4E AM GE2 2022 Paper 2 Mark Scheme

1 **(a)** Differentiate $2x\cos 3x$ with respect to x. [2]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(2x\cos 3x\right)$$
$$= 2x(-3\sin 3x) + 2\cos 3x$$
$$= -6x\sin 3x + 2\cos 3x$$ [B1,B1]: each term

(b) Hence, find $\int x\sin 3x \; \mathrm{d}x$. [3]

$$\int x\sin 3x \; \mathrm{d}x$$
$$= -\frac{1}{6}\left(2x\cos 3x\right) + \frac{1}{3}\int \cos 3x \; \mathrm{d}x$$ [M1]: reverse differentiation
$$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + \mathrm{C}$$ [A1,A1] each term

Deduct 1 mark for missing +C.

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2 **(a)** $a^3+b^3=(a+b)(a^2-ab+b^2)$

[1]

(b)

$$\frac{\left(\sin x+\cos x\right)\left(\sin^2 x-\sin x\cos x+\cos^2 x\right)}{\sin x+\cos x}$$ [M1]: Cubic Identity
$$=1-\sin x\cos x$$ [M1]: simplify
$$=1-\frac{2\sin x\cos x}{2}$$ ~~[A1]: show change of double angle, fraction~~
$$=1-\frac{\sin 2x}{2}$$

(c) Hence, solve the equation $\dfrac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=1-\sin^2 2x$ for $0 \le x \le \pi$. [5]

$$1-\frac{\sin 2x}{2}=1-\sin^2 2x$$ [M1]: Hence
$$2\sin^2 2x-\sin 2x=0$$
$$\sin 2x(2\sin 2x-1)=0$$ [M1]: Factorisation mtd
$\sin 2x=0$ or $\sin 2x=\dfrac{1}{2}$ [M1]: Correct Trigo Equations

$x = 0, \dfrac{\pi}{2}, \pi$ [A1] or $\dfrac{\pi}{12}, \dfrac{5\pi}{12}$ [A1]

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3 **(a)** Express $\dfrac{3x^2-4}{x^2(3x-2)}$ in partial fractions. [4]

Let $\dfrac{3x^2-4}{x^2(3x-2)} = \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{3x-2}$ [B1] Correct form

$3x^2-4 = Ax(3x-2)+B(3x-2)+Cx^2$

Compare constant term, $B = 2$ [M1]

By substitution, $A = 3$, $C = -6$ [M1]

$\dfrac{3x^2-4}{x^2(3x-2)} = \dfrac{3}{x}+\dfrac{2}{x^2}-\dfrac{6}{3x-2}$ [A1]

(b) Hence, evaluate $\displaystyle\int \frac{3x^2-4}{x^2(3x-2)}\,dx$. [3]

$$\int \frac{3}{x}+\frac{2}{x^2}-\frac{6}{3x-2}\,dx$$
$$= 3\ln x-\frac{2}{x}-2\ln(3x-2)+C$$

[B1,B1,B1] each term, penalise 1 mark if missing +C

4 It is given that $\log_2\left(4-x^2\right)-\log_{\sqrt{2}}(x-1)=1$.

(a) Explain clearly why $1<x<2$. [4]

$4-x^2>0$

$(x-2)(x+2)<0$ [M1] and $x-1>0$ [M1]

$-2<x<2$ [A1] $x>1$

Reasonable conclusion using words or number line. [A1]

$\therefore\ 1<x<2$

(b) Hence, solve the equation and show that it has only one solution. [5]

$\log_2\left(4-x^2\right)-\dfrac{\log_2(x-1)}{\log_2\sqrt{2}}=1$ [M1]: Change base

$\log_2\left(4-x^2\right)-\log_2(x-1)^2=1$ [M1]: Power Law

$\log_2\dfrac{4-x^2}{(x-1)^2}=1$ [M1]: Quotient Law

$\dfrac{4-x^2}{(x-1)^2}=2$ [M1]: Convert form/Equivalence

$3x^2-4x-2=0$

$x=\dfrac{4\pm\sqrt{16-4(-6)}}{6}$ [A1]

$=1.72$ or -0.387 (rejected)

5 f(x) is such that $f''(x) = 18x - 4$. Given that f(0) = 1 and f(2) = 9,

(a) find f(x). [4]

$f'(x) = 9x^2 - 4x + c$ [M1]

$f(x) = 3x^3 - 2x^2 + cx + d$ [M1]

show correct usage of notation/substitution [M1]

$d = 1, c = -4$

$f(x) = 3x^3 - 2x^2 - 4x + 1$ [A1]

(b) Show that x + 1 is a factor. [1]

$f(-1)$

$= 3(1)^3 - 2(-1)^2 - 4(-1) + 1$

$=0$

By factor/remainder theorem, x+1 is a factor.

Must state appropriate conclusion to show how the substitution 'show'.

Accept long division+conclusion

(c) Solve f(x) = 0. [4]

Long Division / Selective Expansion [M1]

$(x+1)(3x^2 - 5x + 1) = 0$ [M1]

$x = -1$ or $x = \dfrac{5 \pm \sqrt{13}}{6}$ [M1]

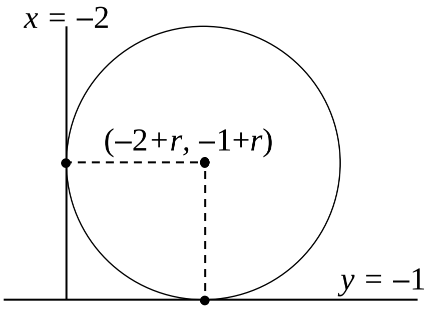
$x = -1$ or 0.232 or 1.43 [A1] : Accept exact surd form.

Penalise for missing method to solve quadratic equation.

6 It is given that $x = -2$ and $y = -1$ are tangents to a circle.
The x-coordinate and y-coordinate of the centre of the circle are positive.
The line $3y = 2x + 5$ is a normal to the circle.

(a) Show that the centre of the circle is (2, 3). [4]

Let radius be r.



Centre = $(-2+r, -1+r)$ [M1]

$3(-1+r) = 2(-2+r) + 5$ -------------------② tangent ⊥ radius [M1]

Solve r [M1]

Substitute back to find centre = (2, 3) [A1]

Alternative method:

Let centre be (a,b). Simultaneous equations centre/radius relationship equation with normal equation.

(b) Find the equation of the circle. [2]

$(x-2)^2 + (y-3)^2 = 16$ [B1, B1 for 16]

7 The height of a coin from the ground, h meters, after it has been flipped in the air for t seconds, can be represented by $h = -6t^2 + 24t + 12$.

(a) By completing the square, find the greatest height which the coin reaches. [3]

$h = -6(t^2 - 4t - 2)$

$= -6\left(t^2 - 4t + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 2\right)$ [M1]

$= 36 - 6(t-2)^2$ [A1]

Greatest h = 36 m [A1]

(b) Find the exact duration of the coin from the time it is flipped in the air till it lands on the ground. [3]

$36 - 6(t-2)^2 = 0$ [M1]

$t = \sqrt{6} + 2$ or $-\sqrt{6} + 2$(rejected) [M1]

Duration $= \left(\sqrt{6} + 2\right)$s [A1]

8 A pot of melted chocolate is cooled from its initial temperature to a temperature of T °C in x minutes and follows the equation of the form $T = A(B^x)$, where A and B are constants.

The freezing point of the chocolate is 17°C. The table below shows the corresponding values of T and x recorded.

x	5	10	15	20	25
T	35.429	20.921	12.353	7.295	4.307

(a) Draw the graph of lg T plotted against x, using a scale of 2 cm for 5 unit on the x-axis and a scale of 1 cm for 0.1 unit on the lg T axis. [3]

x	5	10	15	20	25
lg T	1.55	1.32	1.09	0.86	0.63

B1: table
G1: axes
G1: plot and straight line

(b) Using your graph, state whether the chocolate is frozen at 13 minutes. Justify your answer. [2]

Possible solution 1

At $x = 13$, lg $T = 1.18$, $T = 15.1$°C< 17°C. [M1]

Therefore, it has frozen. [A1]

Possible solution 2

If $T = 17$°C, lg $T = 1.23$. $x = 11.75$ minutes < 13 minutes. [M1]

Therefore, it has frozen. [A1]

(c) estimate the value of each of the constants A and B, [5]

$\lg T = x \lg B + \lg A$ [M1]

$\lg A = 1.78$ (± 0.01) [M1]

$A = 60.3$ (± 2) [A1]

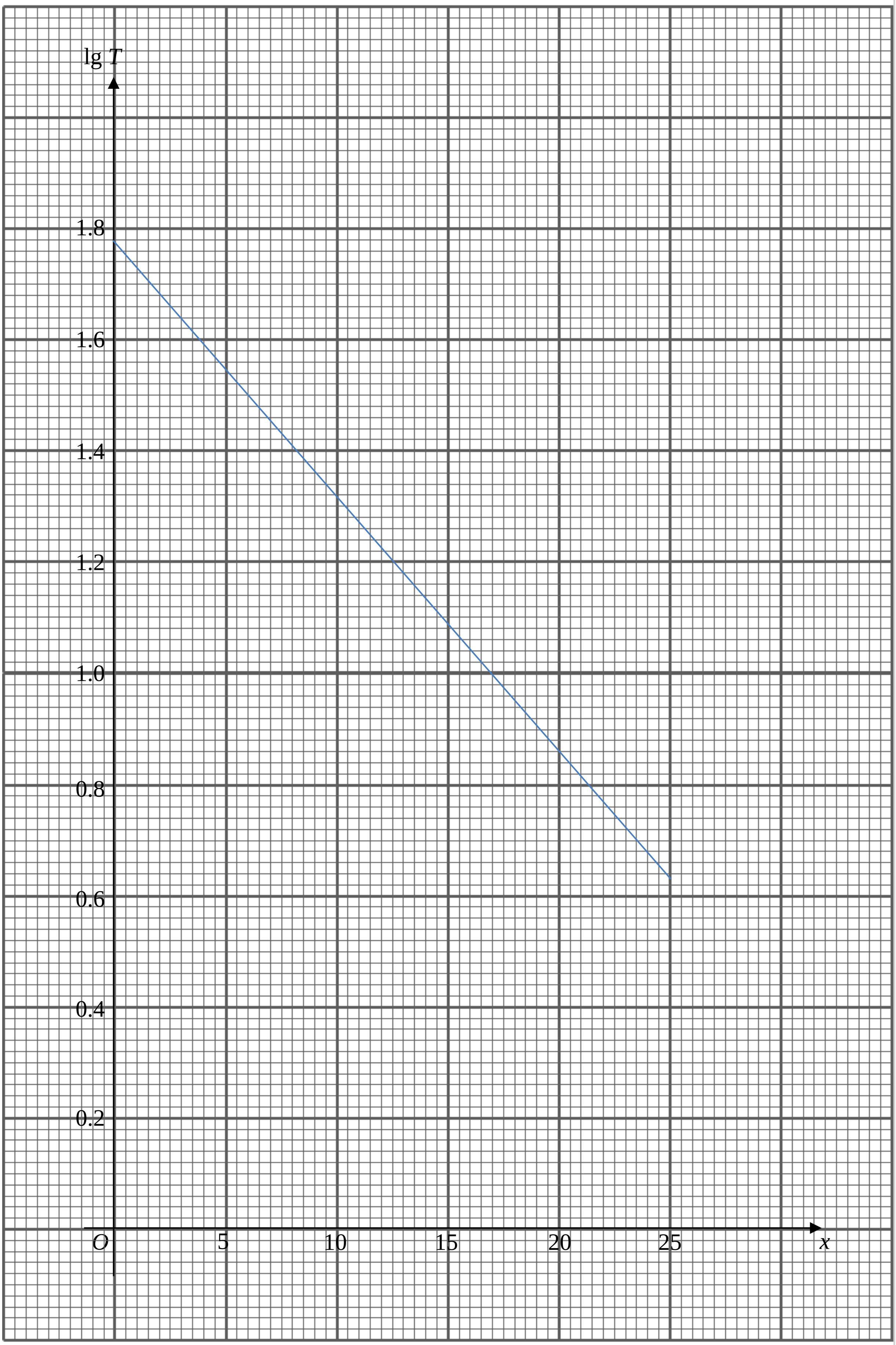
$\lg B$ = gradient = -0.046 [M1]

$B = 0.9$ (± 0.01) [A1]

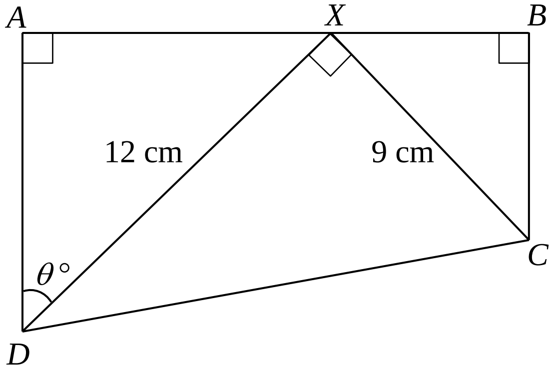
(d) find the amount of time taken for the chocolate to cool to half its original temperature. [2]

$\frac{A}{2} = 30.12798$ [M1]

Read at $\lg T = 1.48(\pm 0.01)$, time = 6.5 minutes (± 0.25) [A1]


lg T
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
O
5
10
15
20
25
x

9 The diagram shows a trapezium $ABCD$. The point X lies on line AB such that DX = 12 cm and CX = 9 cm. $\angle ADX = \theta°$ and $\angle DAX = \angle DXC = \angle XBC = 90°$.



(a) Show that $AB = 9\cos\theta + 12\sin\theta$. [2]

$$\frac{AX}{12} = \sin\theta$$

$$\frac{BX}{9} = \cos\theta$$

[M1] CAH, SOH

$$\therefore AB = AX + BX$$
$$= 12\sin\theta + 9\cos\theta$$

[A1]

(b) Express AB in the form $R\cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum length of AB and its corresponding value of θ. [5]

$\therefore AB$

$= \sqrt{9^2 + 12^2}\cos(\theta - \tan^{-1}\frac{12}{9})$ [M1],[M1]

$= 15\cos(\theta - 53.1°)$ [A1]

Max AB = 15 m [B1]

θ = 53.1° [B1]

9(c) Find the value of θ for which AB = 11 cm. [3]

$15\cos(\theta - \tan^{-1}\frac{12}{9}) = 11$ [M1]

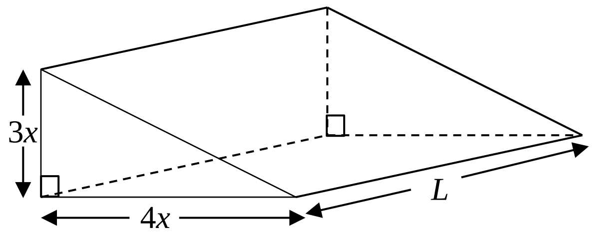
$\cos(\theta - \tan^{-1}\frac{12}{9}) = \frac{11}{15}$

$\alpha = 42.83343°$ [M1]

$\theta - \tan^{-1}\frac{12}{9} = \alpha$ or $360 - \alpha$

$\theta = 96.0°$ (rejected) or $10.3°$ (1dp) [A1]

10 The figure below shows a right-angled triangular prism. The height and base of the triangular faces of the prism are $3x$ meters and $4x$ meters respectively. The length of the prism is L meters.



(a) Given that the volume of the slope is 240 m^3, show that the surface area of the prism, A m^2, is given by

$$A = 12x^2 + \frac{480}{x}.$$ [4]

$\frac{1}{2}(3x)(4x)L = 240$ [M1] form SA equation

$L = \frac{40}{x^2}$ [M1] L the subject

Area $= (3x)(4x) + 5xL + 3xL + 4xL$ [M1] Surface Area

$= 12x^2 + 12x(\frac{40}{x^2})$ [A1] Sub,Simplify

$= 12x^2 + \frac{480}{x}$

(b) Given that x and L can vary, find the value of x for which A has a stationary value and determine whether this value of A is maximum or minimum. [5]

$\frac{dA}{dx} = 24x - \frac{480}{x^2}$ [M1]

Solve $\frac{dA}{dx} = 0$ [M1]

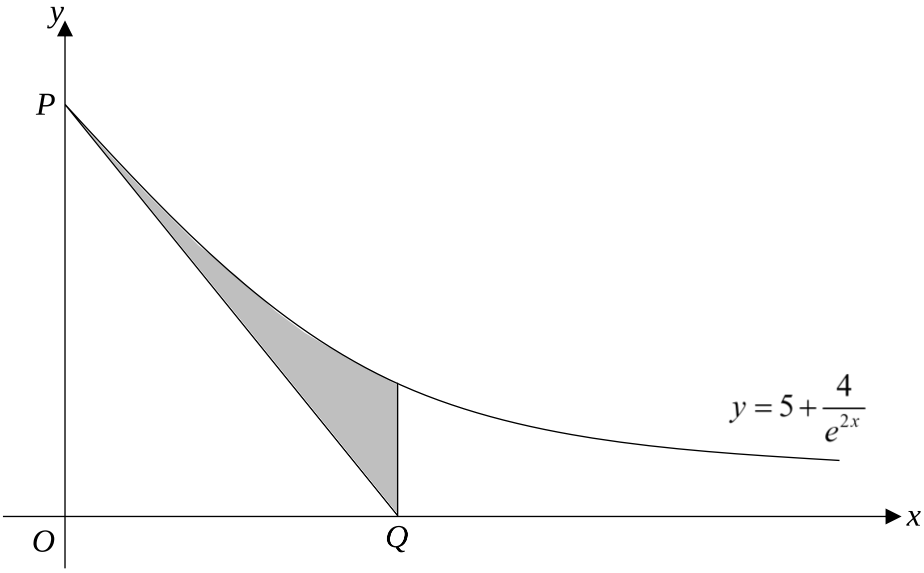
$x = 2.71$ (3sf) [A1]

$\frac{d^2A}{dx^2} = 24 + \frac{960}{x^3}$ [M1]

Since $x > 0$, $\frac{d^2A}{dx^2} > 0$ and surface area is minimum. [A1]

**can substitute x = 2.71 to show too*

11



The diagram shows part of the curve $y = 5 + \frac{4}{e^{2x}}$ intersecting the y-axis at point P.

The tangent to the curve at point P intersects the x-axis at Q.

Find the area of the shaded region. [9]

$\frac{dy}{dx} = -8e^{-2x}$ [M1]

At $P(0, 9)$, gradient $= -8$ [M1]

$y = -8x + 9$ [M1] Tangent PQ

$Q\left(\frac{9}{8}, 0\right)$ [A1]

Area under graph

$= \int_0^{9/8} 5 + 4e^{-2x}\, dx - \frac{1}{2}(9)(\frac{9}{8})$ [M1, M1] Integrate, use triangle/ subtract functions

$= \left[5x - 2e^{-2x}\right]_0^{9/8} - \frac{81}{16}$ [M1]

$= \left(\frac{45}{8} - 2e^{-2(9/8)}\right) - (-2e^0) - \frac{81}{16}$ [M1]

$= 2.35$ units^2 [A1]