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**FAIRFIELD METHODIST SCHOOL (SECONDARY)****PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)****MATHEMATICS****4048/02****Paper 2****Date: 24 August 2022****Duration: 2 hours 30 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 100.

For Examiner's Use

Table of Penalties		Question Number	Parent's / Guardian's Signature	100
Presentation	<input type="checkbox"/> 1 <input type="checkbox"/> 2			
Rounding off	<input type="checkbox"/> 1			

Setter: Mr Joel Li

This question paper consists of 27 printed pages.

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Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

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Answer **all** the questions.

- 1 (a) Solve the inequality $\frac{x}{3} + 2 \leq \frac{3-x}{2}$.

Answer [2]

- (b) It is given that $E = \frac{W}{W+x}$.

- (i) Find the value of E when $W = 30$ and $x = 18$.

Answer $E =$ [1]

- (ii) Express W in terms of E and x .

Answer $W =$ [2]

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1 (c) Solve the equation $\frac{3}{x+1} = 5 - \frac{1}{x-2}$.

Give your solutions correct to 2 decimal places.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [5]

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- 2 (a) Evane plans to travel to the United States of America (USA) for a holiday.
- (i) He goes to the travel fair and is given an early bird discount of 15% for a 7-day tour package that costs \$2888. Calculate the amount of money he has to pay the travel agent after the addition of 7% Goods and Service Tax (GST) on the discounted price.

Answer \$ [3]

- (ii) He then goes to the money changer to change S\$3800 into US dollars. Calculate the amount of US dollars he receives if the exchange rate was US\$1 = S\$1.43. Leave your answer to the nearest dollar.

Answer US\$ [2]

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- 2 (a) (iii) In the following month, he cancelled his holiday plans as he contracted COVID. He changed all his US dollars back at the rate $\text{US\$1} = \text{S\$1.38}$. Calculate the loss as a percentage of the original amount he changed as a result of the new exchange rate.

Answer% [2]

- (b) The marked price of a laptop is \$2300. By selling the laptop at a 10% discount, a merchant makes a loss of 5% on its cost price. Find the cost price of the laptop.

Answer \$ [3]

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- 3 When x copies of a book are produced, the cost, \$ y , of each copy is given by the formula

$$y = 10 + \frac{3600}{x}.$$

x	100	200	300	400	600	900	1200
y	46	28	22	p	16	14	13

- (a) Find the value of p .

Answer $p = \dots\dots\dots$ [1]

- (b) On the grid on page 9, draw the graph of $y = 10 + \frac{3600}{x}$ for $100 \leq x \leq 1200$. [3]

- (c) Use your graph to estimate the number of books to be printed if the cost of producing each book is \$30.

Answer $\dots\dots\dots$ books [1]

- (d) (i) By drawing a tangent, find the gradient of the curve when $x = 400$.

Answer $\dots\dots\dots$ [2]

- (ii) Use your answer to (d)(i) to state briefly what the gradient represents.

Answer $\dots\dots\dots$

$\dots\dots\dots$ [1]

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(e) In order to sell x books, the selling price of each book must be $\$ \left(32 - \frac{x}{50} \right)$.

(i) On the same axes on page 9, draw the graph of $y = 32 - \frac{x}{50}$ to represent the selling price of the books. [2]

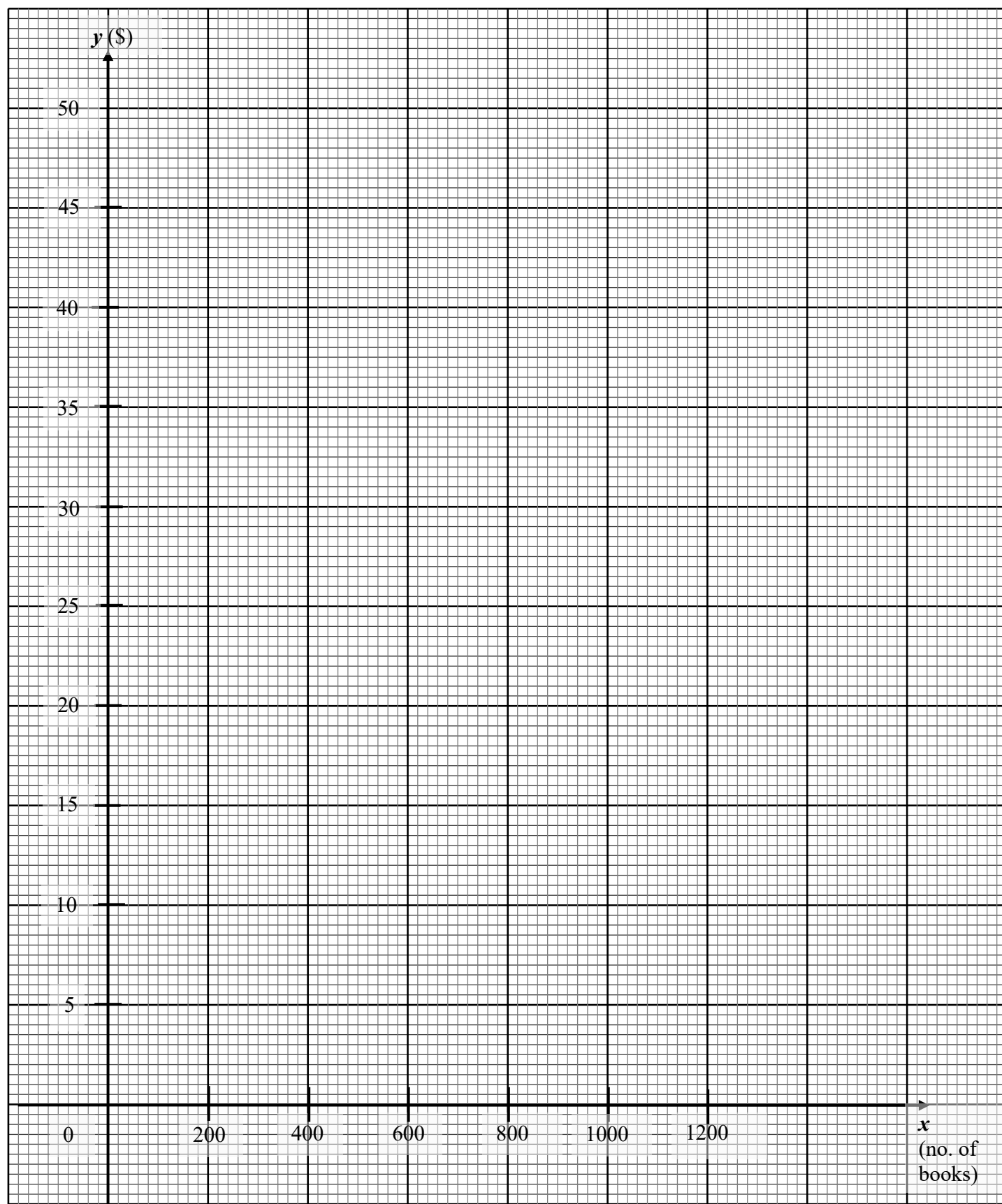
(ii) Using your graphs, find the range of the number of books that should be printed if no loss is to be made, assuming all the books will be sold.

Answer [1]

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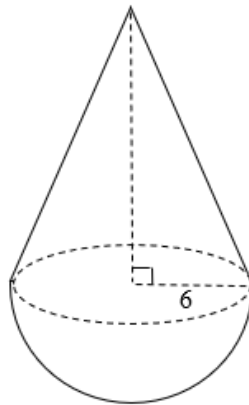
Answer (b)



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- 4 The diagram shows a solid consisting of a right circular cone attached to a hemisphere with a common base which is a circle of radius 6 cm. The volume of the cone is equal to twice the volume of the hemisphere.



- (a) Show that the height of the cone is 24 cm.

Answer

[2]

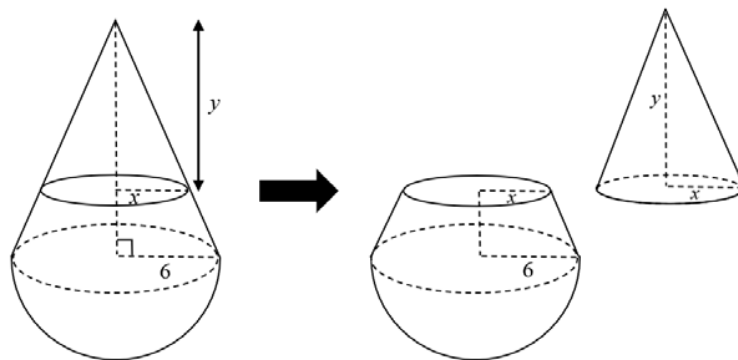
- (b) Find the total surface area of the solid.

Answer cm^2 [3]

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- 4 (c) The same solid is cut into two parts such that the upper part is a cone of base radius x cm and height y cm.



- (i) Find the value of $\frac{x}{y}$.

Answer [1]

- (ii) Find the value of y such that the volume of the upper part is equal to the volume of the lower part.

Answer $y =$ [3]

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5 The points P and Q are $(-1, -5)$ and $(2, 4)$ respectively.

(a) Show that the equation of line PQ is $y = 3x - 2$.

Answer

[2]

(b) A line $3y - 2x = 1$ intersects the line PQ at point S , find the coordinates of S .

Answer $S(\dots\dots\dots, \dots\dots\dots)$ [3]

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- 5 (c) Find the length of PQ .

Answer $PQ = \dots\dots\dots$ units [2]

- (d) Find the equation of the line that passes through P and parallel to the y -axis.

Answer $\dots\dots\dots$ [1]

- (e) Does the point $A(-4, -5)$ lie on the line PQ ? Explain and justify your answer.

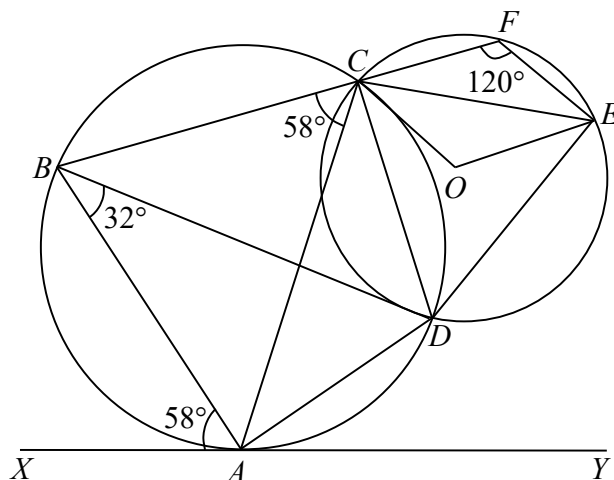
Answer The point $A \dots\dots\dots$ on the line PQ because $\dots\dots\dots$

$\dots\dots\dots$ [2]

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- 6 In the diagram, XAY is a tangent to the circle $ABCD$ at A . O is the centre of the circle $CDEF$ and BCF is a straight line. It is given that $\angle BCA = \angle XAB = 58^\circ$, $\angle ABD = 32^\circ$ and $\angle CFE = 120^\circ$.



- (a) Find angle ACD .
Give a reason for each step of your working.

Answer Angle $ACD = \dots\dots\dots^\circ$ [1]

- (b) Explain why BD is a diameter of circle $ABCD$.

Answer BD is a diameter of circle $ABCD$ because
..... [1]

- (c) Find angle DAY .
Give a reason for each step of your working.

Answer Angle $DAY = \dots\dots\dots^\circ$ [1]

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- 6 (d) Given that $FC = FE$, show that triangle CDE is equilateral.

Give a reason for each step of your working.

Answer

[3]

- (e) (i) Prove that triangle OCE is congruent to triangle FCE .

Answer

.....

.....

.....

.....

..... [3]

- (ii) Hence, what is the special name given to quadrilateral $COEF$?

Answer [1]

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7 (a) Given the sequence 4, 7, 10, 13, 16, ...

(i) Find an expression for the n th term (T_n) of the sequence.*Answer* $T_n = \dots\dots\dots$ [1](ii) Write down the 88th term (T_{88}) of this sequence.*Answer* $T_{88} = \dots\dots\dots$ [1]

(b) Study the following pattern:

Row 1	$1^3 = 1 = \left(\frac{1 \times 2}{2}\right)^2$
Row 2	$1^3 + 2^3 = 9 = (1 + 2)^2 = \left(\frac{2 \times 3}{2}\right)^2$
Row 3	$1^3 + 2^3 + 3^3 = 36 = (1 + 2 + 3)^2 = \left(\frac{3 \times 4}{2}\right)^2$
Row 4	$1^3 + 2^3 + 3^3 + 4^3 = 100 = (1 + 2 + 3 + 4)^2 = \left(\frac{4 \times 5}{2}\right)^2$
\vdots	\vdots
Row n	$1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = q$

(i) Write down Row 5.

Answer Row 5 = $\dots\dots\dots$ $\dots\dots\dots$ [1]

(ii) Calculate the sum of the numbers in Row 10.

Answer $\dots\dots\dots$ [1]

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7 (b) (iii) Express q in terms of n .

Answer $q = \dots\dots\dots$ [1]

(iv) Using your result in (b)(iii) or otherwise, find the value of n when $q = 1296$.

Answer $n = \dots\dots\dots$ [2]

(v) A student found the sum of the numbers in a row to be 600. Give a reason why this answer cannot be accepted.

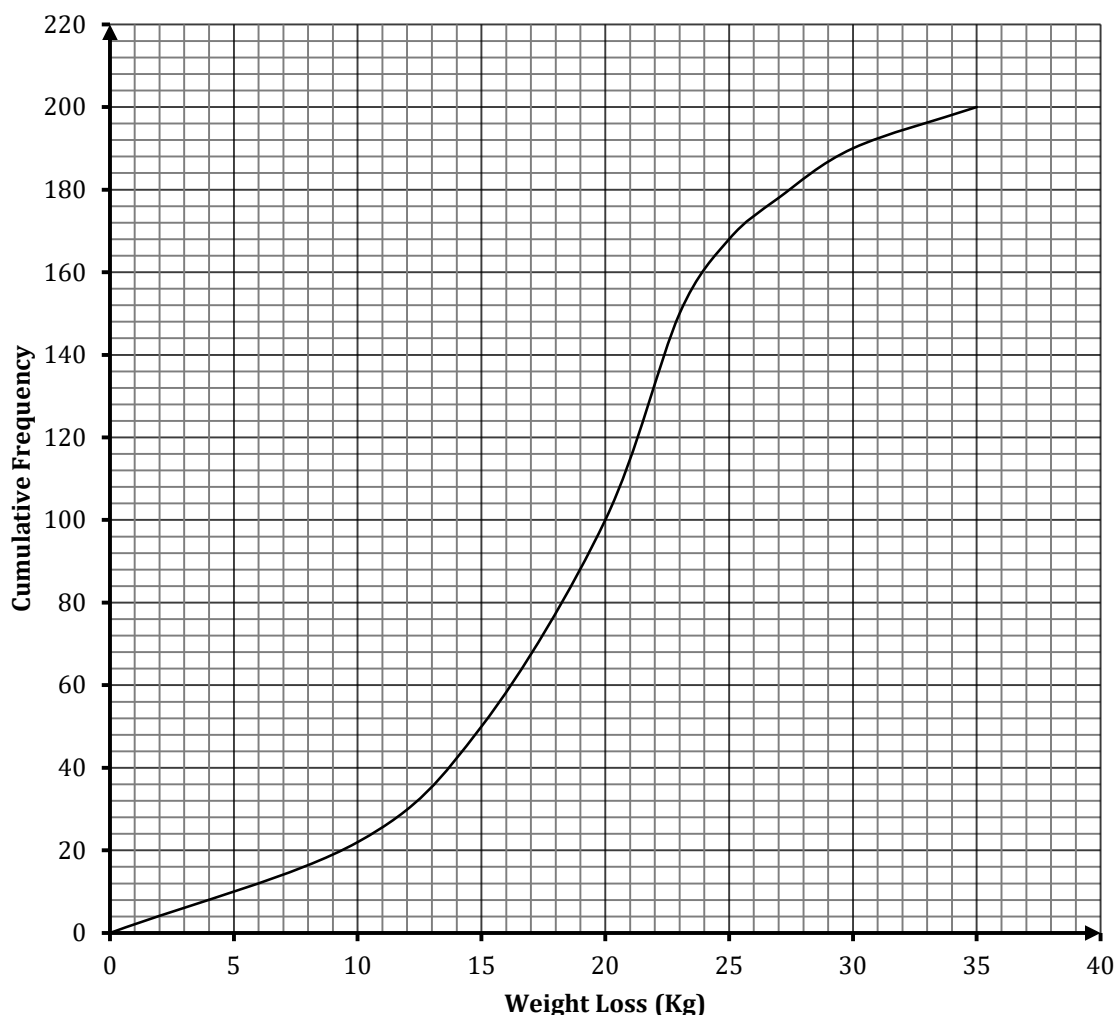
Answer The answer cannot be accepted because

.....[1]

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- 8 The cumulative frequency curve below shows the amount of weight loss, measured to the nearest kg, of 200 men in Alpha Centre.



The corresponding cumulative frequency table for this distribution is as shown below.

Weight loss (x kg)	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30	≤ 35
Cumulative Frequency	22	p	100	168	190	200

- (a) Determine the value of p .

Answer $p = \dots\dots\dots$ [1]

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- 8 (b) Use the curve to estimate
- (i) the median weight loss,

Answer kg [1]

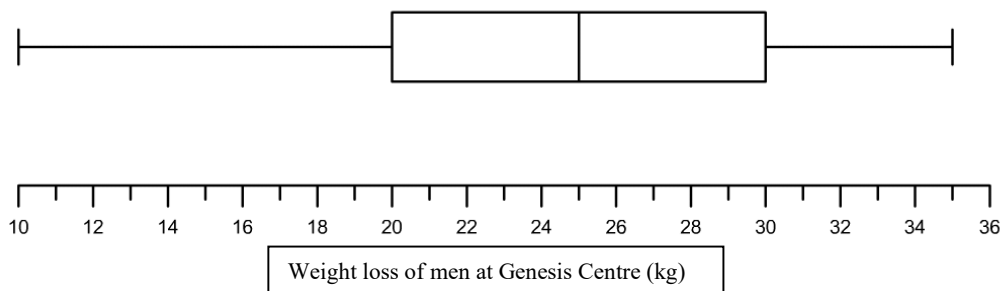
- (ii) the interquartile range of the weight loss,

Answer kg [2]

- (iii) the 44th percentile weight loss.

Answer kg [1]

- (c) A second fitness centre, Genesis Centre also measured the amount of weight loss by 200 of their men. The box-and-whisker diagram below illustrates their weight loss achieved.



- (i) Find the interquartile range of the weight loss.

Answer kg [1]

- (ii) How many men achieved a weight loss of more than 30 kg?

Answer men [1]

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- 8 (c) (iii) A physical instructor claimed that the weight loss program at Alpha Centre is more effective than Genesis Centre. Explain if his statement is true.

Answer
.....
..... [1]

- (d) Charles needs to take a certification test to become a physical instructor. He takes the certification test until he passes. Each time he takes the test, the probability that Charles passes the test is 0.8. Find the probability that Charles

- (i) passes only at the third attempt,

Answer [1]

- (ii) passes in either the first or second attempt.

Answer [2]

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9 (a) The coordinates of points A , B and C are $(4, -16)$, $(1, 5)$ and $(2, 1)$ respectively.

(i) Express \overrightarrow{BC} as a column vector.

Answer $\overrightarrow{BC} = \dots\dots\dots$ [1]

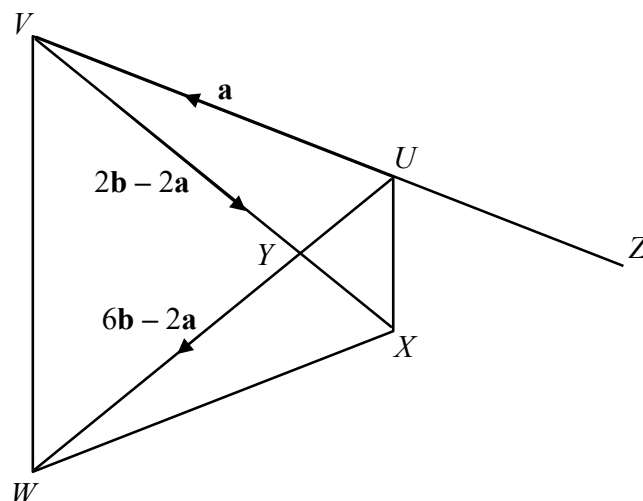
(ii) Find $|\overrightarrow{BC}|$.

Answer $|\overrightarrow{BC}| = \dots\dots\dots$ units [2]

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- 9 (b) In the diagram, $UVWX$ is a trapezium such that $\overrightarrow{UV} = \mathbf{a}$, $\overrightarrow{VX} = 2\mathbf{b} - 2\mathbf{a}$ and $\overrightarrow{UW} = 6\mathbf{b} - 2\mathbf{a}$. UW intersects VX at Y . VU is extended to point Z such that $VU = 2UZ$.



- (i) Express, as simply as possible, in terms of \mathbf{a} and/or \mathbf{b} ,

(a) \overrightarrow{VW} ,

Answer $\overrightarrow{VW} = \dots\dots\dots$ [1]

(b) \overrightarrow{WX} ,

Answer $\overrightarrow{WX} = \dots\dots\dots$ [1]

(c) \overrightarrow{XZ} .

Answer $\overrightarrow{XZ} = \dots\dots\dots$ [1]

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- 9 (b) (ii) Use your answers to part (b)(i)(b) and (b)(i)(c) to explain why W , X and Z lie on a straight line.

Answer
.....
.....
.....[2]

- (iii) Calculate the value of

(a) $\frac{\text{Area of } \triangle ZUX}{\text{Area of } \triangle ZVW}$,

Answer [1]

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9 (b) (iii) (b) $\frac{\text{Area of } \triangle XVW}{\text{Area of } \triangle ZVW},$

Answer [1]

(c) $\frac{\text{Area of } \triangle XVW}{\text{Area of trapezium } UVWX}.$

Answer [2]

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- 10 A class of students plans to sell all-day breakfast sets during the school carnival in order to raise funds for a charity.

Each breakfast set consists of 2 scrambled eggs, 2 slices of toast, 1 sausage, 1 slice of chicken ham and a cup of coffee.

The students estimate that they will sell 300 all-day breakfast sets.

The cost of the ingredients used are as follows.

Item	Description	Unit cost
Eggs	Pasar Fresh Eggs (10 per pack)	\$2.70
	Dason Fresh Eggs (15 per pack)	\$4.55
	Seng Seng Fresh Eggs (10 + 2 per pack)	\$3.70
Bread	Gardenia Enriched Soft White Bread (14 slices)	\$2.70
	Sunshine Enriched Soft White Bread (12 slices)	\$2.50
Sausages	Master Grocer Valley Farm Chicken Frank (10 per pack)	\$5.25
	Grand Chef Sausages (6 per pack) (\$0.35 off per 2 packs)	\$3.20
Ham	FairPrice Baked Ham (10 per pack) (20% off per 3 packs)	\$4.35
	Smart Choice Baked Ham (10 per pack)	\$3.30
Coffee	Nescafe 3 in 1 Instant Coffee (35 per pack) (Buy 5 get 1 free)	\$6.15
	Indocafe 3 in 1 Coffeemix (25 per pack)	\$3.95

- (a) Find the lowest possible total cost of the ingredients required for the 300 all-day breakfast sets.

[Turn over for more working space for Question 10a]

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[Working space for Question 10a]

Answer \$ [5]

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- 10 (b) The school provides \$200 in funding for the students and have set two criteria which every class must meet:

Criteria 1: Up to 40% of the sales can be used to cover for their expenses, while the remaining goes to the charity.

Criteria 2: The students must raise at least \$600 for charity.

Find the minimum price (to the nearest ten cents) they must charge for each breakfast set such that they meet both criteria.

Justify your answer, showing all necessary workings clearly.

State an assumption you have made in your calculations.

[4]

End of paper