Class	Index Number

Name : ______________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2022
Secondary 4

Thursday **ADDITIONAL MATHEMATICS** **4049/02**

18 August 2022 **PAPER 2** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $y = 4x\sin 2x$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4(2x\cos 2x + \sin 2x)$ [2]

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4x(2\cos 2x) + 4\sin 2x$ - M1

$= 8x\cos 2x + 4\sin 2x$

$= 4(2x\cos 2x + \sin 2x)$ - A1

(ii) Given that x is increasing at $\dfrac{2}{5}$ radians per second, find the rate of change of y when $x = \dfrac{\pi}{2}$. [2]

$\dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{\mathrm{d}y}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t}$

$= (8\left(\dfrac{\pi}{2}\right)\cos \pi + 4\sin \pi) \times \dfrac{2}{5}$ - M1

$= -\dfrac{8\pi}{5}$ radians/sec or -5.03 rad/s - A1

2 **(i)** Show that $\dfrac{d}{dx}\left(\dfrac{\ln x^2}{x}\right)=\dfrac{2-2\ln x}{x^2}$. [2]

$$\frac{d}{dx}\left(\frac{\ln x^2}{x}\right) = \frac{(x)\dfrac{2x}{x^2}-\ln x^2}{(x)^2} \qquad \text{- M1}$$

$$= \frac{2-\ln x^2}{x^2} = \frac{2-2\ln x}{x^2} \qquad \text{- A1}$$

(ii) Hence evaluate $\displaystyle\int_1^e \left(\frac{2\ln x}{x^2}\right) dx$. [4]

$$\int_1^e \left(\frac{2}{x^2}-\frac{2\ln x}{x^2}\right) dx = \left[\frac{\ln x^2}{x}\right]_1^e \qquad \text{- M1}$$

$$\int_1^e 2x^{-2}\ dx - \left[\frac{\ln x^2}{x}\right]_1^e = \int_1^e \frac{2\ln x}{x^2}\ dx$$

$$\left[\frac{2x^{-1}}{(-1)} - \frac{\ln x^2}{x}\right]_1^e = \int_1^e \frac{2\ln x}{x^2}\ dx \qquad \text{- M1}$$

$$\left[-\frac{2}{x} - \frac{2\ln x}{x}\right]_1^e = \int_1^e \frac{2\ln x}{x^2}\ dx$$

$$\int_1^e \left(\frac{2\ln x}{x^2}\right) dx = \left[\ \left(\frac{-2}{e} - \frac{2}{e}\right) - \left(\frac{-2}{1} - 0\right)\ \right] \qquad \text{- M1}$$

$$\therefore \int_1^e \left(\frac{2\ln x}{x^2}\right) dx = 2 - \frac{4}{e} \text{ (or 0.528)} \qquad \text{- A1}$$

3 **(a)** Find the coefficient of x^3 in the expansion of $(2-3x)^6$. [2]

x^3 term = $\binom{6}{3}(2)^3(-3x)^3$ - M1

$= 20(8)(-27x^3)$

$= -4320x^3$

Answer : -4320 - A1

(b) **(i)** Write down the first three terms, in ascending powers of x, in the binomial expansion of $(1+ax)^8$, where a is a constant. [2]

$(1+ax)^8 = \binom{8}{0}(1)^8(ax)^0 + \binom{8}{1}(1)^7(ax)^1 + \binom{8}{2}(1)^6(ax)^2 + \dots$ - M1

$(1+ax)^8 = 1 + 8ax + 28a^2x^2 + \dots$ - A1

(ii) Given that the expansion of $(1-3x)^2(1+ax)^8$ in ascending powers of x is $1+10x+bx^2+\dots$, calculate the value of a, and of b. [3]

$(1-3x)^2(1+ax)^8 = (1-6x+9x^2)(1+8ax+28a^2x^2+\dots)$ - M1

coefficient of x = $8a-6$

$10 = 8a - 6$

$16 = 8a$

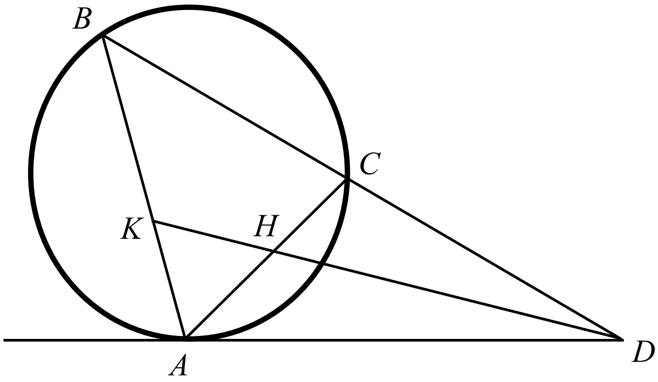
$2 = a$ - A1

coefficient of x^2 = $9-48a+28a^2$

$b = 9 - 48(2) + 28(4)$

$b = 25$ - A1

4 In the diagram, AD is a tangent to the circle at A. BCD is a straight line that intersects the tangent at D. KD bisects $\angle ADC$ and it cuts AC at H and meets AB at K.



Prove that

(i) ΔAHD is similar to ΔBKD. [3]

let $\angle CAD = x$,

$\angle CAD = \angle ABC = x$ (Tangent-Chord Theorem)

$\angle ADH = \angle CDH = y$ (KD bisects $\angle ADC$) - M2

$\therefore \Delta AHD$ and ΔBKD are similar - A1

(ii) $HD \times BD = KD \times AD$, [2]

$\frac{HD}{KD} = \frac{AD}{BD}$ (ΔAHD and ΔBKD are similar) - M1

$HD \times BD = KD \times AD$ (proven) - A1

(iii) ΔAHK is isosceles. [3]

$\angle AHD = 180° - x - y$ (angle sum of triangle)
$\angle AHK = x + y$ (supp angle) - M1

$\angle BKD = 180° - x - y$ (angle sum of triangle) - M1
$\angle AHK = x + y$ (supp angle)

since $\angle AHK = \angle AKH$, ΔAHK is isosceles. - A1

5 **(a)** Show that $\operatorname{cosec} x \sec x \equiv \tan x + \cot x$. [4]

$\tan x + \cot x$

$= \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$ - M1

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ - M1

$= \dfrac{1}{\sin x \cos x}$ - M1

$= \dfrac{1}{\sin x} \times \dfrac{1}{\cos x}$

$= \operatorname{cosec} x \sec x$ - A1

(b) Hence solve the equation $\operatorname{cosec} x \sec x = 2$ for $0 < x < 3\pi$. Leave your answers in terms of π. [3]

$\operatorname{cosec} x \sec x = 2$

$\tan x + \cot x = 2$ - M1

$\tan x + \dfrac{1}{\tan x} = 2$

$\tan^2 x - 2\tan x + 1 = 0$

$(\tan x - 1)^2 = 0$ - M1

$\tan x = 1$

ref. angle, $\alpha = \dfrac{\pi}{4}$

$x = \alpha,\ \pi + \alpha,\ 2\pi + \alpha$

$x = \dfrac{\pi}{4},\ \dfrac{5\pi}{4},\ \dfrac{9\pi}{4}$ - A1

6 Express $\dfrac{7x+4}{(x-2)(x^2+5)}$ in partial fractions. [5]

$$\frac{7x+4}{(x-2)(x^2+5)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+5}$$ - M1

$$\frac{7x+4}{(x-2)(x^2+5)}=\frac{A(x^2+5)}{x-2}+\frac{(Bx+C)(x-2)}{x^2+5}$$

$$7x+4=A(x^2+5)+(Bx+C)(x-2)$$

when $x=2$, $18=9A$
$A=2$ - M1

when $x=0$, $4=10+C(-2)$
$C=3$ - M1

when $x=1$, $11=12+(B+3)(-1)$
$B+3=1$
$B=-2$ - M1

$$\frac{7x+4}{(x-2)(x^2+5)}=\frac{2}{x-2}+\frac{3-2x}{x^2+5}$$ - A1

7 **(a)** Express $2x^2 - 6x + 7$ in the form $a(x+b)^2 + c$, where a, b and c are constants. Without solving, explain why the curve $y = 2x^2 - 6x + 7$ will not intersct the line $y = 2$. [4]

$2x^2 - 6x + 7$

$= 2(x^2 - 3x) + 7$

$= 2\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$ - M1

$= 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + 7$

$= 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$ - M1

Since the coefficient of x^2 is positive, the graph has a minimum point. Given that the minimum value is 2.5, the entire graph lies above $y = 2$. Hence the graph will not intersect the line. - A2

(b) By using a suitable substitution, solve the equation
$\frac{1}{2}\log_3 x = 3 + 2\log_x 81$. [4]

$\frac{1}{2}\log_3 x = 3 + 2\log_x 81$

$\frac{1}{2}\log_3 x = 3 + 2\left(\frac{\log_3 81}{\log_3 x}\right)$ - M1

$\frac{1}{2}\log_3 x = 3 + 2\left(\frac{4}{\log_3 x}\right)$

let $p = \log_3 x$,

$\frac{p}{2} = 3 + \frac{8}{p}$

$p^2 = 6p + 16$

$p^2 - 6p - 16 = 0$ - M1

$(p-8)(p+2) = 0$

$p = 8$ or $p = -2$

$\log_3 x = 8 \qquad \log_3 x = -2$

$x = 3^8 = 6561 \qquad x = 3^{-2} = \frac{1}{9}$ - A2

(c) Solve the simultaneous equations [4]

$$4^x \div 8^y = 32$$
$$\log_2 x - \log_2(y+1) = 1.$$

$4^x \div 8^y = 32$

$2^{2x} \div 2^{3y} = 2^5$

$2x - 3y = 5$ - (1) - M1

$\log_2 x - \log_2(y+1) = 1$

$\log_2\left[\dfrac{x}{y+1}\right] = 1$

$\dfrac{x}{y+1} = 2$

$x = 2y + 2$ - (2) - M1

(2) $\rightarrow$ (1),

$2(2y+2) - 3y = 5$ - M1

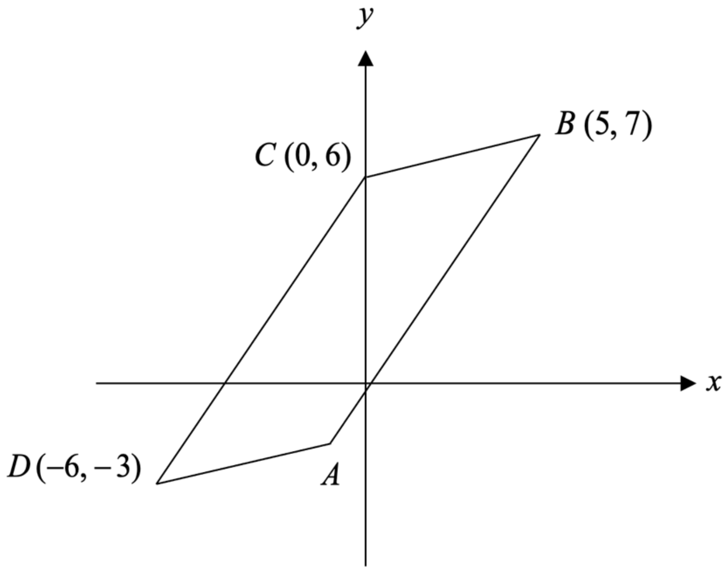
$4y + 4 - 3y = 5$

$y = 1$

$x = 4$ - A1

8 **Solutions to this question by accurate drawing will not be accepted.**

ABCD is a parallelogram where B is $(5, 7)$, C is $(0, 6)$ and D is $(-6, -3)$. The perpendicular from *C* to *AB* meets *DA* extended at a point *E*.



(a) Show that the coordinates of *A* is (−1, −2). [1]

$A(-6+5, -3+1)$

$= A(-1, -2)$ - A1

(b) (i) Find the equations of the line *CE* and line *DA* respectively. [3]

$m_{AB} = \dfrac{7+2}{5+1} = \dfrac{3}{2}$

$m_{CE} = -\dfrac{2}{3}$ - M1

eqn *CE*,

$y - 6 = -\dfrac{2}{3}(x - 0)$

$y = -\dfrac{2}{3}x + 6$ - A1

$m_{DA} = \dfrac{-3+2}{-6+1} = \dfrac{1}{5}$

eqn *DA*,

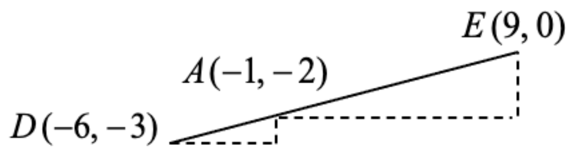
$y + 2 = \dfrac{1}{5}(x + 1)$

$5y = x - 9$ - A1

(ii) **Hence,** find the coordinates of E, and deduce the ratio $DA : AE$. [3]

$y = -\frac{2}{3}x + 6$ --(1)

$5y = x - 9$ -- (2)



(1) $\rightarrow$ (2), $5\left(-\frac{2}{3}x + 6\right) = x - 9$

$-10x + 90 = 3x - 27$ - M1

$117 = 13x$

$x = 9$

From (2), $y = 0$

$E\ (9, 0)$ - M1

$DA : AE = 5 : 10$

$DA : AE = 1 : 2$ - A1

(c) Name the special quadrilateral *BCDE* and explain clearly your reasons for it. [2]

BCDE is a trapezium. *BC* is parallel to *DE* and $BC \neq DE$. - A2

(d) Find the area of quadrilateral *BCDE*. [2]

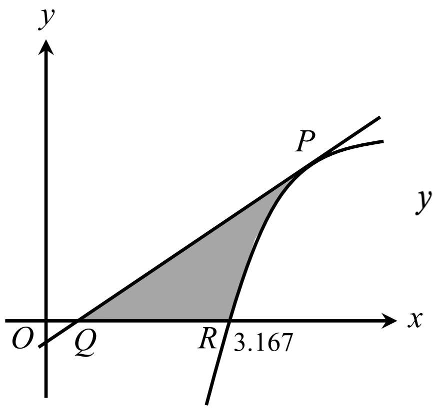
$$A = \frac{1}{2}\begin{vmatrix} 0 & -6 & 9 & 5 & 0 \\ 6 & -3 & 0 & 7 & 6 \end{vmatrix}$$ - M1

$$= \frac{1}{2}\left[(63+30) - (-36-27)\right]$$

$$= \frac{1}{2}[156]$$

$$= 78 \text{ unit}^2$$ - A1

9



The diagram shows part of the curve $y = 3 - \dfrac{4}{2x-5}$, cutting the x-axis at $R(3.167, 0)$. The tangent to the curve at P cuts the x-axis at Q. Given that the normal to the curve at P is parallel to the line $y = -2x+6$, find

(i) the equation of the tangent to the curve at P, [6]

$y = 3 - \dfrac{4}{2x-5}$ -- (1) $\qquad$ $y = -2x+6$

$\dfrac{dy}{dx} = -4(-1)(2x-5)^{-2}(2)$ $\qquad$ $\dfrac{dy}{dx} = \dfrac{1}{2}$ - M1

$= \dfrac{8}{(2x-5)^2}$ - M1

$\dfrac{8}{(2x-5)^2} = \dfrac{1}{2}$

$16 = (2x-5)^2$

$4 = 2x-5 \quad (x > 0)$

$x = 4\dfrac{1}{2}$ - M1

From (1), $y = 2$

$P\left(4\dfrac{1}{2},\ 2\right)$ - M1

equation of tangent at P,

$y - 2 = \dfrac{1}{2}\left(x - 4\dfrac{1}{2}\right)$ - M1

$y = \dfrac{1}{2}x - \dfrac{1}{4}$ or $4y = 2x - 1$ - A1

(ii) the area of the shaded region *RPQ*. [5]

$Q(0.5,0)$ - M1

Area of shaded region $= \frac{1}{2}(4)(2) - \int_{3.167}^{4.5}\left(3 - \frac{4}{2x-5}\right)\mathrm{d}x$ - M2

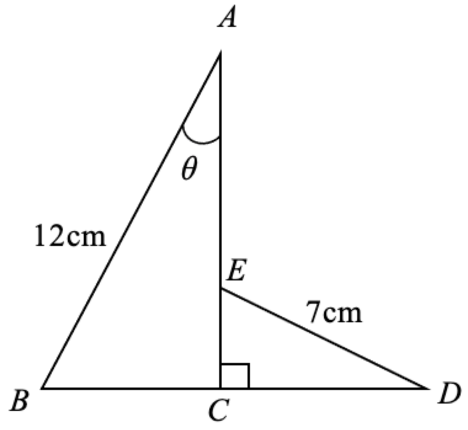
$= 4 - \int_{3.167}^{4.5}\left(3 - 2\times\frac{2}{2x-5}\right)\mathrm{d}x$

$= 4 - \left[3x - 2\ln(2x-5)\right]_{3.167}^{4.5}$ - M1

$= 4 - \left[(13.5 - 2\ln 4) - (9.501 - 2\ln 1.334)\right]$

$= 2.19722... \approx 2.20 \text{ unit}^2$ - A1

10 The diagram shows a quadrilateral $ABCDE$ in which $\angle DCE = 90^0$, $AB = 12$ cm, $DE = 7$ cm and $\angle BAC = \theta$ where $0° < \theta < 90°$. ΔABC is similar to ΔDEC.



(i) Show that the perimeter, P cm, of the quadrilateral is given by $P = 19 + 19\cos\theta + 5\sin\theta$. [3]

$AE = 12\cos\theta - 7\sin\theta$ $\qquad CD = 7\cos\theta$ - M1
$BC = 12\sin\theta$

$P = 12 + 12\sin\theta + 7\cos\theta + 7 + 12\cos\theta - 7\sin\theta$ - M1
$P = 19 + 19\cos\theta + 5\sin\theta$ - A1

(ii) Express P in the form $k + R\cos(\theta - \alpha)$, where k is a constant. [3]

$R = \sqrt{19^2 + 5^2}$ $\qquad \tan\alpha = \dfrac{5}{19}$

$= \sqrt{386}$ - M1 $\qquad \alpha = 14.74356...$ - M1

$P = 19 + \sqrt{386}\cos(\theta - 14.7°)$ - A1

(iii) Find the maximum value of the perimeter, P, and the corresponding value of θ. [2]

max P = 38.6 cm when $\theta - 14.743° = 0°$ - A1

$\theta = 14.7°$ - A1

(iv) Find the value of θ for which $P = 32$. [2]

$$19 + \sqrt{386}\cos(\theta - 14.74356°) = 32$$

$$\cos(\theta - 14.74356°) = \frac{13}{\sqrt{386}}$$ - M1

$$\theta - 14.74356 = 48.5716...$$

$$\theta = 63.3°$$ - A1

11 An experiment was conducted to estimate the population of insects, y, present in a colony x weeks after the start of the experiment.

It is known that the variables x and y can be modelled by the equation $y = kp^{\frac{x}{3}}$, where k and p are constants. The table below shows the values of x and y.

x	2	4	6	8	10
y	1027	1341	1763	2342	3103

(i) Plot $\ln y$ against x on the grid provided and draw a straight line graph. The vertical $\ln y$-axis should start from 6.2 and have a scale of 2cm to 0.2. [3]

x	2	4	6	8	10
$\ln y$	6.934	7.201	7.475	7.759	8.040

$y = kp^{\frac{x}{3}}$ table - M1

$\ln y = \ln\left(kp^{\frac{x}{3}}\right)$ graph - M1

$\ln y = \ln k + \ln p^{\frac{x}{3}}$

$\ln y = \ln k + \frac{x}{3}\ln p$

$\ln y = \left(\frac{\ln p}{3}\right)x + \ln k$ - A1

(ii) Use your graph to estimate the value of k and of p. [3]

$\text{gradient} = \frac{7.9 - 7.06}{9 - 3}$ - M1

$\frac{\ln p}{3} = 0.14$

$p = e^{0.42} = 1.52$ - A1 [1.50 – 1.54]

$\ln k = 6.64$

$k = 765$ - A1 [750 – 781]

(iii) Use your graph to estimate the number of weeks needed for the population of the insect to be doubled from the start of the experiment. [2]

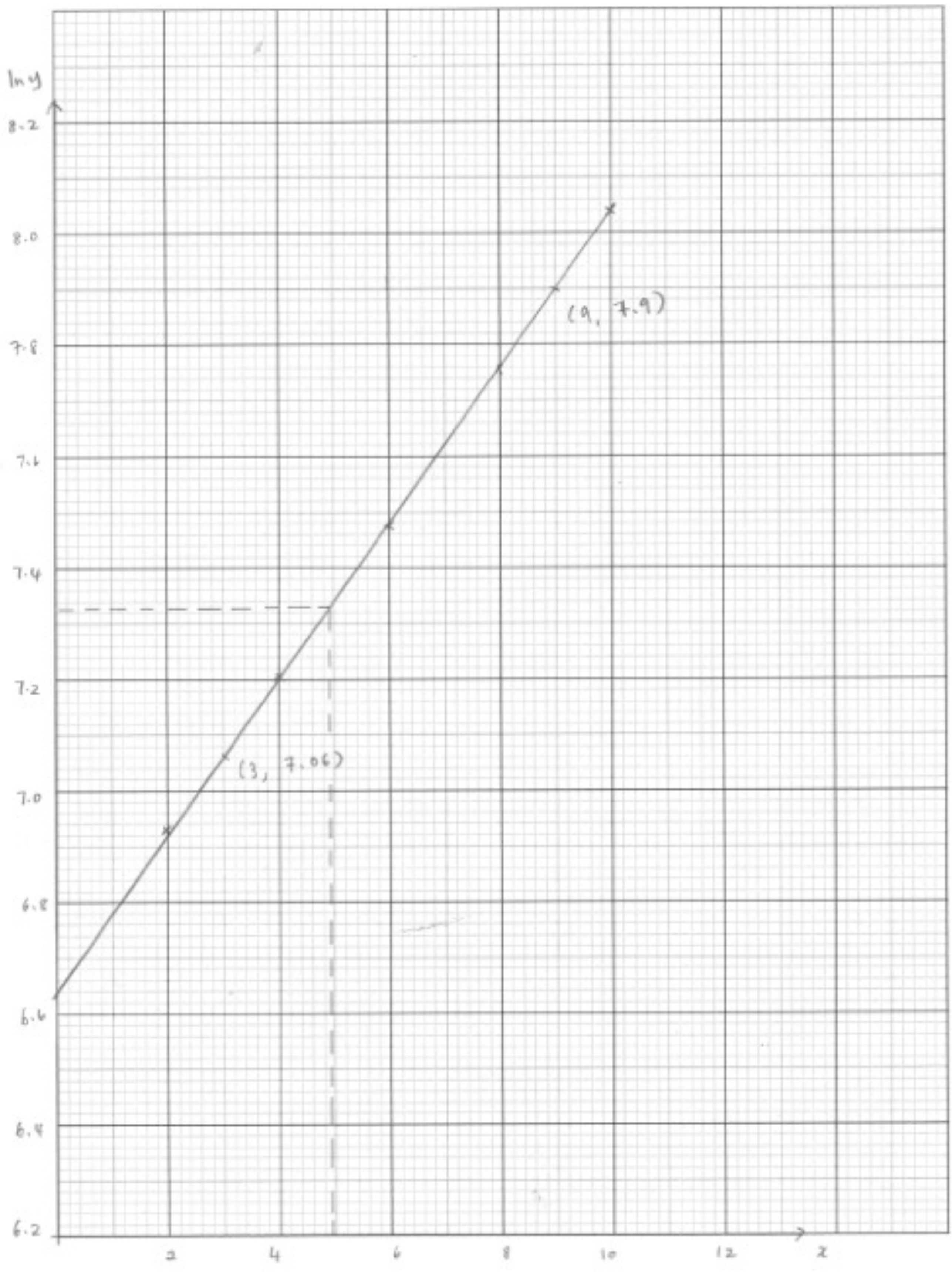
at $x = 0$, population is 765
doubled population = 1530

at ln (1530) = 7.333 - M1 (graph)
$x = 4.95$

no. of weeks = 4.95 – 0 = 4.95 - A1 [4.90 to 5.05]

(iv) Explain why this model cannot be used five years from the start of the experiment. [1]

The model is based on exponential increase and the number of insects cannot increase indefinitely. In 5 years, it will have 4.40×10^{18} insects. - A1



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