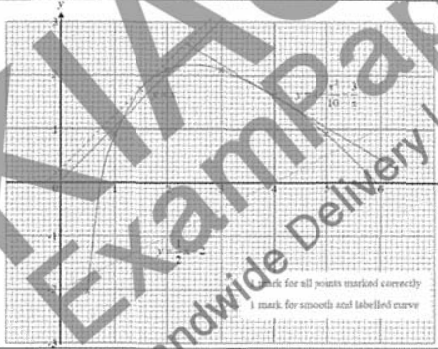



Answer Key

1(a)	$x \leq 3\frac{1}{2}$	6(a)	66°
(b)	$x = 2 \pm \sqrt{3}y$	(b)	33°
(c)	$x = 2.93$ or -0.427	(d)	39°
2(a)	\$5549	7(a)	4782942
(b)	5892500 Yen	(b)	$3^{2n} - 4n + 1$
(c)	S\$1: 94.6 Yen	(c)	$8(3^{2n}) - 4$
(d)	-12.7%	(d)	Since $T_{n+1} - T_n$ is a multiple of 4, it is divisible by 4.
3(a)	4.96 km	8(a)(i)	$3a - 2b$
(b)	75.5 m	(a)(ii)	$6a$
(c)	3 minutes 10 seconds	(a)(iii)	$3(3a - 2b)$
(d)	4.02 km	(a)(iv)	Any reasonable similar triangle test
4	0.9	(a)(v)	0:3
(a)		(b)(i)	$(-1, 3)$
(b)	-0.58 to -0.64	(b)(ii)	-5
(c)(i)	1.4 to 1.6	9(a)(i)	59 minutes
(c)(ii)	0.5 to 0.6 or 5.25 to 5.35	(a)(ii)	25 minutes
5(a)	$\sqrt{45}$ or 6.71 units	(a)(iii)	62
(b)	$y = \frac{1}{2}x + 4$	(b)(i)	$\frac{33}{65}$
(c)	$P(8, 8)$	(b)(ii)	$\frac{32}{65}$
(d)	$(-4, -4)$ and $(-4, -1)$	(b)(iii)	The setup is wrong as more people will win one of the top prizes instead of consolation prizes
(e)	$E(-16, -4)$	10(c)	Total cost = \$474.91 He can achieve his plan

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Name: Marking Scheme	Register No.:	Class:
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CRESCENT GIRLS' SCHOOL SECONDARY FOUR PRELIMINARY EXAMINATION

MATHEMATICS Paper 1

4048/01
22 August 2022
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is **80**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											
Question	12	13	14	15	16	17	18	19	20	21	22
Marks											

Table of Penalties		Qn. No.	Parent's/ Guardian's Signature	80
Presentation	-1			
Accuracy/ Units	-1			

This question paper consists of 18 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 (a) (i) Write 360 as a product of its prime factors.

$$360 = 2^3 \times 3^2 \times 5$$

A1

- (ii) Find the smallest positive integer value of p for which $\sqrt{360 \times p}$ is a whole number.

$$\begin{aligned} p &= 2 \times 5 \\ &= 10 \end{aligned}$$

A1

Answer [1]

Answer $p =$ [1]

- (b) The cube root of q is $2^3 \times 7^2$. Find q as the product of its prime factors.

$$\begin{aligned} q &= (2^3 \times 7^2)^3 \\ &= 2^9 \times 7^6 \end{aligned}$$

A1

Answer $q =$ [1]

- 2 The ratio of males to females adults attending a concert was 3 : 5.
The total number of adults in the concert was 128.

(a) How many female adults were there?

$$\begin{aligned}\text{No. of female adults} &= \frac{128}{8} \times 5 \\ &= 80\end{aligned}$$

A1

Answer [1]

- (b) When some male adults joined the concert later and the ratio of males to females became 4 : 5. How many male adults joined the concert?

Let the number of male adults who came later be x .

$$3 : 5$$

$$= 48 : 80$$

$$\frac{48+x}{80} = \frac{4}{5}$$

$$x = 16$$

M1

A1

Answer [2]

- 3 A map is drawn to a scale of 1 : 250 000.

- (a) The length of a park connector on the map is 3.8 cm.

Calculate the actual length, in kilometres, of the park connector.

$$1 \text{ cm} : 2.5 \text{ km}$$

$$3.8 \text{ cm} : 3.8 \times 2.5$$

$$= 9.5 \text{ km}$$

M1

A1

Answer km [2]

- (b) A park covers an area of 25 km².

Calculate the area, in square centimetres, of the park on the map.

$$1 : 250000$$

$$2.5 \text{ km} : 1 \text{ cm}$$

$$(2.5 \text{ km})^2 : (1 \text{ cm})^2$$

$$6.25 \text{ km}^2 : 1 \text{ cm}^2$$

$$25 \text{ km}^2 : \left(\frac{1}{6.25} \times 25 \right) \text{ cm}^2$$

$$= 4 \text{ cm}^2$$

M1

A1

Answer cm² [2]

- 4 Find the probability that two students both born in September have both their birthdays

(a) fall on 9th of September,

$$\begin{aligned} P(\text{both on 9}^{\text{th}} \text{ Sep}) &= \frac{1}{30} \times \frac{1}{30} \\ &= \frac{1}{900} \end{aligned}$$

M1

A1

Answer [2]

(b) fall on different days.

$$\begin{aligned} P(\text{both on different days}) &= \frac{30}{30} \times \frac{29}{30} \\ &= \frac{29}{30} \end{aligned}$$

M1

A1

Answer [2]

- 5 Write as a single fraction in its simplest form $\frac{4x+1}{3} - \frac{3x-2}{2}$.

$$\begin{aligned} &\frac{4x+1}{3} - \frac{3x-2}{2} \\ &= \frac{2(4x+1) - 3(3x-2)}{6} \\ &= \frac{8x+2-9x+6}{6} \\ &= \frac{8-x}{6} \end{aligned}$$

M1

A1

Answer [2]

- 6 There are five integers.

The mean of these integers is 16, the mode is 13 and the range is 7.

Find the five integers.

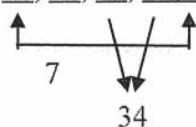
$$\text{Sum} = 16 \times 5$$

$$= 80$$

$$\underline{13, 13, 16, 18, 20}$$

or

$$\underline{13, 13, 15, 19, 20}$$



B2 – all 5

nos correct

B1- any 1 no

wrong

B0 – any 2

nos or more

wrong

Answer [2]

- 7 The dimensions of a wooden cuboid are 35 cm by 18 cm by 12 cm.

The wooden cuboid was cut into as many cubes of sides 3 cm as possible.

- (a) How many cubes were formed?

$$\text{No. of cubes} = 11 \times 6 \times 4$$

$$= 264$$

A1

Answer [1]

- (b) Peter arranged the cubes to form the biggest possible cube.

What is the length of each side of this cube?

$$\sqrt[3]{264} \approx 6.4151$$

$$\text{Length} = 6 \times 3$$

$$= 18 \text{ cm}$$

A1

Answer cm [1]

8

Amount spent per receipt	Member's discount
> \$50	6% off
> \$100	8% off
> \$200	12% off
> \$300	15% off

A popular restaurant offers membership discounts to patrons who pay \$50 per annum to become a member as shown in the table.

Lucy dines in the restaurant once a month and her average bill per meal is about \$75.

- (a) Determine if Lucy should sign up for the membership. Show your working clearly based on a year's spending.

Answer

$$\text{Amount of discount in a year} = 75 \times 12 \times 0.06$$

M1

$$= \$54 > \$50$$

Yes, she should sign up.

A1

[2]

- (b) Alice is already a member of the restaurant. During one of her dining experiences, she received a discount of \$33.30 for her meal. Calculate the cost of the meal before the discount.

Let the amount she spent before discount be

$$\text{Since } 12\% \times 200 = \$24 \text{ and } 15\% \times 300 = 45,$$

M1

the cost of her meal must be between \$200-\$300

$$0.12x = 33.30$$

$$x = 277.50$$

A1

Answer \$..... [2]

9 (a) Solve $5 + \frac{5x-2}{6} = x$.

$$\frac{30+5x-2}{6} = x$$

$$28+5x=6x$$

$$x=28$$

A1

Answer $x = \dots\dots\dots$ [1]

(b) Simplify $\frac{9a^2+b^2}{9a^2-b^2} - \frac{3a-b}{3a+b}$

$$\frac{9a^2+b^2}{(3a+b)(3a-b)} - \frac{3a-b}{3a+b}$$

$$= \frac{9a^2+b^2 - (3a-b)(3a-b)}{(3a+b)(3a-b)}$$

$$= \frac{9a^2+b^2 - 9a^2 + 6ab - b^2}{(3a+b)(3a-b)}$$

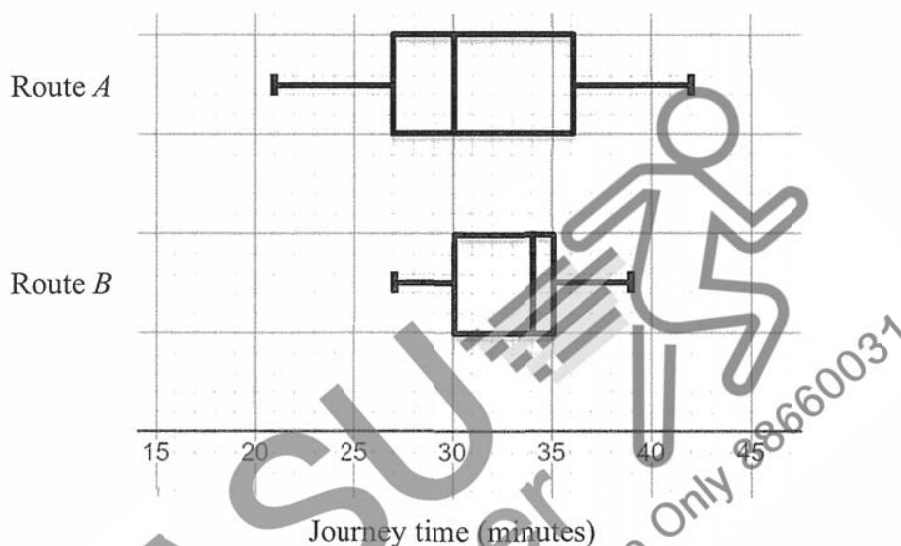
$$= \frac{6ab}{(3a+b)(3a-b)}$$

M1

M1

A1

Answer $\dots\dots\dots$ [3]



John drives from home to office every weekday morning using either Route *A* or Route *B*. For each route, the time taken, in minutes, is recorded for 40 days and plotted in the box and whisker plots above.

Compare the results for both routes. Indicate which route should John take to have a higher probability that

- (a) he will arrive in the office within 30 minutes upon leaving home.

Answer He should take Route **A** because there is a 50% chance that he will be in the office within 30 mins instead of a 25% chance for Route B.

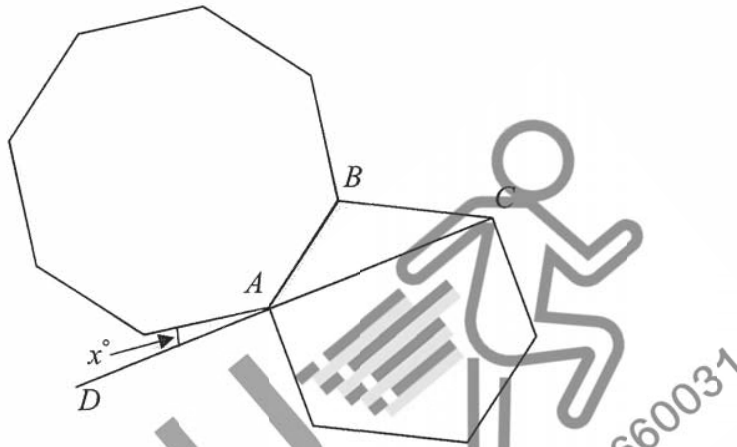
[1]

- (b) he will arrive in the office within 40 minutes upon leaving home.

Answer He should take Route **B** because there is a 100% chance that he will be in the office before 40 mins while he might take longer than 40 mins using Route A.

[1]

11



A regular hexagon and a regular octagon share a side AB .
 DAC is a straight line. Calculate the value of x .

$$\begin{aligned} 1 \text{ int } \angle (\text{hexagon}) &= \frac{(6-2) \times 180^\circ}{6} \\ &= 120^\circ \end{aligned}$$

M1

$$\begin{aligned} 1 \text{ int } \angle (\text{octagon}) &= \frac{(8-2) \times 180^\circ}{8} \\ &= 135^\circ \end{aligned}$$

M1

$$\begin{aligned} \angle BAC &= \frac{180^\circ - 120^\circ}{2} \quad (\text{Base } \angle \text{ of isos triangle}) \\ &= 30^\circ \end{aligned}$$

M1

$$x^\circ = 180^\circ - 135^\circ - 30^\circ$$

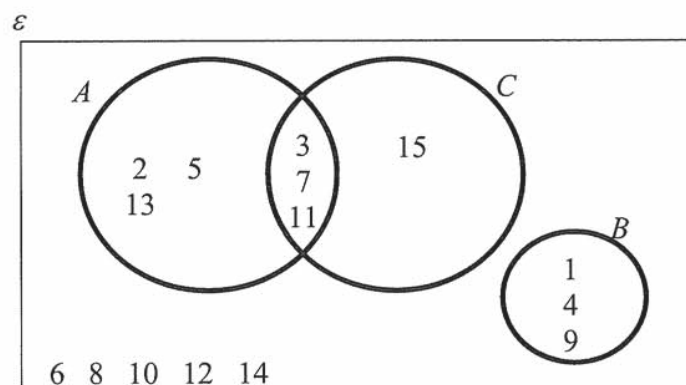
$$x = 15$$

A1 Answer $x = \dots\dots\dots$ [4]

- 12 It is given that $\varepsilon = \{\text{integers } x : 0 < x \leq 15\}$,
 $A = \{\text{prime numbers}\}$,
 $B = \{\text{perfect squares}\}$,
 $C = \{\text{integers with a remainder of 3 when divided by 4}\}$.

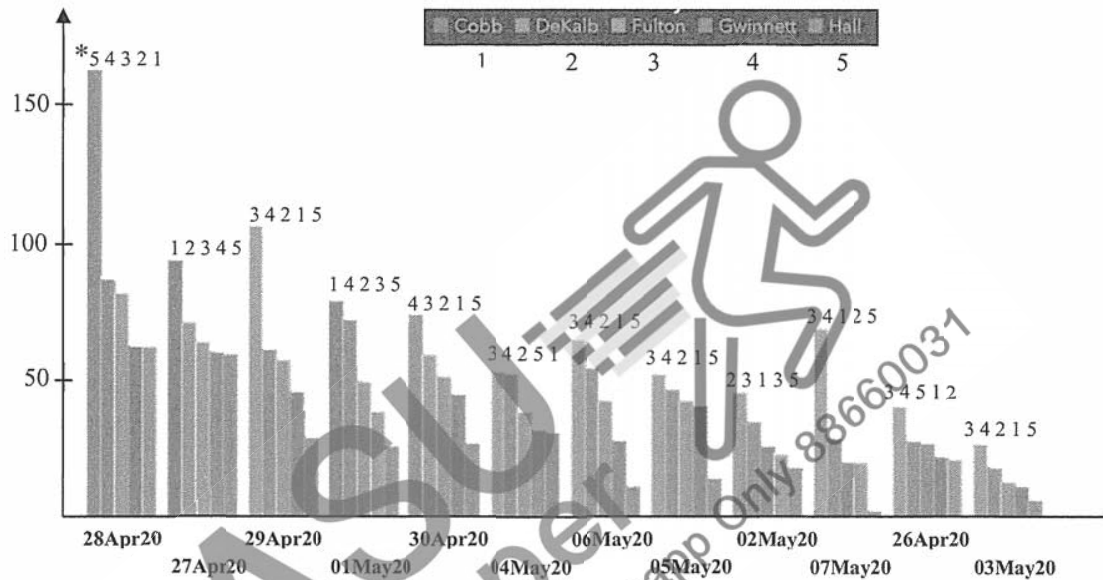
Draw the Venn diagram to illustrate this information.

Answer

B1 – Set A & C B1 – Set B B1 – remaining
numbers in ε

[3]

13



(Source: <https://www.vox.com/covid-19-coronavirus-us-response-trump/2020/5/18/21262265/georgia-covid-19-cases-declining-reopening>)

*(The number above each cluster indicates the county represented for that day, e.g. 1 – Cobb, 2 – DeKalb, 3 – Fulton, 4 – Gwinnett, 5 – Hall)

In May 2020, the US Georgia Department of Public Health posted this graph to show the top 5 counties that had the highest COVID-19 cases in the past 15 days and the number of cases over time.

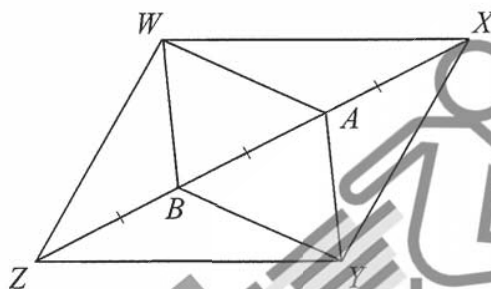
State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

The dates on the x-axis are not in the right chronological order. This leads people to think that the number of COVID-19 cases had decreased over time.

[2]

- 14 In the diagram, $WXYZ$ is a parallelogram and points A and B lie on XZ . It is given that $XA = AB = BZ$.



- (a) Write down the ratio of the area of triangle WAB to area of parallelogram $WXYZ$.

Area of $\triangle WAB$: Area of //gram

$$= 1 : 3 \times 2$$

$$= 1 : 6$$

Answer : [1]

- (b) Show that triangle BWZ is congruent to triangle AYX .

Answer

$$\angle WZB = \angle YXA \text{ (alt } \angle\text{s, } WZ \parallel XY)$$

B1

$$BZ = XA \text{ (given)}$$

$$WZ = YX \text{ (sides of //gram)}$$

B1

$$\triangle BWZ \equiv \triangle AYZ \text{ (SAS)}$$

A1

[3]

15 Given that $(x+y)^2 = 3$ and $6xy = 14$,

(a) find an expression for $x^2 + y^2$.

$$x^2 + 2xy + y^2 = 3$$

$$x^2 + y^2 = 3 - 2xy$$

A1

Answer [1]

(b) Hence or otherwise, find the value of $(3x+2y)^2 + 5y^2$.

$$(3x+2y)^2 + 5y^2$$

$$= 9x^2 + 12xy + 4y^2 + 5y^2$$

$$= 9x^2 + 9y^2 + 12xy$$

$$= 9(x^2 + y^2) + 12xy$$

$$= 9(3 - 2xy) + 12xy$$

$$= 27 - 18xy + 12xy$$

$$= 27 - 6xy$$

$$= 27 - 14$$

$$= 13$$

M1

M1

A1

Answer [3]

16 Given that $\sqrt{3} \times (3^x)^3 = 81 \div 3^{x-0.5}$, find x .

$$3^{\frac{1}{2}} \times 3^{3x} = 3^4 \div 3^{x-\frac{1}{2}}$$

M1

$$\frac{1}{2} + 3x = 4 - \left(x - \frac{1}{2}\right)$$

M1

$$4x = 4$$

$$x = 1$$

A1

Answer $x =$ [3]

- 17 Determine if 2^{400} or 3^{200} is greater. Explain your answer clearly.

Answer

$$2^{400} = (2^2)^{200} \\ = 4^{200}$$

M1

Since $2^{400} = 4^{200} > 3^{200}$, then 2^{400} is greater. A1

[2]

- 18 (a) Factorise $12ax - 3ay + 4x - y$ completely.

$$12ax - 3ay + 4x - y \\ = 3a(4x - y) + (4x - y) \\ = (4x - y)(3a + 1)$$

M1

A1

Answer [2]

- (b) Using factorisation, solve $6x^2 - 19x - 20 = 0$.

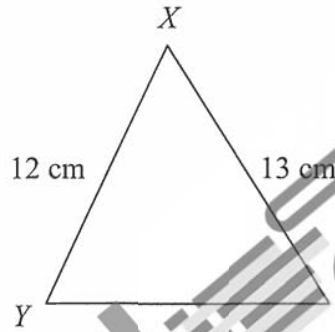
$$(6x + 5)(x - 4) = 0$$

M1

$$x = -\frac{5}{6} \text{ or } x = 4$$

A1

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]



In the triangle, $XY = 12$ cm and $XZ = 13$ cm.

- (a) Find $\frac{\sin \angle XZY}{\sin \angle XYZ}$.

$$\frac{12}{\sin \angle XZY} = \frac{13}{\sin \angle XYZ}$$

$$\frac{\sin \angle XZY}{\sin \angle XYZ} = \frac{12}{13}$$

A1

Answer [1]

- (b) If the area of the triangle XYZ is 69.5 cm^2 , find the two possible values of angle YXZ .

$$0.5(12)(13)\sin \angle YXZ = 69.5 \quad \text{M1}$$

$$\angle YXZ = \sin^{-1}\left(\frac{69.5}{0.5(12)(13)}\right)$$

$$\angle YXZ = 63.0^\circ \text{ (1 d.p.) or } \angle YXZ = 180^\circ - 63.00^\circ$$

$$= 117.0^\circ \text{ (1 d.p.)} \quad \text{A2}$$

Answer $^\circ$ or $^\circ$ [3]

- 20 A foodcourt stall sells bento sets for lunch and dinner on the weekend.
 On one Saturday, the stall sold 65 Bento Lunch *A*, 60 Bento Lunch *B*, x Bento Dinner *C* and 54 Bento Dinner *D*.
 On the same week's Sunday, the stall sold 70 Bento Lunch *A*, 67 Bento Lunch *B*, 66 Bento Dinner *C* and 58 Bento Dinner *D*.

- (a) Represent this information in a 2×4 matrix, **P**, in terms of x .

$$\mathbf{P} = \begin{pmatrix} 65 & 60 & x & 54 \\ 70 & 67 & 66 & 58 \end{pmatrix}$$

A1

Answer **P**

[1]

- (b) The Bento sets are priced at \$8.90, \$9.90, \$10.50 and \$12.50 for Bento Lunch *A*, Bento Lunch *B*, Bento Dinner *C* and Bento Dinner *D* respectively.

Represent this information in a column vector, **Q**.

$$\mathbf{Q} = \begin{pmatrix} 8.90 \\ 9.90 \\ 10.50 \\ 12.50 \end{pmatrix}$$

A1

Answer **Q** =

[1]

- (c) Evaluate **PQ**, in terms of x .

$$\mathbf{PQ} = \begin{pmatrix} 65 & 60 & x & 54 \\ 70 & 67 & 66 & 58 \end{pmatrix} \begin{pmatrix} 8.90 \\ 9.90 \\ 10.50 \\ 12.50 \end{pmatrix}$$

$$\mathbf{PQ} = \begin{pmatrix} 1847.50 + 10.5x \\ 2704.30 \end{pmatrix}$$

A2

Answer **PQ** =

[2]

- (d) Explain what the elements in matrix **PQ** represent.

Answer The elements represent the amount of money collected from the sales of the bento sets for Saturday and Sunday respectively. (B1)

[1]

- (e) If the stall collected a total of \$5129.30 for the sales of all the bento sets on that particular weekend, calculate the value of x .

$$10.5x = 5129.30 - 2704.30 - 1847.50 \quad \text{M1}$$

$$x = 55 \quad \text{A1}$$

Answer $x = \dots\dots\dots$

[2]

- 21 (a) Express $x^2 - 5x + 7$ in the form $(x - a)^2 + b$.

$$\begin{aligned} x^2 - 5x + \left(\frac{5}{2}\right)^2 + 7 - \left(\frac{5}{2}\right)^2 \\ = \left(x - \frac{5}{2}\right)^2 + \frac{3}{4} \end{aligned}$$

M1

A1

Answer [2]

- (b) Explain why the graph $y = x^2 - 5x + 7$ has no real roots.

Answer

$$\begin{aligned} \sqrt{b^2 - 4ac} \\ = \sqrt{(-5)^2 - 4(a)(7)} \\ = \sqrt{-3} \end{aligned}$$

M1

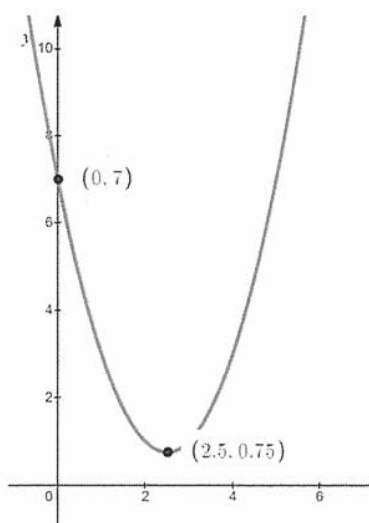
Since $\sqrt{-3}$ is undefined, $y = x^2 - 5x + 7$ has no real roots.

A1

[2]

- (c) On the axes below, sketch the graph $y = x^2 - 5x + 7$.

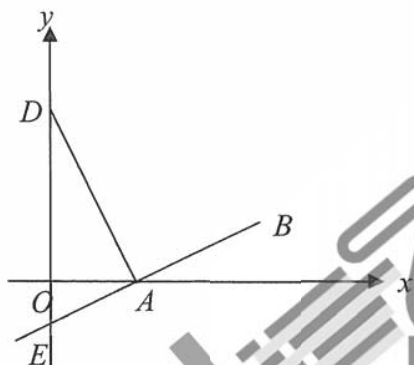
Answer

B1 – Min pt $\left(\frac{5}{2}, \frac{3}{4}\right)$

B1 – shape and y-intercept at (0,7)

[2]

22



In the diagram, D and E are points on the y -axis. A is a point on the x -axis. EAB is a straight line. The equation of the line that passes through the points A and D is $2x + y = 5$.

(a) Find the coordinates of A and D .

$$\text{at } y = 0,$$

$$-2x + 5 = 0$$

$$x = 2.5$$

$$A(2.5, 0)$$

$$\text{at } x = 0,$$

$$y = 5$$

$$D(0, 5)$$

A2

Answer $A = (\dots\dots\dots, \dots\dots\dots)$

$D = (\dots\dots\dots, \dots\dots\dots)$ [2]

(b) Given that the product of the gradients of AD and AB is -1 , find the length of DE .

$$\begin{aligned} \text{Grad of } AD &= \frac{5-0}{0-2.5} \\ &= -2 \end{aligned}$$

M1

Alternative solution:

$$\text{Grad of } AD \times \text{Grad of } AB = -1$$

$$\text{Grad of } AB = -1 \div (-2)$$

$$= \frac{1}{2}$$

M1

$$\text{At } E(0, y)$$

$$\frac{y-0}{0-2.5} = \frac{1}{2}$$

$$y = -1.25$$

M1

$$E = (0, -1.25)$$

$$AE^2 = 2.5^2 + (-1.25)^2 = 7.8125$$

$$AD^2 = 2.5^2 + 5^2 = 31.25$$

$$DE = \sqrt{7.8125 + 31.25}$$

$$= 6.25$$

M1

A1

$$AD = \sqrt{2.5^2 + 5^2} \quad \text{M1}$$

$$= \sqrt{31.25}$$

$$\tan \angle ODA = \frac{2.5}{5} \quad \text{M1}$$

$$\angle ODA = 26.57^\circ \text{ (2 d.p.)} \quad \text{A1}$$

$$DE = \frac{\sqrt{31.25}}{\cos 26.57^\circ} \quad \text{M1}$$

$$= 6.25 \text{ (3 s.f.)} \quad \text{A1}$$

Answer $DE = \dots\dots\dots$ units [5]

END OF PAPER