

Fairfield Methodist School (Secondary)
 Secondary 4 Express / 5 Normal (Academic)
 Preliminary Examinations 2022
 Mathematics Paper 2

Marking Scheme

Qn No.	Workings	Description	Mark Allocation
1a	$\frac{x+6}{3} \leq \frac{3-x}{2}$ $2(x+6) \leq 3(3-x)$ $2x+12 \leq 9-3x$ $5x \leq -3$ $x \leq -\frac{3}{5}$	Remove fraction	M1 A1
1bi	$E = \frac{30}{30+18}$ $E = \frac{5}{8} \text{ or } 0.625$		B1
1bii	$EW + Ex = W$ $EW - W = -Ex$ $W(E-1) = -Ex$ $W = \frac{-Ex}{E-1} \text{ or } \frac{Ex}{1-E}$	Take out W as a common factor	M1 A1
1c	$\frac{3}{x+1} = \frac{5(x-2)-1}{x-2}$ $\frac{3}{x+1} = \frac{5x-10-1}{x-2}$ $3(x-2) = (5x-11)(x+1)$ $3x-6 = 5x^2 + 5x - 11x - 11$ $5x^2 - 9x - 5 = 0$ $x = \frac{9 \pm \sqrt{181}}{10}$ $x = 2.25 \text{ (2d.p.) or } x = -0.45 \text{ (2d.p)}$	Cross multiply or multiply both sides by $(x+1)(x-2)$ Show correct quadratic equation $\sqrt{181}$ must be simplified	M1 M1 M1 A1, A1

Qn No.	Workings	Description	Mark Allocation
2ai	Discounted Price $= 0.85 \times 2888$ $= \$2454.80$ Price inclusive of GST $= 2454.80 \times 1.07$ $= \$2626.64$ (nearest cent)		M1 M1 A1
2aii	Amount of US dollars he receives $= \frac{3800}{1.43}$ $= \text{US}\$2657.34$ $= \text{US}\$2657$ (nearest dollar)		M1 A1
2aiii	Solution 1 Amount of SG dollars he receives $= 2657.34 \times 1.38$ $= \$3667.13$ Percentage Loss $= \frac{3800 - 3667.13}{3800} \times 100\%$ $= 3.4966$ (5 s.f) $= 3.50\%$ (3 s.f) Solution 2 Amount of SG dollars he receives $= 2657 \times 1.38$ $= \$3666.66$ Percentage Loss $= \frac{3800 - 3666.66}{3800} \times 100\%$ $= 3.5089$ (5 s.f) $= 3.51\%$ (3 s.f)		M1 A1 M1 A1
2b	Discounted Price $= 2300 \times 0.9$ $= \$2070$ Cost Price of laptop $= \frac{100}{95} \times 2070$ $= \$2178.95$ (nearest cent)		M1 M1 A1
3a	$p = 19$		B1
3b	Refer to Graph		P2 C1

Qn No.	Workings	Description	Mark Allocation
3c	170, 180 or 190 books (accept any 1)		B1
3di	$\text{Gradient} = \frac{10-19}{800-400} = -0.0225$ <p>Accept (- 0.01 to - 0.04)</p>	<p>Tangent accurately drawn</p> <p>Calculated gradient</p>	<p>M1</p> <p>A1</p>
3dii	Gradient represents the rate of reduction(decrease) of the cost of production per copy of the book.		B1
3ei	Refer to Graph Correct line plotted		P1 L1
3eii	$200 \leq x \leq 900$		B1
4a	<p>Let the height of the cone be h</p> <p>Vol of Cone = 2 x Vol of hemisphere</p> $\frac{1}{3}\pi(6)^2h = 2 \times \frac{2}{3}\pi(6)^3$ $12h = 288$ $h = \frac{288}{12}$ $h = 24$	<p>Equate both volumes</p> <p>Show $\frac{288}{12}$</p>	<p>M1</p> <p>AG1</p>
4b	<p>Let the slant height of the cone be l.</p> <p>By Pythagoras Theorem,</p> $l^2 = 6^2 + 24^2$ $l = \sqrt{612}$ $l = 24.739 \text{ or } 24.738 \text{ (truncate 5 sf)}$ <p>Total Surface Area of Solid</p> $= \pi(6)(24.739) + 2\pi(6)^2$ $= 692.51 \text{ (5 s.f)}$ $= 693 \text{ cm}^2 \text{ (3 s.f) or } 692 \text{ cm}^2 \text{ (truncate 5s.f)}$		<p>M1</p> <p>M1</p> <p>A1</p>
4ci	$\frac{x}{y} = \frac{6}{24} = \frac{1}{4}$ (by similar triangles)		B1
4cii	$\frac{x}{y} = \frac{1}{4}$ $x = \frac{1}{4}y$ <p>Volume of frustum</p> $= \frac{1}{3}\pi(6)^2(24) - \frac{1}{3}\pi x^2 y$ <p>Vol of small cone = Vol of frustum + Vol of hemisphere</p> $\frac{1}{3}\pi x^2 y = \frac{1}{3}\pi(6)^2(24) - \frac{1}{3}\pi x^2 y + \frac{2}{3}\pi(6)^3$	Form Equation	M1

Qn No.	Workings	Description	Mark Allocation
	$\frac{2}{3}\pi x^2 y = \frac{1}{3}\pi(6)^2(24) + \frac{2}{3}\pi(6)^3$ $\frac{2}{3}x^2 y = \frac{1}{3}(6)^2(24) + \frac{2}{3}(6)^3 \text{ (divide both sides by } \pi)$ $\frac{2}{3}x^2 y = 432$ <p>Sub $x = \frac{1}{4}y$ into equation</p> $\frac{2}{3}\left(\frac{1}{4}y\right)^2 y = 432$ $\frac{1}{24}y^3 = 432$ $y^3 = 10368$ $y = 21.8 \text{ (3 s.f)}$	<p>Sub $x = \frac{1}{4}y$ into equation</p>	<p>M1</p> <p>A1</p>
4cii	<p>Alternate Solution 2</p> <p>Let the volume of the top part (cone be V_1), middle frustum be V_2, and hemisphere by V_3.</p> $\frac{V_1 + V_2}{V_3} = \frac{2}{1}$ $\frac{V_1}{V_1 + V_2} = \left(\frac{x}{6}\right)^3$ $\frac{V_1}{V_1 + V_2} = \left(\frac{\frac{1}{4}y}{6}\right)^3 \text{ given } x = \frac{1}{4}y$ $\frac{V_1}{V_1 + V_2} = \frac{y^3}{13824}$ <p>Therefore,</p> $V_1 : V_1 + V_2 : V_3$ $y^3 : 13824$ $2 : 1$ $y^3 : 13824 : \frac{13824}{2}$ $y^3 : 13824 : 6912$ <p>Since</p> $\frac{V_1}{V_2 + V_3} = \frac{1}{1}$ $\frac{y^3}{13824 - y^3 + 6912} = \frac{1}{1}$ $20736 - y^3 = y^3$ $2y^3 = 20736$	<p>Sub $x = \frac{1}{4}y$ into equation</p> <p>Form Equation</p>	<p>M1</p> <p>M1</p>

Qn No.	Workings	Description	Mark Allocation
	$y^3 = 10368$ $y = \sqrt[3]{10368}$ $y = 21.8$ (3 s.f)		A1
4cii	<p>Alternate Solution 3</p> $\text{Vol of Solid} = \frac{1}{3}\pi(6)^2(24) + \frac{2}{3}\pi(6)^3$ $= 432\pi$ $\text{Vol of Upper part} = 432\pi \div 2 = 216\pi$ $\frac{\text{Vol of Upper}}{\text{Vol of Lower}} = \left(\frac{y}{24}\right)^3$ $\left(\frac{y}{24}\right)^3 = \frac{216\pi}{\frac{1}{3}\pi(6)^2(24)}$ $\frac{y}{24} = \sqrt[3]{\frac{216\pi}{288\pi}}$ $y = 24 \sqrt[3]{\frac{216\pi}{288\pi}}$ $y = 21.8 \text{ cm (3.s.f)}$		M2 A1
5a	$\text{Grad of } PQ = \frac{4 - (-5)}{2 - (-1)}$ $= 3$ <p>Sub (2, 4) into $y = 3x + c$</p> $4 = 3(2) + c$ $c = 4 - 6$ $c = -2$ <p>$\therefore y = 3x - 2$ (shown)</p>	<p>Show working for Gradient</p> <p>Show working for y-intercept</p>	B1 AG1
5b	$y = 3x - 2 \quad \text{----- (1)}$ $3y - 2x = 1 \quad \text{----- (2)}$ <p>Sub (1) into (2)</p> $3(3x - 2) - 2x = 1$ $9x - 6 - 2x = 1$ $7x = 7$ $x = 1$ <p>Sub $x=1$ into Eqn (1)</p> $y = 3(1) - 2$ $y = 1$	<p>Elimination or Substitution method</p> <p>Lose 1 mark if x and y values are correct but</p>	M1 A1

Qn No.	Workings	Description	Mark Allocation
	S (1, 1)	state coordinate S wrongly.	A1
5c	Length of PQ $= \sqrt{(4 - (-5))^2 + (2 - (-1))^2}$ $= \sqrt{81 + 9}$ $= \sqrt{90}$ $= 9.4868$ (5 s.f) $= 9.49$ units (3 s.f)		M1 A1
5d	$x = -1$		B1
5e	Substitute $x = -4$ into $y = 3x - 2$ $y = 3(-4) - 2$ $y = -12 - 2$ $y = -14$ The point A does not lie on the line because when $x = -4$, $y = -14 \neq -5$.		M1 A1
6a	$\angle ACD = 32^\circ$ (\angle s in same seg.)		B1
6b	$\angle BCD = 32^\circ + 58^\circ$ $= 90^\circ$ Since it obeys right angle in semicircle property , $\Rightarrow BD$ is a diameter. (shown)	Student should show $\angle BCD = 90^\circ$ and give correct reason. Can accept short form for circle property.	B1
6c	$\angle DAB = 90^\circ$ (right \angle in semicircle) $\angle DAY = 180^\circ - 90^\circ - 58^\circ$ $= 32^\circ$ (adj \angle s on a straight line)	Award 1 mark as long answer is correct and either reason is given	B1
6d	$\angle FCE = \angle FEC = \frac{180^\circ - 120^\circ}{2}$ (base \angle s of isos. Δ) $= 30^\circ$ $\angle ECD = 180^\circ - 90^\circ - 30^\circ$ (adj. \angle s on straight line) $= 60^\circ$ $\angle FED = 90^\circ$ (\angle s in opp. seg.) $\angle CED = 90^\circ - 30^\circ$ $= 60^\circ$ $\therefore \angle CDE = 60^\circ$ Since all 3 angles are 60° $\Rightarrow \Delta CDE$ is an equilateral Δ . (shown)	show $\angle ECD = 60^\circ$ show $\angle FED = 90^\circ$ by stating \angle s in opp. seg. show $\angle CED = 60^\circ$ and conclude ΔCDE is equilateral.	B1 B1 AG1

Qn No.	Workings	Description	Mark Allocation
6d	<p>Alternate Solution</p> $\angle CDE = 180 - 120$ $= 60^\circ$ (\angle s in opposite segments) $\angle FCD = 180 - 90$ $= 90^\circ$ (adj \angle s on straight line) $\angle FED = 180 - 90$ $= 90^\circ$ (\angle s in opposite segments) $\angle FCE = \angle FEC = \frac{180 - 120}{2}$ $= 30^\circ$ (Base \angle s of Isos. Δ) $\angle ECD = 90 - 30$ $= 60^\circ$ $\angle CED = 180 - 60 - 60$ $= 60^\circ$ (sum of \angle s in Δ) <p>Since all 3 angles are 60° $\Rightarrow \Delta CDE$ is an equilateral Δ. (shown)</p>	<p>show $\angle CDE = 60^\circ$</p> <p>show $\angle FED = 90^\circ$ by stating \angles in opp. seg.</p> <p>show $\angle CED = 60^\circ$ and conclude ΔCDE is equilateral.</p>	<p>B1</p> <p>B1</p> <p>AG1</p>
6ei	$\angle COE = 120^\circ$ (\angle at centre = 2 \angle s at circumference) $\Rightarrow \angle OCE = \angle OEC = 30^\circ$ (base \angle s of Isos. Δ) $\angle COE = \angle CFE = 120^\circ$ (A) $\angle OCE = \angle FCE = 30^\circ$ (base \angle s of Isos. Δ) (A) CE is common (S) $\therefore \Delta OCE \equiv \Delta FCE$ (AAS)	<p>Show any two statements with clear reasons given.</p> <p>Correct conclusion and show AAS property</p>	<p>B2</p> <p>B1</p>
6eii	$COEF$ is a rhombus.		B1
7ai	$T_n = 3n + 1$		B1
7aii	$T_{88} = 3(88) + 1$ $= 265$		B1
7bi	$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225 = (1 + 2 + 3 + 4 + 5)^2 = \left(\frac{5 \times 6}{2}\right)^2$		B1
7bii	$\text{Sum} = \left(\frac{10 \times 11}{2}\right)^2 = 3025$		B1
7biii	$q = \left(\frac{n(n+1)}{2}\right)^2$		B1
7biv	$1296 = \left(\frac{n(n+1)}{2}\right)^2$ $36 = \frac{n(n+1)}{2}$ (square root both sides) $72 = n^2 + n$ $n^2 + n - 72 = 0$ $(n - 8)(n + 9) = 0$ $n = 8$ or $n = -9$ (reject)		<p>M1</p> <p>A1</p>

Qn No.	Workings	Description	Mark Allocation
7bv	600 is not a square number. The sum of all numbers in the rows should be a square number.		B1
8a	$p = 50$		B1
8bi	20kg		B1
8bii	Lower Quartile = 15 kg Upper Quartile = 23 kg Interquartile Range = $23 - 15$ = 8 kg		M1 A1
8biii	44 th Percentile = 19 kg		B1
8ci	Interquartile Range = $30 - 20$ = 10 kg		B1
8cii	$\frac{1}{4} \times 200 = 50$ men.		B1
8ciii	The median weight loss of Genesis Fitness Centre is higher than Alpha Fitness Centre ($25\text{kg} > 20\text{kg}$) It is untrue that Alpha Fitness Centre is more effective at weight loss.	State median and comment that it is untrue.	B1
8di	P (Charles fails the test) = $1 - 0.8$ = 0.2 P (Charles passes only at the third attempt) = $0.2 \times 0.2 \times 0.8$ = 0.032		B1
8dii	P(Charles passes in the first attempt) = 0.8 P(Charles passes only at the second attempt) = 0.2×0.8 = 0.16 P(Charles passes in either the first or second attempt) = $0.8 + 0.16$ = 0.96		M1 A1
9ai	$\overline{BC} = \overline{OC} - \overline{OB}$ $\overline{BC} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\overline{BC} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$		B1
9aai	$ \overline{BC} = \sqrt{1^2 + (-4)^2}$ $ \overline{BC} = 4.12$ units		M1 A1

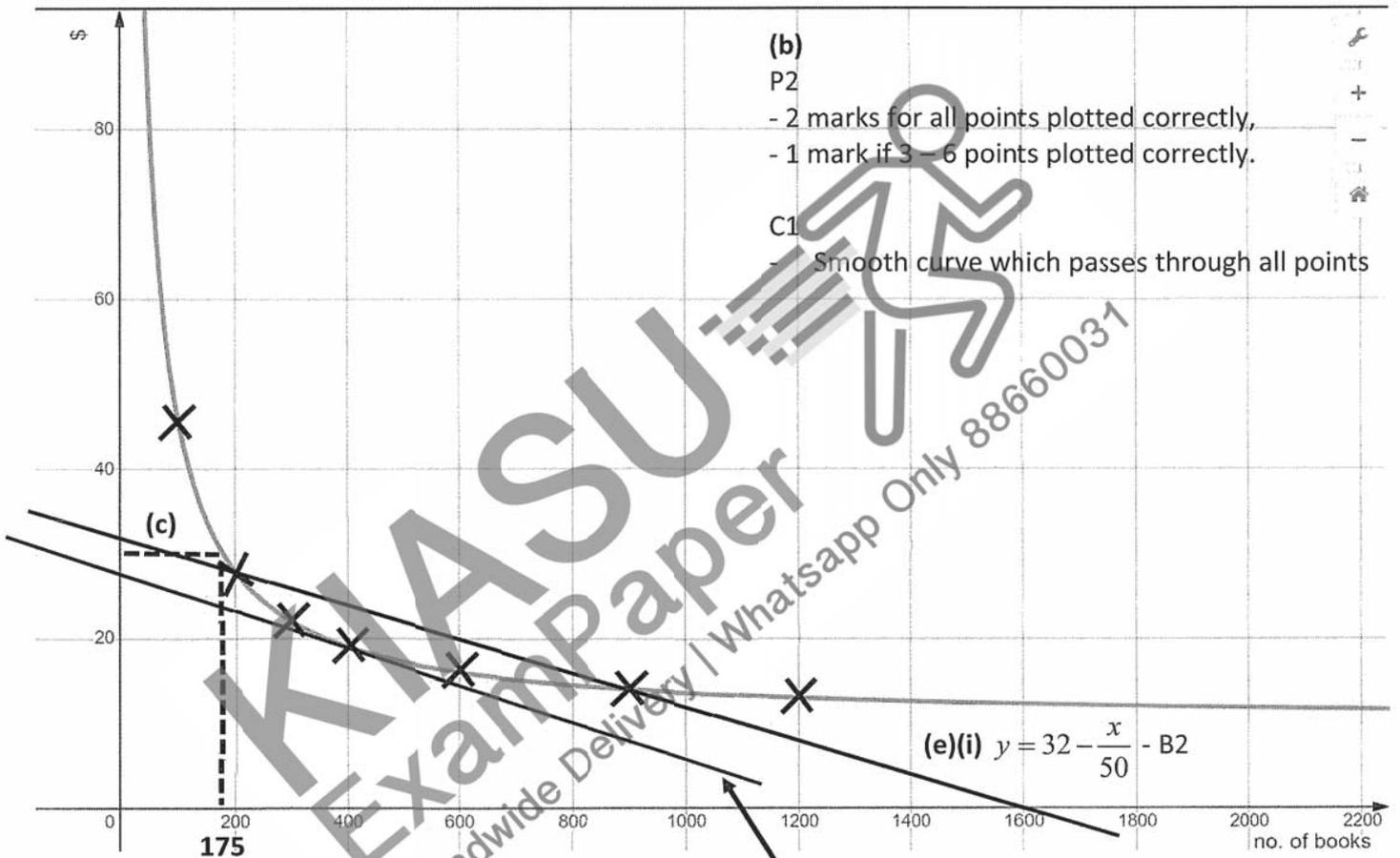
Qn No.	Workings	Description	Mark Allocation
9bia	$\overrightarrow{VW} = \overrightarrow{VU} + \overrightarrow{UV}$ $= -a + (6b - 2a)$ $= 6b - 3a$ Or $3(2b - a)$		B1
9bib	$\overrightarrow{WX} = \overrightarrow{WV} + \overrightarrow{VX}$ $= -(6b - 3a) + 2b - 2a$ $= -4b + a$		B1
9bic	$\overrightarrow{XZ} = \overrightarrow{XV} + \overrightarrow{VZ}$ $= -(2b - 2a) - \frac{3}{2}a$ $= -2b + \frac{1}{2}a$		B1
9bii	$\overrightarrow{WX} = -4b + a = 2\left(-2b + \frac{1}{2}a\right)$ $\overrightarrow{XZ} = -2b + \frac{1}{2}a$ $\therefore \overrightarrow{WX} = 2\overrightarrow{XZ}$ <p>Since $\overrightarrow{WX} = 2\overrightarrow{XZ}$, \overrightarrow{WX} is parallel to \overrightarrow{XZ} with X as the common point. Therefore, the points W, X and Z lie on a straight line.</p>	<p>Show $\overrightarrow{WX} = 2\overrightarrow{XZ}$ or $WX = 2XZ$</p> <p>Write concluding statement</p>	M1 AG1
9biaa	$\frac{\text{Area of } \Delta ZUX}{\text{Area of } \Delta ZVW} = \left(\frac{ZU}{ZV}\right)^2 \quad \frac{VU}{UZ} = \frac{2}{1}$ $= \left(\frac{1}{1+2}\right)^2$ $= \frac{1}{9}$		B1
9biiib	$\frac{\text{Area of } \Delta XVW}{\text{Area of } \Delta ZVW} = \frac{\frac{1}{2} \times XW \times h}{\frac{1}{2} \times ZW \times h} \quad \frac{XW}{ZX} = \frac{2}{1}$ $= \frac{2}{2+1}$ $= \frac{2}{3}$		B1
9biiic	$\overrightarrow{UX} = \overrightarrow{UW} + \overrightarrow{WX}$ $= 6b - 2a - 4b + a$ $= 2b - a$ $\overrightarrow{VW} = 6b - 3a = 3(2b - a)$ $\therefore VW = 3UX$	Show $VW = 3UX$	M1

Qn No.	Workings	Description	Mark Allocation
	$\frac{\text{Area of } \triangle XVW}{\text{Area of trapezium } UVWX} = \frac{\frac{1}{2} \times VW \times h}{\frac{1}{2} \times (UX + VW) \times h}$ $= \frac{VW}{UX + VW}$ $= \frac{3}{1+3}$ $= \frac{3}{4}$		A1
10a	<p><u>Eggs</u> Number of Eggs required = 300 x 2 = 600 eggs Price for 600 eggs: 60 packs of Pasar Eggs: \$2.70 x 60 = \$162 (cheapest) 40 packs of Dason Eggs: \$4.55 x 40 = \$182 50 packs of Seng Seng Eggs: \$3.70 x 50 = \$185 <u>Cheapest cost of 300 Breakfast Set (Eggs) = \$162</u></p> <p><u>Bread</u> Number of slices of bread required = 300 x 2 = 600 slices of bread</p> <p>Price for at least 600 slices of bread: 43 packets of Gardenia Bread: \$2.70 x 43 = \$116.10 (cheaper) 50 packets of Sunshine Bread: \$2.50 x 50 = \$125 <u>Cheapest cost of 300 Breakfast Sets (Bread) = \$116.10</u></p> <p><u>Sausage</u> Number of Sausages required = 300 sausages Price of 300 sausages: 30 packs Master Grocer: \$5.25 x 30 = \$157.50 50 packs of Grand Chef: (\$3.20 x 50) - (\$0.35 x 25) = \$151.25 (cheaper) <u>Cheapest cost of 300 Breakfast Sets (Sausage) = \$151.25</u></p> <p><u>Ham</u> Number of slices of ham required = 300 slices of ham Price of 300 slices of ham: 30 packs of FairPrice Ham: (4.35 x 30) x 0.8 = \$104.40 30 packs of Smart Choice Ham: 3.30 x 30 = \$99 (cheaper) <u>Cheapest cost of 300 Breakfast Sets (Ham) = \$99</u></p>	<p>Find Cost of 43 packets of Gardenia Bread or 60 packs of Pasar Eggs</p> <p>Find Cost of 50 packs Grand Chef Sausage as cheaper option</p> <p>Find Cost of 30 packs of Smart Choice Ham as cheaper option</p>	<p>EB 1</p> <p>S1</p> <p>H1</p>

Qn No.	Workings	Description	Mark Allocation
	<p>Coffee Number of packets required = 300 packets Price of at least 300 packets of coffee 9 packs of Nescafe (inclusive of 1 free): $\\$6.15 \times 8 = \\49.20 12 packs of Indocafe: $\\$3.95 \times 12 = \\47.40 (cheaper) <u>Cheapest cost of 300 Breakfast Sets (Indocafe Coffee)</u> <u>= \$47.40</u></p> <p>Lowest Possible Cost for 300 all day breakfast $= 162 + 116.10 + 151.25 + 99 + 47.40$ $= \\$575.75$</p>	<p>Find Indocafe Coffee as cheaper option</p> <p>Lowest Total Cost</p>	<p>C1</p> <p>A1</p>
10b	<p>Assumption:</p> <ul style="list-style-type: none"> - No cost incurred for cooking. - No food wastage made during the cooking - All 300 sets are sold 	Any valid assumptions	A1
10b	<p>Solution 1 Funded expenses $= \\$575.75 - \\200 $= \\$375.75$</p> <p>Total sales class must make to raise at least \$600 for charity $= \text{funded expenses} + \text{charity amount}$ $= \\$575.75 - \\$200 + \\$600$ $= \\$975.75$</p> <p>To make sure up to 40% of the sales can be used to cover expenses $= \frac{375.75}{975.75} \times 100\%$ $= 38.508\%$ (less than 40%)</p> <p>Minimum amount to charge for each breakfast set $= \frac{975.75}{300}$ $= \\$3.30$ (nearest ten cent)</p>	<p>Total Sales (with criteria 2)</p> <p>Check that the total sales meet the criteria 1</p> <p>Correct minimum price</p>	<p>S1</p> <p>C1</p> <p>P1</p>

Qn No.	Workings	Description	Mark Allocation
10b	<p>Solution 2</p> <p>Funded expenses = \$575.75 – \$200 = \$375.75</p> <p>Total sales class must make to if used 40% to cover expenses = $\frac{100}{40} \times 375.75$ = \$939.375</p> <p>Amount left for charity = \$939.375 – \$375.75 = \$563.625 (not enough for charity)</p> <p>Additional amount needed to raise at least \$600 for charity = \$600 – \$563.625 = \$36.375</p> <p>Adjusted total sales class = \$939.375 + \$36.375 = \$975.75</p> <p>Minimum amount to charge for each breakfast set = $\frac{975.75}{300}$ = \$3.30 (nearest ten cent)</p>	<p>Total Sales (with criteria 1)</p> <p>Check that the total sales meet the criteria 2</p> <p>Correct minimum price</p>	<p>S1</p> <p>C1</p> <p>P1</p>

Question 3



(d)(i) – accurate tangent drawn (M1)

$$\text{Gradient} = \frac{10 - 19}{800 - 400} = -0.0225 \text{ - A1}$$

(e)(ii) $200 \leq x \leq 900$ - B1

End of marking scheme