HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2022 **4E5N**

ADDITIONAL MATHEMATICS

Paper 2

MARKING SCHEME

1	**(a)**		$b^2 - 4ac = (2a)^2 - 4(1)(2a^2 - 3b)$ $= 4a^2 - 8a^2 + 12b$ $= 12b - 4a^2$ $= 4(3b - a^2)$ Since $3b - a^2 > 0,$ hence $\mathrm{b}^2 - ac > 0.$ Two real and distinct roots.
	(b)		Subst $y = 5 - 2x$ into $y^2 + y - 3x = 9$ $(5-2x)^2 + 5 - 2x - 3x = 9$ $25 - 20x + 4x^2 + 5 - 5x = 9$ $4x^2 - 25x + 21 = 0$ $(4x - 21)(x - 1) = 0$ $x = \frac{21}{4}$ or $x = 1$
	(c)	**(i)**	$h = -\frac{1}{5}t^2 + 4t + 2$ $= -\frac{1}{5}(t^2 - 20t - 10)$ $= -\frac{1}{5}[(t-10)^2 - 100 - 10]$ $= -\frac{1}{5}[(t-10)^2 - 110]$ $= -\frac{1}{5}(t-10)^2 + 22$
		(ii)	Max height = 22 m
2			$(10 + 2\sqrt{3})\pi = \frac{1}{3}\pi r^2(3 - \sqrt{3})$ $r^2 = \frac{30 + 6\sqrt{3}}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$ $= \frac{90 + 30\sqrt{3} + 18\sqrt{3} + 18}{9 - 3}$ $= \frac{108 + 48\sqrt{3}}{6} = 18 + 8\sqrt{3}$ $l^2 = 18 + 8\sqrt{3} + (3 - \sqrt{3})^2$ $= 18 + 8\sqrt{3} + 9 - 6\sqrt{3} + 3$ $= 30 + 2\sqrt{3}$

3	**(a)**	$\left(x^2-\frac{1}{3x}\right)^9$ $T_r=\binom{9}{r}\left(x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$ Powers of $x=18-2r-r$ $=18-3r$ $=3(6-r)$ Multiple of 3
	(b)	$\left(x^2-\frac{1}{3x}\right)^9(1+6x^3+9x^6)$ Term with $x^{-6}, r=8\to\frac{9}{3^8x^6}$ Term with $x^{-3}, r=7\to-\frac{36}{3^7x^3}$ Term with $x^0, r=6\to\frac{84}{3^6}$ Product $=\frac{84}{3^6}(1)-\frac{36}{3^7x^3}(6x^3)+\frac{9}{3^8x^6}(9x^6)=\frac{28}{243}-\frac{8}{81}+\frac{1}{81}=\frac{7}{243}$
4	**(a)**	$MidptAB=\left(\frac{5-11}{2},\frac{7+15}{2}\right)=(-3,11)$ $GradAB=\frac{15-7}{-11-5}=-\frac{1}{2}$ $GradPB=2$ $EqnPB: y=2x+c$ $11=2(-3)+c$ $c=17$ $Eqn: y=2x+17$
	(b)	$y=2x+17------(1)$ $y=-2x-3------(2)$ $2x+17=-2x-3$ $x=-5$ $y=7$ $C(-5,7)$ $Radius=\sqrt{(5+5)^2+(0)}=10$ $EqnCircle: (x+5)^2+(y-7)^2=100$

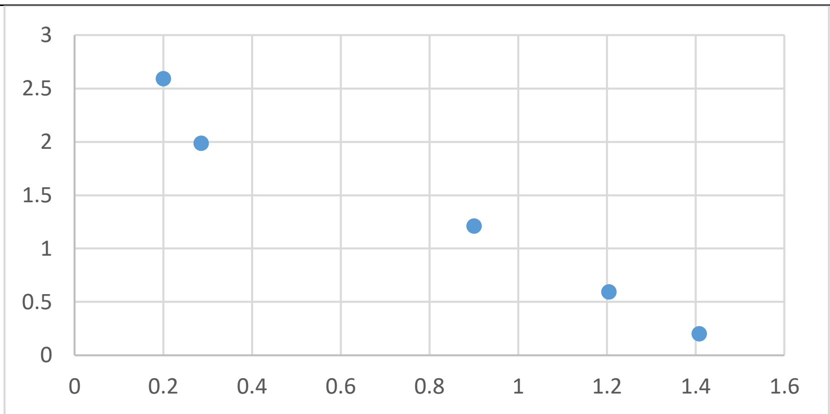
	(c)	AD is diameter $C(-5,7)$ is midpoint of AD $\left(\frac{5+x}{2},\frac{7+y}{2}\right)=(-5,7)$ $x=-15$ $y=7$ $D(-15,7)$
	(d)	$-15<k<5$
	(e)	$(x-5)^2+(y-7)^2=100$
5	**(a)**	$y=\frac{1-\sin x}{\cos x}$ $\frac{dy}{dx}=\frac{\cos x(-\cos x)-(1-\sin x)(-\sin x)}{\cos^2 x}$ $=\frac{-\cos^2 x+\sin x-\sin^2 x}{\cos^2 x}$ $=\frac{\sin x-1}{\cos^2 x}$ $=\tan x\sec x-\sec^2 x$
	(b)	$\int \tan x\sec x-\sec^2 x\ dx=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\ dx-\int \sec^2 x\ dx=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\ dx-\tan x=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\ dx=\frac{1-\sin x}{\cos x}+\tan x+c$ $\int_0^{\frac{\pi}{4}} \tan x\sec x\ dx=\left[\frac{1-\sin x}{\cos x}+\tan x\right]_0^{\frac{\pi}{4}}$ $=\left[\frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}+1\right]-[1+0]$ $=\sqrt{2}-1+1-1=\sqrt{2}-1$

6	**(a)**		$\int_0^5 3f(x)\ dx + \int_5^3 x - kf(x)\ dx = 8$ $3(8) - \int_3^5 x - kf(x)\ dx = 8$ $24 - \int_3^5 x\ dx + \int_3^5 kf(x)\ dx = 8$ $24 - \left[\frac{x^2}{2}\right]_3^5 + k\int_3^5 f(x)\ dx = 8$ $24 - \left(\frac{25}{2} - \frac{9}{2}\right) + k(4) = 8$ $4k = -8$ $k = -2$
	(b)	**(i)**	$\frac{-14x^2+14x-3}{x(2x-1)^2} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$ $-14x^2+14x-3 = A(2x-1)^2 + Bx(2x-1) + Cx$ Let $x = 0, -3 = A$ Let $x = \frac{1}{2}, -\frac{14}{4} + 7 - 3 = \frac{1}{2}C \to C = 1$ Let $x = 1, -14+14-3 = A+B+C$ $-3 = -2 + B$ $B = -1$ $\frac{-14x^2+14x-3}{x(2x-1)^2} = -\frac{3}{x} - \frac{1}{2x-1} + \frac{1}{(2x-1)^2}$
		(ii)	$\int \frac{-14x^2+14x-3}{x(2x-1)^2}\ dx$ $= \int -\frac{3}{x} - \frac{1}{2x-1} + \frac{1}{(2x-1)^2}\ dx$ $= -3\ln x - \frac{1}{2}\ln(2x-1) + \frac{(2x-1)^{-1}}{2(-1)} + c$ $= -3\ln x - \frac{1}{2}\ln(2x-1) - \frac{1}{2(2x-1)} + c$
7	**(a)**		$PQ = 100\sin\theta - 40\cos\theta$ $QR = 100\cos\theta + 40\sin\theta$ $P = 100 + 40 + 100\sin\theta - 40\cos\theta + 100\cos\theta + 40\sin\theta$ $= 140 + 140\sin\theta + 60\cos\theta$

	(b)	$P = 140 + 140\sin\theta + 60\cos\theta$ $60\cos\theta + 140\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{23200}$ $\alpha = 66.8°$ $P = 140 + \sqrt{23200}\cos(\theta - 66.8°)$
	(c)	$250 = 140 + \sqrt{23200}\cos(\theta - 66.8°)$ $\cos(\theta - 66.8°) = \frac{110}{\sqrt{23200}}$ Basic angle = 43.764° (Q1, 4) $\theta - 66.8° = -43.764°$ or $43.764°$ $\theta = 23.0°$
8.	**(a)**	$\frac{d^2y}{dx^2} = 6e^{3x} - x$ $\frac{dy}{dx} = \frac{6e^{3x}}{3} - \frac{x^2}{2} + c$ $5 = 2 + c$ $c = 3$ $\frac{dy}{dx} = \frac{6e^{3x}}{3} - \frac{x^2}{2} + 3$ $y = \frac{2e^{3x}}{3} - \frac{x^3}{6} + 3x + c$ $\frac{2}{3}e^6 = \frac{2}{3}e^6 - \frac{4}{3} + 6 + c$ $c = -\frac{14}{3}$ $y = \frac{2e^{3x}}{3} - \frac{x^3}{6} + 3x - \frac{14}{3}$

<table>
<tr><td></td><td>(b)</td><td colspan="2">$\frac{dy}{dx}=2e^{3x}-\frac{x^2}{2}+3$
When $x=2$
$\frac{dy}{dx}=2e^6+1$
$y=(2e^6+1)x+c$
$\frac{2}{3}e^6=(2e^6+1)(2)+c$
$\frac{2}{3}e^6=4e^6+2+c$
$c=-\frac{10}{3}e^6-2$
$y=(2e^6+1)x-\frac{10}{3}e^6-2$</td></tr>
<tr><td>9.</td><td>(a)</td><td colspan="2">$\lg y=\lg a+b\lg x$
$\lg y=b\lg x+\lg a$

$Y=\frac{1}{3}X+c$
$7=\frac{1}{3}(6)+c$
$c=5$

$b=\frac{1}{3}$
$\lg a=5$
$a=10^5$</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td>

Axis M1, Points and line M1

Incorrect value: $y=3.5$ A1</td></tr>
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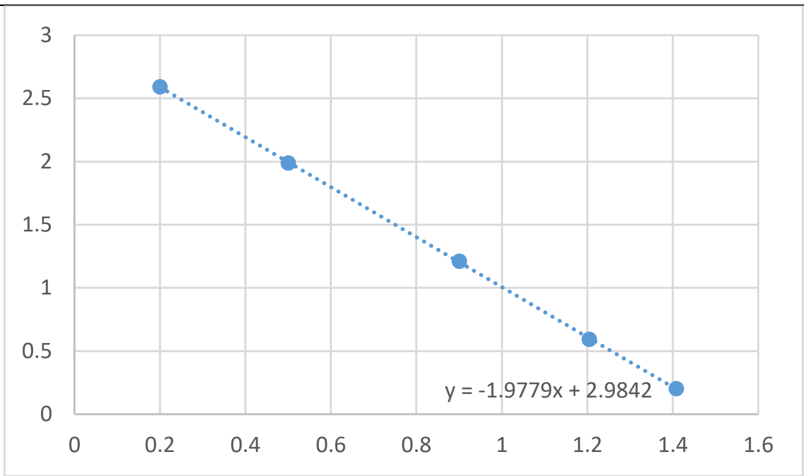
		(ii)	 $\frac{1}{y}=\frac{1}{2}$ Correct value of $y=2$
		(iii)	$y=\frac{p}{x^2+k}$ $x^2y+ky=p$ $x^2y=-ky+p$ $x^2=\frac{p}{y}-k$ Gradient $= p=-2$ Vertical intercept $=-k=3$ $k=-3$
10	**(a)**	$v=\frac{1}{2}t^2-t-4$ $s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t+c$ When $s=0, t=0$ therefore $c=0$ $s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t$ At max velocity, $\frac{dv}{dt}=0$ $t-1=0$ $t=1$ Displacement $=\frac{1}{6}-\frac{1}{2}-4=-\frac{13}{3}$	
	(b)	$t^2-2t-8=0$ $(t-4)(t+2)=0$ $t=4$ or -2	

	(c)	$s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t$ $t=0, s=0$ $t=4, s=-13\frac{1}{3}$ $t=6, s=-6$ Total distance travelled $=13\frac{1}{3}+7\frac{1}{3}=20\frac{2}{3}$ m

-End of Paper-