HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2022 **4E5N**

ADDITIONAL MATHEMATICS

Paper 1

MARKING SCHEME

1.	**(a)**		$p^2-4(1)(-4p+9)<0$ $p^2+16p-36<0$ $(p-2)(p+18)<0$ $-18<p<2$
	(b)		$x^2+4x-16+9=6x-8$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$ Since there is only one solution, $y=6x-8$ is a tangent. Or use b^2-4ac.
2.	**(a)**	**(i)**	$f(-3)=2(-3)^3+a(-3)^2+b(-3)-6=0$ $-54+9a-3b-6=0$ $9a-3b=60$ $3a-b=20$ $f'(x)=6x^2+2ax+b$ $f'(1)=6(1)^2+2a(1)+b=1$ $2a+b=-5$ $5a=15$ $a=3$ $b=-11$
		(ii)	$2x^3+3x^2-11x-6=(x+3)(2x^2+px-2)$ $3p-2=-11$ $p=-3$ $2x^3+3x^2-11x-6=(x+3)(2x^2-3x-2)$ $2x^3+3x^2-11x-6=(x+3)(2x+1)(x-2)$

		(iii)	$2x^3+3x^2-11x-6=(x+3)(2x+1)(x-2)=0$ $x=-3,-\frac{1}{2},2$ $2+3y-11y^2-6y^3=0$ $\frac{1}{y}=-3,-\frac{1}{2},2$ $y=-\frac{1}{3},-2,\frac{1}{2}$
	(b)	$54x^6-16y^3$ $=2(27x^6-8y^3)$ $=2[(3x^2)^3-(2y)^3]$ $=2(3x^2-2y)(9x^4+6x^2y+4y^2)$	
3.	**(a)**	$V=\pi r^2h+\frac{2}{3}\pi r^3$ $60\pi=\pi r^2h+\frac{2}{3}\pi r^3$ $60\pi-\frac{2}{3}\pi r^3=\pi r^2h$ $h=\frac{60}{r^2}-\frac{2}{3}r$	
	(b)	$SA=2\pi rh+\pi r^2+2\pi r^2$ $C=3(2\pi rh+\pi r^2)+4(2\pi r^2)$ $=6\pi r\left(\frac{60}{r^2}-\frac{2}{3}r\right)+11\pi r^2$ $=\frac{360\pi}{r}-4\pi r^2+11\pi r^2$ $=\frac{360\pi}{r}+7\pi r^2$	
	(c)	$\frac{dC}{dr}=-\frac{360\pi}{r^2}+14\pi r=0$ $\frac{360\pi}{r^2}=14\pi r$ $r^3=\frac{360}{14}$ $r=2.9516$	

	(d)	When $r = 2.9516$, $C = \dfrac{360\pi}{2.9516} + 7\pi(2.9516)^2 = 574.76$ $\dfrac{d^2C}{dr^2} = \dfrac{720\pi}{r^3} + 14\pi > 0$ \$5.75 is a minimum value which is > \$5.60. He should not continue to make this product.
4.	**(a)**	$2^{4p+1} + 20(4^{p-1}) = 3$ $2(4^p)^2 + 5(4^p) = 3$ $2k^2 + 5k - 3 = 0$ $(2k-1)(k+3) = 0$ $k = \dfrac{1}{2}, -3$ $4^p = \dfrac{1}{2}$ or -3 (rej) $p = -\dfrac{1}{2}$
	(b)	$2u^2 + 5u - k = 0$ If no solution, $b^2 - 4ac < 0$ $25 + 8k < 0$ $k < -\dfrac{25}{8}$ OR $2^{4p+1} + 20(4^{p-1}) = k$ $2(4^p)^2 + 5(4^p) = k$ Since $4^p > 0$ for all p, $2(4^p)^2 + 5(4^p)$ is always positive for all p. Therefore, no solution for $k < -3\dfrac{1}{8}$.

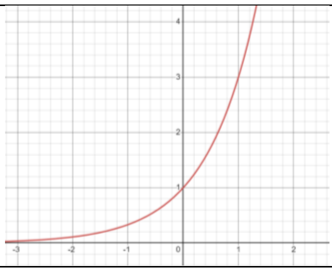
5	**(a)**	$\dfrac{(\log_y x)^2}{\log_x y}+64=0$ $\dfrac{(\log_y x)^2}{\dfrac{\log_x x}{\log_x y}}=-64$ $(\log_y x)^3=-64$ $\log_y x=-4$ $x=y^{-4}$ $y=x^{-\frac{1}{4}}$
	(b)	$2\log_2(x-1)-\log_2 x=3$ $\log_2\dfrac{(x-1)^2}{x}=3$ $\dfrac{x^2-2x+1}{x}=8$ $x^2-10x+1=0$ $x=0.101$ (rej) or 9.90

6.	**(a)**	Max $=9.9$ m Min $=0.3$ m
	(b)	$k\pi$ cycles in 2π hours 1 cycle in $\dfrac{2}{k}$ hours 2 cycles in $\dfrac{4}{k}$ hours $\dfrac{4}{k}=24$ $k=\dfrac{1}{6}$ Note: Do not accept $24k\pi=4\pi$ as working.

	(c)		$4.8\sin\frac{\pi t}{6}+5.1>2$ $4.8\sin\frac{\pi t}{6}>-3.1$ $\sin\frac{\pi t}{6}>-\frac{31}{48}$ Basic Angle = 0.702 (Q3,4) $\frac{\pi t}{6}=\pi+0.702,2\pi-0.702$ $t=7.34,10.66$ $0<t<7.34,10.66<t<12$
7.	**(a)**	**(i)**	$LHS=\frac{\cos x}{\sin x}-2\sin x\cos x$ $=\frac{\cos x-2\sin^2 x\cos x}{\sin x}$ $=\frac{\cos x(1-2\sin^2 x)}{\sin x}$ $=\cot x\cos 2x$
		(ii)	$4(\cot x-\sin 2x)=\cos 2x$ $4(\cot x\cos 2x)=\cos 2x$ $\cos 2x(4\cot x-1)=0$ $\cos 2x=0$ or $\cot x=\frac{1}{4}$ $\alpha=\pi$ (Q1,4) or $\tan x=4$ $2x=\frac{\pi}{2},\frac{3\pi}{2}$ or $\alpha=1.3258$ (Q1,3) $x=\frac{\pi}{4},\frac{3\pi}{4}$ or $x=1.33,4.47$ (rej) Ans: $x=\frac{\pi}{4},\frac{3\pi}{4},1.33$
	(b)		$\sin A\cos B+\cos A\sin B=\frac{6}{7}$ $\sin A\cos B+\frac{2}{7}=\frac{6}{7}$ $\sin A\cos B=\frac{4}{7}$ $\frac{\sin A\cos B}{\cos A\sin B}=\frac{4}{7}\div\frac{2}{7}$ $\frac{\tan A}{\tan B}=2$

8.	**(a)**	$y=\dfrac{2}{\sqrt{3x+1}}=2(3x+1)^{-\frac{1}{2}}$ $\dfrac{dy}{dx}=-3(3x+1)^{-\frac{3}{2}}$ $\dfrac{dy}{dt}=\dfrac{dy}{dx}\cdot\dfrac{dx}{dt}$ $-3=-3(3x+1)^{-\frac{3}{2}}(0.125)$ $8=(3x+1)^{-\frac{3}{2}}$ $3x+1=\dfrac{1}{4}$ $x=-\dfrac{1}{4},\ y=4$	M1 M1 M1 M1 A1
	(b)	$y=2x\ln x$ $\dfrac{dy}{dx}=2x\left(\dfrac{1}{x}\right)+2\ln x=2+2\ln x$ For increasing functions, $1+\ln x>0$ $\ln x>-1$ $x>\dfrac{1}{e}$	M1 A1
9.	**(a)**	$y=(2x+1)^{-1}$ $\dfrac{dy}{dx}=-2(2x+1)^{-2}$ $x=0,\dfrac{dy}{dx}=-2$ Eqn of tangent: $y=-2x+c$ $1=-2(0)+c$ $c=1$ $y=-2x+1$	M1 M1 A1
	(b)	Where tangent intersects the x-axis, $y=0$ $0=-2x+1$ $x=\dfrac{1}{2}$ $Area=\int_0^1\dfrac{1}{2x+1}\ dx-\dfrac{1}{2}\left(\dfrac{1}{2}\right)1$ $=\left[\dfrac{1}{2}\ln(2x+1)\right]_0^1-\dfrac{1}{4}$ $=\dfrac{1}{2}\ln 3-\dfrac{1}{4}$	M1 M1 M1 A1

10.	**(a)**	Let $\angle FBC = x$, $\angle CAB = x$ (tangent chord theorem) $\angle FED = x$ (angles in same segment) By the converse of alternate angles, since $\angle CBA = \angle FED$, AB is parallel to DE.	 M1 M1 A1
	(b)	ΔBCF and ΔEDF	B1 B1
	(b)	ΔABF is similar to ΔBCF. Therefore $\frac{AF}{BF} = \frac{BF}{CF}$ $AF \times CF = BF^2$ (shown)	M1 M1
11.	**(a)**	Grad $PR = \frac{10}{-5} = -2$ Grad $SQ = \frac{1}{2}$ Eqn SQ: $y = \frac{1}{2}x + c$ $7 = \frac{1}{2}(3) + c$ $c = \frac{11}{2}$ Eqn SQ: $y = \frac{1}{2}x + \frac{11}{2}$	M1 M1 A1
	(b)	Grad $PR = \frac{10}{-5} = -2$ Eqn: $y = -2x + c$ $7 = -2(3) + c$ $c = 13$ $y = -2x + 13$	M1 A1
	(c)	$Area\ PQR = \frac{1}{2}\begin{vmatrix} -3 & 2 & 3 & -3 \\ 9 & -1 & 7 & 9 \end{vmatrix}$ $= \frac{1}{2}[(3+14+27)-(18-3-21)]$ $= \frac{1}{2}(44-(-6))$ $= 25$	M1 A1

12	**(a)**		B1
	(b)	$\log_9(5-x)=\dfrac{x}{2}$ $(5-x)^2=3^{2x}$ $5-x=3^x$ Eqn straight line: $y=5-x$	M1 A1
	(c)	1 solution	B1