Name:________________________________ Class:____________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2022

MATHEMATICS **9758/01**
Higher 2
16 September 2022

Paper 1 **3 hours**

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given by [ ] at the end of each question or part question.

For Candidate's Use	For Examiner's Use
Question Number	**Marks Obtained**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	/ 100

This document consists of **5** printed pages.

[Turn over

1 A function f is defined by $\mathrm{f}(x) = ax^3 + bx^2 + cx + d$. The graph of $y = \mathrm{f}(x)$ passes through $(1, -19)$ and has a maximum point $(-1,\ 13)$. Given that $\int_{-1}^{0} \mathrm{f}(x)\ \mathrm{d}x = 9.5$, find the values of a, b, c and d. [5]

2 Given that $\mathrm{f}(x) = \dfrac{ax^2 + 3ax + 10}{x+2}, x \in \mathbb{R}, x \neq -2$, where a is a constant.

(i) Given that $a = 3$, solve $\mathrm{f}(x) \geq 2x + 6$. [3]

(ii) Find the set of values of a such that $\mathrm{f}'(x) > 0$ for all real values of x, $x \neq -2$. [4]

3 A curve C has equation $y = \dfrac{1}{x^2 - 6ax}$, where $a > 0$.

(i) Sketch the curve C and give the equations of any asymptotes and the coordinates of any turning points in terms of a where appropriate. [4]

(ii) Describe the transformation that maps the graph of C onto the graph of $y = \dfrac{1}{x^2 - 9a^2}$. [2]

4 **(i)** Express $\dfrac{1}{(2r+1)(2r+3)}$ in the form $\dfrac{A}{2r+1} + \dfrac{B}{2r+3}$ where A and B are constants to be determined. [2]

(ii) Hence find the sum of the series

$$\frac{1}{(3)(5)} + \frac{1}{(5)(7)} + \frac{1}{(7)(9)} + \frac{1}{(9)(11)} + \frac{1}{(11)(13)} + \ldots \frac{1}{(2n+1)(2n+3)},$$

giving your answer in the form $k - \mathrm{f}(n)$, where k is a constant and $\mathrm{f}(n)$ is a function in n to be determined. [3]

(iii) Give a reason why the series

$$\frac{1}{(3)(5)} + \frac{1}{(5)(7)} + \frac{1}{(7)(9)} + \frac{1}{(9)(11)} + \frac{1}{(11)(13)} + \ldots$$

converges and write down the value of the sum to infinity. [2]

(iv) Hence find

$$\frac{1}{(10)(14)} + \frac{1}{(14)(18)} + \frac{1}{(18)(22)} + \frac{1}{(22)(26)} + \ldots.$$ [3]

5 **(a)** Find $\int 3\sin x\cos 3x \, \mathrm{d}x$. [2]

(b) Use the substitution $\theta = \sqrt{x}$ to find the exact value of $\int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\pi}} \theta^3 \sin\left(\theta^2\right) \mathrm{d}\theta$. [5]

6 **(i)** By means of the substitution $u = xy$, express the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + y - 2\left(xy\right)^2 = 0$$

into the form $\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{f}(u)$, where $\mathrm{f}(u)$ is a function in u to be found. [2]

(ii) Hence find the general solution of y in terms of x. [3]

(iii) Find the equation of the solution curve that passes through $\left(1, \frac{1}{2}\right)$. [1]

(iv) State a particular solution for which the solution curve has no stationary point. [1]

7 The equation of a curve C is $2x^3 + y^3 - 3xy = k$, where k is a constant.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [2]

It is given that C has a tangent which is parallel to the y-axis.

(ii) Show that the y-coordinate of the point of contact of the tangent with C must satisfy

$$ay^6 + by^3 - k = 0,$$

where the constants a and b are to be determined. [3]

(i) Hence, find the values of k when the line $x = 1$ is a tangent to the curve C. [3]

8 Given that $y = \frac{\ln\sqrt{1-x}}{2+x}$, where $-1 \le x < 1$, show that

$$2y + 2(2+x)\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{1}{1-x} = 0.$$ [2]

(i) By further differentiation, find the Maclaurin's series for y up to and including the term in x^3. [5]

(ii) Verify that the same result can be obtained if the standard series expansions are used. [3]

(iii) By substituting $x = -1$ to your result, find an approximate value for ln 2, giving your answer to 4 decimal places. [2]

9 **(a)** The functions f and g are defined by

$$\mathrm{f} : x \to \frac{x-2}{x+2}\ ,\ x \in \mathbb{R},\ x \neq -2\ ,$$
$$\mathrm{g} : x \to\ -x^2\ ,\ x \in \mathbb{R},\ x < -\sqrt{2}.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(ii) Find an expression for fg(x) and hence, or otherwise, find $(\mathrm{fg})^{-1}(3)$. [4]

(b) It is given that

$$\mathrm{h}(x) = \begin{cases} -4x-12 & \text{for } -4 \le x \le -2, \\ x|x| & \text{for } -2 < x \le 2 \end{cases}$$

and that $\mathrm{h}(x) = \mathrm{h}(x+6)$ for all real values of x.

(i) Evaluate $\mathrm{h}(-4)$ and $\mathrm{h}(12)$. [2]

(ii) Sketch the graph of $y = \mathrm{h}(x)$ for $-4 \le x \le 6$ and explain why h has no inverse. [4]

10 A customer owes a bank \$15 000. In the middle of every month, the customer pays \$$x$ to the bank where $x \le 1000$. At the end of every month, the bank adds interest at a rate of 4% of the remaining amount still owed. This process continues every month until the money owed is repaid in full.

(i) Find the value of x, for which the customer still owes \$15 000 at the end of the first month. [3]

(ii) Show that the amount owed at the end of the nth month is

$$(1.04)^n(15\ 000) - kx(1.04^n - 1),$$

where k is a constant to be determined. [4]

(iii) Find the amount that the customer still owes the bank at the beginning of the 13th month if x = 1000. [2]

(iv) Find the least number of months required to repay the loan, given that x = 800. [3]

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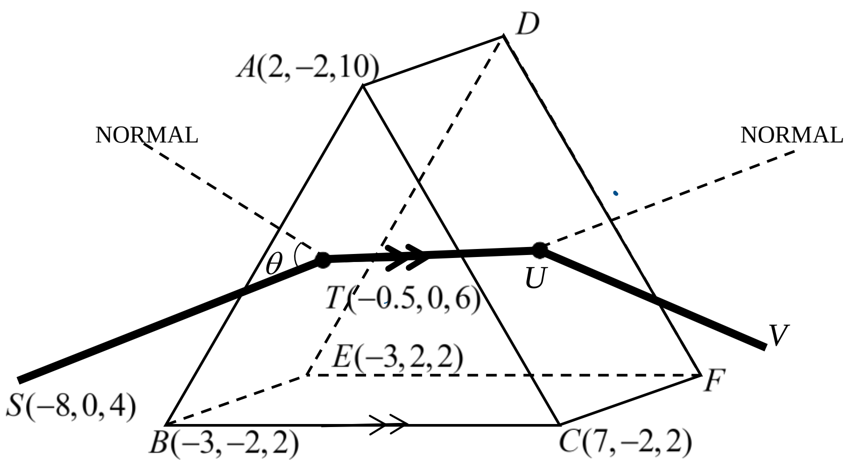

A light ray passes from air into a material made into a triangular prism *ABCFDE* with triangular sides *ABC* and *DEF* and rectangular sides *ABED*, *ACFD* and *BCFE*. The coordinates of the vertices *A*, *B*, *C* and *E* are shown in the diagram. A ray of light is sent from a monochromatic light source at point $S(-8, 0, 4)$ to enter the prism at point $T(-0.5, 0, 6)$. It then emerges at point *U* and is picked up by a sensor at point *V*. The refracted ray *TU* is parallel to the side *BC* and the equation of the plane *ADFC* is given by $8x + 5z = 66$.

(i) Find a vector equation of the plane *ABED* in scalar product form. [3]

(ii) Find the angle of incidence θ, the acute angle *ST* makes with the normal of the plane *ABED*. [3]

(iii) Find the coordinates of *U*. [4]

(iv) Find the shortest distance from *T* to the plane *ADFC*. [3]