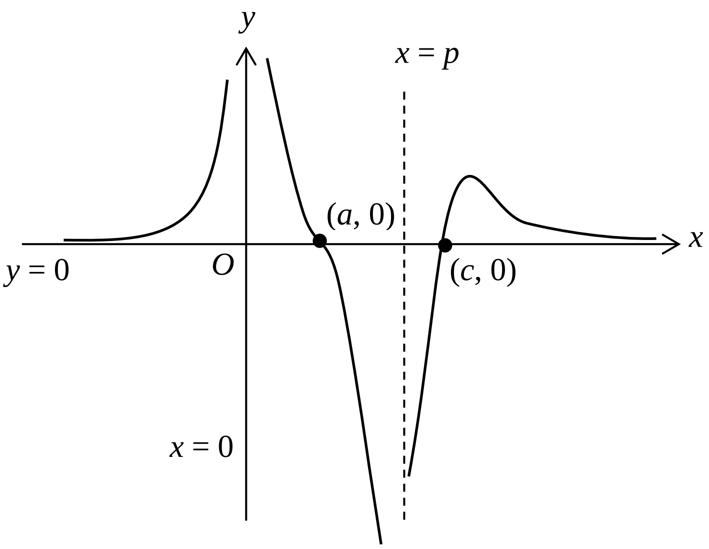
2022 JC2 H2 Paper 2 Suggested Solution:

1. (i) $y = \mathrm{f}'(x+1)$

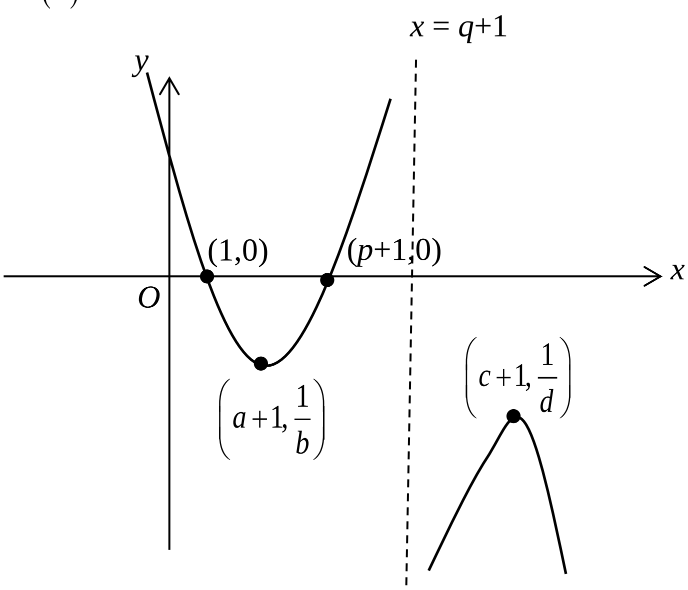


Asymptotes: $x = 0$, $x = \mathrm{p}$ and $y = 0$

x-intercepts: $(a,0)$, $(c,0)$

Shape

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$



Asymptotes: $x = q+1$

x-intercepts: $(1,0),(p+1,0)$

Shape & turning point (with relative positions of $1/b$ vs $1/d$)

2. (i)

Given that $a_1 = 2$ and $a_2 = 4$,

$$4 = 2(2) - 3(2) + K \quad \Rightarrow \quad K = 6$$

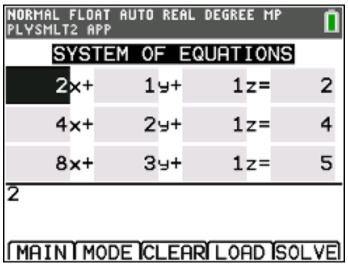
$$\therefore\ a_3 = 2a_2 - 3(3) + K$$
$$= 2(4) - 3(3) + 6 \quad = 5$$

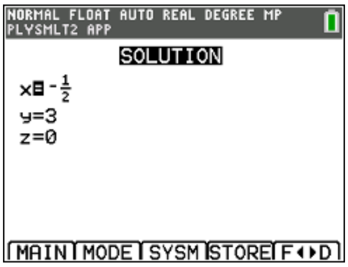
(ii)

$\because\ \ a_n = p(2^n) + qn + r,$

$$\left.\begin{array}{lll} a_1 = 2 & \Rightarrow & p(2^1) + q(1) + r = 2 \\ a_2 = 4 & \Rightarrow & p(2^2) + q(2) + r = 4 \\ a_3 = 5 & \Rightarrow & p(2^3) + q(3) + r = 5 \end{array}\right\}$$

Solving this system of linear equations :





$\therefore\ \ p = -\frac{1}{2}$, $q = 3$, and $r = 0$.

Alternative Method (**not** recommended) :

For any integer $n \geq 2$,

$$a_n = p(2^n) + qn + r$$
$$2a_{n-1} - 3n + 6 = p(2^n) + qn + r$$

$\because\ a_{n-1} = p(2^{n-1}) + q(n-1) + r$,

$$\Rightarrow\ 2\left(p(2^{n-1}) + q(n-1) + r\right) - 3n + 6 = p(2^n) + qn + r$$

$$p(2^n) + 2q(n-1) + 2r - 3n + 6 = p(2^n) + qn + r$$

$$p(2^n) + \underline{(2q-3)}n + \underline{(-2q+2r+6)} = p(2^n) + \underline{q}n + \underline{r}\ , \quad \text{for any } n \in \mathbb{Z},\ n \geq 2$$

Comparing the linear term in n and the constant on both sides of the equation produces

$$\underline{2q-3} = \underline{q} \quad \text{and} \quad \underline{-2q+2r+6} = \underline{r}$$
$$q = 3, \qquad r = 2q - 6$$
$$= 2(3) - 6 = 0$$

$$a_1 = 2 \quad \Rightarrow \quad p(2^1) + q(1) + r = 2$$
$$p = \tfrac{1}{2}\left(2 - q - r\right)$$
$$= \tfrac{1}{2}\left(2 - 3\right) = -\tfrac{1}{2}$$

(iii) $\therefore\ a_n = -\frac{1}{2}(2^n) + 3n$.

$$\sum_{n=1}^{N} a_n = \sum_{n=1}^{N} -\frac{1}{2}(2^n) + 3n$$

$$= -\frac{1}{2}\sum_{n=1}^{N} 2^n + 3\sum_{n=1}^{N} n$$

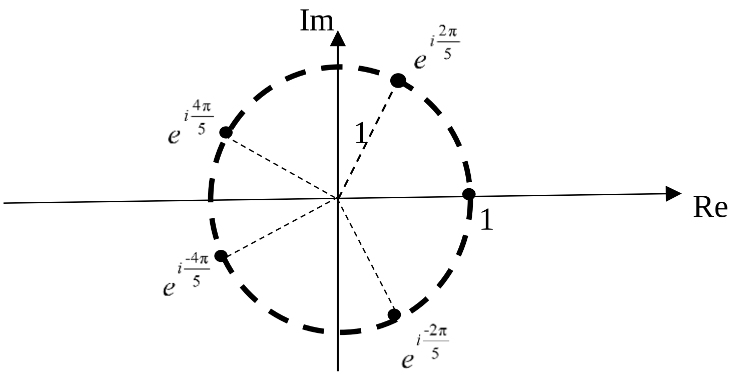
$$= -\frac{1}{2}\left(2^1 + 2^2 + 2^3 + \ldots + 2^N\right) + 3\left(1 + 2 + 3 + \ldots + N\right)$$

$$= -\frac{1}{2}\left(\frac{2^1(2^N - 1)}{2-1}\right) + 3\left(\frac{N}{2}(1+N)\right)$$

$$= -(2^N - 1) + \frac{3}{2}N(1+N)$$

3.

(i)

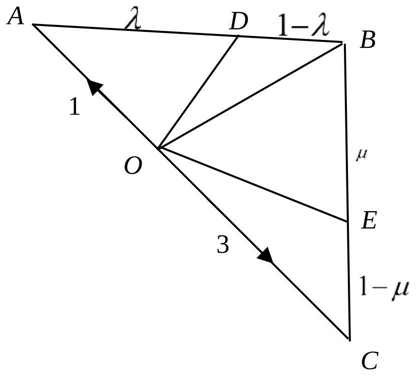


(ii) $(z - e^{i\phi})(z - e^{-i\phi}) = z^2 - \left(e^{i\phi} + e^{-i\phi}\right)z + 1$, with $\left(e^{i\phi} + e^{-i\phi}\right) = 2\cos\phi$.

(iii) $z^5 - 1 = (z-1)(z - e^{i\frac{2\pi}{5}})(z - e^{-\frac{i2\pi}{5}})(z - e^{i\frac{4\pi}{5}})(z - e^{-\frac{i4\pi}{5}})$

$$= (z-1)\left[z^2 - \left(2\cos\frac{2\pi}{5}\right)z + 1\right]\left[z^2 - \left(2\cos\frac{4\pi}{5}\right)z + 1\right]$$

4. (a)



Since O divides the line segment AC in the ratio 1:3, so $\overrightarrow{OC} = -3\overrightarrow{OA} = -3\mathbf{a}$

$$\mathbf{d} = \lambda\mathbf{b} + (1-\lambda)\mathbf{a}$$

$$\begin{aligned}\mathbf{e} &= \mu\mathbf{c} + (1-\mu)\mathbf{b} \\ &= \mu(-3\mathbf{a}) + (1-\mu)\mathbf{b} \\ &= -3\mu\mathbf{a} + (1-\mu)\mathbf{b}\end{aligned}$$

Area of triangle ODE $= \frac{1}{2}|\mathbf{d}\times\mathbf{e}|$

$$\begin{aligned}&= \tfrac{1}{2}\left|\left[\lambda\mathbf{b} + (1-\lambda)\mathbf{a}\right]\times\left[-3\mu\mathbf{a} + (1-\mu)\mathbf{b}\right]\right| \\ &= \tfrac{1}{2}\left|\left[(1-\lambda)\mathbf{a} + \lambda\mathbf{b}\right]\times\left[-3\mu\mathbf{a} + (1-\mu)\mathbf{b}\right]\right| \\ &= \tfrac{1}{2}\left|(1-\lambda)\mathbf{a}\times(1-\mu)\mathbf{b} + \lambda\mathbf{b}\times -3\mu\mathbf{a}\right| \\ &= \tfrac{1}{2}\left|(1-\lambda)(1-\mu)\mathbf{a}\times\mathbf{b} + (-3\mu)(\lambda)\mathbf{b}\times\mathbf{a}\right| \\ &= \tfrac{1}{2}\left|(1-\lambda)(1-\mu)\mathbf{a}\times\mathbf{b} + (3\mu)(\lambda)\mathbf{a}\times\mathbf{b}\right| \\ &= \tfrac{1}{2}\left|(1-\lambda)(1-\mu) + (3\mu)(\lambda)\right|\left|\mathbf{a}\times\mathbf{b}\right| \\ &= \tfrac{1}{2}\left|(1-\lambda) + (-\mu+\lambda\mu) + (3\lambda\mu)\right|\left|\mathbf{a}\times\mathbf{b}\right| \\ &= \tfrac{1}{2}\left|1-\lambda-\mu+4\lambda\mu\right|\left|\mathbf{a}\times\mathbf{b}\right|\end{aligned}$$

Therefore, $k = \frac{1}{2}|1-\lambda-\mu+4\lambda\mu|$

(b)

$$\mathbf{s}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$$

$$\mathbf{s}\times\mathbf{b} = \mathbf{a}\times\mathbf{b}$$

$$\mathbf{s}\times\mathbf{b} - \mathbf{a}\times\mathbf{b} = \mathbf{0}$$

$$(\mathbf{s}-\mathbf{a})\times\mathbf{b} = \mathbf{0}$$

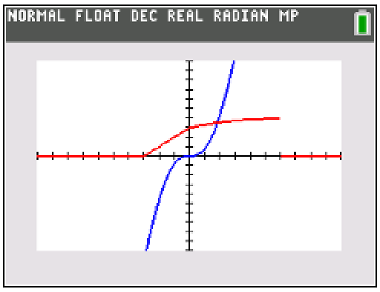
$\mathbf{s}-\mathbf{a}\ //\ \mathbf{b}$ or $(\mathbf{s}-\mathbf{a}) = \mathbf{0}$

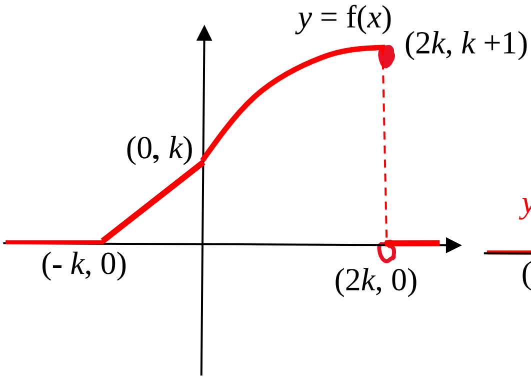
$\mathbf{s}-\mathbf{a} = k\,\mathbf{b}, k\in\mathbb{R}$ $\quad$ $\mathbf{s} = \mathbf{a}$

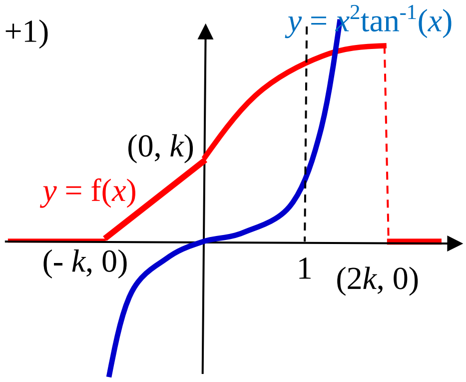
$\mathbf{s} = \mathbf{a} + k\,\mathbf{b}$

S lies on a line that passes through the point A and the line is parallel to the vector $\mathbf{b}$.

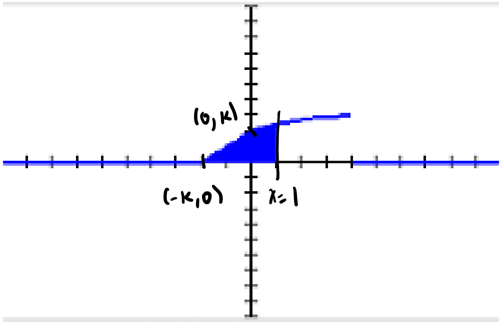
5. (i) and (iii)







(ii)



Required Area

$$= \int_{-k}^{0} (x+k)dx + \int_{0}^{1} (k+1) - \frac{(2k-x)^2}{4k^2} dx$$

$$= \left[\frac{x^2}{2} + kx\right]_{-k}^{0} + \left[(k+1)x - \frac{(2k-x)^3}{4k^2(3)(-1)}\right]_{0}^{1}$$

$$= 0 - \left[\frac{k^2}{2} - k^2\right] + \left[(k+1) - \frac{(2k-1)^3}{4k^2(3)(-1)}\right] - \left[-\frac{(2k)^3}{4k^2(3)(-1)}\right]$$

$$= \frac{k^2}{2} + \frac{k}{3} + 1 + \frac{(2k-1)^3}{12k^2}$$

OR

Required Area

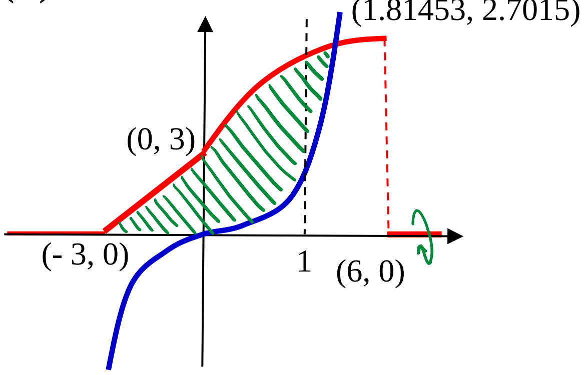
$$= \frac{1}{2}(k)(k) + \int_{0}^{1} (k+1) - \frac{(2k-x)^2}{4k^2} dx$$

$$= \frac{k^2}{2} + \left[(k+1)x - \frac{(2k-x)^3}{4k^2(3)(-1)}\right]_{0}^{1}$$

$$= \frac{k^2}{2} + \left[(k+1) - \frac{(2k-1)^3}{4k^2(3)(-1)}\right] - \left[-\frac{(2k)^3}{4k^2(3)(-1)}\right]$$

6. $= \frac{k^2}{2} + \frac{k}{3} + 1 + \frac{(2k-1)^3}{12k^2}$

(iv)



Point of intersection btw C_1 and C_2 is at $x = 1.81453$

Volume of *S*

$$= \pi \left\{ \int_{-3}^{0} (x+3)^2 dx + \int_{0}^{1.81453} \left[(3+1) - \frac{(2(3)-x)^2}{4(3)^2} \right]^2 dx - \int_{0}^{1.81453} \left[x^2 \tan^{-1}(x) \right]^2 dx \right\}$$

= 77.54 (2 d.p.) unit^3

OR

Volume of *S*

$$= \frac{1}{3}\pi(3)^2(3) + \pi \left\{ \int_{0}^{1.81453} \left[(3+1) - \frac{(2(3)-x)^2}{4(3)^2} \right]^2 dx - \int_{0}^{1.81453} \left[x^2 \tan^{-1}(x) \right]^2 dx \right\}$$

= 77.54 (2 d.p.) unit^3

6. (i)

$\because$ A and C are independent ,

$$\begin{aligned} P(A\cap C) &= P(A)\times P(C) \\ &= 0.4\times 0.5 = 0.2 \end{aligned}$$

<u>Alternative</u> :

$\because$ A and C are independent ,

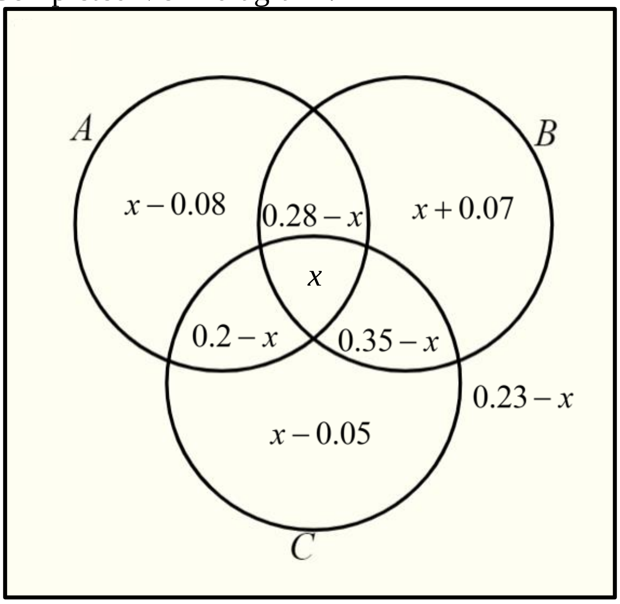
$$P(A|C) = P(A)$$

$$\frac{P(A\cap C)}{P(C)} = P(A) \Rightarrow \frac{P(A\cap C)}{0.5} = 0.4$$

$$P(A\cap C) = 0.2$$

(ii)

Completed Venn diagram :



$$\begin{aligned} P(A\cap B'\cap C) &= P(A\cap C) - P(A\cap B\cap C) \\ &= 0.2 - x \end{aligned}$$

$\because$ A and B are independent ,

$$\begin{aligned} P(A\cap B) &= P(A)\times P(B) \\ &= 0.4\times 0.7 = 0.28 \end{aligned}$$

$$\begin{aligned} P(A\cap B\cap C') &= P(A\cap B) - P(A\cap B\cap C) \\ &= 0.28 - x \end{aligned}$$

Similarly,

$\because$ B and C are independent ,

$$\begin{aligned} P(B\cap C) &= P(B)\times P(C) \\ &= 0.7\times 0.5 = 0.35 \end{aligned}$$

$$\begin{aligned} P(A'\cap B\cap C) &= P(B\cap C) - P(A\cap B\cap C) \\ &= 0.35 - x \end{aligned}$$

Next,

$$\begin{aligned} P(A\cap B'\cap C') &= P(A) - P(A\cap B\cap C') - P(A\cap B'\cap C) - P(A\cap B\cap C) \\ &= 0.4 - (0.28-x) - (0.2-x) - x \\ &= x - 0.08 \end{aligned}$$

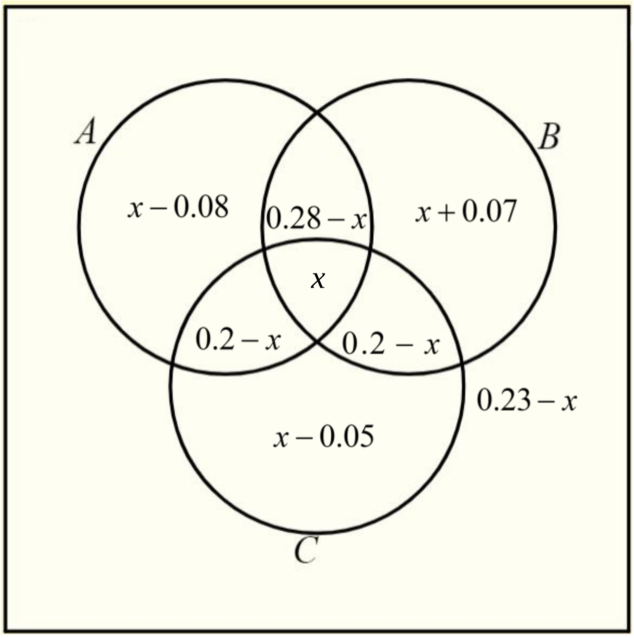
$$\begin{aligned} P(A'\cap B\cap C') &= P(B) - P(A\cap B\cap C') - P(A'\cap B\cap C) - P(A\cap B\cap C) \\ &= 0.7 - (0.28-x) - (0.35-x) - x \\ &= x + 0.07 \end{aligned}$$

$$\begin{aligned}\mathrm{P}(A'\cap B'\cap C) &= \mathrm{P}(C)-\mathrm{P}(A'\cap B\cap C)-\mathrm{P}(A\cap B'\cap C)-\mathrm{P}(A\cap B\cap C)\\ &= 0.5-(0.35-x)-(0.2-x)-x\\ &= x-0.05\end{aligned}$$

Finally,

$$\begin{aligned}\mathrm{P}(A'\cap B'\cap C') &= 1-\mathrm{P}(A)-\mathrm{P}(A'\cap B\cap C')-\mathrm{P}(A'\cap B'\cap C)-\mathrm{P}(A'\cap B\cap C)\\ &= 1-0.4-(x+0.07)-(x-0.05)-(0.35-x)\\ &= 0.23-x\end{aligned}$$

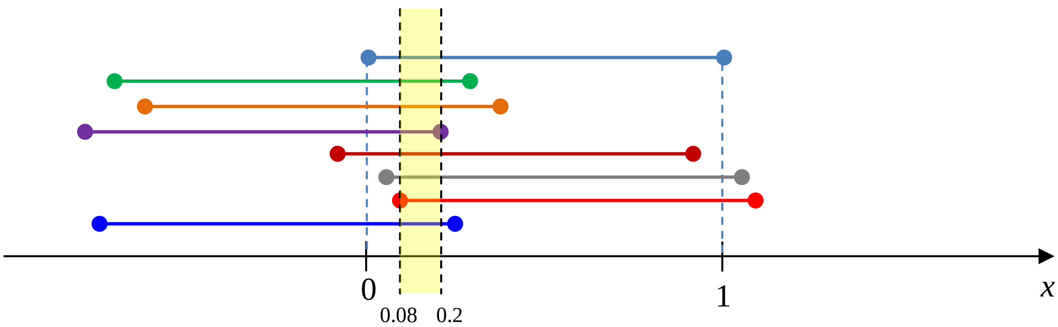
(iii) From the completed Venn diagram in part **(ii)** :



The expression for each of the eight regions in the Venn diagram is a probability figure, which must lie in the interval [0, 1].

$$0 \le x \le 1$$

$$\begin{aligned}
0\le 0.28-x\le 1 &\Rightarrow 0.28\ge x\ge -0.72\\
0\le 0.35-x\le 1 &\Rightarrow 0.35\ge x\ge -0.65\\
0\le 0.2-x\le 1 &\Rightarrow 0.2\ge x\ge -0.8\\
0\le x+0.07\le 1 &\Rightarrow -0.07\le x\le 0.93\\
0\le x-0.05\le 1 &\Rightarrow 0.05\le x\le 1.05\\
0\le x-0.08\le 1 &\Rightarrow 0.08\le x\le 1.08\\
0\le 0.23-x\le 1 &\Rightarrow 0.23\ge x\ge -0.77
\end{aligned}$$



Taking the common intersection for the range of possible values produces

$$0.08\le x\le 0.2$$

$$\therefore\quad 0.08\le \mathrm{P}(A\cap B\cap C)\le 0.2$$

<u>Alternative Solution</u> (also accepted) :

$\because\ P(A \cap B' \cap C') \ \geq\ 0$

$$0.2 - x \geq 0$$

$$x \leq 0.2$$

Max. possible $P(A \cap B \cap C) = 0.2$

$\because\ P(A' \cap B \cap C) \ \geq\ 0$

$$x - 0.08 \geq 0$$

$$x \geq 0.08$$

Min. possible $P(A \cap B \cap C) = 0.08$

$\therefore\quad 0.08 \leq P(A \cap B \cap C) \leq 0.2$

7.

(i)

$$p+q+\frac{2}{5}=1 \Rightarrow p+q=\frac{3}{5}$$

$$P(Y=4)=\frac{3}{25}$$

$$P(X_1=1, X_2=2)+P(X_1=2, X_2=0)=\frac{3}{25}$$

$$q\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{5}=\frac{3}{25}$$

$$q\frac{1}{5}=\frac{2}{25}$$

$$q=\frac{2}{5}$$

$$p+\frac{2}{5}=\frac{3}{5} \Leftrightarrow p=\frac{1}{5}$$

(ii)

x	-1	0	1	2
$P(X=x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

$$\mathrm{E}(X)=\frac{-1+2+2}{5}=\frac{3}{5}$$

$$\mathrm{E}(X^2)=\frac{1+2+4}{5}=\frac{7}{5}$$

$$\begin{aligned}\mathrm{Var}(X)&=\mathrm{E}(X^2)-\left[\mathrm{E}(X)\right]^2\\&=\frac{7}{5}-\left(\frac{3}{5}\right)^2\\&=\frac{35}{25}-\frac{9}{25}\\&=\frac{26}{25}\end{aligned}$$

$$\mathrm{Var}(X)=\frac{26}{25}$$

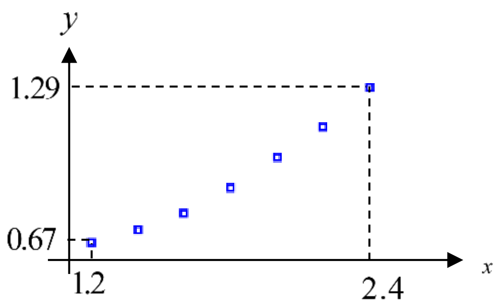
(iii)

$$\begin{aligned}\mathrm{Var}(Y)&=\mathrm{Var}(2X_1+X_2)\\&=2^2\,\mathrm{Var}(X_1)+\mathrm{Var}(X_2)\\&=2^2\,\mathrm{Var}(X)+\mathrm{Var}(X)\\&=5\mathrm{Var}(X)\\&=\frac{26}{5}\end{aligned}$$

8. (i) From data, $\bar{x} = 1.8$ and $\sum y = 5.49 + a$

$$\bar{y} = 0.517857(1.8) - 0.003571$$
$$= 0.9285716$$
$$a = 7(0.9285716) - 5.49$$
$$= 1.01 \text{ (shown)}$$

(ii)



(iii)(a) 0.986
(iii)(b) 0.996

(iv) $\ln y = cx + d$ is a better model. From the scatter diagram in part **(ii)**, y is increasing at an increasing rate as x increases. From part **(iii)**, the product moment correlation coefficient is closer to 1 between $\ln y$ and x than between y and x.

9.

(i)

Number of different teams of 2 setters, 3 spikers and 1 libero

$= {}^5C_2 \times {}^7C_3 \times {}^3C_1$

$= 1050$

(ii)

5 setters select 2
7 spikers select 3
3 libero select 1

One setter and one spiker are classmates

Method 1 (direct approach)

Case 1 : Setter classmate selected but not spiker classmate.

No. of teams $= {}^4C_1 \times {}^6C_3 \times {}^3C_1 = 240$

Since setter classmate included, only need to select 1 more setter from remaining 4 setters.

Since spiker classmate excluded, can select 3 only from remaining 6 spikers.

Case 2 : Spiker classmate selected but not setter classmate.

No. of teams $= {}^4C_2 \times {}^6C_2 \times {}^3C_1 = 270$

Since setter classmate excluded, can select 2 from remaining 4 setters.

Since spiker classmate included, only need to select 2 more from remaining 6 spikers.

Required no. of different teams $= 240 + 270 = 510$.

Method 2 (Complement)

Number of teams = all possible teams – include both classmates – exclude both classmates

$= 1050 - {}^4C_1 \times {}^6C_2 \times {}^3C_1 - {}^4C_2 \times {}^6C_3 \times {}^3C_1$

From (i)

Since setter classmate included, only need to select 1 more setter from remaining 4 setters.

Since spiker classmate included, only need to select 2 more spikers from remaining 6 spikers.

Since setter classmate excluded, can only select 2 setters from remaining 4 setters.

Since spiker classmate excluded, can only select 3 spikers from remaining 6 setters.

$= 1050 - 180 - 360$

$= 510$

Method 3 (Complement)

Number of teams

= (setter classmate excluded and spiker classmate may or may not be included) – (spiker classmate excluded and setter classmate may or may not be included) – (exclude both classmates)

$$= {}^4C_2 \times {}^7C_3 \times {}^3C_1 + {}^5C_2 \times {}^6C_3 \times {}^3C_1 - 2\times\left({}^4C_2 \times {}^6C_3 \times {}^3C_1\right)$$

Since setter classmate excluded, can only select 2 setters from remaining 4 setters.

Since spiker classmate may or may not be included, 3 spikers from 7 spikers.

Since setter classmate may or may not be included, select 2 setters from 5 setters.

Since spiker classmate excluded, can select 3 spikers from remaining 6 spikers.

Case of both classmates excluded is included in both the previous two cases. Hence must remove double counting twice.

Since both classmates excluded, select 2 setters from remaining 4 setters.

Since both classmates excluded, select 3 spikers from remaining 6 spikers.

$$= 630+600-2(360)$$

$$=510$$

(iii)

One setter classmate and one spiker classmate removed

Since coach already decided one spiker to play as either spiker or setter, **no need** to select this spiker.

4 setters select 2
6 spikers select 3
2 libero select 1

Method 1 (Direct approach)

After the two classmates are advised to rest, along with the new decision from the coach, the CCA has 4 setters, 5 spikers, 1 setter/spiker (denoted as *A*) and 3 liberos.

Case 1 : (A does not play)
Select 2 setters, 3 spikers, and 1 libero.

$$\text{No. of teams} = {}^4C_2 \times {}^5C_3 \times {}^1C_0 \times {}^3C_1 = 180$$

Case 2 : (A play as spiker)
Select 2 setters, 2 spikers, *A* as spiker, and 1 libero.

$$\text{No. of teams} = {}^4C_2 \times {}^5C_2 \times {}^1C_1 \times {}^3C_1 = 180$$

Case 3 : (A play as setter)
Select 1 setter, 3 spikers, *A* as setter and 1 libero.

$$\text{No. of teams} = {}^4C_1 \times {}^5C_3 \times {}^1C_1 \times {}^3C_1 = 120$$

Required no. of different teams $= 180+180+120 = 480$.

Method 2 (Direct approach)

Number of teams

= (spiker plays as spiker but may or may not be included in team) + (spiker plays as setter and included in team)

$$= {}^{4}C_{2} \times {}^{6}C_{3} \times {}^{3}C_{1} + {}^{4}C_{1} \times {}^{5}C_{3} \times {}^{3}C_{1}$$

Since spiker play as spiker and may or may not be included, select 2 setters from 4 setters.

Since spiker play as spiker and may or may not be included, select 3 spikers from 6 spikers.

Since spiker plays as setter and included, select remaining 1 setters from 4 setters.

Since spiker play as setter and included, select 3 spikers from remaining 5 spikers.

$$= 360 + 120$$

$$= 480$$

10.

(i) $np = 4.5$ ----- (1)

$np(1-p) = 3.15$ ----- (2)

Solving, $n = 15$ and $p = 0.3$

(ii) $\mathrm{P}(X \geq 5) = 1 - \mathrm{P}(X \leq 4)$

$= 0.485$

(iii) $\mathrm{P}(X = 10) > \mathrm{P}(X = 9)$

$$\frac{n!}{10!(n-10)!}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{n-10} > \frac{n!}{9!(n-9)!}\left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{n-9}$$

$$\frac{n!}{10!(n-10)!}\left(\frac{1}{2}\right)^{n} > \frac{n!}{9!(n-9)!}\left(\frac{1}{2}\right)^{n}$$

$$\frac{1}{10} > \frac{1}{(n-9)}$$

$$10 < n-9$$

$$n > 19$$

$\mathrm{P}(X = 10) > \mathrm{P}(X = 11)$

$$\frac{n!}{10!(n-10)!}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{n-10} > \frac{n!}{11!(n-11)!}\left(\frac{1}{2}\right)^{11}\left(\frac{1}{2}\right)^{n-11}$$

$$\frac{n!}{10!(n-10)!}\left(\frac{1}{2}\right)^{n} > \frac{n!}{11!(n-11)!}\left(\frac{1}{2}\right)^{n}$$

$$\frac{1}{(n-10)} > \frac{1}{11}$$

$$n-10 < 11$$

$$n < 21$$

Combining, $19 < n < 21$

Hence, $n = 20$

11.

(i) Let $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$

Method 1 (using symmetry)

$$\mu = \frac{240+260}{2}$$
$$= 250$$
$$\mathrm{P}(X > 260) = 0.15$$
$$\mathrm{P}\left(Z > \frac{260-250}{\sigma}\right) = 0.15$$
$$\frac{10}{\sigma} = 1.03643338$$
$$\sigma = 9.64847 = 9.65 \text{ (3 sf)}$$

Method 2

$$\mathrm{P}(X < 240) = 0.15$$
$$\mathrm{P}\left(Z < \frac{240-\mu}{\sigma}\right) = 0.15$$
$$\frac{240-\mu}{\sigma} = -1.03643338$$
$$240 - \mu = -1.03643338\sigma \text{ ------ (1)}$$

$$\mathrm{P}(X < 260) = 0.85$$
$$\mathrm{P}\left(Z < \frac{260-\mu}{\sigma}\right) = 0.85$$
$$\frac{260-\mu}{\sigma} = 1.03643338$$
$$260 - \mu = 1.03643338\sigma \text{ ------ (2)}$$

Solving (1) and (2), $\mu = 250$ and $\sigma = 9.65$ (3 sf)

(ii) **(ii)** $\mathrm{E}(X - Y) = 50$

$$\mathrm{Var}(X - Y) = 109.09 = \left(10.444^2\right)$$
$$X - Y \sim N(50,\ 109.09)$$
$$\mathrm{P}(|X - Y| \le 45) = \mathrm{P}(-45 < X - Y < 45)$$
$$= 0.31607$$
$$= 0.316 \text{ (3 sf)}$$

(iii) Let T be the rv denoting the marks used for the calculation

$T = 0.6X + 0.4Y$

$\mathrm{E}(T) = 0.6(250) + 0.4(200) = 230$

$\mathrm{Var}(T) = 0.6^2\,\mathrm{Var}(X) + 0.4^2\,\mathrm{Var}(Y)$

$= 36.073$

$T \sim N(230,\ 36.073)$

Let the minimum marks be *a*.

$\mathrm{P}(T \geq a) = 0.1$

$a = 237.697 = 237.70$

(iv) The scores of the two components are independent.

12.

(i) A random sample of 120 patients means that they are taken from a population where each patient has an equal and independent chance of being selected.

(ii) $\bar{x} = \dfrac{\sum(x-7)}{120} + 7 = 7.2$

$s^2 = \dfrac{1}{120-1}\left(\sum(x-7)^2 - \dfrac{\left(\sum(x-7)\right)^2}{120}\right) = \dfrac{1}{119}\left(14908 - \dfrac{(24)^2}{120}\right) \quad = 125 \,(3 \text{ s.f.})$

(iii) H_0: $\mu = 7$

H_1: $\mu > 7$

Since sample size of 120 patients is large,

Under H_0, $\bar{X} \sim \mathrm{N}\left(7, \dfrac{125.2369748}{120}\right)$ approximately by CLT.

p-value $= 0.422393669$ or test-statistic $= 0.1957736936$

At 5% level of significance, since p-value $= 0.422 > 0.05$, do not reject H_0 and conclude that there this is insufficient evidence for the claim that the average recovery period has been understated.

Or

At 5% level of significance, since test-statistic $= 0.196 < 1.6449$ (critical value), do not reject H_0 and conclude that there this is insufficient evidence for the claim that the average recovery period has been understated.

(iv) The test would no longer be valid because the sample size 15 is small and the distribution of the recovery period is not given be to normal, therefore Central Limit Theorem could not be used to approximate $\bar{X}$.

(v) H_0: $\mu = 7$

H_1: $\mu \neq 7$

Since sample size of 120 patients is large,

Under H_0, $\bar{X} \sim \mathrm{N}\left(7, \dfrac{1^2}{120}\right)$ approximately by CLT.

test-statistic $= \dfrac{\bar{x}-7}{\frac{1}{\sqrt{120}}}$

At 1% level of significance, since H_0 is rejected,

$\dfrac{\bar{x}-7}{\frac{1}{\sqrt{120}}} \leq -2.5758$ or $\dfrac{\bar{x}-7}{\frac{1}{\sqrt{120}}} \geq 2.5758$

$0 \leq \bar{x} \leq 6.76$ or $\bar{x} \geq 7.24$