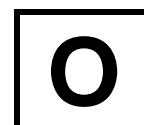
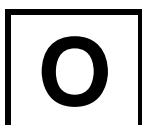


Index Number	Class	Name	Calculator Model
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CHIJ ST JOSEPH'S CONVENT PRELIMINARY EXAMINATION



MATHEMATICS

4048/02

Paper 2

Secondary 4 Express / 4 Normal Academic (O Level) /
5 Normal Academic

25 August 2021
2 hours 30 minutes

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your index number, name and class in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of the marks for this paper is **100**.

FOR EXAMINER'S USE
100

This document consists of **33** printed pages.

Setter: Ms Ong Suat King & Mrs Margaret Ng

[Turn over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions

1 (a) Simplify $\frac{2x-3y}{6x^2-5xy-6y^2}$.

Answer [2]

(b) Express as a single fraction in its simplest form $\frac{1}{y-2x} - \frac{x+3}{4x^2-y^2}$.

Answer [3]

- (c) Given that $\frac{1}{3}xy = 2\sqrt{(1-x)y}$, express y in terms of x .

Answer $y = \dots\dots\dots$ [3]

- 2 Mr Lee paid \$100 for his water bills in January when the rate was \$ x per cubic metres.

In December, the price of water was increased by 5 cents per cubic metres. By cutting down on usage, Mr Lee still managed to pay \$100 for his water bill in December.

- (a) Write down an expression, in terms of x , for the amount of water used by Mr Lee in January.

Answer [1]

- (b) Write down an expression, in terms of x , for the amount of water used by Mr Lee in December.

Answer [1]

- (c) If the amount of water used in December was 2 m^3 less than the amount used in January, form an equation in x and show that it reduces to $20x^2 + x - 50 = 0$.

Answer (in answer space) [3]

- (d) Solve the equation and find the price of water per cubic metres in January.

Answer \$/ m³ [3]

- (e) Using your result obtained in part (d), calculate how much water Mr Lee used in December leaving your answer correct to 2 decimal places.

Answer m³ [2]

- 3 The diagram below shows a series of pattern formed by dots and squares of side of 1 cm.

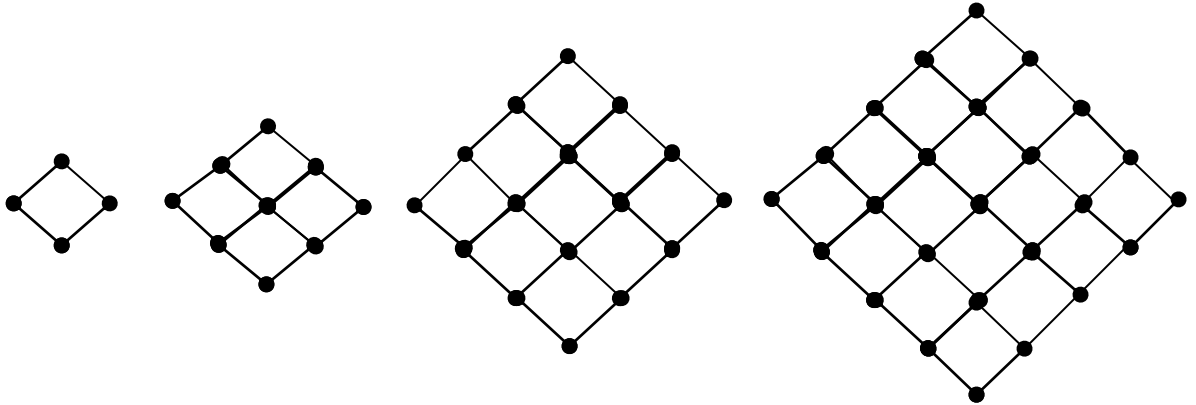


Figure 1

Figure 2

Figure 3

Figure 4

The total number of dots and number of small squares of each figure is shown in the table below.

Figure	Total number of small squares (S)	Total number of dots (D)	Number of match sticks used (M)
1	1	4	4
2	4	9	12
3	9	16	24
4	16	25	40
5	a	b	c
\vdots	\vdots	\vdots	\vdots
n	x	y	z

By considering the diagrams above and the pattern developed,

- (a) find the values of a , b and c ,

Answer $a = \dots\dots\dots$

$b = \dots\dots\dots$

$c = \dots\dots\dots$ [3]

- (b) find the total number of dots needed to form Figure 25,

Answer [1]

- (c) express y in terms of n ,

Answer $y =$ [1]

- (d) express z in terms of n ,

Answer $z =$ [1]

- (e) find the value of D when $M = 5100$.

Answer $D =$ [2]

- 4 The workers of a company can take public transport to travel from the company to the nearest Mass Rapid Transit (MRT) station. They can travel by basic bus, Light Rail Transit (LRT) or express bus.

When paying with a fare card, the fares are \$0.70 for basic bus, \$0.80 for LRT and \$1.40 for express bus.

This information can be represented by the matrix $F = \begin{pmatrix} 0.70 \\ 0.80 \\ 1.40 \end{pmatrix}$.

When paying by cash, the fares for basic bus and LRT increase by 85% and the fare for express bus increases by 60% .

- (a) The elements of the matrix S , where $S = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} F$, represents the fares for basic bus, LRT and express bus when paid by cash.

- (i) Find the values of a , b and c .

Answer $a = \dots\dots\dots$

$b = \dots\dots\dots$

$c = \dots\dots\dots [1]$

- (ii) Hence evaluate S .

Answer $S = \dots\dots\dots$ [2]

- (b) The workers work in three shifts.

The table below shows the mode of transport they chose to travel to the MRT station after completing their shifts each day.

	Basic bus	LRT	Express bus
Shift 1	25	14	0
Shift 2	18	20	19
Shift 3	13	20	7

It is given that the matrix T represents the **total fare** paid by the workers each day using cash.

By using a product of three matrices, find T and then evaluate it.

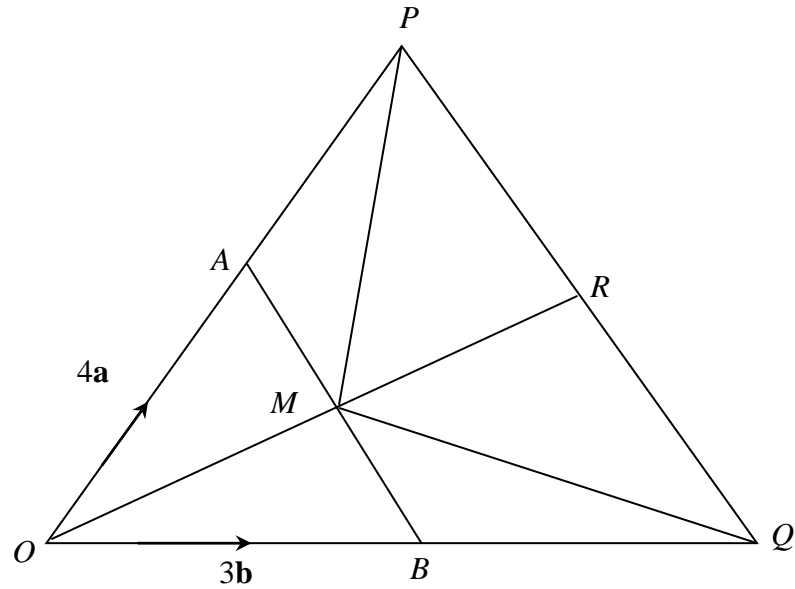
Answer (*in answer space*) [3]

- (c) There are x number of workers who do not take public transport.
Two workers are selected at random, one after the other.
The probability that the first worker is from shift 2 or 3 and chooses the LRT,
and the second worker is from shift 2 and chooses the basic bus is $\frac{12}{239}$.

Find the total number of workers in the company.

Answer [4]

- 5 In the diagram, $\vec{OA} = 4\mathbf{a}$, $\vec{OB} = 3\mathbf{b}$ and $\vec{OA} = \frac{4}{7}\vec{OP}$. Given that M is the midpoint of AB , B is the midpoint of OQ and R is on PQ .



- (a) Express each of the following in terms of \mathbf{a} and/or \mathbf{b} .

(i) \vec{AB} ,

Answer [1]

(ii) \vec{AM} ,

Answer [1]

(iii) $\overrightarrow{OM},$

Answer [1]

(iv) $\overrightarrow{PQ},$

Answer [1]

(v) $\overrightarrow{MQ},$

Answer [1]

(vi) $\overrightarrow{MP}.$

Answer [1]

(b) Find the value of

(i) $\frac{\text{area of } \triangle OAM}{\text{area of } \triangle AMP},$

Answer [1]

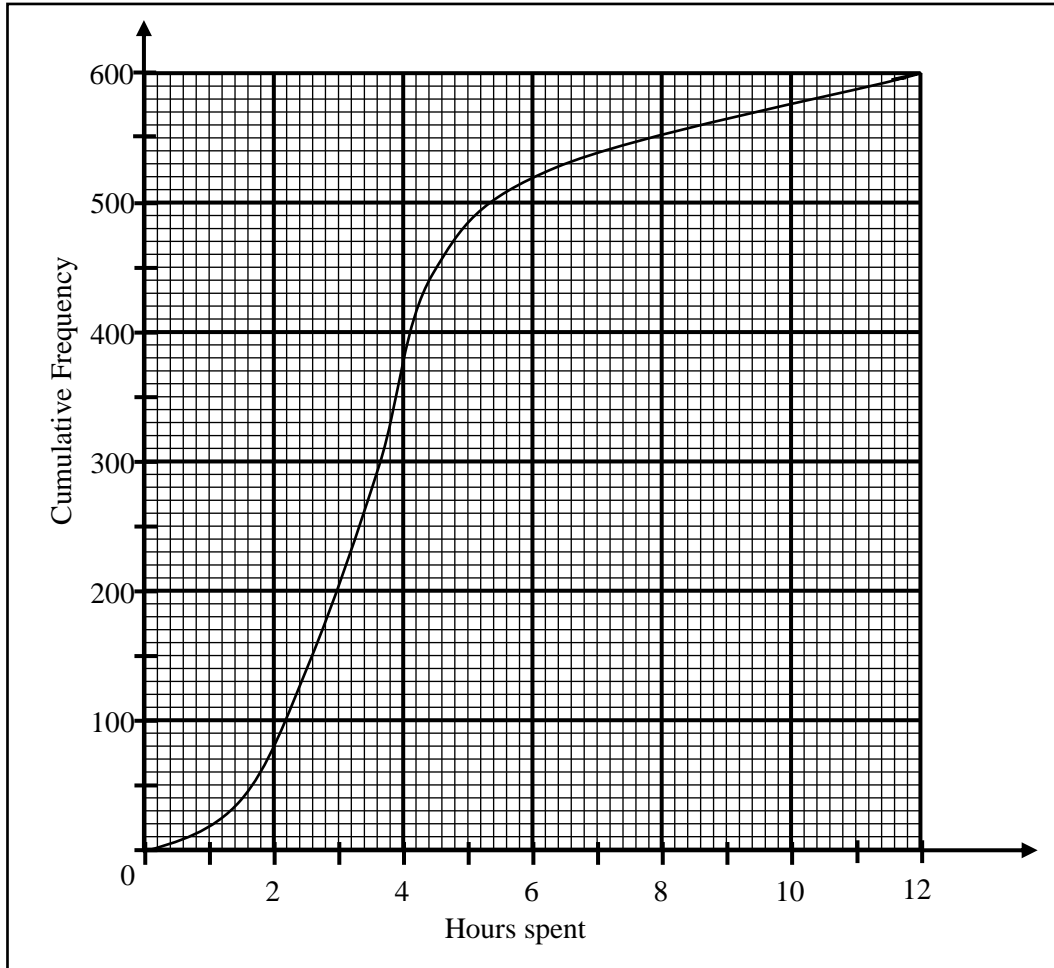
(ii) $\frac{\text{area of } \triangle OMP}{\text{area of } \triangle OMB}.$

Answer [1]

(c) Given that $\frac{PR}{RQ} = \frac{7}{8}$ and $\overrightarrow{OM} = h\overrightarrow{OR}$, find h .

Answer [3]

- 6 A survey on the daily time spent on the smart phones was carried out.
The amount of time spent by 600 people in Group A is given in the following cumulative frequency curve.



(a) Use the graph to find

(i) the median,

Answer h [1]

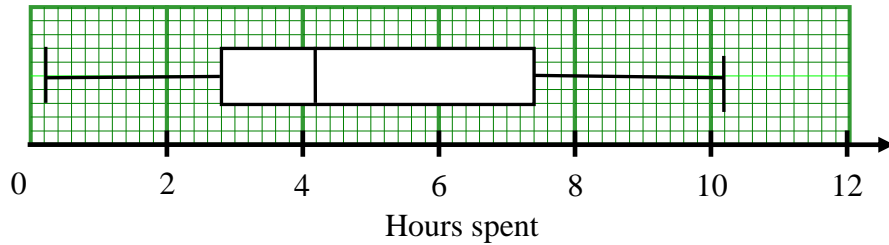
- (ii) the interquartile range,

Answer h [2]

- (iii) the value of x if 10% of the people surveyed had spent at least x hours daily on their smart phones.

Answer [2]

In Group B, the daily time spent on the smart phones of 600 people are represented by the box and whisker diagram below.



(b) Use the box and whisker diagram to find

(i) the median,

Answer h [2]

(ii) the interquartile range.

Answer h [2]

(c) (i) “On average, people in Group A on daily basis, spend more time on their smart phones than people in Group B”

Do you agree or disagree? Give a reason for your answer.

Answer (in answer space) [1]

- (ii) Which group is more consistent on the usage of the smart phones on a daily basis? Give a reason for your answer.

Answer(*in answer space*)..... [1]

7

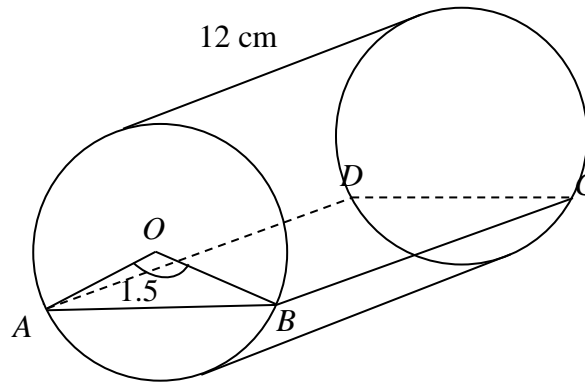


Figure 1

- (a) Figure 1 shows a cylindrical container of length 12 cm. The circular end of the container has a centre O . 240 cm^3 of water is poured into the cylindrical container to fill the container up to the level such that $\widehat{AOB} = 1.5$ radians as shown in Figure 1.
- (i) Show that the radius of the cylindrical container is 8.92 cm.

Answer (in answer space) [3]

- (ii) Calculate the surface area of the water that is exposed inside the container.

Answer cm^2 [3]

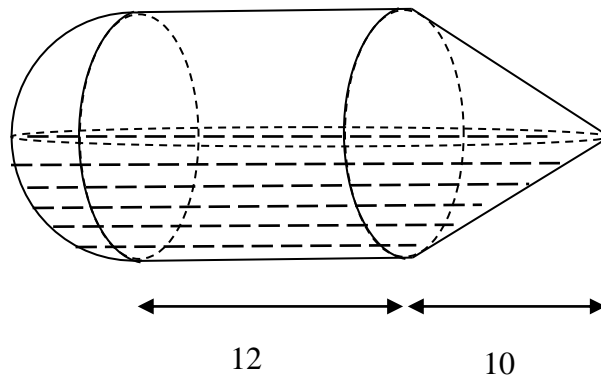


Figure 2

- (b) The container is modified as shown in Figure 2 with a cone and hemisphere attached at the two ends of the container. Water is poured into the container up to the level shown in Figure 2, which is exactly at the halfway level. Calculate the area of the container that is in contact with water.

Answer cm^2 [4]

8 Mrs Tan wants to save for her retirement and she has a few options.

Option A

Deposit \$100 000 and after 5 years receive \$800 per month for the next 10 years. At the end of the 15 year tenure, receive a lump sum of \$80 000.

Option B

Deposit \$100 000 and after 5 years receive 20% of the amount deposit. The remaining amount will earn a compound interest of 7% per half yearly for the next 10 years.

Option C

Deposit a certain sum of money for 15 years with compound interest of 5% per annum so that she can receive \$180 000 after 15 years.

- (a) Calculate the total amount she will receive and the total amount of interest earned for Option A after 15 years.

Answer Amount = \$.....

Interest Earned = \$..... [3]

- (b) Calculate the total amount she will receive and the interest earned for Option B after 15 years.

Answer Amount = \$.....

Interest Earned = \$..... [4]

- (c) Calculate the amount of money she needs to deposit at the beginning in order for her to receive \$180 000 at the end of 15 years for Option C.

Answer \$ [2]

- 9 The table below gives some values of x and the corresponding values of y , correct to 2 decimal places where

$$y = 2x^2 + \frac{20}{x}.$$

x	1	1.25	1.5	2	2.5	3	3.5	4
y	22	19.13	17.83	18	20.5	24.67	30.21	a

- (a) Calculate the value of a .

Answer [1]

- (b) By using the graph paper on page 25 and taking 4 cm to represent 1 unit on the x -axis and 2 cm to represent 5 units on the y -axis, draw the graph of

$$y = 2x^2 + \frac{20}{x} \text{ for } 1 \leq x \leq 4. \quad [3]$$

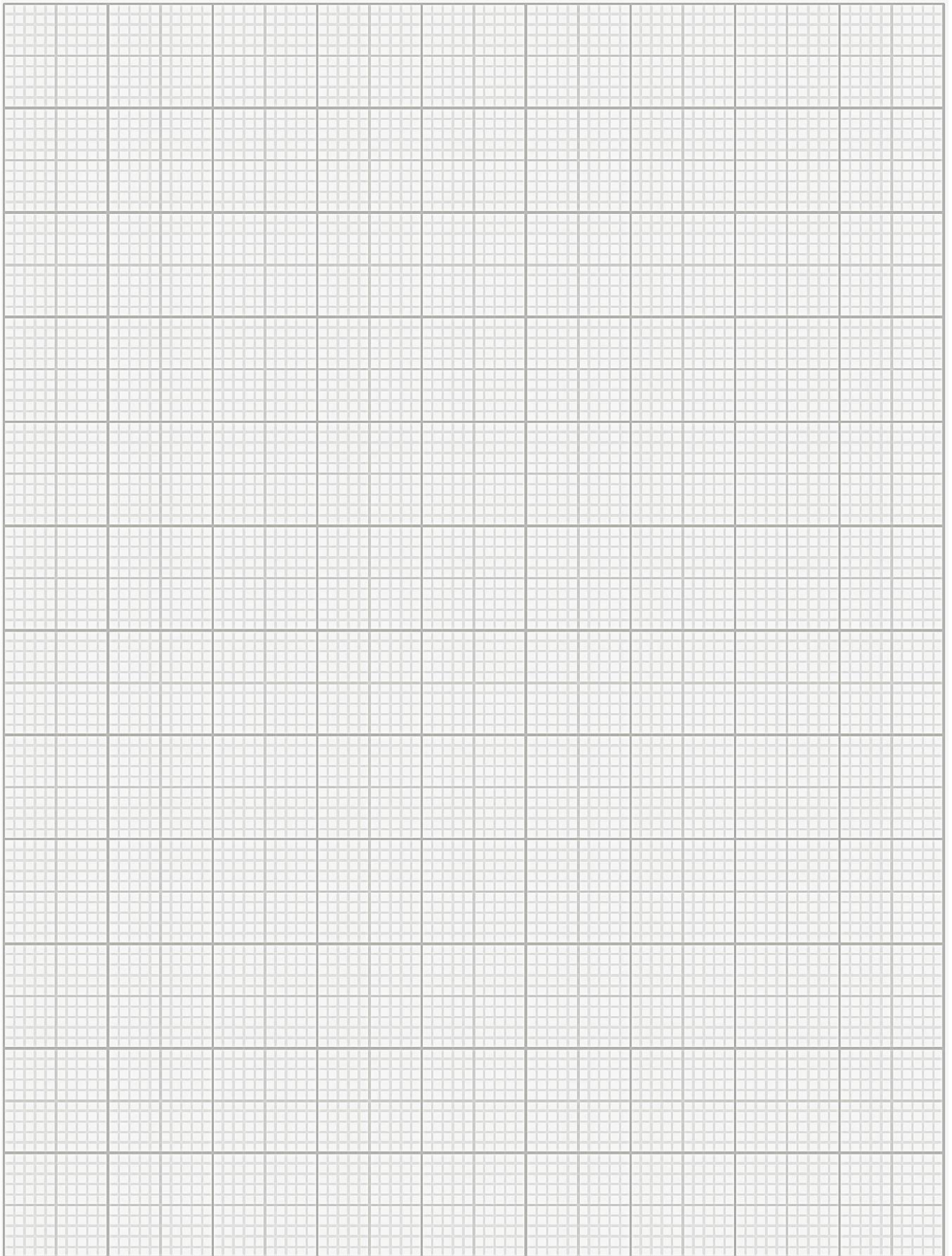
- (c) Using your graph, find

(i) the solutions of $x^2 + \frac{10}{x} - 10 = \frac{1}{2}x$,

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (ii) the value of k if the line $y = k$ is a tangent to the curve,

Answer $k = \dots\dots\dots$ [1]



- (iii) the value of x whereby the gradient is 5,

Answer $x = \dots\dots\dots$ [2]

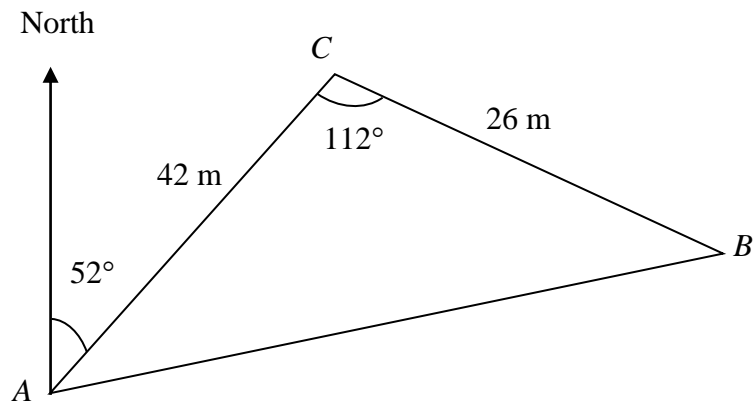
- (iv) the range of values of x for $x + \frac{10}{x^2} - \frac{10}{x} < 1$.

Answer $\dots\dots\dots$ [2]

- (d) The solutions of the equation $2x^3 + Ax^2 + Bx + C = 0$ are points of intersection between $y = 2x^2 + \frac{20}{x}$ and the line $y = 5x + 1$. Find the values of A , B and C .

Answer $A = \dots\dots\dots, B = \dots\dots\dots, C = \dots\dots\dots$ [2]

- 10 A, B, C are three points on level ground. The bearing of C from A is 052° and angle $ACB = 112^\circ$. $AC = 42$ m and $BC = 26$ m.



- (a) Stating your reasons clearly, calculate

- (i) the bearing of C from B ,

Answer $^\circ$ [2]

- (ii) the length of AB ,

Answer m [2]

- (iii) the area of triangle ABC .

Answer m^2 [1]

- (b) John walks from A to B , calculate
- (i) the distance from C when he is nearest to it,

Answer m [2]

- (ii) the largest angle of elevation of a 5 m flagpole erected at C .

Answer ° [2]

- (c) Mary walks from B to A and looks up at the same flagpole erected at C .

- (i) At which point will there be the smallest angle of elevation? Justify your answer.

Answer Point because
 [1]

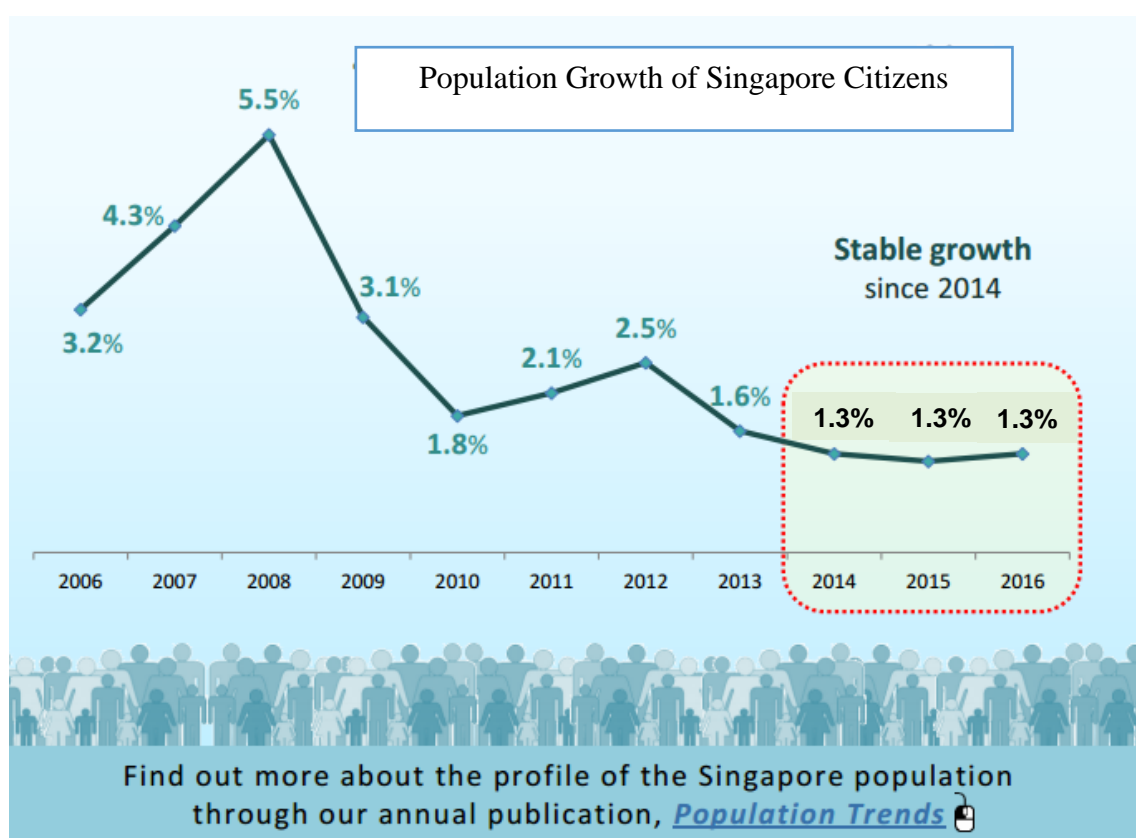
- (ii) Find this angle.

Answer ° [1]

- 11 Singapore has an ageing population and it is projected that in 2030, about 1 in 4 Singapore Citizens will be aged 65 and above. The Agency for Integrated Care (AIC) intends to build sufficient elderly care centres to provide simple medical care to 10% of this elderly population who have difficulty travelling to a medical centre.

The information AIC requires are provided below.

Population and Vital Statistics (2016)	
Land area (square km)	719.2
Total Population (thousand)	5,607.3
Singapore Citizens (thousand)	3,933.6



Distribution of Land Usage in 2030 (Top Five)	
Defence Requirements	19%
Residential	17%
Industry and Commerce	17%
Land Transport Infrastructure	13%
Parks and Nature Reserves	9%

- (a) Determine, in 2030,
- (i) the total land area used for residential purposes,

Answerkm² [1]

- (ii) the number of Singapore Citizens.

Answer [2]

- (b) To provide accessible medical services, each elderly care centre will service a circular catchment area of maximum radius 1 km and can serve a maximum of 1000 elderly.

Showing your calculations clearly, estimate the number of elderly living in one catchment area in 2030, and determine if AIC needs to build more centres by 2030.

Answer Average number of elderly in one catchment area is

.....

.....

.....[6]

- (c) State a reasonable assumption made in your calculations.

Answer
.....
.....
.....[1]

END OF PAPER 2

Answers for checking

Qn	Solution
1(a)	$\frac{1}{3x+2y}$
1(b)	$\frac{-3x-y-3}{(2x-y)(2x+y)}$
1(c)	$y = \frac{36-36x}{x^2}$
2a)	$\frac{100}{x}$
b)	$\frac{100}{x+0.05}$
d)	1.56 or -1.61 Therefore price of water in January = \$1.56/m ³
e)	62.3 m ³
3a)	a = 25, b = 36, c = 60
b)	676
c)	$y = (n+1)^2$
d)	$z = n + n(2n+1) / 2n + 2n^2 / 2n(n+1)$
e)	2601
4(ai)	a = 1.85, b = 1.85, c = 1.6
4(aii)	$S = \begin{pmatrix} 1.30 \\ 1.48 \\ 2.24 \end{pmatrix}$
4(b)	$T = (210.68)$
4(c)	141
5(ai)	$-4\tilde{a} + 3\tilde{b}$
5(aii)	$-2\tilde{a} + \frac{3}{2}\tilde{b}$
5(aiii)	$2\tilde{a} + \frac{3}{2}\tilde{b}$

5(aiv)	$-7a + 6b$
5(av)	$-2a + \frac{9}{2}b$
5(avi)	$5a - \frac{3}{2}b$
5(bi)	$\frac{4}{3}$
5(bii)	$\frac{7}{4}$
5(c)	$h = \frac{15}{28}$
6ai) ii) iii) bi) ii) 7aii) b)	median = 3.6 to 3.7 Interquartile range = 1.9 to 2.0 $x = 7$ to 7.2 median = 4.2 Interquartile range = $7.4 - 2.8 = 4.6$ 146 cm ² 774 cm ²
8a)	\$76 000

b)	$\text{Amount received after 5 years} = \frac{20}{100} \times 100000$ $= \$20\,000$ <table> <tr> <th>Interpretation 1</th><th>Interpretation 2</th></tr> <tr> <td> $= 80000 \left(1 + \frac{14}{100}\right)^{10}$ $= \\$296577.71$ Tot Amt $= 20000 + 296577.71$ $= \\$316577.71$ Interest $= \\$316577.71 - 100000$ $= \\$216577.71$ </td><td> $= 80000 \left(1 + \frac{7}{100}\right)^{20}$ $= \\$309574.76$ Tot Amt $= 20000 + 309574.76$ $= \\$329574.76$ Interest $= \\$329574.76 - 100000$ $= \\$229574.76$ </td></tr> </table>	Interpretation 1	Interpretation 2	$= 80000 \left(1 + \frac{14}{100}\right)^{10}$ $= \$296577.71$ Tot Amt $= 20000 + 296577.71$ $= \$316577.71$ Interest $= \$316577.71 - 100000$ $= \$216577.71$	$= 80000 \left(1 + \frac{7}{100}\right)^{20}$ $= \$309574.76$ Tot Amt $= 20000 + 309574.76$ $= \$329574.76$ Interest $= \$329574.76 - 100000$ $= \$229574.76$
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c)	\$86 583.08				
9a)	$a = 37$				
b)	Refer to graph				
ci)	$x = 1.05$ or 2.8				
cii)	$k = 17 - 17.5$				
ciii)	$x = 2.2$ to 2.3				
civ)	$1 < x < 3.1$				
d)	$A = -5, B = -1, C = 20$				
10(ai)	300°				
10(aii)	57.1 m				
10(aiii)	506 m^2				
10(bi)	17.7 m				
10(bii)	15.7°				
10(ci)	Point A because she will furthest away from the flagpole at C.				
10 (cii)	6.8°				
11(ai)	122 km^2				
11(aii)	4710000				
11(b)	3028.49 As the number of elderly needing the centre services far exceeds the current capacity of 1000 , the AIC needs to build more centres .				
11(c)	<u>Possible assumptions</u> Total land area remains constant. Even distribution of elderly in all residential areas. Population density is the same for all areas. Stable population growth continues until 2030.				