CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS 9758/01

Paper 1

30 Aug 2022

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks													
Total	3	4	5	6	8	9	9	9	10	12	12	13	100

This document consists of **23** printed pages and **1** blank page.

1 By considering the derivative as a limit, $\mathrm{f}'(x)=\lim_{\delta x\to 0}\left(\frac{\mathrm{f}(x+\delta x)-\mathrm{f}(x)}{\delta x}\right)$, show that the derivative of $\frac{1}{x^2}$ is $-\frac{2}{x^3}$. [3]

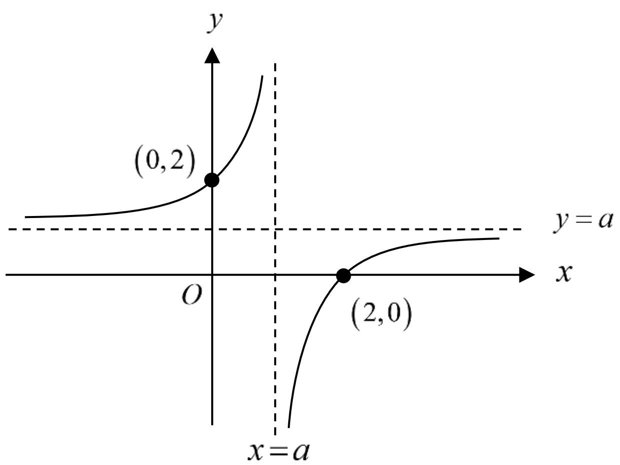
2 A quadratic curve has its maximum point at $\left(\frac{1}{4},\frac{57}{8}\right)$ and has gradient -3 at the point where $x=1$. Find the equation of the curve. [4]

3 **(i)** The complex number z has $|z|=2$ and $\arg(z)=-\frac{5\pi}{6}$. Find the exact values of the modulus r and argument θ of $\frac{\mathrm{i}}{z}$ where $r>0$ and $-\pi<\theta\le\pi$. [3]

(ii) Hence express $\left(\frac{\mathrm{i}}{z}\right)^*$ in the form a + bi exactly. [2]

4 **(a)** Describe a pair of transformations which transforms the graph of $\frac{y^2}{4}+x^2=1$ on to the graph of $y^2+(x-1)^2=1$. [2]

(b)



The diagram shows a sketch of the curve $y=\mathrm{f}(x)$. The curve cuts the axes at $(2,0)$ and $(0,2)$. The lines $y=a$ and $x=a$ are asymptotes to the curve, where $0<a<2$.

(i) Sketch the curve $y=|\mathrm{f}(x+a)|$, stating in terms of a, the equations of any asymptotes and the coordinates of any point(s) of intersection with the axes. [3]

(ii) Write down the roots to the equation $\mathrm{f}(|x|)=0$. [1]

5 It is given that $y=\sqrt{1+\cos^2 x}$.

(i) Show that $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}+\sin 2x=0$. By repeated differentiation of this result, find the Maclaurin expansion for y up to and including the term in x^2. [4]

(ii) Using the standard results given in the List of Formulae (MF26), verify that the series expansion for y up to and including the term in x^2 is the same as that obtained in part **(i)**. Find the range of values of x for which the expansion is valid. [4]

6 **(i)** Integrate the following with respect to x

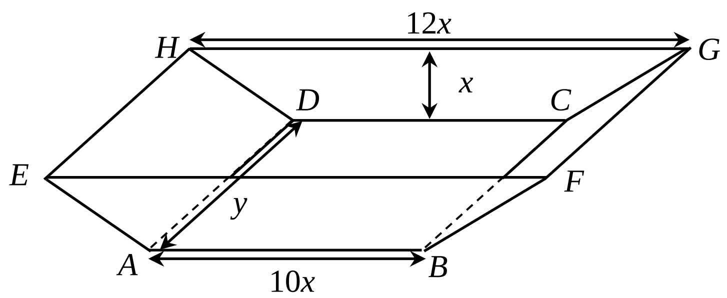
(a) $\dfrac{x^3}{\left(1+x^4\right)^2}$, [3]

(b) $\dfrac{2\tan x}{1-\tan^2 x}$. [2]

(ii) Find $\int_0^{\pi}\cos\left[\left(a+1\right)x\right]\sin ax\,\mathrm{d}x$ in terms of a, where a is a positive integer. [4]

7 The diagram below shows the design of a baking tray with a horizontal rectangular base $ABCD$, measuring $10x$ cm by y cm. The faces $ABFE$ and $DCGH$ are isosceles trapeziums which are parallel to each other such that CG, DH, AE and BF are of equal length.

The lengths of the edges EF and HG are each $12x$ cm and the faces $ADHE$ and $BCGF$ are identical rectangles. The height of the tray is x cm and the capacity of the tray is 1980 cm^3. It is assumed that the tray has negligible thickness.



(i) Show that the inner surface area, A cm^2 of the tray is given by

$$A=22x^2+\frac{360}{x}\left(5+\sqrt{2}\right).$$ [5]

(ii) Find, using differentiation, the value of x for which A is a minimum. Deduce the value of A corresponding to this value of x. [4]

8 The function f is defined by

$$\mathrm{f} : x \mapsto \frac{1}{1-\mathrm{e}^{-x}} \text{ for } x \in \mathbb{R},\ x \neq 0.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [1]

(ii) Explain why the function f^{-1} exists, and state its domain. [2]

The function g is defined by

$$\mathrm{g} : x \mapsto 2\ln|x-4| \text{ for } x \in \mathbb{R},\ x \neq 3,\ x \neq 4,\ x \neq 5.$$

(iii) Show that $\mathrm{fg}(x) = \dfrac{(x-4)^2}{(x-3)(x-5)}$, and state the range of fg. [3]

(iv) Solve the inequality $\mathrm{fg}(x) < 0$. [3]

9 **(a)** The water is leaking out of a hole at the side of an upright cylindrical barrel.
The height of the water surface h metres above the hole t seconds after the water has started leaking is modelled by

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -A\sqrt{h}\text{, where } A \text{ is a positive constant.}$$

Initially, the water surface is 1 metre above the hole.

(i) Verify that the solution to this differential equation is $h = \left(1 - \dfrac{1}{2}At\right)^2$. [1]

The water stops leaking after 20 seconds when $h = 0$.

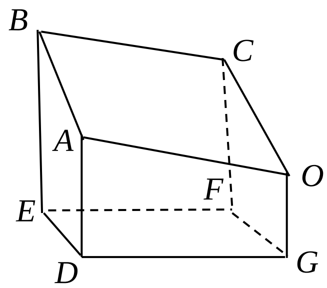
(ii) Find the value of A and hence find the time when h is 0.5 metres. [2]

(b) Determine the general solution to the second order differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{1}{\sqrt{3-t^2}}\ ,$$

expressing x explicitly in terms of t. [7]

10 The diagram below shows the structure of a building.



The slanted rooftop is modelled by the plane $OABC$ where O is taken as the origin. The horizontal ground is modelled by the plane $DEFG$ which has a normal vector in the direction of $\mathbf{k}$. It is given that the position vectors of points A, B and C are $-7\mathbf{j}+\mathbf{k}$, $3\mathbf{i}-9\mathbf{j}+3\mathbf{k}$ and $4\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ respectively where the units are in metres.

(i) Find the cartesian equation of the rooftop. [3]

(ii) Find the acute angle between the rooftop and the horizontal ground. [2]

(iii) Find the area of the slanted rooftop. [3]

A point H has position vector $-2\mathbf{i}-5\mathbf{j}+9\mathbf{k}$.

(iv) find the coordinates of the point on the rooftop which is nearest to H. [4]

11 A new coating is applied to TX tennis balls to improve its elasticity. To examine its effect, a TX tennis ball is dropped vertically from a fixed height H cm onto the floor. The height reached after each bounce decreased by 5 cm from the height reached by the previous bounce.

Find, in terms of H and n,

(i) the height reached by the tennis ball after the $(n-1)$th bounce, [1]

(ii) the total distance travelled by the tennis ball just before it rebounds off the floor for the nth bounce. [5]

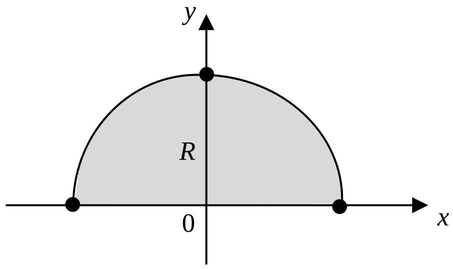
(iii) State an assumption used in your calculation for part **(ii).** [1]

The time interval between any successive bounces is also measured. Given that the time interval between the first and second bounce is 1.2 seconds, and the time interval between subsequent consecutive bounces is 0.75 times of the time interval of the previous bounce, find

(iv) the least integer m such that it takes less than 0.02 seconds between the mth and $(m + 1)$th bounce, [3]

(v) total time taken before the tennis ball comes to a stop given that it takes 2 seconds for the tennis ball to first hit the ground. [2]

12 The figure below shows part of the cross-sectional region, R, of a toy 3D printed by a company.



The region R is bounded by the x-axis and the curve C with parametric equations

$$x = 2r\cos\theta, \qquad y = 2r\sin\theta, \qquad \text{for } 0 \le \theta \le \pi,$$

where r is a non-zero constant.

(i) Find the coordinates of the points of intersection of C with the axes. [2]

(ii) Find, using integration, the exact area of the region R. [4]

If a curve defined by parametric equations, $x = \mathrm{f}(\theta)$, $y = \mathrm{g}(\theta)$, $\alpha \le \theta \le \beta$ is rotated completely about the x-axis, then the area of the resulting surface is given by

$$S = \int_{\alpha}^{\beta} 2\pi y \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta.$$

(iii) The toy is obtained by rotating the region R completely about the x-axis. Find, using integration, the exact surface area of the toy in terms of r. [5]

(iv) Another toy is defined by a second curve Γ with the following parametric equations

$$x = 8\sin\theta, \qquad y = 8\cos\theta, \qquad \text{for } 0 \le \theta \le \pi.$$

Part of the cross-sectional region of this toy, T, is bounded by the curve Γ and the y-axis.

Using the result in part **(iii)**, deduce the exact surface area of the new toy when the region T is rotated completely about the y-axis. [2]