CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

9758/02

Paper 2

14 Sep 2022

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

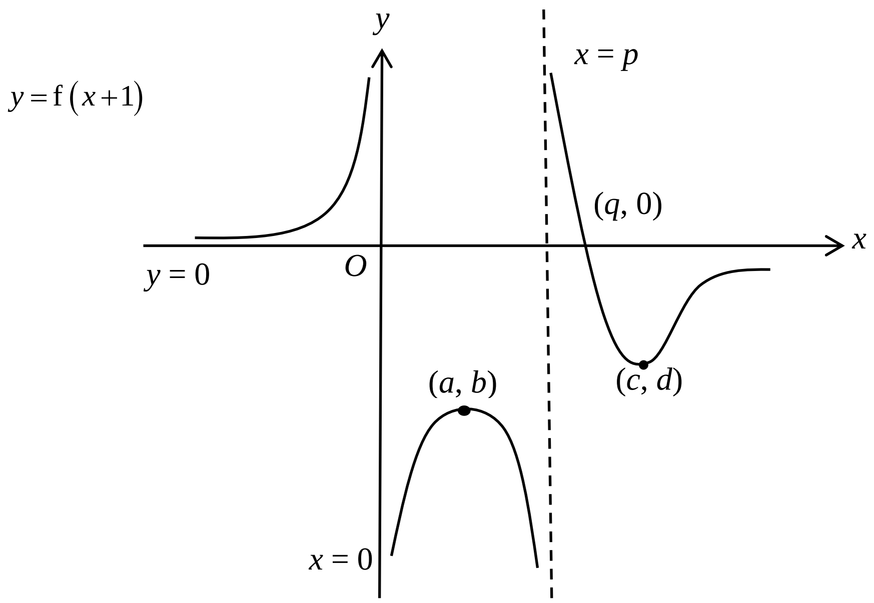
The number of marks is given in brackets [ ] at the end of each question or part question.

Question	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks													
Total	6	7	7	8	12	5	7	7	8	9	12	12	100

This document consists of **27** printed pages and **1** blank page.

Section A: Pure Mathematics [40 marks]

1 The graph of $y = \mathrm{f}(x+1)$ has two stationary points at (a, b) and (c, d), where $-1 < b < d < 0$. It intersects the x-axis at $x = q$ and has asymptotes $x = 0$, $x = p$ and $y = 0$ as shown in the diagram below.



On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}'(x+1)$, [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

2 A sequence $a_1,\ a_2,\ a_3,\ \ldots$ is such that $a_n = 2a_{n-1} - 3n + K$, where K is a constant, and $n \geq 2$.

(i) Given that $a_1 = 2$ and $a_2 = 4$, find K and a_3. [2]

It is known that the nth term of this sequence is given by

$$a_n = p(2^n) + qn + r\ ,$$

where p, q and r are constants.

(ii) Find p, q and r. [3]

(iii) Find $\sum_{n=1}^{N} a_n$ in terms of N. [2]

3 It is known that the five distinct roots of the equation $z^5 = 1$ are 1, $e^{i\frac{2\pi}{5}}, e^{-i\frac{2\pi}{5}}, e^{i\frac{4\pi}{5}}$ and $e^{-i\frac{4\pi}{5}}$.

(i) Mark the five points corresponding to these roots on an Argand diagram, indicating clearly their positions relative to the origin. [3]

(ii) Show that $(z-e^{i\phi})(z-e^{-i\phi}) = z^2-(2\cos\phi)z+1$. [1]

(iii) Hence express z^5-1 as a product of one linear factor and two quadratic factors, where all coefficients are real. [3]

4 **(a)** Referred to the origin O, the fixed points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors. It is given that O divides the line segment AC in the ratio 1:3. Point D divides the line segment AB in the ratio of $\lambda : 1-\lambda$. Point E divides the line segment BC in the ratio $\mu : 1-\mu$.

Show that the area of triangle ODE can be written as $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be found in terms of μ and λ. [5]

(b) A variable point S has position vector $\mathbf{s}$ relative to the origin O. Given that $\mathbf{s}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$, describe geometrically the set of all possible positions of the point S. [3]

5 A curve C_1 has equation $y = \mathrm{f}(x)$, where $k > 1$ and

$$\mathrm{f}(x) = \begin{cases} x+k & \text{for } -k < x \le 0, \\ (k+1) - \dfrac{(2k-x)^2}{4k^2} & \text{for } 0 < x \le 2k, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Sketch the curve C_1. [2]

(ii) The region R is bounded by the curve C_1, the line $x = 1$ and the x-axis. Find the exact area of R, leaving your answer in terms of k. [6]

(iii) On the same diagram on page 10, sketch the curve C_2 with equation $y = x^2\tan^{-1}(x)$. [1]

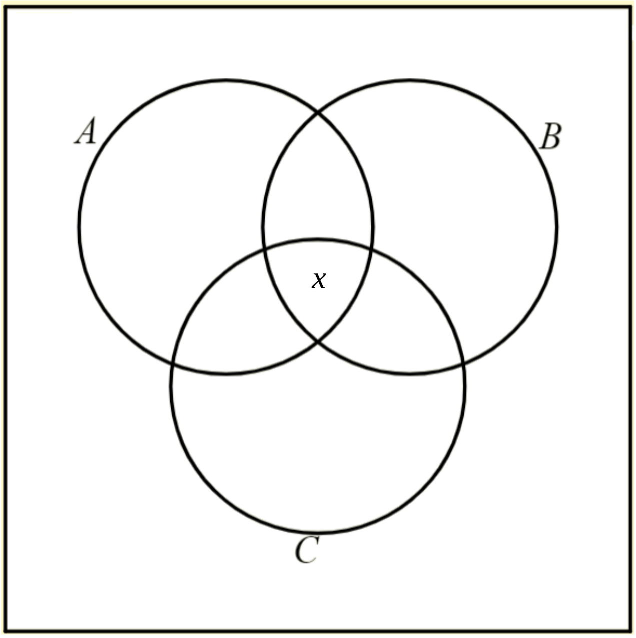
(iv) The region S is bounded by the curves C_1 and C_2, and the x-axis. Given that $k = 3$, determine the volume of solid of revolution formed when S is rotated 2π radians about the x-axis, leaving your answer to 2 decimal places. [3]

Section B: Probability and Statistics [60 marks]

6 The events A, B and C are such that $\mathrm{P}(A) = 0.4$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(C) = 0.5$.
It is given that events A and B are independent, events A and C are independent, and events B and C are independent.

(i) Find $\mathrm{P}(A \cap C)$. [1]

(ii) Let $\mathrm{P}(A \cap B \cap C)$ be denoted by x.



In terms of x, complete the given Venn diagram to show the probability in each of the seven remaining regions. [2]

(iii) Hence find the range of possible values of $\mathrm{P}(A \cap B \cap C)$. [2]

7 The probability distribution for the random variable X is shown in the table.

x	-1	0	1	2
$P(X = x)$	p	$\frac{1}{5}$	q	$\frac{1}{5}$

Another random variable Y is defined by $Y = 2X_1 + X_2$, where X_1 and X_2 are independent observations of X.

(i) Given that $\mathrm{P}(Y = 4) = \frac{3}{25}$, find the values of p and q. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [2]

(iii) Find Var (Y). [2]

8 An investigation of the effect of a fertiliser on the growth of malt is carried out. The table below shows how the amount of fertiliser applied, x, resulted in the average growth of malt, y, where x and y are measured in suitable units.

x	1.2	1.4	1.6	1.8	2.0	2.2	2.4
y	0.67	0.72	0.79	0.89	a	1.13	1.29

It is given that the least squares regression line of y on x is $y = 0.517857x - 0.003571$, correct to 6 decimal places.

(i) Show that $a = 1.01$, correct to 3 significant figures. [2]

(ii) Draw a scatter diagram to illustrate the data. [2]

(iii) Find the product moment correlation coefficient between

(a) y and x, [1]

(b) $\ln y$ and x. [1]

(iv) Use your answers to parts **(ii)** and **(iii)** to explain which of $y = ax + b$ or $\ln y = cx + d$ is a better model. [1]

9 A Junior College Volleyball CCA team is sent to compete in a volleyball match. The team consists of 2 setters, 3 spikers (for attack) and 1 libero (for defence). The CCA has 5 members who can play as setters, 7 members who can play as spikers, and 3 members who can play the libero.

(i) How many different teams can be formed? [2]

In the CCA, one of the setters is the classmate of one of the spikers.

(ii) How many different teams can be formed, if the team includes exactly one of the two classmates? [3]

The two classmates suffered injuries and are not able to play. The coach decides that one of the remaining spikers can play as either a spiker or a setter.

(iii) How many different teams can now be formed? [3]

10 The random variable X has distribution $\mathrm{B}(n, p)$. It is given that the mean of X is 4.5 and the variance of X is 3.15.

(i) Find the values of n and p. [3]

(ii) Using the values of n and p found in part **(i)**, find $\mathrm{P}(X \geq 5)$. [2]

Do not use a calculator in answering this part.

It is given instead that the mode of X is 10 and $p = \frac{1}{2}$.

(iii) By considering $\mathrm{P}(X = 10) > \mathrm{P}(X = 9)$ and $\mathrm{P}(X = 10) > \mathrm{P}(X = 11)$, write down an inequality involving n and find the value of n. [4]

11 In this question you should state the parameters of any normal distributions you use.

A company conducts an aptitude test to recruit potential employees. There are two sections to the test, a problem-solving component and a numeracy component.

It is found that the test scores of the problem-solving component is normally distributed; 15% of the candidates obtained scores of less than 240, and 85% of the candidates had scores less than 260.

(i) Find the mean and standard deviation of the scores for the problem-solving component. [3]

The scores of the numeracy component follow the distribution $N(200, 4^2)$. A candidate is randomly selected.

(ii) Find the probability that the difference in scores of the problem-solving component of a randomly selected candidate and numeracy component of another randomly selected candidate scores is within 45 marks. [4]

The recruitment score is calculated based on the sum of 60% of the problem-solving component and 40% of the numeracy component. Candidates whose recruitment scores are in the top 10% of the group will be recruited.

(iii) Find the minimum recruitment score a candidate needs to attain for him to be recruited, leaving your answer correct to 2 decimal places. [4]

(iv) State an assumption for the calculations in parts **(ii)** and **(iii)** to be valid. [1]

12 A virologist claims that due to the introduction of vaccine, the recovery period from a certain type of flu-like virus is on average 7 days. However, an analyst who works with such patients on a day-to-day basis believes that the average recovery period quoted by the virologist is understated. He carries out a survey to investigate this by recording the recovery period of 120 randomly chosen patients. The recovery period of each of these 120 patients are denoted by x days. The summarised results are given by

$$\sum(x-7)=24 \text{ and } \sum(x-7)^2=14908.$$

(i) Explain in context, the meaning of a random sample of 120 patients. [1]

(ii) Calculate the unbiased estimates of the population mean and variance of the recovery period of the patients. [2]

(iii) Carry out a test at 5% significance level, on the claim that the average recovery period of 7 days is understated. [4]

(iv) Explain if the test in part **(iii)** would still be valid if the analyst had asked for recovery periods from a random sample of 15 patients. [1]

(v) Suppose instead, the analyst wants to test if the average recovery period differs from 7 days. The analyst collects data from another random sample of 120 patients. Given that the population standard deviation is now known to be 1 day, find the range of values of the sample mean, $\bar{x}$, such that the null hypothesis is rejected at 1% level of significance. [4]