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**JURONG SECONDARY SCHOOL
2022 GRADUATION EXAMINATION 2
SECONDARY 4 EXPRESS**

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS   4049/01

PAPER 1   **29 August 2022**
Candidates answer on the Question Paper.   **2 hours 15 minutes**
Additional Materials : Writing Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
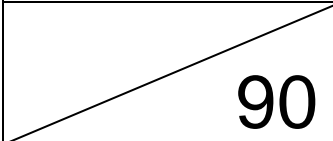The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

90

This document consists of **17** printed pages including this page.

[Turn Over]

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1   It is given that $\sin A = 0.3$, where A is obtuse.
Find the following trigonometric ratios.

(a) $\sec A$,   [2]

(b) $\cos 2A$,   [2]

(c) $\tan(A + 45°)$.   [4]

2 **(a)** Find the range of values of k such that $3x^2 - 5x + k$ is always positive. [2]

(b) Hence, solve the inequality $\dfrac{x^2 - x - 2}{3x^2 - 5x + 4} < 0$. [3]

3 It is given that $f(x) = Ax(e^{kx})$, where A and k are constants.

Find the exact values of A and k such that $f'(x) + 2ke^{kx} + 6f(x) = 0$. [6]

4 A curve has equation $y = \dfrac{4}{\sqrt{x+3}}$. A point (x, y) is moving along the curve.

Find the coordinates of the point at the instant where the y-coordinate is decreasing at a rate twice of the rate of increase of the x-coordinate. [5]

5 A metal cube is heated to a temperature of 212°C before being dropped into a liquid. As the cube cools, its temperature T°C , t minutes after it enters the liquid is given by $T = P + 180e^{-kt}$, where P and k are constants.
It is recorded that when $t = 5$, $T = 185$.

**(a)** Find the value of P and of k. [4]

(b) By sketching the graph of T against t, explain why T cannot be 30. [3]

[Turn Over]

6 It is given that the first three terms, in ascending powers of x, of the binomial expansion of $(2+ax)^6$ are $64-960x+bx^2$ respectively.

(a) Find the value of a and of b. [3]

(b) Using the values found in part **(a)**, find the coefficient of x^3 in the expansion of $(1+3x^2)^5(2+ax)^6$. [4]

7 In the diagram below, the line *OP* makes an angle of $\alpha°$ with the positive *x*-axis such that $\tan \alpha = 0.2$ and *Q* lies on the *x*-axis.
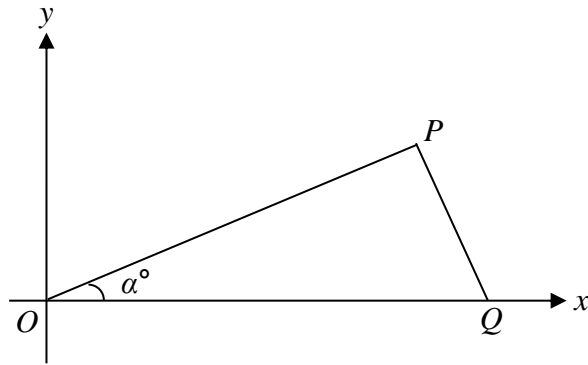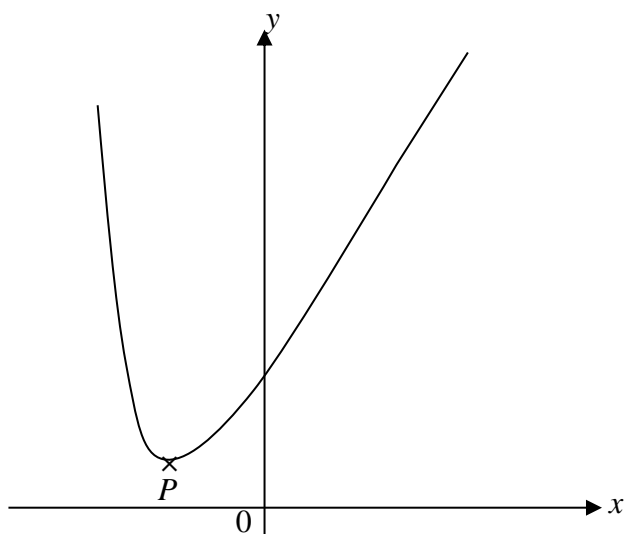

(a) Given that $OP = \dfrac{\sqrt{26}}{2}$ units, show that $P = (2.5, 0.5)$. [3]

(b) Given that the area of $\triangle OPQ$ is 0.65 units², find the coordinates of *Q*. [2]

[Turn Over]

(c) Explain, with calculations, why ΔOPQ is a right-angled triangle. [3]

(d) Find the coordinates of R such that $OPQR$ is a rectangle. [2]

8 The diagram below shows part of the graph of $y = \dfrac{x^2 + 2x + 5}{x + 3}$.

Find the coordinates of the minimum point P. [7]

9 **(a)** It is given that $\dfrac{2^x \times 32(2^x)}{8^{x+1}} = \dfrac{9(5^{2x}) - 5^{2x+1}}{5^x - 5^{x-1}}$.

Evaluate 10^x without using a calculator. [5]

(b) Solve $\sqrt{x+7} - x - 1 = 0$. [3]

10 A particle moves in a straight line so that its velocity, v m/s is given by $v = t^2 - 4t + 3$, where t is the time in seconds after passing a fixed point O.

    (a) Find the acceleration of the particle when $t = 1$.   [2]

    (b) Find the value(s) of t when the particle comes to instantaneous rest.   [2]

    (c) Find the displacement(s) of the particle at the instant when it comes to rest.   [3]

    (d) Find the average speed of the particle for the first 4 seconds.   [3]

11 The diagram below shows a portion a plastic fibre cable, which allows light waves to pass through. The path of the light wave can be modelled by a trigonometric function.
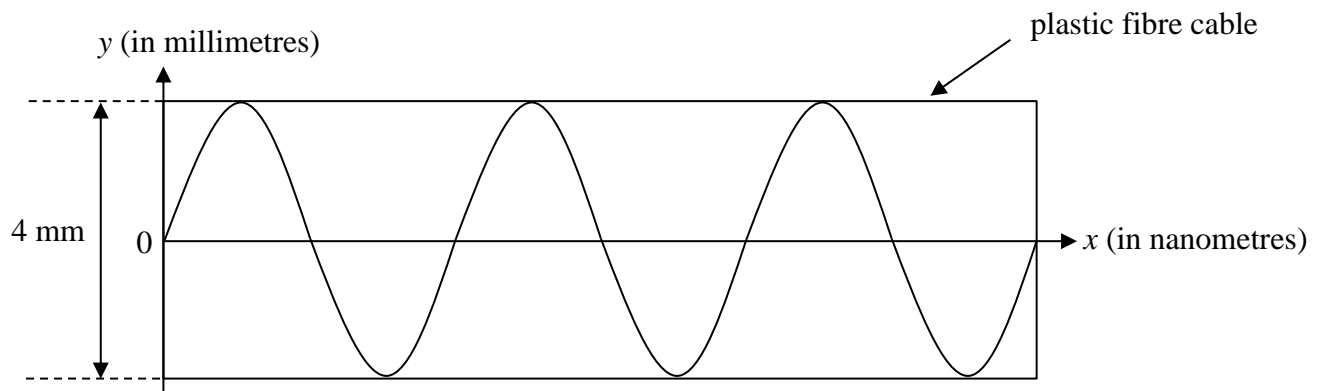

(a) It is given that the diameter of the cable is 4 millimetres.
Find the amplitude of the light wave. [1]

(b) It is given further that the period of the light wave is 500 nanometres.
Find the length of the portion of cable shown in the diagram. [1]

(c) Which of the following can be a suitable model for the light wave?

$$y = 2\sin(\pi x) \qquad y = 2\cos(\pi x) \qquad y = 2\sin(\frac{\pi}{250}x)$$

Explain your answer. [3]

12   **(a)**   Sketch the graph of $y = -\sin x + 1$ and $y = 3\cos 2x$ for $0° \le x \le 360°$ on the same axes.   [4]

   (b)   Hence, state the number of solutions to the equation $-\sin x + 1 = 3\cos 2x$ for $0° \le x \le 360°$.   [1]

[Turn Over]

13 In the diagram below, *TAX* and *TCY* are tangents to the circle at *A* and *C* respectively. *AC* meets *TD* at *E* and *D* is on *BC* such that *TD* is parallel to *AB*.
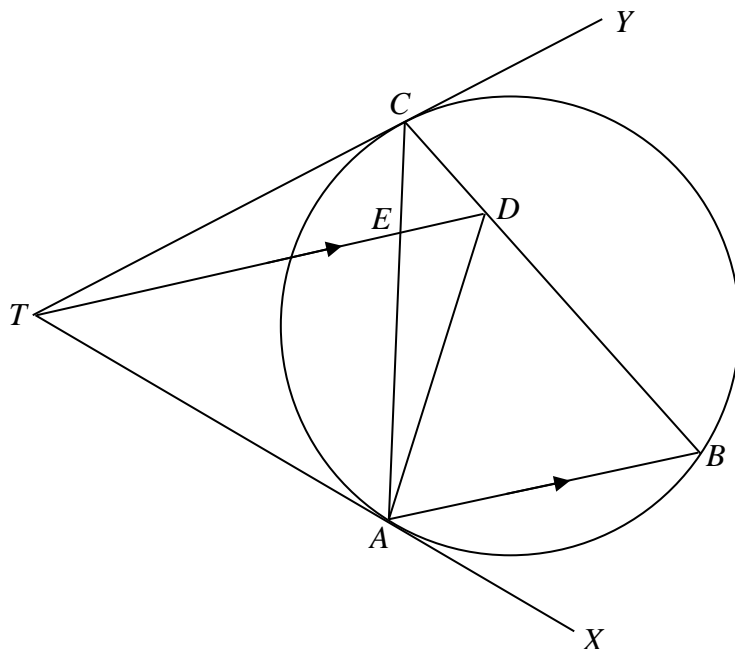
(a) Prove that angle *ACB* is equal to angle *ATD*. [2]

(b) Explain why a circle can be drawn passing through the points *T*, *A*, *D* and *C*. [1]

(c) Hence, prove that *CE* × *EA* = *DE* × *TE*. [4]

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