

Name: \_\_\_\_\_

Class: \_\_\_\_\_



# JURONG PIONEER JUNIOR COLLEGE

## JC2 Preliminary Examination 2022

### MATHEMATICS Higher 2

**9758/02**
**20 September 2022**
**Paper 2**
**3 hours**

Candidates answer on the Question Paper.

Additional materials:      List of Formulae (MF 26)

#### READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.  
 Write in dark blue or black pen on both sides of the paper.  
 You may use an HB pencil for any diagrams or graphs.  
 Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.  
 Give non-exact numerical answers correct to 3 significant figures,  
 or 1 decimal place in the case of angles in degrees, unless a different  
 level of accuracy is specified in the question.  
 You are expected to use an approved graphing calculator.  
 Unsupported answers from a graphing calculator are allowed unless  
 a question specifically states otherwise.  
 Where unsupported answers from a graphing calculator are  
 not allowed in a question, you are required to present the  
 mathematical steps using mathematical notations and not calculator  
 commands.  
 You are reminded of the need for clear presentation in your answers.

The number of marks is given by [    ] at the end of each question or  
 part question.

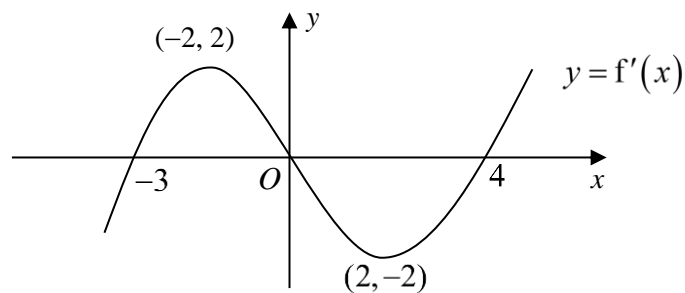
For Candidate's Use	For Examiner's Use
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
<b>Total Marks</b>	/ 100

 This document consists of **8** printed pages.

**[Turn over**

## Section A: Pure Mathematics [40 marks]

- 1 (a) The diagram shows the **derivative** graph of  $y = f(x)$ .



Justifying your answers, find the range of values of  $x$  for which the graph  $y = f(x)$  is

- (i) decreasing, [2]
  - (ii) increasing and concave downwards. [2]
- (b) [It is given that the volume and surface area of a sphere of radius  $r$  is  $\frac{4}{3}\pi r^3$  and  $4\pi r^2$  respectively.]
- Air is pumped into a spherical balloon at a constant rate of  $12\pi \text{ cm}^3$  per second.
- (i) Find the rate of increase of the balloon's surface area when the volume is  $\frac{256}{3}\pi \text{ cm}^3$ . [4]
  - (ii) Show that the rate of increase of the balloon's radius is inversely proportional to the surface area of the balloon. [1]

- 2 (a) With reference to the origin  $O$ , the points  $A$ ,  $B$  and  $X$  are  $\overrightarrow{OA} = \mathbf{a}$ ,  $\overrightarrow{OB} = \mathbf{b}$  and  $\overrightarrow{OX} = \frac{1}{8}\mathbf{a} + \frac{3}{8}\mathbf{b}$ . The point  $Y$  lies on  $AB$  such that  $O$ ,  $X$  and  $Y$  are collinear. Express  $\overrightarrow{OY}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$  and find the ratio of  $AY : YB$ . [5]
- (b) The points  $P$ ,  $Q$  and  $R$  have position vectors  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$  respectively.  $P$  and  $Q$  are fixed and  $R$  varies. Describe geometrically the set of possible positions of the point  $R$  such that
- (i)  $(\mathbf{r} - \mathbf{p}) \times \mathbf{q} = \mathbf{0}$ , [2]
- (ii)  $(\mathbf{r} - \mathbf{p}) \cdot \mathbf{q} = 0$ . [2]

3 **Do not use a calculator in answering this question.**

- (a) If  $z = 1 + i$  is a root of the equation  $z^4 + 4z^2 - 8z + 12 = 0$ , find the other roots. [4]
- (b) By expressing in the exponential form or otherwise, show that

$$\frac{1 + \sin \frac{3\pi}{8} + i \cos \frac{3\pi}{8}}{1 + \sin \frac{3\pi}{8} - i \cos \frac{3\pi}{8}} = \cos \frac{\pi}{8} + i \sin \frac{\pi}{8}. \quad [3]$$

Hence find the two smallest positive integer values of  $n$  for which

$$\left( \frac{1 + \sin \frac{3\pi}{8} + i \cos \frac{3\pi}{8}}{1 + \sin \frac{3\pi}{8} - i \cos \frac{3\pi}{8}} \right)^n - i = 0. \quad [3]$$

- 4 A curve  $C$  is given by the parametric equations

$$x = 2 + 2\sin \theta, \quad y = 2\cos \theta + \sin 2\theta, \quad \text{for } -\pi < \theta \leq \pi.$$

- (i) Sketch the curve, indicating clearly the coordinates of the axial intercepts. [2]
- (ii) Find the exact area bounded by the curve. [5]
- (iii) Verify that  $y = x \cos \theta$ .

Deduce that the Cartesian equation of the curve  $C$  is

$$4y^2 = 4x^3 - x^4. \quad [3]$$

- (iv) Find the volume of the solid of revolution formed when the curve  $C$  is rotated  $\pi$  radians about the  $x$ -axis. [2]

### Section B: Probability and Statistics [60 marks]

- 5 A bag contains 2 fair tetrahedral dice. The first die has faces labelled 1, 1, 2, and 3 and the second die has faces labelled 1, 2, 3 and 3. A die is taken at random from the bag and thrown. The score,  $W$  is defined as follows:

If the first die is picked and thrown, the score is defined as twice the number which appears on its base.

If the second die is picked and thrown, the score is the number which appears on its base.

- (i) Show that  $P(W = 2) = \frac{3}{8}$  and find the probability distribution of  $W$ . [3]
- (ii) Find  $E(W)$  and  $\text{Var}(W)$ . [4]

- 6 A computer is used to generate codes consisting of four letters followed by two digits. Each of the four letters generated is equally likely to be any of the twenty-six letters of the alphabet “A – Z”. Each of the two digits generated is equally likely to be any of the ten digits “0 – 9”.

Find the probability that a randomly chosen code has

- (i) four different letters and two different digits, [2]
- (ii) two different consonants and two different vowels, where the consonants and vowels alternate, [3]
- (iii) two letters the same, two letters different and two digits the same. [3]

- 7 (a) Draw separate scatter diagrams, each with 8 data points, all in the first quadrant which represent the situation where the product moment correlation coefficient between variables  $x$  and  $y$  is

(i) between  $-0.8$  and  $-0.5$ ,

(ii) 0. [2]

- (b) In a chemical reaction, the concentration,  $y$  grams/ litre of a particular reactant at time  $x$  minutes is given in this table. The product moment correlation coefficient for this data is  $-0.9811$ .

$x$ (min)	5	10	15	20	25	30	35	40
$y$ (g/ litre)	6	5.75	5.66	5.51	5.39	5.31	5.26	5.15

(i) Sketch a scatter diagram of  $y$  against  $x$  for the data given in the table. [1]

(ii) A student attempts to model the relationship between  $y$  and  $x$  with a straight line, explain whether this is likely to provide a good model. [1]

(iii) By using the model  $\frac{1}{y} = ax + b$ , where  $a$  and  $b$  are constants to be found, write down the equation for the relationship between  $x$  and  $y$ . State the product moment correlation coefficient for this model. [3]

(iv) Using the equation found in (iii), estimate the time taken for the concentration to reach 5.4 g / litre. Comment on whether we should use an equation of the form  $x = \frac{c}{y} + d$  to find the estimate instead. [2]

- 8** On average, 72 out of 100 students in a school complete online Mathematics homework by the deadline. The number of students in a class of  $n$  students who complete online Mathematics homework by the deadline is denoted by  $M$ . State, in context, two assumptions needed for  $M$  to be well-modelled by a binomial distribution. [2]

Assume now that  $M$  has a binomial distribution.

- (a) By taking  $n = 25$ ,
- (i) find the probability that fewer than 15 students in a class complete the online Mathematics homework by the deadline. [2]
  - (ii) 10 classes with  $n$  students each are chosen at random from the school. Find the probability that exactly 3 of these classes have fewer than 15 students who complete the online Mathematics homework by the deadline. [2]
  - (iii) 30 classes with  $n$  students each are chosen at random from the school. Find the probability that the mean number of students who complete the online Mathematics homework by the deadline per class is more than 19. [3]
- (b) Find the least value of  $n$  such that there is greater than 70% chance that at least 20 students in a class complete the online Mathematics homework by the deadline. [3]

- 9 A company administers two aptitude tests to the job applicants. One test measures verbal ability while the other measures written ability. Based on past experience, the verbal ability score  $V$  and the written ability score  $W$  are independent and normally distributed with means and standard deviations shown in the following table:

Test score	Mean	Standard deviation
$V$	56	8
$W$	60	12

- (i) A female applicant and a male applicant are randomly chosen. Find the probability that the female's verbal score is less than 55 and the male's written score is more than 55. [2]
- (ii) Three females and one male are randomly chosen. Find the probability that the females' total verbal score is within 15 marks of thrice the male's written score. [4]
- (iii) Five job applicants' verbal scores and six job applicants' written scores are observed. Given that  $M$  is the average score of these applicants, find  $E(M)$  and  $\text{Var}(M)$ . Hence find the probability that  $M$  is less than 60. [3]
- (iv) The company's manager set a criterion for an interview based on a combined score  $T$  where  $T = 2V + W$ . Find the value of  $t$  if 35% of the job applicants have a combined score exceeding  $t$ . [3]

- 10** The Electronic Road Pricing (ERP) system is the primary method of regulating traffic in Singapore. ERP rates are determined based on traffic conditions. If the average traffic speed rises above 45 km/h on expressways, ERP charges at that gantry will be reduced. Conversely, ERP rates will be increased if traffic moves slower than the average speed of 45 km/h on expressways.

- (a) The authority reviewed the traffic conditions on a particular expressway by measuring the speeds of 150 randomly selected cars as they pass a speed camera. The speed,  $x$  km/h was recorded. The results are summarised by

$$\sum (x - 40) = 585, \quad \sum (x - 40)^2 = 10998.$$

- (i) Find the unbiased estimates of the population mean and variance of the speed of a car on the expressway. [3]
- (ii) Test, at the 5% level of significance, whether the ERP rate needs to be increased for this expressway. [4]
- (iii) Explain the meaning of ' $p$ -value' in the context of the question in (ii). [1]
- (b) A new speed camera is installed along the expressway and it is now known that the population standard deviation of the speed of a car along this expressway is 7.79 km/h. A large random sample of  $n$  cars is observed and the average speed of 45.9 km/h is recorded. A new test is carried out at 5% level of significance. Find the least value of  $n$  that results in the reduction of the ERP rate along this expressway. [4]