O	JURONG SECONDARY SCHOOL 2022 GRADUATION EXAMINATION 2 SECONDARY 4 EXPRESS/ SECONDARY 5 NORMAL ACADEMIC

CANDIDATE NAME	

CLASS		INDEX NUMBER	

MATHEMATICS **4048/02**

PAPER 2 **24 August 2022**
2 hour 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of **26** printed pages including this page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** It is given that $y = \frac{5x-2}{3x+1}$.

(i) Find y when $x = 7$.

Answer $y =$ …….. [1]

(ii) Express x in terms of y.

Answer $x =$ …….. [2]

(b) Solve $\frac{x-5}{3} - \frac{x+3}{2} = -4$.

Answer $x =$ …….. [2]

(c) Solve these simultaneous equations.

$$3x - 2y = 10$$
$$2x + y = 23$$

Answer $x =$ ……..

$y =$ ……..

[3]

(d) Simplify $\dfrac{4x^2 - 9}{6x^2 - x - 12}$.

Answer ……..

[3]

2 Mrs Teo is a Mathematics tutor.
She offers tutorial sessions for Lower Secondary and Upper Secondary students on weekdays and on weekends.
Each student attends one session a week for 4 sessions in every month.

The matrix **N** shows the number of students she tutors every month.

$$\mathbf{N} = \begin{array}{c} \begin{array}{cc} \text{Lower Secondary} & \text{Upper Secondary} \end{array} \\ \begin{pmatrix} 7 & 6 \\ 4 & 3 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{Weekday} \\ \text{Weekend} \end{array}$$

(a) Evaluate the matrix $\mathbf{M} = 4\mathbf{N}$.

Answer …….. [1]

(b) Mrs Teo charges \$70 for each Lower Secondary session and \$80 for each Upper Secondary session.

Represent the session charges in a 2×1 column matrix **C**.

Answer $\mathbf{C} =$ [1]

(c) Evaluate the matrix $\mathbf{P} = \mathbf{M}\mathrm{C}$.

Answer $\mathbf{P} =$ [1]

(d) State what the elements of **P** represent.

Answer ……..……..……..……

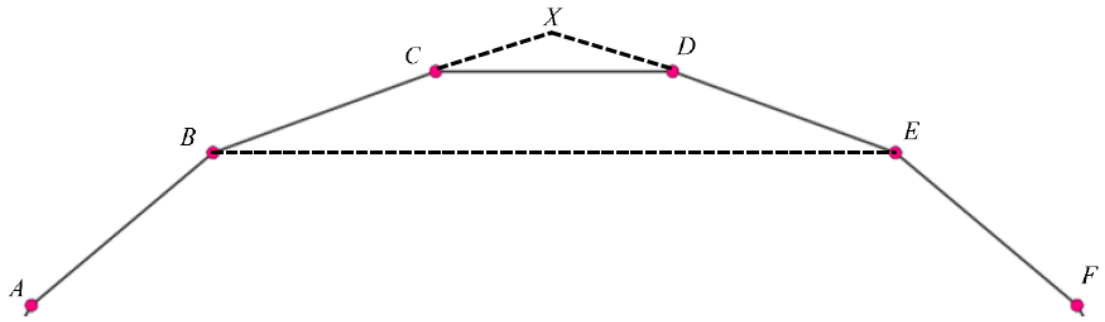
……..……..……..…… [1]

Mrs Teo wants to increase her tuition fees for the weekend sessions by 10% for the last 3 months before examinations start.
The number of students registered for the weekday sessions are 10 Lower Secondary students and 8 Upper Secondary students. On weekends she has 3 Lower Secondary students and 2 Upper Secondary students.

(e) Calculate the total amount of money she earns during this 3 month period.

Answer …….. [4]

3



The diagram below shows part of a regular 18-sided polygon, $ABCDEF\ldots\ldots$
The lines BC and ED are extended to meet at the point X such that $XC = XD$.

(a) Find $\angle XCD$.

Answer …….. [1]

(b) Find $\angle CXD$.

Answer …….. [1]

(c) Explain why $BX = EX$.

Answer

[2]

(d) Show that ΔXCD and ΔXBE are similar.

Answer

[2]

(e) What can we conclude about the side *CD* and the line *BE*?

Answer ……..……..……..…… [1]

4 The first four terms in a sequence of numbers are given below.

$T_1 = 5^2 - 18 = 7$

$T_2 = 6^2 - 22 = 14$

$T_3 = 7^2 - 26 = 23$

$T_4 = 8^2 - 30 = 34$

(a) Find T_5.

Answer …….. [1]

(b) Show that the nth term of the sequence, T_n, is given by $n^2 + 4n + 2$.

Answer

[2]

(c) Determine and explain if 962 is a term of the sequence.

Answer

[2]

(d) Find and simplify an expression, in terms of n, for $T_n - T_{n-1}$.

Answer …….. [3]

(e) Explain why the difference between consecutive terms in the sequence is always odd.

Answer ……..……..……..……

……..……..……..……

……..……..……..…… [1]

5 The variables x and y are connected by the equation

$$y = x + \frac{5}{x^2} - 3$$

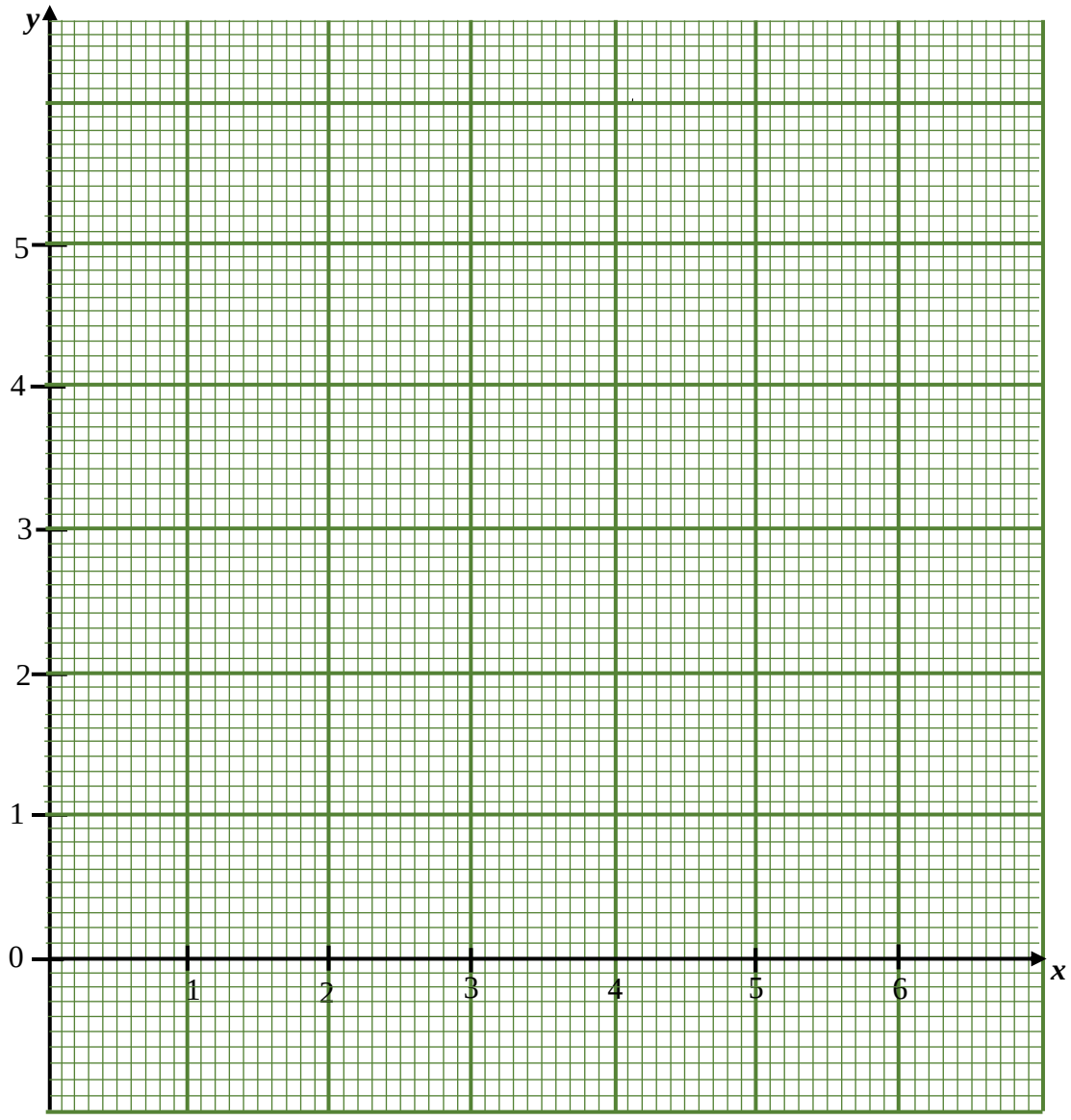
Some corresponding values of x and y are given in the table below.

x	1	1.5	2	2.5	3	4	5	6
y	3	0.7	0.3	0.3	0.6	1.3	p	3.1

(a) Find the value of p.

Answer p = …….. [1]

(b) On the grid, draw the graph of $y = x + \frac{5}{x^2} - 3$ for $1 \leq x \leq 6$. [3]



(c) The equation $x+\frac{5}{x^2}-3=0$ has no solution.

Explain how this can be seen from your graph.

Answer ……..……..……..……

……..……..……..…… [1]

(d) By drawing a tangent, find the gradient of the curve when $x=1.5$.

Answer …….. [2]

(e) **(i)** On the same axes, draw the line with gradient 2 that passes through the point with coordinates $(0.5, 2)$. [2]

(ii) Write down the equation of this line.

Answer …….. [1]

(iii) Show, with mathematical justification, that the equation $5 - 4x^2 - x^3 = 0$ can be solved by finding the points of intersection of the straight line and the curve.

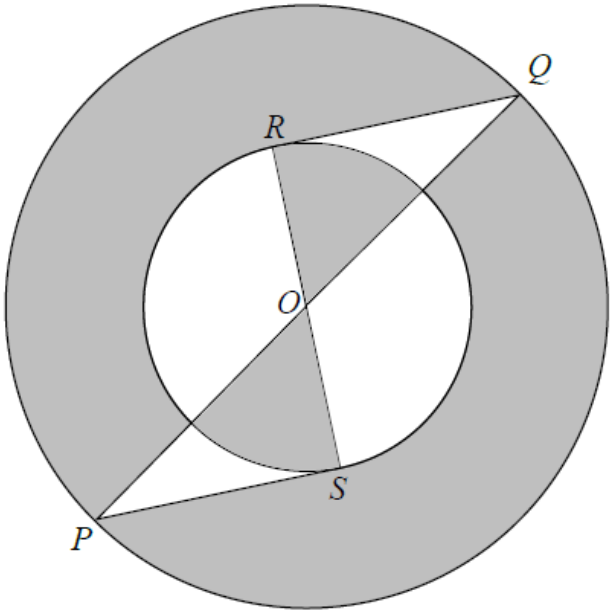
Answer

[2]

(iv) Use your graphs to solve the equation $5 - 4x^2 - x^3 = 0$.

Answer …….. [1]

6



The diagram shows two concentric circles with centre *O*.
PQ is the diameter of the larger circle and *RS* is the diameter of the smaller circle.
PS and *RQ* are tangents to the smaller circle.

(a) Show that the triangle *PSO* is congruent to triangle *QRO*.
Give a reason for each statement you make.

Answer

[2]

(b) The radius of the larger circle is 8 cm and the radius of the smaller circle is 4.36 cm.

(i) Calculate the area of triangle *QRO*.

Answer …….. cm^2 [3]

(ii) Given that angle *QOR* is 0.995 radians, calculate the shaded area.

Answer …….. cm^2 [4]

7 The seating capacity of an auditorium is 300. The seats are categorised as 1, 2, 3 and 4. The costs of the tickets in the categories are in the table below.

CATEGORY	TICKET PRICES
CAT 1	S$ 348
CAT 2	S$ 288
CAT 3	S$ 228
CAT 4	S$ 148

(a) The number of seats are distributed in the ratio $1:2:4:5$ for CAT 1 to 4 respectively. If all the seats are sold out, calculate the ticket sales for category 4.

Answer $ …….. [2]

The auditorium has 2 entrances, front entrance *F* and back entrance *B*.
Entrance *F* is able to evacuate *x* people out of the auditorium per second.

(b) (i) Write down an expression, in terms of *x*, for the time taken in seconds for all 300 people to evacuate if only entrance *F* is open.

Answer ……. s [1]

(ii) Entrance *F* is able to let approximately 2 more people evacuate per second compared to entrance *B*.

Write down an expression, in terms of *x*, for the time taken in seconds for all 300 people to evacuate the auditorium if only entrance *B* is open.

Answer …….. s [1]

(iii) Usually, only one entrance is open at any time.

It takes approximately 3 seconds more for all the people to evacuate the auditorium using entrance *B* as compared to entrance *F*.

Form an equation in x and show that it reduces to $x^2 - 2x - 200 = 0$.

Answer

[3]

(iv) Solve the equation $x^2 - 2x - 200 = 0$.

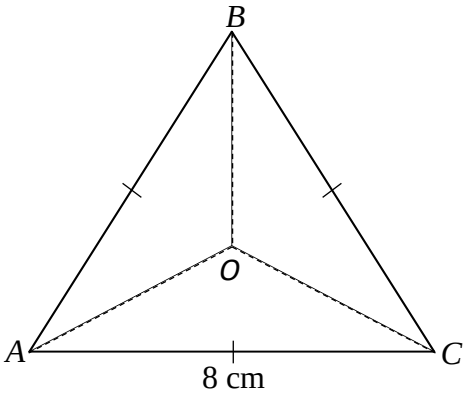
Answer $x =$ …… or …….. [2]

(v) Find the number of people that can be evacuated in **1 minute** if only entrance *F* is opened.

Answer ……. people [2]

8 An equilateral triangle *ABC*, has sides of 8 cm.

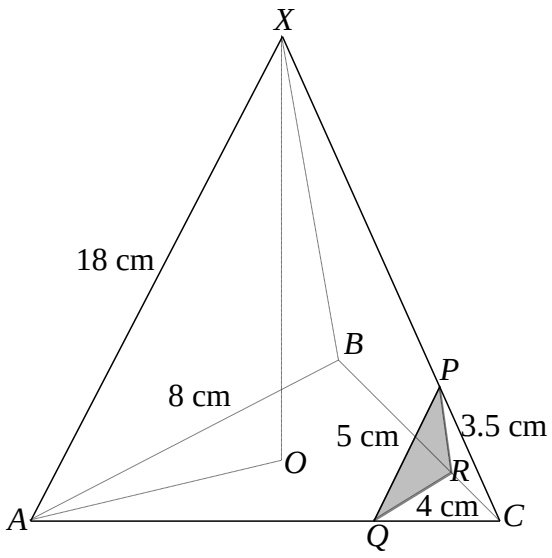
O is the centre of the equilateral triangle.



(a) Show that the length of *OA* is 4.619 cm, correct to 3 decimal places.
Answer

[2]

The triangle *ABC* forms the base of a pyramid.
The vertex, *X* is vertically above *O* and *AX* = 18 cm.



(b) Calculate the **total** surface area of the pyramid.

Answer …….. cm^2 [3]

(c) Part of the pyramid is cut off at triangle *PQR* such that *P* is a point on *XC*, *Q* is a point on *AC* and *R* is a point at *BC*.
PQ = 5 cm, *QR* = 4 cm and *PR* = 3.5 cm.
Find angle *PQR*.

Answer ……°. [2]

(d) The ratio of $AQ : QC = 3 : 1$.
Calculate the angle of elevation of X from Q.

Answer ……..° [4]

9 (a) A group of 50 patients had their blood pressure taken.

The results are shown in the table below.

Blood Pressure (mmHg)	$100 < x \leq 120$	$120 < x \leq 140$	$140 < x \leq 160$	$160 < x \leq 180$
Frequency	27	12	6	5

(i) State the median class of blood pressure.

Answer …….. [1]

(ii) A blood pressure of 140 mmHg or higher indicates high blood pressure.
Find the percentage of patients who may have high blood pressure.

Answer ……. % [1]

(iii) Calculate the standard deviation of the blood pressures.

Answer ……. mmHg [2]

(iv) The standard deviation of the blood pressures taken by a second group of patients was 22.1 mmHg.

Comment on one difference between the two distributions, making reference to this information.

Answer ……..……..……..……...

……..……..……..……

……..……..……..…… [1]

(b) A drawer contains 2 blue socks and 6 white socks.
Two socks are taken from the drawer at random without replacement.
If the two socks are different colours, then a third sock is taken from the drawer.
Otherwise, no third sock is taken.

(i) Draw a tree diagram to show the probabilities of the possible outcomes.

Answer

[2]

(ii) Find, as a fraction in its simplest form, the probability that

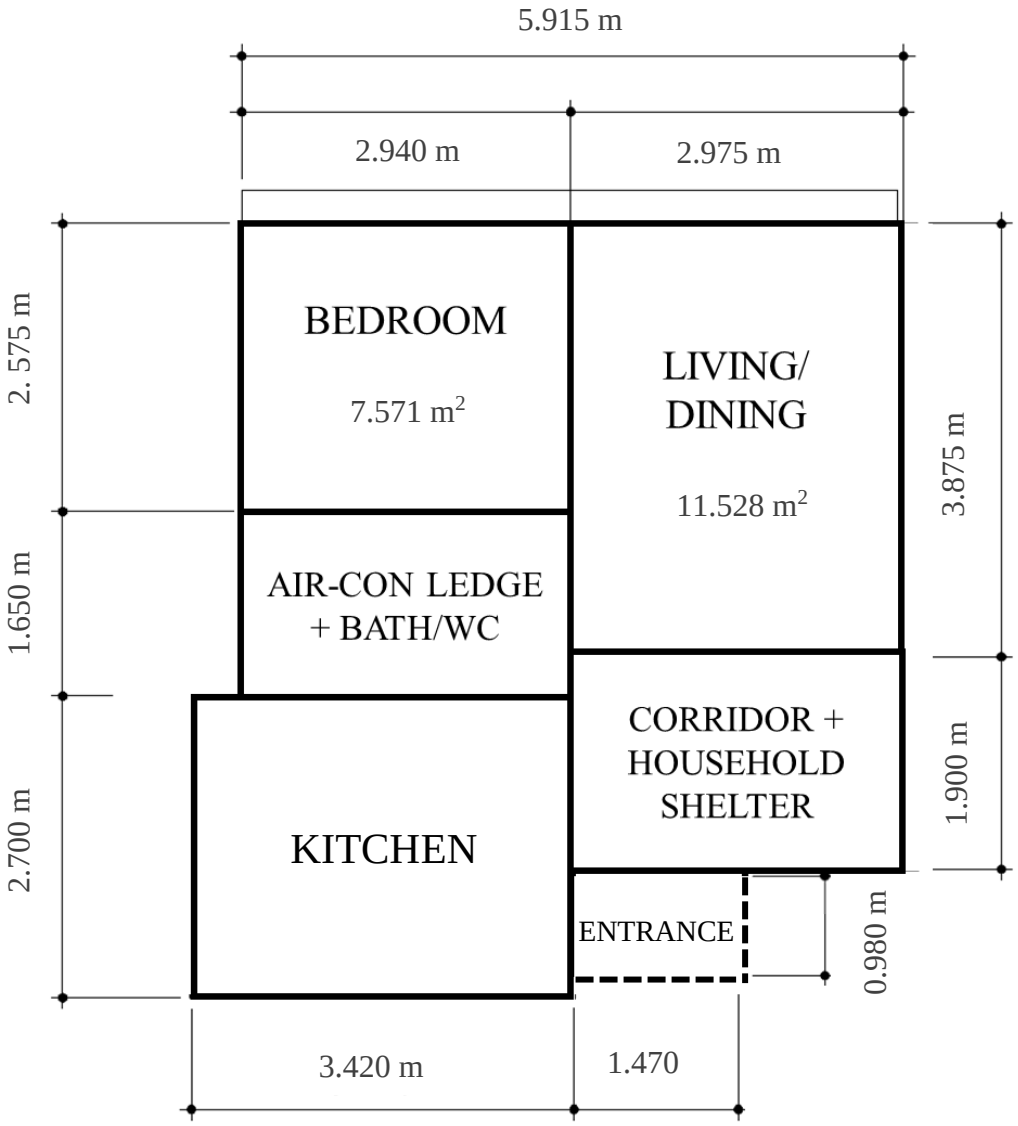
(a) the first two socks taken are the same colour,

Answer …….. [2]

(b) a third sock is taken and it is the same colour as the first sock.

Answer …….. [2]

10 Mr Graham bought a 2-room flat in Sembawang.
The floor plan can be modelled as five rectangles as shown below.

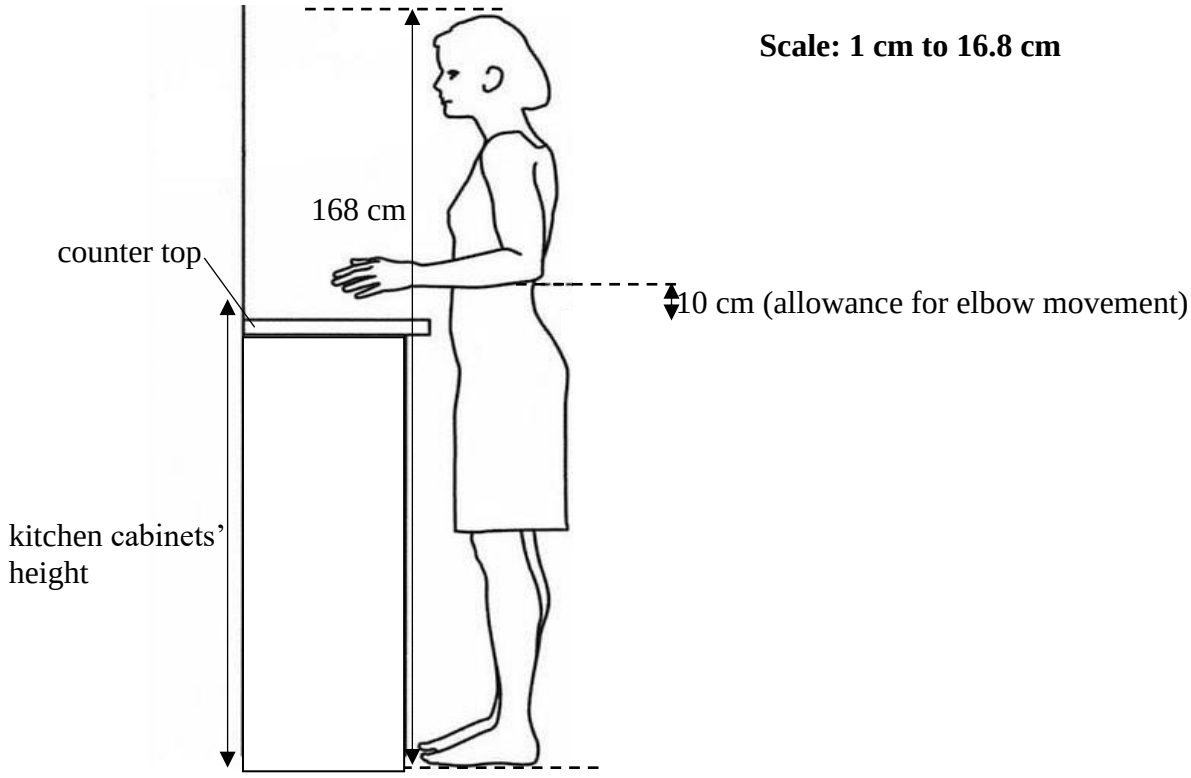


(a) Calculate the total floor area of Mr Graham's kitchen.

Answer ..... m^2 [1]

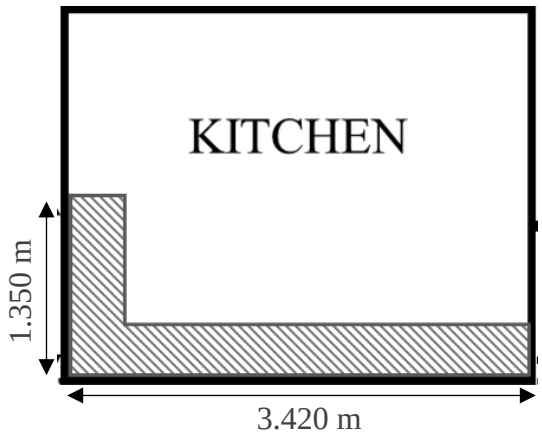
(b) The optimal kitchen cabinets' height, including the counter top, is calculated based on the concept of ergonomics. Ergonomics is the process of designing products that fit the people who use them.
Using the diagram as reference, kitchen cabinet designers view that optimal cabinet height should be the distance between one's elbow and the ground, with 10 cm allowance for elbow movement.
The diagram below shows a drawing of Mrs Graham whose height is 168 cm.

Calculate the range of optimal kitchen cabinets' heights, including the counter top, for Mrs Graham.



Answer …….. cm [2]

(c) Mr Graham plans to install a combination of kitchen cabinets along the L-shaped shaded area in the kitchen.



In the combination, he requires one of the cabinets to have a sink, a corner cabinet and at least one of the other cabinets to have wire basket shelves.
He wants to customise the height of the cabinets to suit Mrs Graham's height.
He also wishes to install a counter top for the top of the cabinet, considering to use either granite, ceramic or acrylic as its material.
He has a budget of $2200 to purchase the kitchen cabinets and L-shaped counter top.
Using the information in the tables given on the next page and your answer in **(b)**, propose a possible combination that will best suit Mr Graham's requirements that will optimise storage and space while maximising the use of his budget.

Answer

[7]

Table 1: Kitchen Cabinets

Cabinet	**1**	**2**	**3**	**4**	**5**	**6**
Description	Cabinet with sink, 1 drawer and 1 door (no shelves)	Corner cabinet	Cabinet with 4 drawers	Cabinet with 2 shelves and 2 doors	Cabinet with 1 door and wire basket shelves	Cabinet with 1 drawer, 3 shelves and 1 door.
Cost	$156	$166	$260	$290	$166	$193
Width	50 cm	60 cm	90 cm	80 cm	60 cm	70 cm
Depth	60 cm	60 cm	60 cm	60 cm	60 cm	60 cm

Table 2: Height Customisation for Kitchen Cabinet

Height	**Cost per cabinet**
50 cm	Free
up to 75 cm	$15
up to 105 cm	$25

Table 3: Materials for L-Shaped Counter Top

Materials	**Overall Cost**
Granite	$1200
Ceramic	$800
Acrylic	$600

End of Paper