Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E5N | Preliminary Examination 2022 | 4E5N

ADDITIONAL MATHEMATICS

4049/01

Paper 1

31 August 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number in the spaces provided at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use
90

This document consists of **16** printed pages including the cover page.

Mathematical Formula

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)....(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\tan^2 A + 1 = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. A curve has the equation $y = x^2 + px - 4p + 9$, where p is a constant.

(a) Find the range of values of p such that $x^2 + px - 4p + 9$ is always positive for all real values of x. [3]

(b) In the case where $p = 4$, show that the line $y = 6x - 8$ is a tangent to the curve. [3]

2. **(a)** The function $f(x) = 2x^3 + ax^2 + bx - 6$ is divisible by $x+3$ and $f'(x)$ leaves a remainder of 1 when divided by $x-1$.

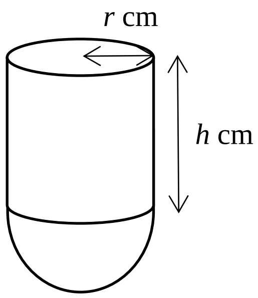
(i) Find the values of *a* and *b*. [3]

(ii) Using the integer values of *a* and *b* found in part **(i)**, factorise $f(x)$ completely. [3]

(iii) Hence, solve $2+3y-11y^2-6y^3=0$. [2]

(b) Fully factorize the expression $54x^6-16y^3$. [2]

3. A factory manufactures miniature containers that are made up of a closed circular cylinder and a hemisphere which have the same radius. The radius of the cylinder is r cm and its height is h cm. The volume of the container is $60\pi\ \text{cm}^3$.



(a) Express h in terms of r. [2]

(b) The container is made up of two different types of thin metal sheets with negligible thickness. The cost of the cover and curved surface of the cylinder is 3 cents per cm^2 and that of the hemisphere is 4 cents per cm^2. The total cost of materials to make the container is C cents.

Show that $C = 7\pi r^2 + \dfrac{360\pi}{r}$. [3]

(c) Given that r can vary, find the value of r which gives a stationary value of C. [2]

(d) Find the nature of this stationary value and explain if the factory owner should continue to produce this container if he wants to keep the cost below $5.60. [3]

4. **(a)** Solve the equation $2^{4p+1} + 20(4^{p-1}) = 3$. [4]

(b) Explain why $2^{4p+1} + 20(4^{p-1}) = k$ has no solution if $k < -3\frac{1}{8}$. [3]

5. **(a)** Given that $\dfrac{(\log_y x)^2}{\log_x y} + 64 = 0$, express y in terms of x. [3]

(b) Solve the equation $\log_2 x + 3 = 2\log_2(x-1)$. [4]

6. The height of water in a harbour at time t hours after midnight on a certain day is modelled by the formula $h = 4.8\sin k\pi t + 5.1$ where $0 \le t \le 24$.

(a) State the maximum and minimum levels of the tide. [2]

(b) A tide cycle starts at high tide and ends when it is high tide again. Given that there are 2 tide cycles in 24 hours, show that $k = \dfrac{1}{6}$. [1]

(c) Boats can come into the harbour when the tide is above 2 m. Find the range of times at which the boats can come into the harbour in the first 12 hour cycle. [4]

7. **(a)** **(i)** Prove the identity $\cot x - \sin 2x = \cot x \cos 2x$. [3]

(ii) Hence, solve the equation $4\cot x - 4\sin 2x = \cos 2x$ for $0 \le x \le \pi$. [4]

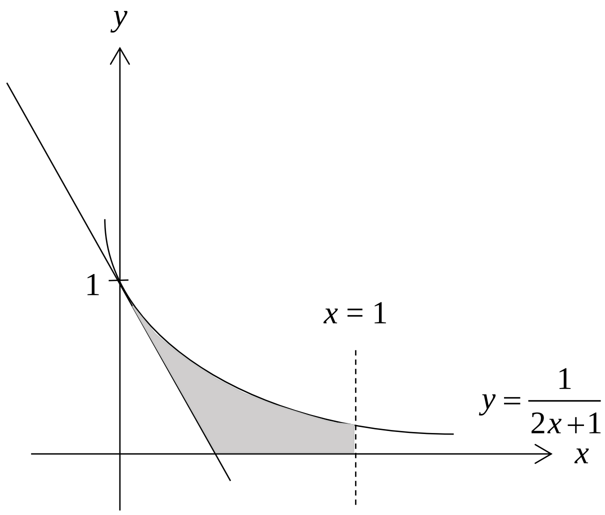
(b) A and B are acute angles such that $\sin(A+B) = \frac{6}{7}$ and $\sin B \cos A = \frac{2}{7}$.

Without calculating angles A and B, find $\frac{\tan A}{\tan B}$. [4]

8. **(a)** A particle moves along the curve $y = \dfrac{2}{\sqrt{3x+1}}$ in such a way that the y-coordinate of the particle is decreasing at a constant rate of 3 units per second. Find the y-coordinate of the particle at the instant when the x-coordinate is increasing at 0.125 units per second. [5]

(b) Show that $y = x \ln x^2$ is an increasing function when $x > \dfrac{1}{e}$. [2]

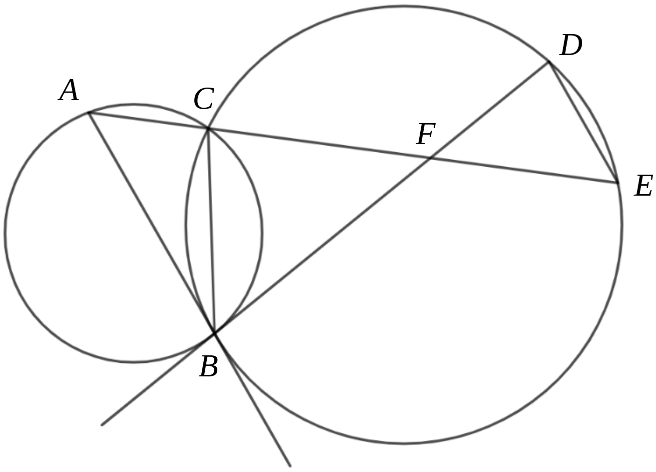
9. The diagram below shows the graph of $y = \dfrac{1}{2x+1}$ and the tangent to the curve where the curve intersects the y-axis.



(a) Find the equation of the tangent. [3]

(b) Calculate the area of the shaded region. [4]

10. In the diagram below, BD is a tangent to circle ACB and AB is a tangent to circle $BCDE$. $ACFE$ is a straight line.

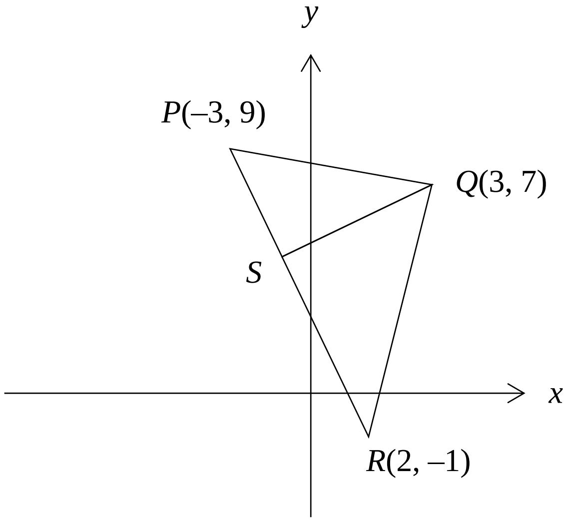


(a) Show that AB is parallel to DE. [3]

(b) Name two triangles that are similar to triangle ABF. [2]

(c) Show that $AF \times CF = BF^2$. [2]

11. The diagram below, which is not drawn to scale, shows a triangle PQR with vertices $P(-3,\ 9)$, $Q(3,\ 7)$ and $R(2,\ \ -1)$. The point S which lies on PR is the foot of the perpendicular line from Q.



(a) Find the equation of QS. [3]

(b) Find the equation of a line that is parallel to PR and passes through Q. [2]

(c) Find the area of triangle *PQR*. [2]

12. **(a)** Sketch the graph of $y = 3^x$. [1]

(b) In order to solve the equation $\log_9(5-x) = \frac{x}{2}$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = 3^x$. Find the equation of the straight line. [2]

(c) State the number of solutions for the equation $\log_9(5-x) = \frac{x}{2}$. [1]

-End of Paper-