(A+B)$ . [2]

4 (a) Solve the equation  $2^{2x+1} - 17(2^x) + 21 = 0$ .

[4]

- (b) Explain why the equation  $2^{2^{x+1}} - 17(2^x) + p = 0$  has no solution if  $p > 36\frac{1}{8}$ . [2]

5 The mass,  $m$  grams, of a radioactive substance remaining,  $t$  days after being measured is given by  $m = 10e^{-0.01t} + 0.2$ .

(a) Find the initial mass.

[1]

(b) Sketch the graph  $m = 10e^{-0.01t} + 0.2$  for  $t \geq 0$ .

[2]

- (c) Find the least number of days it takes before the amount of substance is reduced to 5% of its initial mass. [3]

- (d) Explain why the mass of the radioactive substance can never be less than 0.2 g. [2]

6 The polynomial  $2x^3 - 3ax^2 - 2ax + b$  has a factor  $2x - 1$  and leaves a remainder of  $-8$  when divided by  $x - 1$ .

(a) Find the values of  $a$  and of  $b$ .

[4]

- (b) Using the values of  $a$  and  $b$  in part (a), factorise the polynomial completely. [3]

7 A circle,  $C_1$  has a diameter  $AB$  where  $A$  is the point  $(6, -2)$  and  $B$  is the point  $(12, 6)$ .

(a) Find the equation of  $C_1$ . [2]

(b) Show that the equation of the tangent to the circle at  $A$  is  $4y + 3x = 10$ . [2]

- (c) Another circle,  $C_2$ , has its centre at  $A$ . Given that the area of  $C_2$  is  $\frac{1}{9}$  that of  $C_1$ , find the equation of  $C_2$ .

[2]

8 The function  $y = 1 - 0.5 \cos(bx)$  is defined for  $0 \leq x \leq \pi$  and  $b$  is a positive integer.

(a) State the amplitude. [1]

(b) It is given that the period of  $y$  is  $\frac{2\pi}{3}$ . Find the value of  $b$ . [1]

(c) Using your answer in part (b), sketch the graph of  $y = 1 - 0.5 \cos(bx)$ , for  $0 \leq x \leq \pi$ . [3]

- (d) Hence, explain how the solutions of the equation  $\pi \cos(bx) + 3x = 0$  can be obtained from the graph sketched in part (c). [3]

9 Solve  $3 \cos 2x = 2 \sin^2 2x$  for  $0^\circ \leq x \leq 360^\circ$ .

[5]

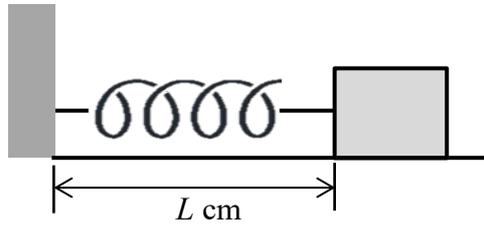
10 (a) Given that  $y = \frac{x}{3x-4}$ , find  $\frac{dy}{dx}$ .

[2]

(b) Hence, show that  $y$  is a decreasing function for all real values of  $x$ .

[2]

- 11 In the diagram below, the length,  $L$  cm, of a spring at time,  $t$  seconds is given by the equation  $L = 20 + 3 \sin\left(4t - \frac{\pi}{2}\right)$ .



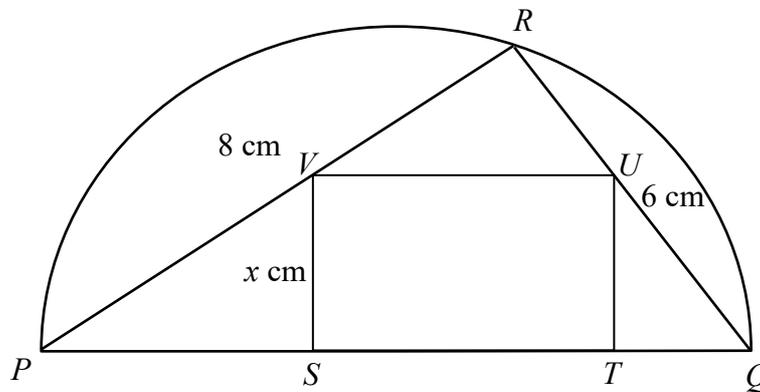
- (a) Show that the shortest length of the spring is 17 cm.

[2]

(b) Find the time when the spring first reaches 22 cm.

[3]

- 12 The diagram below shows a semicircle with diameter  $PQ$ , with the point  $R$  on the circumference of the semicircle. A rectangle  $STUV$  is drawn within the triangle  $PQR$  where  $ST$  lies on  $PQ$ .  $U$  and  $V$  are points on  $QR$  and  $PR$  respectively. It is given that  $PR = 8$  cm and  $QR = 6$  cm.



- (a) Find the perpendicular distance from  $R$  to  $PQ$ .

[2]

- (b) It is given that  $SV$  is  $x$  cm. Show that the area of rectangle  $STUV$ ,  $A$  m<sup>2</sup> is given by

$$A = 10x - \frac{25}{12}x^2.$$

[3]

- (c) Calculate the value of  $x$  for which  $A$  has a stationary value.

[2]

**13** A function  $f(x)$  is defined for all real values of  $x$  such that  $f''(x) = 4e^{-2x}$ . It is given that the gradient of the curve  $y = f(x)$  is 3 when  $x = 0$  and the curve passes through the point  $(2, e^{-4})$ .

**(a)** Find an expression for  $f(x)$ .

[5]

- (b) Find the coordinates of the stationary point of the curve and determine the nature of this stationary point.

[4]

**14** A particle moves in a straight line such that,  $t$  seconds after leaving a fixed point  $O$ , its velocity,  $v \text{ ms}^{-1}$ , is given by  $v = t^2 - 6t + 5$ . The particle comes to instantaneous rest, firstly at  $A$  and then at  $B$ .

**(a)** Find an expression, in terms of  $t$ , for the distance of the particle from  $O$  at time  $t$ . [2]

**(b)** Find the total distance travelled by the particle in the first 5 seconds after passing  $O$ . [4]

- (c) Given that  $C$  is a point at which the particle has zero acceleration, determine, with working, whether  $C$  is nearer to  $O$  or to  $B$ .

[3]

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