Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2022 4E5N

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

14 September 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number in the spaces provided at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use
90

This document consists of **17** printed pages including the cover page.

Mathematical Formula

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$1 + \cot^2 A = \operatorname{cosec}^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1. **(a)** It is given that a and b are positive constants such that $3b > a^2$. Explain why $y = x^2 + 2ax + 2a^2 - 3b$ has two x-intercepts for all real values of x. [3]

(b) The curve $y^2 + y - 3x = 9$ and the line $2x = 5 - y$ intersect at the points P and Q. Find the x-coordinate of P and Q. [3]

(c) An object is projected upwards from a mini launcher. The height, h m of the object above the ground at the time t seconds is given by

$h=-\frac{1}{5}t^2+4t+2$.

(i) Express $h=-\frac{1}{5}t^2+4t+2$ in the form $h=-a(t-b)^2+c$. [3]

(ii) Using your answer in **(i)**, explain whether the object can reach a height of 25 meters when projected from the mini laucher. [1]

2. A right circular cone has a vertical height of $\left(3-\sqrt{3}\right)$ cm and a slant height of l cm. The volume of the cone is $\left(10+2\sqrt{3}\right)\pi$ cm^3. **Without using a calculator,** express l^2 in the form $a+b\sqrt{3}$, where a and b are constants.
[Volume of cone = $\frac{1}{3}\pi r^2 h$]

[5]

3 **(a)** By considering the general term in the binomial expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, explain why the powers of x are always divisible by 3. [3]

(b) Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{3x}\right)^9 (1+3x^3)^2$. [5]

4. The points $A(5, 7)$ and $B(-11, 15)$ lie on a circle C. The centre of the circle O lies on the line $y = -2x - 3$.

(a) Find the equation of the perpendicular bisector of AB. [3]

(b) Find the equation of the circle. [4]

(c) Point D lies on the circle such that angle ABD is 90°. Find the coordinates of D, show your working and explanation clearly. [2]

(d) Given that the circle intersects the line $x = k$ at two points, state the range of values of k. [1]

(e) The circle C_2 is a reflection of C in the y-axis. Find the equation of C_2. [1]

5. The equation of a curve is $y = \dfrac{1-\sin x}{\cos x}$.

(a) Show that $\dfrac{dy}{dx} = \tan x \sec x - \sec^2 x$. [2]

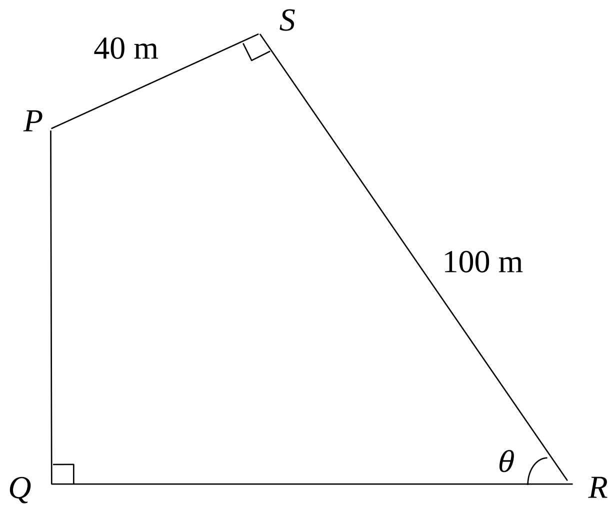
(b) Hence find $\int_0^{\frac{\pi}{4}} \tan x \sec x \; dx$, giving your answer in the exact value. [5]

6. **(a)** Given that $\int_0^3 f(x)\ dx = \int_3^5 f(x)\ dx = 4$, find the value of k for which $\int_0^5 3f(x)\ dx + \int_5^3 x - kf(x)\ dx = 8$. [4]

(b) **(i)** Express $\frac{-14x^2+14x-3}{x(2x-1)^2}$ in partial fractions. [5]

(ii) Hence find $\int \frac{-14x^2+14x-3}{x(2x-1)^2}\,dx$. [3]

7. The diagram below shows a fenced field *PQRS*. It is given that *PS* = 40 m, *SR* = 100 m, both angles *PQR* and *PSR* are right angles and angle *QRS* = θ, where θ is an acute angle in degrees.



(a) Show that the perimeter, *P* m, of the fence *PQRS* is given by $P = 140 + 140\sin\theta + 60\cos\theta$. [3]

(b) Express P in the form $140 + R\cos(\theta - \alpha)$, where $R > 0$ and $0° \le \theta \le 90°$. [3]

(c) Find the value of θ when $P = 250$. [2]

8. A curve is such that $\dfrac{d^2y}{dx^2} = 6e^{3x} - x$ and passes through the point $\left(2,\ \dfrac{2}{3}e^6\right)$.

(a) Given that $\dfrac{dy}{dx} = 5$ when $x = 0$, find the equation of the curve. [5]

(b) Find the equation of the tangent at the point $\left(2,\ \dfrac{2}{3}e^6\right)$ leaving your answer in the exact form. [3]

9. **(a)** Two variables x and y are related by the equation $y = ax^b$ where a and b are constants. When the graph of lg y against lg x is drawn, a straight line is obtained. Given that this line passes through the point (6, 7) and has a gradient of $\frac{1}{3}$, find the value of a and of b. [4]

(b) The table shows some experimental values of x and y which are known to be related by the equation $y = \frac{p}{x^2 + k}$. One of the values of y was recorded wrongly.

x	1.61	1.41	1.10	0.77	0.45
y	5.00	3.50	1.11	0.83	0.71

(i) On the grid on the next page, plot x^2 against $\frac{1}{y}$ and hence determine which value of y, in the table above, is the incorrect recording. [3]

(ii) Using the graph, estimate a value of y to replace the incorrect recoding of y found in part **(i)**. [2]

(iii) Use your graph to estimate the value of p and k. [3]

10. A particle X travels in a straight line so that t seconds after leaving a fixed point O, its velocity v m/s, is given by $v = \frac{1}{2}t^2 - t - 4$.

(a) Calculate the displacement of the particle X when the velocity is minimum. [4]

(b) Find the time when the particle turns. [2]

(c) Find the total distance travelled in the first 6 seconds. [3]

-End of Paper-