Section A: Pure Mathematics [40 marks]

1(a) Method 1: use standard expansion

$$\mathrm{f}(x)=(4-3x)^{\frac{1}{2}}$$
$$=4^{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{\frac{1}{2}}$$
$$=2\left(1+\left(\frac{1}{2}\right)\left(-\frac{3x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{3x}{4}\right)^2+\ldots\right)$$
$$=2-\frac{3}{4}x-\frac{9}{64}x^2+\ldots$$

Validity range: $\left|-\frac{3}{4}x\right|<1\Rightarrow|x|<\frac{4}{3}\Rightarrow-\frac{4}{3}<x<\frac{4}{3}$

Method 2: use repeated differentiation

$$\mathrm{f}(x)=(4-3x)^{\frac{1}{2}}$$
$$\mathrm{f}'(x)=\frac{1}{2}(4-3x)^{-\frac{1}{2}}(-3)=-\frac{3}{2}(4-3x)^{-\frac{1}{2}}$$
$$\mathrm{f}''(x)=\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)(4-3x)^{-\frac{3}{2}}(-3)=-\frac{9}{4}(4-3x)^{-\frac{3}{2}}$$

When $x=0$,

$$\mathrm{f}(0)=(4-3(0))^{\frac{1}{2}}=2$$
$$\mathrm{f}'(0)=-\frac{3}{2}(4-3(0))^{-\frac{1}{2}}=-\frac{3}{4}$$
$$\mathrm{f}''(0)=-\frac{9}{4}(4-3(0))^{-\frac{3}{2}}=-\frac{9}{32}$$

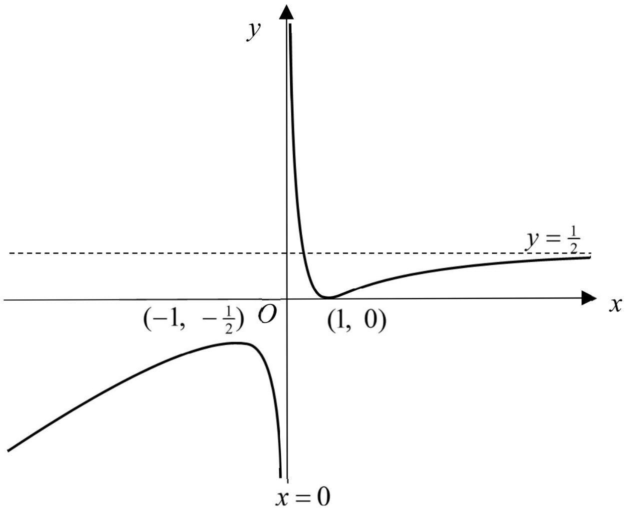
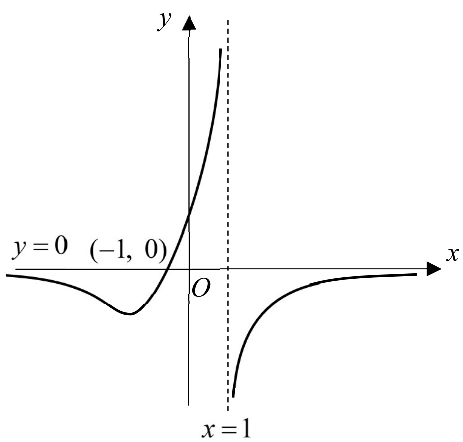
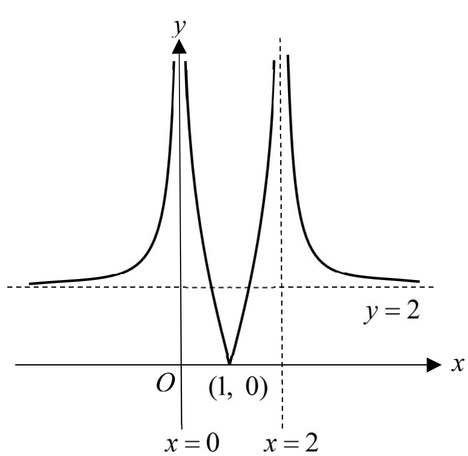
Then $\mathrm{f}(x)=2+\left(-\frac{3}{4}\right)x+\frac{\left(-\frac{9}{32}\right)}{2!}x^2+\ldots=2-\frac{3}{4}x-\frac{9}{64}x^2+\ldots$

Validity range: $\left|-\frac{3}{4}x\right|<1\Rightarrow|x|<\frac{4}{3}\Rightarrow-\frac{4}{3}<x<\frac{4}{3}$

(b) Sub $x=\frac{1}{4}$ into the expansion:

$$\sqrt{4-3\left(\frac{1}{4}\right)}=2-\frac{3}{4}\left(\frac{1}{4}\right)-\frac{9}{64}\left(\frac{1}{4}\right)^2+\ldots=2-\frac{3}{16}-\frac{9}{1024}+\ldots$$
$$\sqrt{\frac{13}{4}}\approx\frac{2048-3\times64-9}{1024}=\frac{1847}{1024}$$
$$\sqrt{13}\approx2\times\frac{1847}{1024}=\frac{1847}{512}\text{ (shown)}$$

(c) The value obtained by substituting $x=\frac{1}{13}$ will be a better approximation for $\sqrt{13}$ because $x=\frac{1}{13}$ is closer to 0 than $x=\frac{1}{4}$.

2(a)(i)	
(a)(ii)	
(a)(iii)	
(b) $0 \le a \le 2$	

3(a) $S_n = \ln\left(\dfrac{e^n}{3^{n^2}}\right)$

$$\begin{aligned} u_n &= S_n - S_{n-1} \\ &= \ln\left(\frac{e^n}{3^{n^2}}\right) - \ln\left(\frac{e^{n-1}}{3^{(n-1)^2}}\right) \\ &= \ln\left(\frac{e^n}{3^{n^2}} \times \frac{3^{(n-1)^2}}{e^{n-1}}\right) \\ &= \ln\left(\frac{3^{-2n+1}}{e^{-1}}\right) \\ &= 1 + (1-2n)\ln 3 \quad \text{(Shown)} \end{aligned}$$

(b) $u_n - u_{n-1} = \left[1 + (1-2n)\ln 3\right] - \left[1 + (1-2(n-1))\ln 3\right]$

$= -2\ln 3$ which is a constant independent of n

Hence, the sequence is an arithmetic progression.

(c) First term $= 1 - \ln 3$

Common difference $= -4\ln 3$

Sum of the first ten odd-numbered terms

$$= \frac{10}{2}[2(1-\ln 3) + 9(-4\ln 3)]$$
$$= 10 - 190\ln 3$$

(d) Method 1 – use $r = \dfrac{T_n}{T_{n-1}}$

$$\begin{aligned} r &= \frac{e^{u_n}}{e^{u_{n-1}}} \\ &= e^{u_n - u_{n-1}} \\ &= e^{-2\ln 3} \quad \text{(from (b))} \\ &= 3^{-2} \\ &= \frac{1}{9} \end{aligned}$$

Method 2

$$r = \frac{e^{1-3\ln 3}}{e^{1-\ln 3}} = e^{-2\ln 3} = \frac{1}{9}$$

(e) We want $\left|S_n - S_\infty\right| \le 10^{-8}$.

$$\left|\frac{a(1-r^n)}{1-r} - \frac{a}{1-r}\right| \le 10^{-8}$$

$$\Rightarrow \left|\frac{-ar^n}{1-r}\right| \le 10^{-8}$$

$$\Rightarrow \frac{e^{1-\ln 3}\left(\frac{1}{9}\right)^n}{1-\frac{1}{9}} \le 10^{-8}$$

$$\Rightarrow \frac{3e}{8}\left(\frac{1}{9}\right)^n \le 10^{-8}$$

Method 1 (algebraic)

$$n \ge \frac{\ln\frac{8(10^{-8})}{3e}}{\ln\frac{1}{9}} = 8.39233808$$

Least $n = 9$.

Method 2 (use GC)

n	$\frac{3e}{8}\left(\frac{1}{9}\right)^n$
8	2.368×10^{-8} $(>10^{-8})$
9	2.631×10^{-9} $(<10^{-8})$
10	2.923×10^{-10} $(<10^{-8})$

Least $n = 9$.

4(a) Method 1 (find p and q first)

Since $1-2\sqrt{3}\mathrm{i}$ is a root of $z^4 + pz^3 + 5z^2 + qz - 26 = 0$, then

$$\left(1-2\sqrt{3}\mathrm{i}\right)^4 + p\left(1-2\sqrt{3}\mathrm{i}\right)^3 + 5\left(1-2\sqrt{3}\mathrm{i}\right)^2 + q\left(1-2\sqrt{3}\mathrm{i}\right) - 26 = 0$$

Using G.C.,

$$73 + 152.420\mathrm{i} + p\left(-35+31.177\mathrm{i}\right)$$
$$+5\left(-11-6.9282\mathrm{i}\right) + q\left(1-2\sqrt{3}\mathrm{i}\right) - 26 = 0$$

Comparing real and imaginary parts,

$$73 - 35p - 55 + q - 26 = 0 \Rightarrow -35p + q = 8$$

$$152.420 + 31.177p - 34.641 - 2\sqrt{3}q = 0$$
$$\Rightarrow 31.177p - 2\sqrt{3}q = -117.779$$

Using G.C., $p = 1.00,\ q = 43.0$ (3 s.f.)

Hence, the equation is $z^4 + 1.00z^3 + 5z^2 + 43.0z - 26 = 0$

Using G.C., the roots of the equation are

$1 + 3.46\mathrm{i},\ 1 - 3.46\mathrm{i},\ -3.56$ and 0.562.

<u>Method 2 (find factors first)</u>

Since the coefficients of the polynomial are real and $z = 1 - 2\sqrt{3}\mathrm{i}$ is a root, by Conjugate Root Theorem, $z = 1 + 2\sqrt{3}\mathrm{i}$ is also a root.

$$\begin{aligned}
&z^4 + pz^3 + 5z^2 + qz - 26 \\
&= \left[z - \left(1 + 2\sqrt{3}\mathrm{i}\right)\right]\left[z - \left(1 - 2\sqrt{3}\mathrm{i}\right)\right]\left(z^2 + az + b\right) \\
&= \left[(z-1) - 2\sqrt{3}\mathrm{i}\right]\left[(z-1) + 2\sqrt{3}\mathrm{i}\right]\left(z^2 + az + b\right) \\
&= \left[(z-1)^2 - \left(2\sqrt{3}\mathrm{i}\right)^2\right]\left(z^2 + az + b\right)
\end{aligned}$$

$$\begin{aligned}
&z^4 + pz^3 + 5z^2 + qz - 26 \\
&= \left[z - \left(1 + 2\sqrt{3}\mathrm{i}\right)\right]\left[z - \left(1 - 2\sqrt{3}\mathrm{i}\right)\right]\left(z^2 + az + b\right) \\
&= \left[(z-1) - 2\sqrt{3}\mathrm{i}\right]\left[(z-1) + 2\sqrt{3}\mathrm{i}\right]\left(z^2 + az + b\right). \\
&= \left[(z-1)^2 - \left(2\sqrt{3}\mathrm{i}\right)^2\right]\left(z^2 + az + b\right)
\end{aligned}$$

$$\begin{aligned}
&= \left(z^2 - 2z + 1 + 12\right)\left(z^2 + az + b\right) \\
&= \left(z^2 - 2z + 13\right)\left(z^2 + az + b\right) \\
&= z^4 + z^3(-2 + a) + z^2(13 - 2a + b) + z(13a - 2b) + 13b
\end{aligned}$$

Comparing the constant terms:

$13b = -26$

$b = -2$

Comparing the coefficients of z^2

$13 - 2a + b = 5$

$13 - 2a - 2 = 5$

$2a = 6$

$a = 3$

Comparing the coefficients of z^3

$p = -2 + a$

$p = 1$

Comparing the coefficients of z

$q = 13a - 2b$

$q = 39 + 4 = 43$

$\therefore$ The other roots are $-3.56,\ 0.562$ and $1 + 2\sqrt{3}\mathrm{i}$.

(b)(i) $w = \dfrac{\mathrm{i}^3}{\left(-\sqrt{3} + \mathrm{i}\right)^4}$

$$|w| = \left|\frac{\mathrm{i}^3}{\left(-\sqrt{3} + \mathrm{i}\right)^4}\right| = \frac{|\mathrm{i}|^3}{\left|\left(-\sqrt{3} + \mathrm{i}\right)\right|^4} = \frac{1^3}{\left(\sqrt{3+1}\right)^4} = \frac{1}{16}$$

$$\arg(w) = \arg\left(\frac{\mathrm{i}^3}{\left(-\sqrt{3}+\mathrm{i}\right)^4}\right)$$
$$= \left[3\arg(\mathrm{i}) - 4\arg\left(-\sqrt{3}+\mathrm{i}\right)\right]$$
$$= \left[3\left(\frac{\pi}{2}\right) - 4\left(\frac{5\pi}{6}\right)\right]$$
$$= -\frac{11\pi}{6}$$
$$\therefore \arg(w) = \frac{\pi}{6}$$

Method 2 (change to exponential form first)

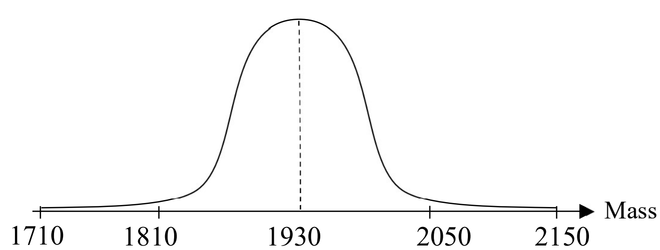
$$w = \frac{\mathrm{i}^3}{\left(-\sqrt{3}+\mathrm{i}\right)^4} = \frac{\left(\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}\right)^3}{\left(2\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}\right)^4} = \frac{\mathrm{e}^{\frac{3\pi}{2}\mathrm{i}}}{2^4\mathrm{e}^{\frac{20\pi}{6}\mathrm{i}}}$$
$$= \frac{1}{16}\mathrm{e}^{\frac{3\pi}{2}\mathrm{i} - \frac{20\pi}{6}\mathrm{i}} = \frac{1}{16}\mathrm{e}^{-\frac{11\pi}{6}\mathrm{i}}$$
$$|w| = \frac{1}{16}$$
$$\arg(w) = -\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$$

(b)(ii)

$$\arg\left(\frac{\mathrm{i}w^n}{w^*}\right) = \arg(\mathrm{i}) + n\arg(w) - \arg(w^*)$$
$$= \frac{\pi}{2} + \frac{n\pi}{6} - \left(-\frac{\pi}{6}\right)$$
$$= \frac{\pi}{2} + \frac{n\pi}{6} + \frac{\pi}{6}$$
$$\frac{\pi}{2} + \frac{n\pi}{6} + \frac{\pi}{6} = (2k+1)\left(\frac{\pi}{2}\right)$$
$$\frac{\pi}{2} + \frac{n\pi}{6} + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$$
$$n+1 = 6k$$
$$n = 6k-1$$

Since n is positive, the smallest possible value is $n = 5$ (when $k = 1$).

Section B: Probability and Statistics [60 marks]

5(a)	Let the mass in grams of a randomly chosen orange be X. $X \sim \mathrm{N}(150,\ 14^2)$ $\mathrm{P}(X < 180)$ $= 0.98394$ ≈ 0.984
(b)	Let the mass in grams of a randomly chosen kiwi be Y. $Y \sim \mathrm{N}(70,\ 8^2)$ Let the mass in grams of a randomly chosen empty basket be W. $W \sim \mathrm{N}(750,\ 168)$ Let $M = X_1 + \ldots + X_6 + Y_1 + \ldots + Y_4 + W$ $M \sim \mathrm{N}(1930,\ 1600)$ $\mathrm{P}(\lvert M - 1930 \rvert < 25)$ $= \mathrm{P}(-25 < M - 1930 < 25)$ $= \mathrm{P}(1905 < M < 1955)$ $= 0.46803$ ≈ 0.468
(c)	 $\mathrm{P}\big(1930 - 3(40) < M < 1930 + 3(40)\big)$ $= \mathrm{P}\big(1810 < M < 2050\big) \approx 0.997$ [Note that $\mathrm{P}\big(M < 1810\big), \mathrm{P}\big(M > 2050\big)$ **is very small.]**
(d)	$\mathrm{P}(M_1 > 2000) \times \mathrm{P}(M_2 < 1900) \times \mathrm{P}(1900 \le M_3 \le 2000) \times 3!$ $= 0.039944$ ≈ 0.0399

6(a) Let X be the number of cracked screen protectors, out of 20.

$X \sim \mathrm{B}(20,\ 0.17)$

$\mathrm{P}(X \le 3) \approx 0.55041$ (5 s.f.)

(b)

X	$\mathrm{P}(X = x)$
2	0.19189
3	0.23582
4	0.20528

Most probable number of cracked screen protectors $= 3$

(c) Let Y be the number of boxes that contain no more than 3 cracked screen protectors, out of 8.

$Y \sim \mathrm{B}(8,\ 0.55041)$

$\text{Mean} = 8(0.55041) = 4.40328$

$\text{Variance} = 8(0.55041)(1 - 0.55041) \approx 1.9797$

Since n is large, by Central Limit Theorem,

$\overline{Y} \sim \mathrm{N}\left(4.40328,\ \dfrac{1.9797}{n}\right)$ approximately.

$\mathrm{P}(\overline{Y} \le 5) \ge 0.99$

n	$\mathrm{P}(\overline{Y} \le 5)$
30	$0.98991 < 0.99$
31	$0.99089 > 0.99$
32	$0.99178 > 0.99$

From GC, least n is 31.

(d)

$$\begin{aligned}
&\mathrm{P}(\text{ready for local sales} \mid \text{not ready for overseas sales}) \\
&= \frac{\mathrm{P}(\text{ready for local sales AND not ready for overseas sales})}{\mathrm{P}(\text{not ready for overseas sales})} \\
&= \frac{\mathrm{P}(Y \ge 4 \ \cap \ Y < 6)}{\mathrm{P}(Y < 6)} \\
&= \frac{\mathrm{P}(Y = 4) + \mathrm{P}(Y = 5)}{\mathrm{P}(Y \le 5)} \\
&= 0.66682 \\
&\approx 0.667
\end{aligned}$$

7(a) $\mathrm{P}(W=1)=2\left(\frac{n}{n+m+1}\times\frac{m}{n+m}\right)$

$=\frac{2nm}{(n+m+1)(n+m)}$ (shown)

$\mathrm{P}(W=0)=\frac{m(m-1)}{(n+m+1)(n+m)}$

$\mathrm{P}(W=2)=\frac{n(n-1)}{(n+m+1)(n+m)}$

$\mathrm{P}(W=10)=\frac{2m}{(n+m+1)(n+m)}$

$\mathrm{P}(W=11)=\frac{2n}{(n+m+1)(n+m)}$

(b)

$$\begin{aligned}\mathrm{E}(W)&=\frac{1}{(n+m+1)(n+m)}\left[2nm+2n(n-1)+20m+22n\right]\\&=\frac{1}{(n+m+1)(n+m)}\left[2nm+2n^2-2n+20m+22n\right]\\&=\frac{1}{(n+m+1)(n+m)}\left[2n(n+m)+20(n+m)\right]\\&=\frac{2(n+10)}{n+m+1}\end{aligned}$$

For the game master to expect to make a profit,

$$\begin{aligned}\mathrm{E}(W)&<1\\\frac{2(n+10)}{n+m+1}&<1\\2n+20&<n+m+1\\m-n&>19\text{ (shown)}\end{aligned}$$

(c)

w	0	1	2	10	11
y	4	3	2	6	7
$\mathrm{P}(W=w)$	$\frac{1560}{2550}$	$\frac{800}{2550}$	$\frac{90}{2550}$	$\frac{80}{2550}$	$\frac{20}{2550}$

Using GC,

$\mathrm{E}(Y)=3.70196=3.70$ (3 s.f.)

$\mathrm{Var}(Y)=0.56215=0.562$ (3 s.f.)

8(a) No. of ways

$$= ({}^{9}C_{2}\,{}^{7}C_{2}\,{}^{5}C_{5} \times \frac{3!}{2!}) + ({}^{9}C_{2}\,{}^{7}C_{3}\,{}^{4}C_{4} \times 3!) + ({}^{9}C_{3}\,{}^{6}C_{3}\,{}^{3}C_{3})$$
$$= 2268 + 7560 + 1680$$
$$= 11508$$

(b) Case 1: Triangle formed from 1 point from each side $= {}^{2}C_{1} \times {}^{3}C_{1} \times {}^{4}C_{1} = 24$

Case 2: Triangle formed from 2 points on XZ + 1 other $= {}^{4}C_{2} \times {}^{5}C_{1} = 30$

Case 3: Triangle formed from 2 points on YZ + 1 other $= {}^{3}C_{2} \times {}^{6}C_{1} = 18$

Case 4: Triangle formed from 2 points on XY + 1 other $= {}^{2}C_{2} \times {}^{7}C_{1} = 7$

Total no. of ways $= 24 + 30 + 18 + 7 = 79$

<u>Method 2 (complement)</u>

$$\text{No. of ways } = \begin{pmatrix} \text{no. of ways} \\ \text{of choosing 3 bulbs} \\ \text{without restrictions} \end{pmatrix} - \begin{pmatrix} \text{no. of ways} \\ \text{of choosing 3 bulbs} \\ \text{from side } YZ \end{pmatrix} - \begin{pmatrix} \text{no. of ways} \\ \text{of choosing 3 bulbs} \\ \text{from side } XZ \end{pmatrix}$$
$$= {}^{9}C_{3} - {}^{3}C_{3} - {}^{4}C_{3}$$
$$= 84 - 1 - 4$$
$$= 79$$

(c)
$$\text{No. of ways} = \begin{pmatrix} \text{no. of ways} \\ \text{without restrictions} \end{pmatrix} - \begin{pmatrix} \text{no. of ways} \\ \text{red or blue} \\ \text{is used} \end{pmatrix} - \begin{pmatrix} \text{no. of ways} \\ \text{blue or green} \\ \text{is used} \end{pmatrix} - \begin{pmatrix} \text{no. of ways} \\ \text{red or green} \\ \text{is used} \end{pmatrix}$$
$$+ \begin{pmatrix} \text{no. of ways} \\ \text{only red} \\ \text{is used} \end{pmatrix} + \begin{pmatrix} \text{no. of ways} \\ \text{only blue} \\ \text{is used} \end{pmatrix} + \begin{pmatrix} \text{no. of ways} \\ \text{only green} \\ \text{is used} \end{pmatrix}$$
$$= 3^9 - 3(2^9) + 3$$
$$= 19683 - 1536 + 3$$
$$= 18150$$

<u>Alternative method</u>

$$\text{No. of ways } = \begin{pmatrix} \text{no. of ways} \\ \text{without restrictions} \end{pmatrix} - \begin{pmatrix} \text{no. of ways} \\ \text{only 1 colour} \\ \text{is used} \end{pmatrix} - \begin{pmatrix} \text{no. of ways} \\ \text{exactly 2 colours} \\ \text{are used} \end{pmatrix}$$
$$= 3^9 - 3 - {}^{3}C_{2}(2^9 - 2)$$
$$= 19683 - 3 - 1530$$
$$= 18150$$

(d) No. of ways $=(\text{Total Area})-(\text{Area 1 and 2})-(\text{Area 2 and 3})+(\text{Area 2})$

$$=\begin{pmatrix}\text{no. of ways}\\\text{without}\\\text{restrictions}\end{pmatrix}-\begin{pmatrix}\text{no. of ways}\\\text{2 }A\text{'s are}\\\text{together}\end{pmatrix}-\begin{pmatrix}\text{no. of ways}\\\text{2 }B\text{'s are}\\\text{together}\end{pmatrix}+\begin{pmatrix}\text{no. of ways}\\\text{2 }A\text{'s \& 2 }B\text{'s}\\\text{are together}\end{pmatrix}$$

$$=\frac{(9-1)!}{2!2!2!}-\frac{(8-1)!}{2!2!}-\frac{(8-1)!}{2!2!}+\frac{(7-1)!}{2!}$$

$$=5040-1260-1260+360$$

$$=2880$$

Alternative 1 (complement)

$$=(\text{Total Area})-(\text{Area 1})-(\text{Area 3})-(\text{Area 2})$$

$$=\begin{pmatrix}\text{no. of ways}\\\text{without}\\\text{restrictions}\end{pmatrix}-\begin{pmatrix}\text{no. of ways}\\\text{A's are separate}\\\text{B's are together}\end{pmatrix}-\begin{pmatrix}\text{no. of ways}\\\text{B's are separate}\\\text{A's are together}\end{pmatrix}-\begin{pmatrix}\text{no. of ways}\\\text{2 }A\text{'s \& 2 }B\text{'s}\\\text{are together}\end{pmatrix}$$

$$=\frac{(9-1)!}{2!2!2!}-\left(\frac{(6-1)!}{2!}\times {}^{6}C_{2}\right)-\left(\frac{(6-1)!}{2!}\times {}^{6}C_{2}\right)-\frac{(7-1)!}{2!}$$

$$=5040-900-900-360$$

$$=2880$$

Alternative 2 (complement)

$$=\begin{pmatrix}\text{no. of ways}\\\text{A's are separate}\\\ \end{pmatrix}-\begin{pmatrix}\text{no. of ways}\\\text{A's are separate and}\\\text{2B's are together}\end{pmatrix}$$

$$=\left(\frac{(7-1)!}{2!2!}\times {}^{7}C_{2}\right)-\left(\frac{(6-1)!}{2!}\times {}^{6}C_{2}\right)$$

$$=3780-900$$

$$=2880$$

Alternative 3 – Use the slotting method by putting in the 2 A's first, followed by the 2 B's.

No. of ways $=\begin{pmatrix}\text{no. of ways}\\\text{2 }A\text{'s are separated}\\\text{initially}\end{pmatrix}+\begin{pmatrix}\text{no. of ways}\\\text{2 }A\text{'s are together}\\\text{initially}\end{pmatrix}$

$$=\frac{(5-1)!}{2!}\times {}^{5}C_{2}\times {}^{7}C_{2}+\frac{(5-1)!}{2!}\times {}^{5}C_{1}\times {}^{6}C_{1}$$

$$=12\times 10\times 21+12\times 5\times 6$$

$$=2880$$

9(a) From GC, $\bar{x} = 3088$ (exact)

$$\begin{aligned} s^2 &= 56.64509393^2 \\ &= 3208.666667 \\ &\approx 3210 \text{ (3 s.f.)} \end{aligned}$$

(b) Since $n = 13$ is not large, we need to assume that the population fan speed is normally distributed.

Let X denote the fan speeds in RPM and μ denote the population mean.

To test $\mathrm{H}_0 : \mu = 3100$ against $\mathrm{H}_1 : \mu \neq 3100$ at 10% level of significance.

$$\bar{X} = \frac{X_1 + X_2 + ... + X_{13}}{13}$$

Under H_0, $\bar{X} \sim \mathrm{N}\left(3100, \frac{30^2}{13}\right)$.

Using $\bar{X}$	Using p-value	Using z-value
Critical region: $\bar{x} < 3086.3$ or $\bar{x} > 3113.7$ Since $\bar{x} = 3088$ does not lie in the critical region,	For $\bar{x} = 3088$, $p\text{-value} = 0.14924$. Since $p\text{-value} > 0.10$,	Critical region: $z > 1.6449$ or $z < -1.6449$ For $\bar{x} = 3088$, $z = \dfrac{3088 - 3100}{\sqrt{900/13}}$. Since $z = -1.4422$ does not lie in the critical region,

we do not reject H_0 and conclude that there is insufficient evidence at the 10% level of significance to support the claim that the population mean fan speed is different from 3100 RPM.

(c) The p-value of 0.14924 means that, given that the population mean is indeed 3100 RPM, the probability of obtaining a sample mean that is less than or equal to 3088 RPM, or more than or equal to 3112 RPM, is 0.14924.

[Mathematically, $P\left(\bar{X} \leq 3088 \cup \bar{X} \geq 3112 \mid \mu = 3100\right) = 0.14924$]

(d) From **(b)**, $p\text{-value} = 0.14924$.

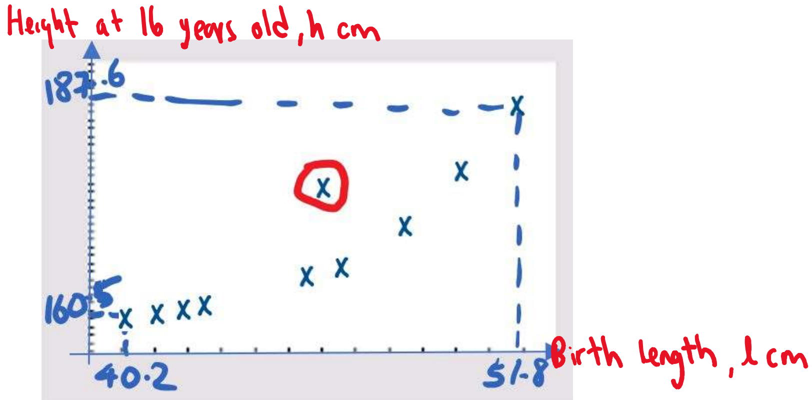
Hence, least level of significance = 15%.

(e) Let Y denote the standby-time in hours.

To test $\mathrm{H}_0 : \mu = 36$ against $\mathrm{H}_1 : \mu < 36$ at 5% level of significance.

Under H_0, we have $\bar{Y} \sim \mathrm{N}\left(36, \frac{9}{50}\right)$.

Critical region: $\bar{y} < 35.302 \quad \Rightarrow \quad \bar{y} < 35.3$ (3 s.f.)

10(a)	Scatter diagram: 
(b)	From the scatter diagram, the points appear to lie on a curve rather than a straight line, so $h=a+bl$ may not model the relationship well.
(c)	$h=a+bl$: $r=0.93877$; $h=a+bl^3$: $r=0.95886$ Since $\|r\|$ between h and l^3 is closer to 1 than that between l and h, this indicates a stronger linear correlation between l^3 and h, so $h=a+bl^3$ is a better model. Equation is $h=135.96+0.00033807l^3$, i.e. $h=136+0.000338l^3$ to 3 s.f.
(d)	No, the regression line should not be used as there is no clear independence of either variable/ no clear independent and dependent variable. We should use the regression line of l^3 on h to do the estimation.
(e)	for $l=46.0$, $h=168.9$ to 1 d.p The estimate is reliable because 46.0 is in the range $40.2 \le l \le 51.8$ so we are doing an interpolation, and r is close to 1 so there is a strong positive linear correlation between h and l^3.
(f)	Possible explanations: - Diet - Lifestyle - Exercise - Nutrition - Halfway through growth spurt