| O | JURONG SECONDARY SCHOOL<br>2022 GRADUATION EXAMINATION 2<br>SECONDARY 4 EXPRESS |
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| CANDIDATE NAME | |
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| CLASS | | INDEX NUMBER | |
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## ADDITIONAL MATHEMATICS 4049/01

PAPER 1
Candidates answer on the Question Paper.
Additional Materials : Writing Paper (1 sheet)

**29 August 2022**
**2 hours 15 minutes**

### READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

| For Examiner's Use |
| --- |
| 90 |

This document consists of **15** printed pages including this page.

**[Turn Over]**

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** It is given that $\sin A = 0.3$, where $A$ is obtuse.
Find the following trigonometric ratios.

**(a)** $\sec A$, [2]

| Solution | Mark |
|---|---|
| $-\frac{10\sqrt{91}}{91}$ | B1 correct value of $\cos A$<br>B1 change $\sec A$ to $1/\cos A$ |

**(b)** $\cos 2A$. [2]

| Solution | Mark |
|---|---|
| $\cos 2A = 2\cos^2 A - 1$<br>$= 2(1 - 0.3^2) - 1$<br>$= 0.82$ | M1 double angle formula<br><br>A1 |

**(c)** $\tan(A + 45°)$. [4]

| Solution | Mark |
|---|---|
| $\tan(A + 45°) = \frac{\tan A + \tan 45°}{1 - \tan A \tan 45°}$<br>$= \frac{\frac{3}{-\sqrt{91}} + 1}{1 + \frac{7}{\sqrt{51}}}$<br>$= \frac{\sqrt{91} - 3}{\sqrt{91} + 3} \times \frac{\sqrt{91} - 3}{\sqrt{91} - 3}$<br>$= \frac{50 - 3\sqrt{91}}{41}$ | M1 addition formula<br>B1 correct value for $\tan 45°$<br><br>M1 rationalisation<br>A1 |

**2** **(a)** Find the range of values of $k$ such that $3x^2 - 5x + k$ is always positive. [2]

| Solution | Mark |
|---|---|
| $(-5)^2 - 4(3)(k) < 0$<br>$k > \frac{25}{12}$ | M1<br>A1 |

**(b)** Hence, solve the inequality $\frac{x^2-x-2}{3x^2-5x+4} < 0$. [3]

| Solution | Mark |
|---|---|
| $k = 4 > \frac{25}{12}$, hence $3x^2 - 5x + 4 > 0$<br>$(x + 1)(x - 2) < 0$<br><br>$-1 < x < 2$ | M1 using part (a)<br>M1 correct quadratic inequality in factorised form<br>A1 |

**3** It is given that $f(x) = Ax(e^{kx})$, where $A$ and $k$ are constants.
Find the value of $A$ and of $k$ such that $f'(x) + 2ke^{kx} + 6f(x) = 0$ [6]

| Solution | Mark |
|---|---|
| $f'(x) = Akxe^{kx} + Ae^{kx}$ | M1 product rule |
| $Akxe^{kx} + Ae^{kx} + 2ke^{kx} + 6Axe^{kx} = 0$ | M1 substitution |
| $A + 2k = 0$ --------------- (1) | M1 |
| $Ak + 6A = 0$ ------------ (2) | M1 |
| From (2) $A(k + 6) = 0$, $A = 0$ (rej) or $k = -6$ | A1 |
| sub $k = -6$ into (1), $A = 12$ | A1 |

**4** A curve has equation $y = \frac{4}{\sqrt{x+3}}$. A point $(x, y)$ is moving along the curve.

Find the coordinates of the point at the instant where the $y$-coordinate is decreasing at a rate twice of the rate of increase of the $x$-coordinate. [5]

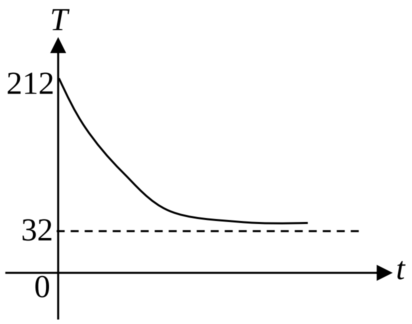
| Solution | Mark |
|---|---|
| $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$<br>$\frac{dy}{dx} = -2$<br>$4 \times \left(-\frac{1}{2}\right)(x+3)^{-\frac{3}{2}} = -2$<br>$x + 3 = 1$<br>$(-2, 4)$ | M1 correct formula for connected rate of change<br>M1 correct value of $\frac{dy}{dx}$<br>M1 correct derivative<br>A1 correct values of $x$ and $y$<br>A1 coordinate form |

**5** A metal cube is heated to a temperature of 212°C before being dropped into a liquid. As the cube cools, its temperature $T$°C, $t$ minutes after it enters the liquid is given by $T = P + 180e^{-kt}$, where $P$ and $k$ are constants.
It is recorded that when $t = 5$, $T = 185$.

**(a)** Find the value of $k$ and of $P$. [4]

| Solution | Mark |
|---|---|
| $P = 212 - 180 = 32$<br>$32 + 180e^{-5k} = 185$<br>$e^{-5k} = 0.85$<br>$-5k = \ln 0.85$<br>$k = 0.0325$ (3 s.f.) | B1<br>M1 substitution<br>M1 isolating the exp term<br><br>A1 |

**(b)** By sketching the graph of $T$ against $t$, explain why $T$ cannot be 30. [3]

| Solution | Mark |
|---|---|
| <br><br>From the graph, the graph is completely above $T = 32$, hence, $T$ cannot be 30. | B1 correct shape<br>B1 correct $y$-intercept and horizontal asymptote<br><br>-1 if $t < 0$ is included<br><br>-1 if no labelling of axis<br><br>B1 accept any reasonable answer |

**6** It is given that the first three terms, in ascending powers of $x$, of the binomial expansion of $(2 + ax)^6$ are $64 - 960x + bx^2$.

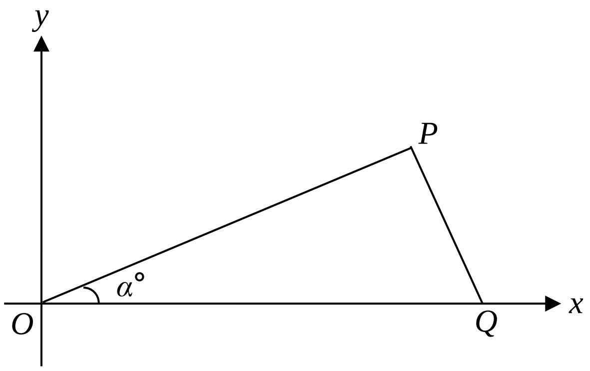
**(a)** Find the value of $a$ and of $b$. [3]

| Solution | Mark |
|---|---|
| $2^6 + \binom{6}{1} 2^5(ax) + \binom{6}{2} 2^4(ax)^2 = 32 - 960x + bx^2$<br>$192a = -960$, $b = 240a^2$<br>$a = -5$, $b = 6000$ | M1 binomial expansion<br><br>M1 compare coefficient<br>A1 |

**(b)** Using the values found in part **(a)**, find the coefficient of $x^3$ in the expansion of $(1 + 3x^2)^5(2 + ax)^6$. [4]

| Solution | Mark |
|---|---|
| $(1 + 15x^2 + \ldots)(32 - 960x + 6000x^2 - 20000x^3 + \ldots)$<br><br><br>coefficient of $x^3 = 1(-20000) + (15)(-960)$<br>$= -34400$ | M1 binomial expansion of $(1 + 3x^2)^5$<br>M1 correct $x^3$ in $(2 + ax)^6$<br>M1<br>A1 |

**7** In the diagram below, the line $OP$ makes an angle of $\alpha°$ with the positive $x$-axis such that $\tan\alpha = 0.2$ and $Q$ lies on the $x$-axis.




**(a)** Given that $OP = \frac{\sqrt{26}}{2}$ units, show that $P = (2.5, 0.5)$. [3]

| Solution | Mark |
|---|---|
| Let $OP = (x, y)$<br>$\sqrt{x^2 + y^2} = \frac{\sqrt{26}}{2}$ ---------------- (1)<br>$\frac{y}{x} = 0.2$ ------------------------ (2)<br>$x^2 + (0.2x)^2 = 6.5$<br>$x = 2.5, y = 0.5$ | M1 using length of $OP$<br>M1 using gradient<br>A1 with working |

**(b)** Given that the area of $\Delta OPQ$ is 0.65 units$^2$, find the coordinates of $Q$. [2]

| Solution | Mark |
|---|---|
| $\frac{1}{2} \times OQ \times 0.5 = 0.65$<br>$OQ = 2.6$<br>$Q = (2.6, 0)$ | M1<br>A1 |

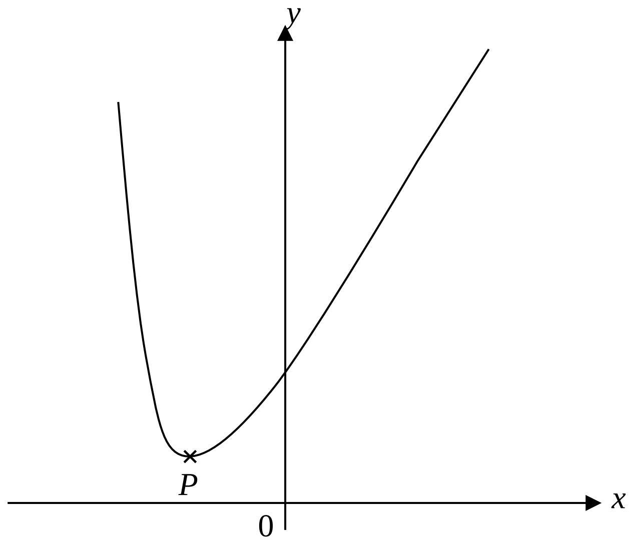
**(c)** Explain, with calculations, why $\Delta OPQ$ is a right-angle triangle. [3]

| Solution | Mark |
|---|---|
| gradient of $PQ = \frac{0-0.5}{2.6-2.5} = -5$<br>gradient of $PQ$ × gradient of $OP = -5 \times 0.2 = -1$<br>Hence, $OP$ is perpendicular to $PQ$, $\Delta OPQ$ is a right-angle triangle | M1<br>M1<br>A1 must identify the perpendicular lines/right angle |

**(d)** Find the coordinates of $R$ such that $OPQR$ is a rectangle. [2]

| Solution | Mark |
|---|---|
| Let $R = (x, y)$<br>$\left(\frac{0+2.6}{2}, 0\right) = \left(\frac{x+2.5}{2}, \frac{y+0.5}{2}\right)$<br>$R = (0.1, -0.5)$ | M1<br>A1 |

**8** The diagram below shows part of the graph of $y = \frac{x^2+2x+5}{x+3}$.




Find the coordinates of the minimum point $P$. [7]

| Solution | Mark |
|---|---|
| $\frac{dy}{dx} = \frac{(x+3)(2x+2)-(x^2+2x+5)(1)}{(x+3)^2}$ | M1 quotient rule |
| $\frac{2x^2+8x+6-x^2-2x-5}{(x+3)^2} = 0$ | M1 $\frac{dy}{dx} = 0$ |
| $x^2 + 6x + 1 = 0$ | |
| $x = \frac{-6\pm\sqrt{6^2-4(1)(1)}}{2}$ | M1 quadratic formula |
| $= -3 \pm 2\sqrt{2}$ | A1 accept -0.17157 and -5.8284 |
| When $x = -3 - 2\sqrt{2}$, $y$ is a maximum | M1 1st or 2nd derivative test |
| When $x = -3 + 2\sqrt{2}$, $y$ is a minimum | A1 nature of s.p. |
| $P$ = (–0.172, 1.66) | A1 |

**9** **(a)** It is given that $\frac{2^x \times 32(2^x)}{8^{x+1}} = \frac{9(5^{2x}) - 5^{2x+1}}{5^x - 5^{x-1}}$.

Evaluate $10^x$ without using a calculator. [5]

| Solution | Mark |
|---|---|
| $\frac{2^x \times 2^5 \times 2^x}{2^{3x+3}} = \frac{9 \times 5^{2x} - 5^{2x} \times 5}{5^x - 5^x \times \frac{1}{5}}$ | M1 change to common base |
| $\frac{2^{2x+5}}{2^{3x+3}} = \frac{4 \times 5^{2x}}{\frac{4}{5} \times 5^x}$ | M1 factorisation of RHS |
| $\frac{4}{2^x} = 5^x \times 5$ | M1 simplification to one term on each side |
| $\frac{4}{5} = 5^x \times 2^x$ | M1 isolate $2^x$ and $5^x$ |
| $10^x = 0.8$ | A1 |

**(b)** Solve $\sqrt{x+7} - x - 1 = 0$. [3]

| Solution | Mark |
|---|---|
| $\sqrt{x+7} = x + 1$ | |
| $x + 7 = x^2 + 2x + 1$ | M1 getting rid of square root |
| $x^2 + x - 6 = 0$ | |
| $(x - 2)(x + 3) = 0$ | M1 method to solve eqn |
| $x = 2$ or $-3$ (reject as $x + 1 > 0$) | A1 with rejection |

**10** A particle moves in a straight line so that its velocity, $v$ m/s is given by $v = t^2 - 4t + 3$, where $t$ is the time in seconds after passing a fixed point $O$.

**(a)** Find the acceleration of the particle when $t = 1$. [2]

| Solution | Mark |
|---|---|
| $a = 2t - 4$<br>when $t = 1$, $a = -2$ m/s$^2$ | M1<br>A1 |

**(b)** Find the value(s) of $t$ when the particle comes to instantaneous rest. [2]

| Solution | Mark |
|---|---|
| $(t-1)(t-3) = 0$<br>$t = 1$ or $3$ | M1 $v = 0$<br>A1 |

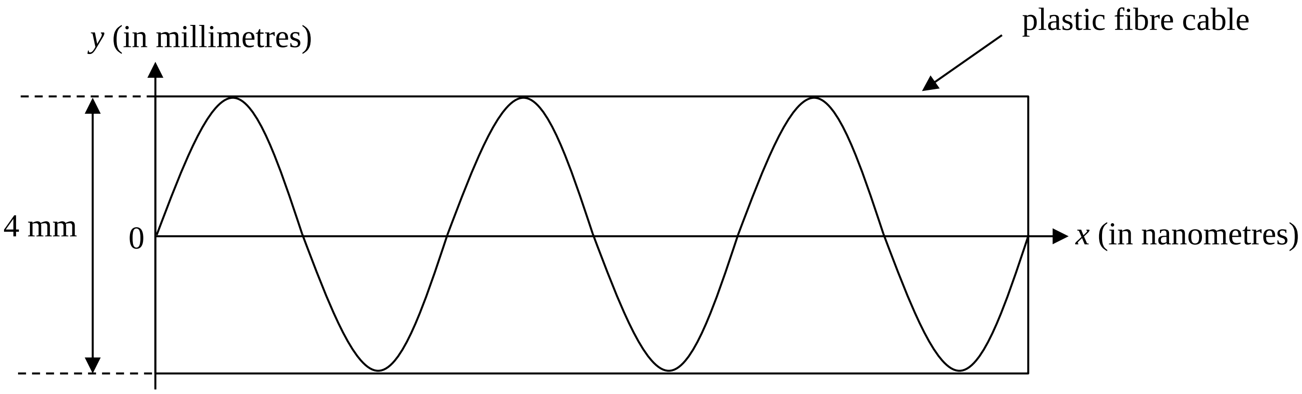
**(c)** Find the displacement(s) of the particle at the instant when it comes to rest. [3]

| Solution | Mark |
|---|---|
| $s = \frac{1}{3}t^3 - 2t^2 + 3t + c$<br>$s = 0$ when $t = 0$, $c = 0$<br>$s = \frac{1}{3}t^3 - 2t^2 + 3t$<br>When $t = 1$, $s = \frac{4}{3}$ m<br>When $t = 3$, $s = 0$ m | M1 indefinite integral of $v$<br><br><br>B1 correct expression for $s$<br>A1 for both values<br>-1 if no unit |

**(d)** Find the average speed of the particle for the first 4 seconds. [3]

| Solution | Mark |
|---|---|
| When $t = 4$, $s = \frac{4}{3}$ m<br>Total distance travelled = 4 m<br>average speed = 1 m/s | M1 find $s$ when $t = 4$<br>M1 total distance<br>A1 |

**11** The diagram below shows a portion a plastic fibre cable, which allows light waves to pass through. The path of the light wave can be modelled by a trigonometric function.




**(a)** It is given that the diameter of the cable is 4 millimetres.
Find the amplitude of the light wave. [1]

| Solution | Mark |
|---|---|
| 2 mm | B1 |

**(b)** It is given further that the period of the light wave is 500 nanometres.
Find the length of the portion of cable shown in the diagram. [1]

| Solution | Mark |
|---|---|
| 1500 nanometers | B1 |

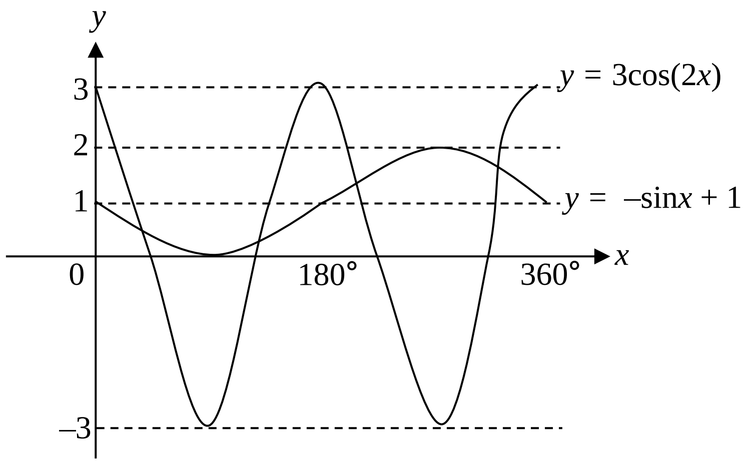
**(c)** Which of the following can be a suitable model for the light wave?

$y = 2\sin(\pi x)$ $\qquad$ $y = 2\cos(\pi x)$ $\qquad$ $y = 2\sin\left(\frac{\pi}{250}x\right)$

Explain your answer. [3]

| Solution | Mark |
|---|---|
| $y = 2\sin\left(\frac{\pi}{250}x\right)$<br>The graph starts from the centre position 0, hence it is a sine graph.<br>The coefficient of $x$, $b$ is such that $\frac{2\pi}{b} = 500$, hence $b = \frac{\pi}{250}$ | B1<br><br>B1 choice of trigo ratio<br>B1 coefficient of $x$ |

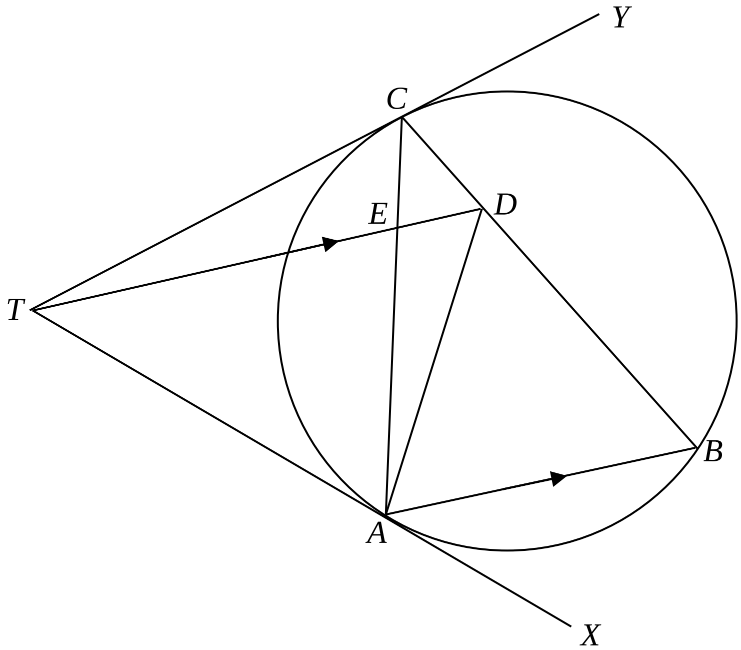
**12** **(a)** Sketch the graph of $y = -\sin x + 1$ and $y = 3\cos(2x)$ for $0° \le x \le 360°$ on the same axes. [4]

| Solution | Mark |
| --- | --- |
|  | B1, B1 each amplitude and max/min value<br><br>B1, B1 each correct period, correct shape |

**(b)** Hence, state the number of solutions to the equation $-\sin x + 1 = 3\cos(2x)$ for $0° \le x \le 360°$. [1]

| Solution | Mark |
| --- | --- |
| 4 | B1 |

**13** In the diagram below, *TAX* and *TCY* are tangents to the circle at *A* and *C* respectively. *AC* meets *TD* at *E* and *D* is on *BC* such that *TD* is parallel to *AB*.




**(a)** Prove that angle *ACB* is equal to angle *ATD*. [2]

| Solution | Mark |
| --- | --- |
| $\measuredangle ATD = \measuredangle XAB$ (corresponding angles, *AB* // *TD*)<br>$= \measuredangle ACB$ (angles in alternate segments) | B1<br>B1<br>-1 if no reason/wrong reason |

**(b)** Explain why a circle can be drawn passing through the points *T*, *A*, *D* and *C*. [1]

| Solution | Mark |
| --- | --- |
| Angles in the same segment | B1 |

**(c)** Hence, prove that *CE* × *EA* = *DE* × *TE*. [4]

| Solution | Mark |
| --- | --- |
| $\measuredangle ATE = \measuredangle DCE$ (from part a)<br>$\measuredangle TEA = \measuredangle CED$ (vertically opposite angles)<br>$\Delta ATE$ is similar to $\Delta DCE$ (AA similarity test)<br>$\frac{TE}{CE} = \frac{EA}{ED}$<br>*CE* × *EA* = *DE* × *TE* | M1 two reasons<br><br>A1 similar triangles with test<br>M1<br>A1 |

---------------------------------------------END OF PAPER---------------------------------------------