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JURONG SECONDARY SCHOOL
2022 GRADUATION EXAMINATION 2
SECONDARY 4 EXPRESS

CANDIDATE NAME			
CLASS		**INDEX NUMBER**	

ADDITIONAL MATHEMATICS — 4049/02

Paper 2 — **25 August 2022**

Candidates answer on the Question Paper. — **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **18** printed pages including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Differentiate $2x\cos 3x$ with respect to x. [2]

(b) Hence, find $\int x\sin 3x \, \mathrm{d}x$. [3]

2 **(a)** Factorise $a^3 + b^3$. [1]

(b) Show that $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \dfrac{\sin 2x}{2}$. [2]

(c) Hence, solve the equation $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin^2 2x$ for $0 \le x \le \pi$. [5]

3 **(a)** Express $\dfrac{3x^2-4}{x^2(3x-2)}$ in partial fractions. [4]

(b) Hence, evaluate $\displaystyle\int \dfrac{3x^2-4}{x^2(3x-2)}\,\mathrm{d}x$. [3]

4 It is given that $\log_2\left(4-x^2\right)-\log_{\sqrt{2}}(x-1)=1$.

(a) Explain clearly why $1<x<2$. [4]

(b) Hence, solve the equation and show that it has only one solution. [5]

5 f(x) is such that $f''(x) = 18x - 4$. Given that f(0) = 1 and f(2) = 9,

(a) find f(x). [4]

(b) Show that $x + 1$ is a factor. [1]

(c) Solve $f(x) = 0$. [4]

6 It is given that $x = -2$ and $y = -1$ are tangents to a circle.
The x-coordinate and y-coordinate of the centre of the circle are positive.
The line $3y = 2x + 5$ is a normal to the circle.

(a) Show that the centre of the circle is (2, 3). [4]

(b) Find the equation of the circle. [2]

7 The height of a coin from the ground, h meters, after it has been flipped in the air for t seconds, can be represented by $h = -6t^2 + 24t + 12$.

(a) By completing the square, find the greatest height which the coin reaches. [3]

(b) Find the exact duration of the coin from the time it is flipped in the air till it lands on the ground. [3]

8 A pot of melted chocolate is cooled from its initial temperature to a temperature of T °C in x minutes and follows the equation of the form $T = A(B^x)$, where A and B are constants. The freezing point of the chocolate is 17°C. The table below shows the corresponding values of T and x recorded.

x	5	10	15	20	25
T	35.429	20.921	12.353	7.295	4.307

(a) Draw the graph of lg T plotted against x, using a scale of 2 cm for 5 unit on the x-axis and a scale of 1 cm for 0.1 unit on the lg T-axis. [3]

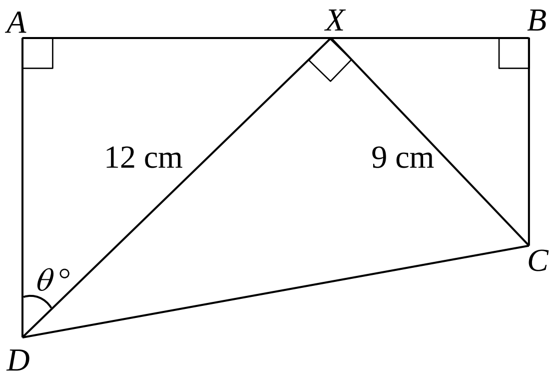
Using your graph,

(b) state whether the chocolate is frozen at 13 minutes and justify your answer, [2]

(c) estimate the value of each of the constants A and B, [5]

(d) find the amount of time taken for the chocolate to cool to half its original temperature. [2]

9 The diagram shows a trapezium *ABCD*. The point *X* lies on line *AB* such that $DX = 12$ cm and $CX = 9$ cm. $\angle ADX = \theta°$ and $\angle DAX = \angle DXC = \angle XBC = 90°$.

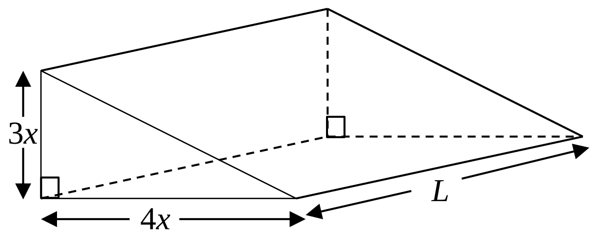


(a) Show that $AB = 9\cos\theta + 12\sin\theta$. [2]

(b) Express *AB* in the form $R\cos(\theta - \alpha)$, where *R* and α are constants, and hence state the maximum length of *AB* and its corresponding value of θ. [5]

(c) Find the value of θ for which $AB = 11$ cm. [3]

10 The figure below shows a right-angled triangular prism.
The height and base of the triangular faces of the prism are $3x$ meters and $4x$ meters respectively. The length of the prism is L meters.

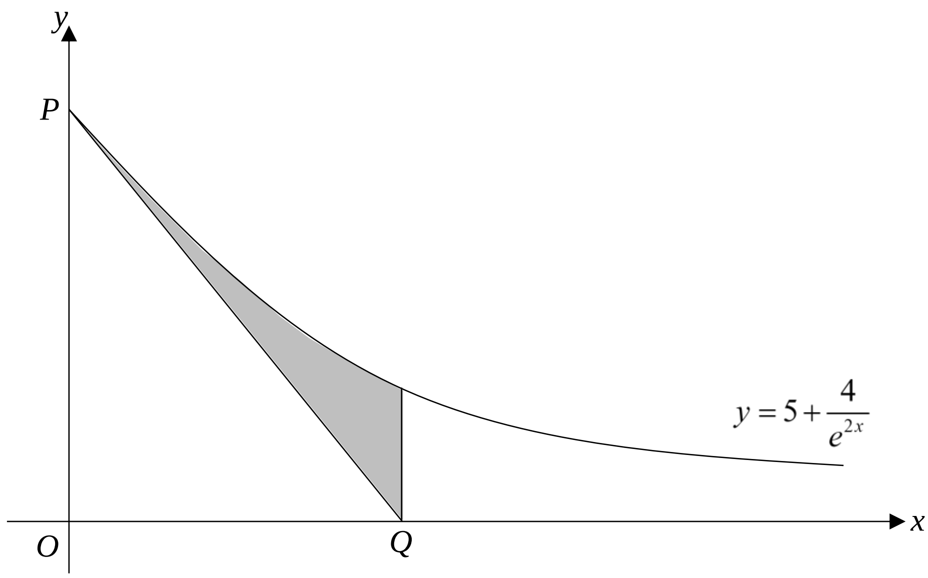


(a) Given that the volume of the prism is 240 m^3, show that the surface area of the prism, A m^2, is given by

$$A = 12x^2 + \frac{480}{x}.$$ [4]

(b) Given that x and L can vary, find the value of x for which A has a stationary value and determine whether this value of A is maximum or minimum. [5]

11



The diagram shows part of the curve $y = 5 + \frac{4}{e^{2x}}$ intersecting the *y*-axis at point *P*.

The tangent to the curve at point *P* intersects the *x*-axis at *Q*.

Find the area of the shaded region. [9]

End of Paper