HUA YI SECONDARY SCHOOL

4E

Preliminary Examination 2022

MATHEMATICS

Paper 2

Candidates answer on the Answer Space provided.

4E

4048/2

2022
2 h 30 min

Mark Scheme

[Turn Over

Setter: Ms Lee Hui Ling

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^{n}$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^{2}$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^{2}h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^{3}$$

$$\text{Area of triangle } ABC \ \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^{2}\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

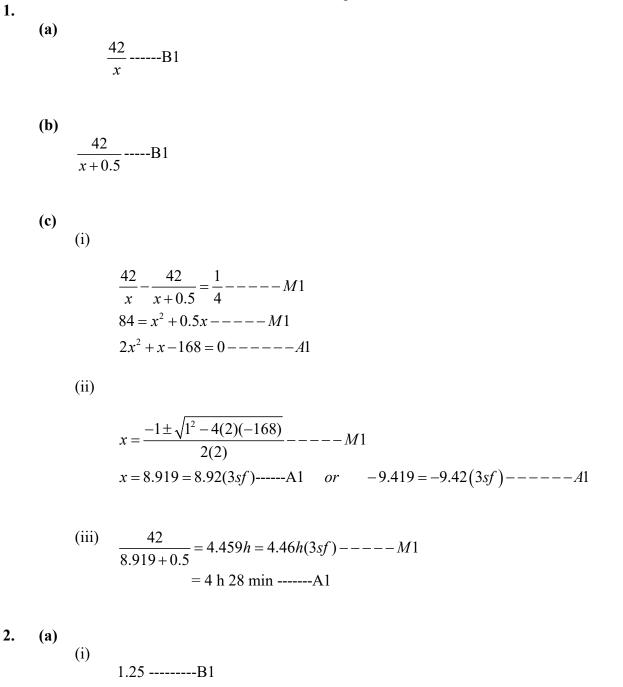$$a^{2} = b^{2} + c^{2} - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^{2}}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^{2}}$$

Answer **all** the questions.

1.

(a)

$$\frac{42}{x} \text{ ------B1}$$

(b)

$$\frac{42}{x+0.5} \text{ -----B1}$$

(c)

(i)

$$\frac{42}{x} - \frac{42}{x+0.5} = \frac{1}{4} ----- M1$$

$$84 = x^2 + 0.5x ----- M1$$

$$2x^2 + x - 168 = 0 ------ A1$$

(ii)

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-168)}}{2(2)} ----- M1$$

$$x = 8.919 = 8.92(3sf)\text{------A1} \quad or \quad -9.419 = -9.42\left(3sf\right) ------ A1$$

(iii) $\frac{42}{8.919+0.5} = 4.459h = 4.46h(3sf) ----- M1$

$$= 4 \text{ h } 28 \text{ min -------A1}$$

2. **(a)**

(i)

1.25 ---------B1

(ii)

Find area of triangle AOC: 0.5 (12)(8)(sin 1.25) = 45.55 ----M1 ecf
Find area of sector AOB: 0.5 (12^2)(1.25) = 90 ------M1 ecf
Area of shaded region = 44.45 ----------A1

(iii)

Find AC using cosine rule = 12.14 ---------M1
Perimeter = 31.1 (3sf) --------A1

(b)

(i)

32+38 = 70 (isosceles triangle AOD and AOB)

(ii)

140 (angle at center = twice angle at circumference)

(iii)
Find DCB using opposite angle of cyclic quad (M1 ecf)

Angle ODC = 180 – angle DCB (interior angles, // lines)
=180-110
=70-------A1

3.

(a)   .

$AD^2 = 4^2 + 12^2$ --------M1
$AD = 12.649$ (shown) -----A1

(b)

Area of ABCD = 96 -------M1
Lateral area = (10+12+6+12.649) x 16 =650.384 --------M1

Surface area = 842.384
=842 (3sf) -----A1

(c)

Volume of prism = 96 (16) -----M1
= 1536 -----A1

4.

(a)

3**c** - 3**a** -----**B1**

(b)

3**a** – 1.5**c** -----**B1**

(c)

$$\overrightarrow{AK} = 1.5\mathbf{c} + \frac{1}{3}(3\mathbf{a} - 1.5\mathbf{c}) - - - -M1$$

$$= \mathbf{c} + \mathbf{a}$$

$$\overrightarrow{AK} = 2\overrightarrow{KP} - - - - -M1$$

This implies that AK// KP and they have a common point K. Hence A, K and P lies on a straight line. -----A1

(d)

1/ 2 ------B1

(e)

1/6 -----B1

5.

(a)

Form equation : Area of trapezium $0.5(20 + 55)v = 450$ ---------M1

Get $v = 12$ -----A1

(b)

Acceleration = 1.2 ------M1 or use similar triangle

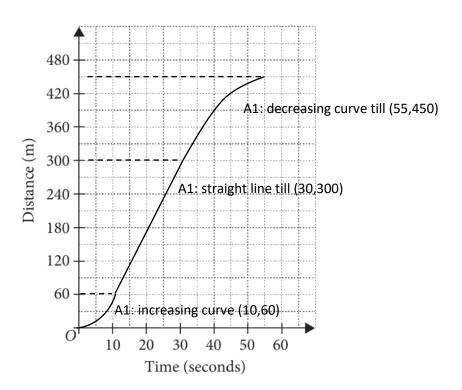Speed at t = 8 , 1.2(8)= 9.6 -----A1

(c)

[1]

The motorcyclist is travelling at a constant speed . ------B1

(d)

0.48 ------B1

(e)

A1: decreasing curve till (55,450)

A1: straight line till (30,300)

A1: increasing curve (10,60)

6.

(a)
$p = 3.4$ -----B1

(b) See graph
(c) Draw any line that cuts y- axis at -2 and intersect the curve at 2 points for $-4 \le x \le 4$.

One possible line is the line that passes through (2,-1.4) and (0, -2)

$$m = \frac{-1.4-(-2)}{2-0} = 0.3 ----- A1$$

$0.3 \le m \le 1.95$

(d)

Get y = 4 ----M1
The line y = 4 intersect the curve at 1 point, hence the equation only has one solution------A1

(e)
B1 for graph

(f)
Form $\frac{x^3}{5} - 2x + 1 = -\frac{x}{2} + 2$ ----M1

Balance equation -------M1
Get $2x^3 - 15x - 10 = 0$ ------A1

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7. **(a)**

Time taken = 11/3 h or 1 h 20 min -----M1

Time expected to arrive at B = 0005 or 12:05am ----A1

(b)

Use cosine rule :
PQ² = 300²+120²-2(300)(120)cos 116 -------M1

PQ = 368.73
=369 (3sf)

(c)
Form sine rule equation or cosine rule to find angle QPB-----M1
find angle QPB------M1 (ecf)

bearing of Q from P = 064 - 017.00 =047.0----A1

8. **(a)**

(i)

11.625---B1

(ii)

It was because we do not have the actual timing for each customer. ---B1

(iii)

5.35 -----B1

(iv)

I would go to Shop B although the the mean is slightly higher than shop A. But the smaller SD suggest that the more consistency in the waiting time. -----B1

Or

I would go to Shop A as the mean is smaller, meaning on average I will have a shorter waiting time. ----B1

Any reasoning that is logicial.

(v)

The mean will <u>remain the same</u> and the SD <u>will decrease.</u> B1 each

(b)

(i)

52 ---B1

(ii)

66----B1

(iii)

44 -----B1

(iv)

180-75 -------M1 (Find 75)
=105 ------B1

9. **(a)**

$$\begin{pmatrix} 84 & 90 & 56 \\ 92 & 60 & 61 \end{pmatrix} \text{---B1}$$

(b) $\begin{pmatrix} 1812 & 886 \\ 1673 & 853 \end{pmatrix}$ -----B1

(c) 48.9% -----B1

(d) (1742.5   869.5)--------A1

(e) The average sale for the 2 days is \$1742.50 and the average profit is \$ 869.50. ---B1

10. **(a)**

(i) Find gradient or y intercept correct ---M1
$$y = \frac{3}{8}x + 3\frac{1}{4} --- A1$$

She is not correct as the y intercept should be 3.25.

(ii) Use Pythagoras' thm or formula ------M1
AB = 5 units

(iii) -4/5 -----B1

(iv) x = -1 or 5

(b) (i)

$$\frac{130000000}{55} ----M1$$
$$= 2.37 \times 10^6 --------- A1$$

(ii)
$(1.3 \times 10^8) \div 70 \div 10 \div 150$----M1
$= 1238$ ---- A1

11 **(a)**

$$\text{interest} = \frac{2}{100} \times 40000 \times \frac{30}{12} = 2000 ---M1$$

monthly instalment $= 42000 \div 30 = 1400$---A1

(b)

$$\frac{5100}{85} \times 15 ----M1 \quad (\text{ 85\% is \$5100, find 15\%})$$
$$= 900------A1$$

(c) Teaching staff : 12 (2)(4)(\$70) + 14(2)(4)(\$90)= (\$6720 + \$10080) = \$16 800
M1
Total staff cost : 16 800 + 1800 + 45(5)(4)=\$19500

Rental and printing cost = 8400

Total operational cost per month = \$19500 + 8000+400 =27900 M1

Total fee collected (assuming each class has the minimum number of students)
= 12(5)(\$200) + 14(5)(\$300) ---M1
=\$33 000

(\$52800 if they find 8 students per class) Can find for other number as well but they have to state.

Min Profit per month = \$5100 ---M1

Her target of a minimum of \$5000 per month can be reached as her minimum profit per month is \$5100. –A1 justification

\$5100 x 6 months = \$30 600 < \$45 100 + 2000 interest ----M1

She might miss her target of recovering her start up cost within 6 months as the total profit for 6 months assuming she get the minimal number of students per class is less than the start up cost.--_A1