

Answer **all** the questions.

- 1 (a) Solve the inequality  $1 - \frac{2x-1}{2} \geq \frac{x+1}{3} - x$ . [3]

$$1 - \frac{2x-1}{2} \geq \frac{x+1}{3} - x$$

$$\frac{6}{6} - \frac{3(2x-1)}{6} \geq \frac{2(x+1)}{6} - \frac{6x}{6}$$

$$6 - 3(2x-1) \geq 2(x+1) - 6x$$

$$6 - 6x + 3 \geq 2x + 2 - 6x$$

M1 for correct common denominator

M1 for correct expansion

$$x \leq 3\frac{1}{2}$$

A1 for correct answer

- (b) Given that  $x = \frac{3y-4}{x-4}$ , express  $x$  in terms of  $y$ . [3]

$$x = \frac{3y-4}{x-4}$$

$$x^2 - 4x = 3y - 4$$

$$x^2 - 4x + 4 = 3y$$

$$(x-2)^2 = 3y$$

$$x-2 = \pm\sqrt{3y}$$

$$x = 2 \pm \sqrt{3y}$$

$$x = \frac{3y-4}{x-4}$$

$$x^2 - 4x + (4-3y) = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4-3y)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{4(3y)}}{2}$$

$$= 2 \pm \sqrt{3y}$$

M1 for correct cross multiplication

M1 for correct substitution

M1 for correct substitution

A1 for correct answer

- (c) Solve the equation  $\frac{1}{3x^2-x-2} = \frac{4x}{3x+2} + \frac{2}{1-x}$ , giving your answers to 2 decimal places. [5]

$$\frac{1}{3x^2-x-2} = \frac{4x}{3x+2} + \frac{2}{1-x}$$

$$\frac{1}{(3x+2)(x-1)} = \frac{4x}{3x+2} - \frac{2}{x-1}$$

$$1 = 4x(x-1) - 2(3x+2)$$

$$1 = 4x^2 - 4x - 6x - 4$$

$$4x^2 - 10x - 5 = 0$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(-5)}}{2(4)}$$

$$x = 2.93 \text{ or } -0.427$$

M1 for correct factorization and change of sign

M1 for removal of denominator

M1 for correct equation

M1 for correct substitution of values into equation

A1 for correct answer

- 2 John took up a job offer in Japan. His annual salary package at that time was 5 500 000 Yen for his one-year contract.

The exchange rate between Singapore dollars and Japanese Yen at that time was S\$1 = 82.6 Yen.

- (a) Calculate his average monthly salary in Singapore dollars.  
Give your answer correct to the nearest dollar. [2]

$$\begin{aligned}\text{Monthly salary} &= \frac{5500000}{82.6} \div 12 && \text{M1 for correct working} \\ &= \$5548.83 \\ &\approx \$5549 && \text{A1 for correct answer}\end{aligned}$$

Upon completion of his contract, his company decided to extend his contract by another year. His new annual salary package entitled him to a pay rise of 3.5% based on his past annual salary package plus 200 000 Yen as gratuity if he completes his entire contract.

- (b) Assuming he completes his entire contract, calculate his revised annual salary package in Yen. [2]

$$\begin{aligned}\text{Annual salary package} &= 5500000 \times 1.035 + 200000 && \text{M1 for correct working} \\ &= 5892500 \text{ Yen} && \text{A1 for correct answer}\end{aligned}$$

Based on the exchange rate between Singapore dollars and Japanese Yen at that time, John realised that his new average monthly salary in Singapore dollars was S\$5 191.

- (c) Calculate the exchange rate between Singapore dollars and Japanese Yen at that time.  
Give your answer to one decimal place. [2]

$$\begin{aligned}\text{Let the exchange rate for S\$1 be } x \text{ Yen} \\ 5191 &= \frac{5892500}{x} \div 12 && \text{M1 for correct working} \\ \frac{5892500}{x} &= 62292 \\ x &= 94.594 \\ &= 94.6 && \text{A1 for correct answer}\end{aligned}$$

$\therefore$  S\$1:94.6 Yen

- (d) Calculate the percentage decrease in the value of Yen against the Singapore dollar. [3]

On 1<sup>st</sup> January 2021

\$1 = 82.6 Yen

1 Yen = \$0.012107

On 1<sup>st</sup> June 2022

\$1 = 94.6 Yen

1 Yen = \$0.010571

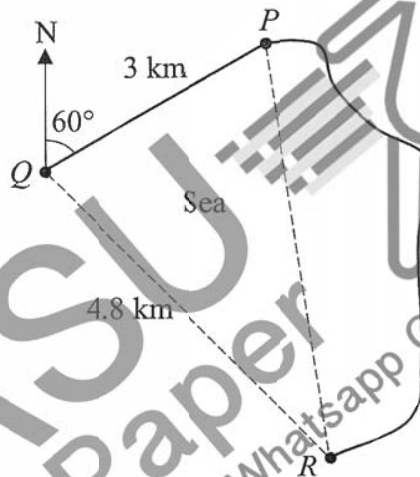
M1 for changing to 1 Yen

$$\begin{aligned}\text{Percentage change} &= \left( \frac{0.010571 - 0.012107}{0.012107} \right) \times 100\% && \text{M1 for correct working} \\ &= -12.7\% && \text{A1 for correct answer}\end{aligned}$$

or

$$\begin{aligned}\text{Percentage change} &= \left( \frac{\frac{1}{94.6} - \frac{1}{82.6}}{\frac{1}{82.6}} \right) \times 100\% && \text{M2 for correct working} \\ &= -12.7\% && \text{A1 for correct answer}\end{aligned}$$

- 3 The diagram below shows a rescue tower at point  $P$  and a jetty at point  $Q$  along the shore of a bay on a level ground. A lighthouse is on top of a hill at point  $R$ . Rescue tower  $P$  is 3 km along a straight shoreline at a bearing of  $060^\circ$  from jetty  $Q$ . Lighthouse  $R$  is 4.8 km southeast of Jetty  $Q$ .



- (a) Show that the distance between rescue tower  $P$  and lighthouse  $R$  is approximately 4.96 km. [3]

$$\begin{aligned}\angle PQR &= 135^\circ - 60^\circ \\ &= 75^\circ\end{aligned}$$

M1 for correct angle

$$PR^2 = 3^2 + 4.8^2 - 2(3)(4.8)\cos 75^\circ$$

$$PR = \sqrt{3^2 + 4.8^2 - 2(3)(4.8)\cos 75^\circ}$$

M1 for correct substitution

$$= \sqrt{24.5861}$$

$$= 4.9584$$

$$\approx 4.96 \text{ km}$$

A1 for correct answer



- (b) The angle of depression from the top of the hill at  $R$  to the jetty  $Q$  is  $1.2^\circ$ .  
The angle of depression from the top of the lighthouse at  $R$  to the jetty  $Q$  is  $2.1^\circ$ .  
Calculate the height of the lighthouse, in metres, correct to the nearest one decimal place. [3]

Let height of hill be  $H$  m.

Let height of lighthouse be  $L$  m.

$$\tan 1.2^\circ = \frac{H}{4800}$$

$$H = 4800 \tan 1.2^\circ$$

$$= 100.5456 \text{ m}$$

M1 for height of hill

$$\tan 2.1^\circ = \frac{L + 100.5456}{4800}$$

M1 for correct equation for height of lighthouse

$$L + 100.5456 = 4800 \tan 2.1^\circ$$

$$L = 4800 \tan 2.1^\circ - 100.5456$$

$$= 75.4624$$

$$\approx 75.5 \text{ m}$$

A1 for correct answer

- (c) A boat travelling from  $Q$  to  $R$  capsized at point  $X$ , which is nearest to rescue tower  $P$ .  
A rescue boat was dispatched immediately from  $P$ , travelling at 55 km/h.  
Calculate the time it will take to reach the capsized boat.  
Express your answer in terms of minutes and seconds. [3]

Let distance travelled be  $d$

Let time taken be  $t$

$$\sin 75^\circ = \frac{d}{3}$$

$$d = 2.8978 \text{ km}$$

M1 for correct distance

$$t = \frac{2.8978}{55}$$

$$t = \frac{2.8978}{55}$$

$$= 0.05269 \text{ hour}$$

M1 for correct for correct time

$$= 3 \text{ minutes } 10 \text{ seconds}$$

A1 for correct time in mins and secs

- (d) How far away was the boat from lighthouse  $R$  when it capsized? [2]

$$\cos 75^\circ = \frac{QX}{3}$$

$$QX = 3 \cos 75^\circ$$

M1 for correct distance for  $QX$

$$\text{Distance from } R = 4.8 - 3 \cos 75^\circ$$

$$= 4.0235$$

$$= 4.02 \text{ km (3sf)}$$

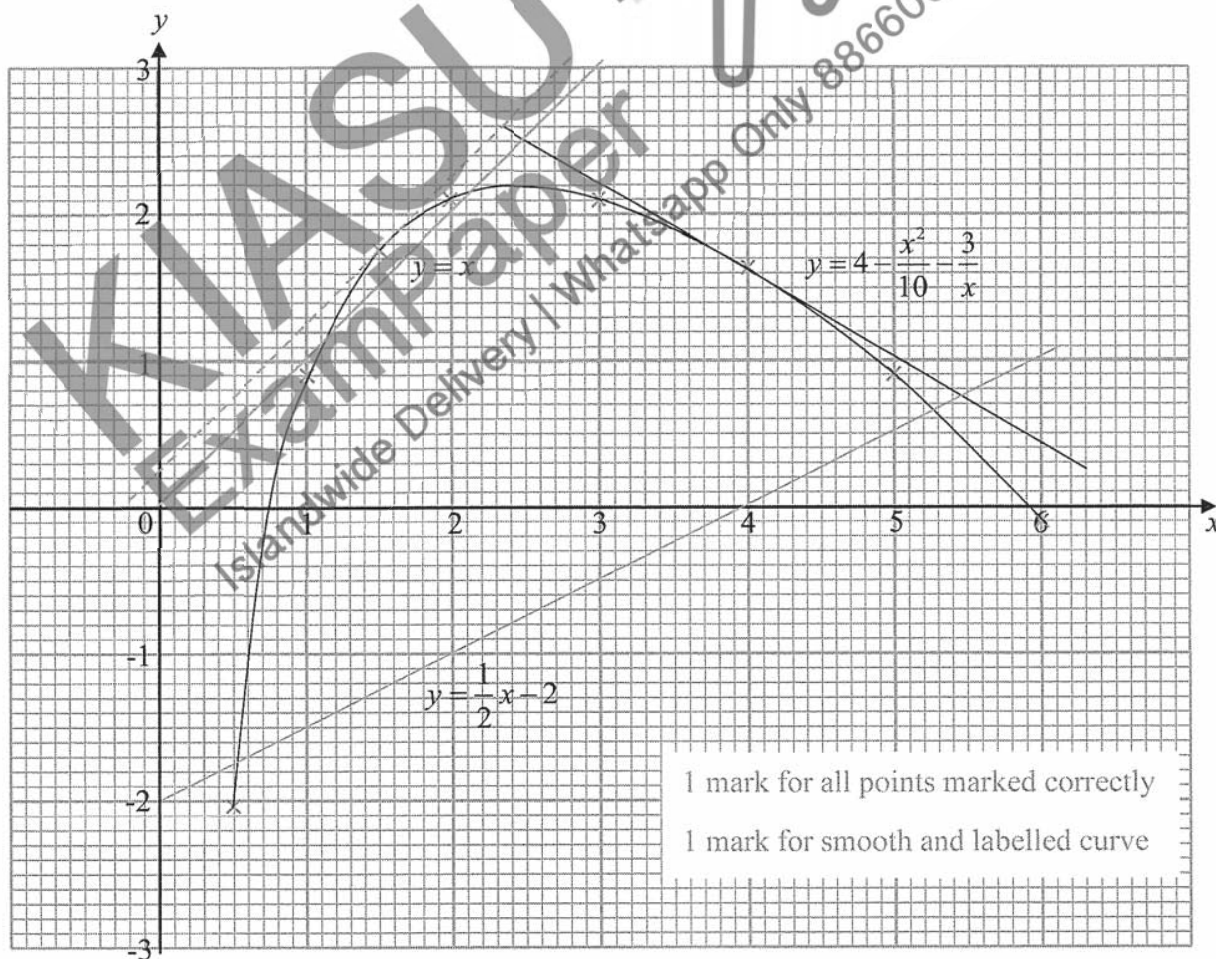
A1 for correct answer

- 4 Complete the table of values for  $y = 4 - \frac{x^2}{10} - \frac{3}{x}$ .

$x$	0.5	1	1.5	2	3	4	5	6
$y$	-2.03	0.9	1.78	2.1	2.1	1.65	0.9	-0.1

[1]

- (a) On the grid, draw the graph of  $y = 4 - \frac{x^2}{10} - \frac{3}{x}$  for  $0.5 \leq x \leq 6$ .



[2]

- (b) By drawing a tangent, find the gradient at the point when  $x = 4$ . [2]

acceptable range for gradient =  $-0.58$  to  $-0.64$

1 mark for correct tangent drawn

1 mark for correct gradient

- (c) Use your graph to find  
(i) the  $x$ -coordinate of the point on the curve at which the gradient is 1, [2]

1 mark for any correct line drawn and labelled

acceptable range for  $x = 1.4$  to  $1.6$

1 mark for correct point

- (ii) the values of  $x$ , in the range of  $0.5 \leq x \leq 6$  for which  $x^3 + 5x^2 - 60x + 30 = 0$  [3]

$$x^3 + 5x^2 - 60x + 30 = 0$$

$$\frac{x^3}{10x} + \frac{5x^2}{10x} - \frac{60x}{10x} + \frac{30}{10x} = 0$$

$$\frac{x^3}{10x} + \frac{5x^2}{10x} - \frac{60x}{10x} + \frac{30}{10x} = 0$$

$$\frac{x^2}{10} + \frac{1}{2}x - 6 + \frac{3}{x} = 0$$

$$\frac{1}{2}x - 6 = -\frac{x^2}{10} - \frac{3}{x}$$

$$\frac{1}{2}x - 2 = 4 - \frac{x^2}{10} - \frac{3}{x}$$

$$\text{Draw } y = \frac{1}{2}x - 2$$

1 mark for correct equation

$$x = 0.5 \text{ to } 0.6 \text{ or } 5.25 \text{ to } 5.35$$

1 mark for correct line drawn and labelled

1 mark for 2 correct points

- 5 The points  $A$  and  $B$  are  $(-4, 2)$  and  $(2, 5)$  respectively.

- (a) Find the length of the line segment  $AB$ . [2]

$$\begin{aligned}\text{Length of } AB &= \sqrt{(5-2)^2 + (2-(-4))^2} && \text{M1 for correct working} \\ &= \sqrt{(3)^2 + (6)^2} \\ &= \sqrt{45} \text{ units or } 6.71 \text{ units (3sf)} && \text{A1 for correct answer}\end{aligned}$$

- (b) Find the equation of the line  $AB$ . [2]

$$\begin{aligned}\frac{y-5}{x-2} &= \frac{5-2}{2-(-4)} && \text{M1 for correct working} \\ \frac{y-5}{x-2} &= \frac{1}{2} \\ 2y-10 &= x-2 \\ \therefore y &= \frac{1}{2}x + 4 && \text{A1 for correct answer}\end{aligned}$$

- (c)  $P$  is a point on  $AB$  produced such that  $P$  is equidistant from the two axes. Find the coordinates of  $P$ . [2]

$$\begin{aligned}\text{Since } P(x, x), \\ x &= \frac{1}{2}x + 4 && \text{M1 for correct working} \\ x &= 8 \\ \therefore P(8, 8) &&& \text{A1 for correct answer}\end{aligned}$$

- (d) Given that  $C$  is  $(2, -4)$  and length of  $AB$  and  $CD$  are equal, find the two possible coordinates of  $D$  if it lies on the vertical line passing through  $A$ . [2]

$$\begin{aligned}D_1(x, y) &= (2-6, -4-3) \\ &= (-4, -4) && \text{B1 for correct answer} \\ D_2(x, y) &= (2-6, -4+3) \\ &= (-4, -7) && \text{B1 for correct answer}\end{aligned}$$

- (e) A trapezium  $ABCE$ , with  $AE$  parallel to  $BC$ , has an area of  $81 \text{ units}^2$ . Find the coordinates of  $E$ .

[2]

$$\frac{1}{2}(AE + BC)(6) = 81$$

M1 for correct working

$$\frac{1}{2}(AE + 9)(6) = 81$$

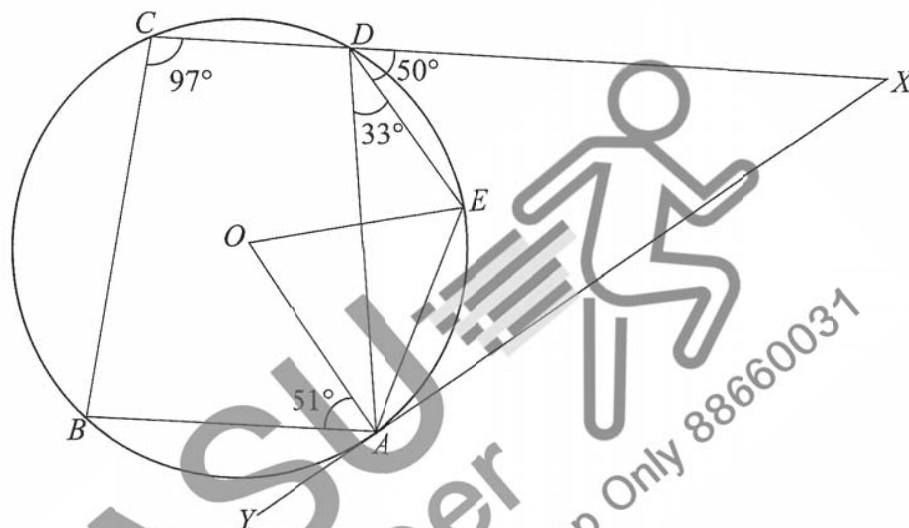
$$AE = 18 \text{ units}$$

$$\therefore E(-4, -16)$$

A1 for correct answer

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6



The diagram shows a circle  $ABCDE$  with centre  $O$ .

$XY$  is a tangent to the circle at  $A$ .

$CDX$  is a straight line.

Angle  $ADE = 33^\circ$ , angle  $BAO = 51^\circ$ , angle  $BCD = 97^\circ$  and angle  $EDX = 50^\circ$ .

- (a) Find angle  $AOE$ . State the reason clearly.

[1]

$$\begin{aligned}\angle AOE &= 33^\circ \times 2 \text{ (angle at the centre twice angle on the circumference)} \\ &= 66^\circ\end{aligned}$$

B1 for correct answer

- (b) Find angle  $XAE$ .

Give a reason for each step of your working.

[2]

$$\begin{aligned}\angle OAE &= \frac{180^\circ - 66^\circ}{2} \text{ (isosceles triangle)} \\ &= 57^\circ\end{aligned}$$

M1 for correct angle

$$\angle XAO = 90^\circ \text{ (radius } \perp \text{ tangent)}$$

A1 for correct answer

$$\begin{aligned}\angle XAE &= 90^\circ - 57^\circ \\ &= 33^\circ\end{aligned}$$

Alternatively,

$$\angle XAE = 33^\circ \text{ (angle in alternate segment)}$$

A2 for correct answer  
with correct reason

- (c) Explain why  $BA$  is parallel to  $CX$ .

Give a reason for each step of your working.

[2]

Answer

Method 1

$$\angle CDA = 180^\circ - 33^\circ - 50^\circ \text{ (}\angle\text{s on a st. line)}$$

$$= 97^\circ$$

$$\angle CBA = 180^\circ - 97^\circ \text{ (}\angle\text{s in opposite segments)}$$

$$= 83^\circ$$

$$\text{Since } \angle XCB + \angle CBA = 97^\circ + 83^\circ \text{ (int } \angle\text{s)}$$

$$= 180^\circ$$

$$\therefore BA \text{ is parallel to } CX$$

M1 for correct angle  
A1 for showing correct proof

Method 2

$$\angle CBA = 180^\circ - 50^\circ - 33^\circ \text{ (ext } \angle \text{ of cyclic quad)}$$

$$= 83^\circ$$

$$\text{Since } \angle XCB + \angle CBA = 97^\circ + 83^\circ \text{ (int } \angle\text{s)}$$

$$= 180^\circ$$

$$\therefore BA \text{ is parallel to } CX$$

M1 for correct angle  
A1 for showing correct proof

- (d) Hence, or otherwise, find angle  $AXD$ . State the reasons clearly.

[3]

$$\angle DAB = 180^\circ - 97^\circ \text{ (}\angle\text{s in opposite segments)}$$

$$= 83^\circ$$

$$\angle DAO = 83^\circ - 51^\circ$$

$$= 32^\circ$$

M1 for correct angle

$$\angle XAD = 90^\circ - 32^\circ$$

$$= 58^\circ$$

M1 for correct angle

$$\angle AXD = 180^\circ - 58^\circ - 83^\circ \text{ (sum of angles in a triangle)}$$

$$= 39^\circ$$

A1 for correct answer

- 7 The first four terms in a sequence of numbers,  $T_1, T_2, T_3, T_4, \dots$ , are given below.

$$T_1 = 3^2 - 3$$

$$T_2 = 3^4 - 7$$

$$T_3 = 3^6 - 11$$

$$T_4 = 3^8 - 15$$

- (a) Write down an expression for  $T_7$  and evaluate it. [2]

$$T_7 = 3^{14} - 27 \quad \text{M1 for correct expression}$$

$$= 4\,782\,942 \quad \text{A1 for correct value}$$

- (b) Find an expression, in terms of  $n$ , for the  $n^{\text{th}}$  term,  $T_n$  of the sequence. [2]

$$T_n = 3^{2n} - (4n - 1) \quad \text{M1 for either getting } 3^{2n} \text{ or } (4n - 1) \text{ correct}$$

$$= 3^{2n} - 4n + 1 \quad \text{A1 for correct expression}$$

- (c) Find, and simplify, an expression, in terms of  $n$ , for  $T_{n+1} - T_n$ . [2]

$$T_{n+1} - T_n = 3^{2(n+1)} - 4(n+1) + 1 - (3^{2n} - 4n + 1) \quad \text{M1 for correct substitution}$$

$$= 3^2 (3^{2n}) - 4n - 4 + 1 - 3^{2n} + 4n - 1$$

$$= 8(3^{2n}) - 4 \quad \text{A1 for correct expression}$$



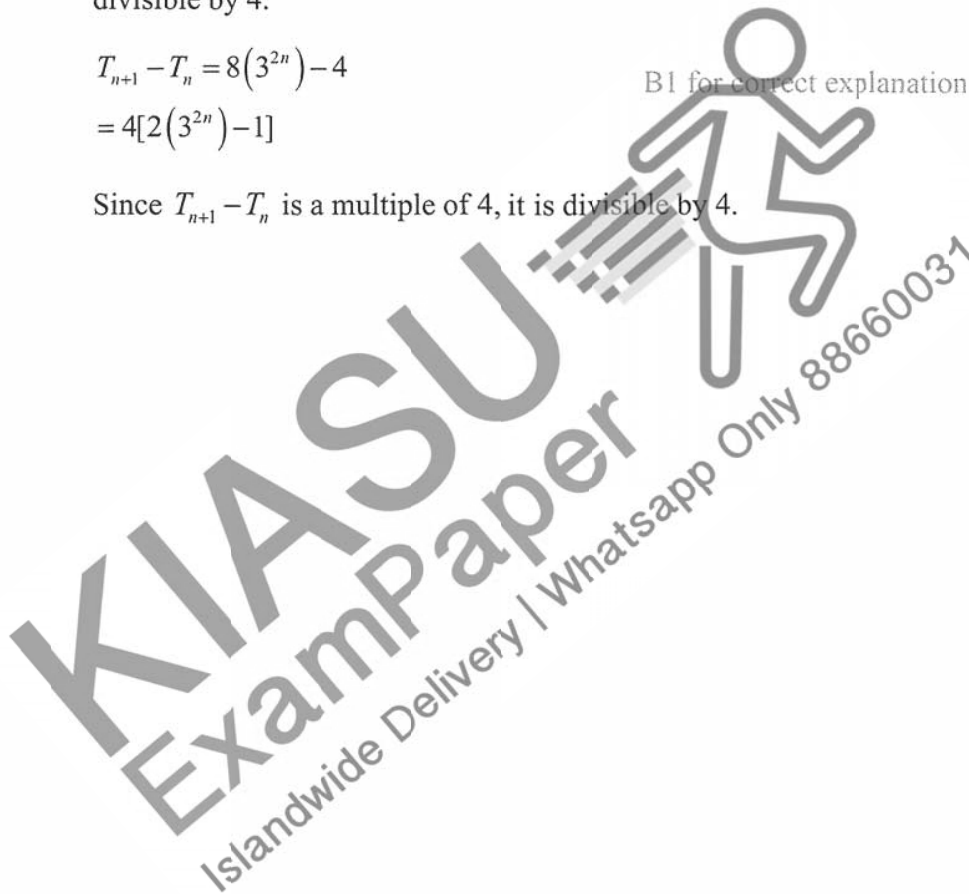
- (d) Hence explain why the difference between the two consecutive terms is always divisible by 4. [1]

$$T_{n+1} - T_n = 8(3^{2n}) - 4$$

$$= 4[2(3^{2n}) - 1]$$

B1 for correct explanation

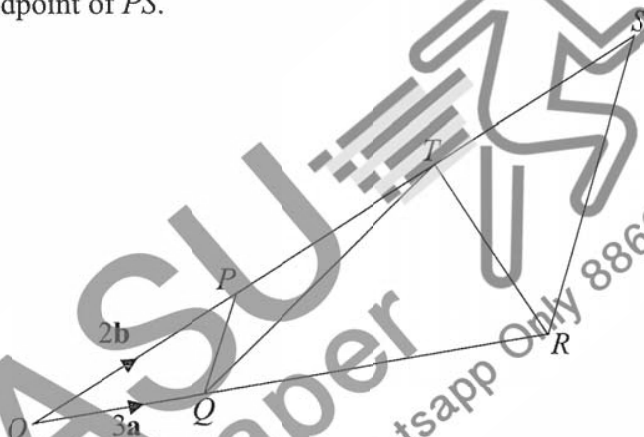
Since  $T_{n+1} - T_n$  is a multiple of 4, it is divisible by 4.



- 8 (a) In the diagram,  $ORS$  is a triangle where  $Q$  is a point on  $OR$ .  
 $P$  and  $T$  are points on  $OS$ .

$$\overrightarrow{OQ} = 3\mathbf{a}, \overrightarrow{OP} = 2\mathbf{b}, \overrightarrow{TQ} = 3\mathbf{a} - 4\mathbf{b}, \overrightarrow{TR} = 9\mathbf{a} - 4\mathbf{b}.$$

$T$  is the midpoint of  $PS$ .



Express and simplify your answers in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ,

- (i)  $\overrightarrow{PQ}$ , [1]

$$\overrightarrow{PQ} = 3\mathbf{a} - 2\mathbf{b}$$

B1 for correct answer

- (ii)  $\overrightarrow{QR}$ , [2]

$$\overrightarrow{QR} = \overrightarrow{QT} + \overrightarrow{TR}$$

$$= -3\mathbf{a} + 4\mathbf{b} + 9\mathbf{a} - 4\mathbf{b}$$

M1 for correct working

$$= 6\mathbf{a}$$

B1 for correct answer

- (iii)  $\overrightarrow{SR}$ . [2]

$$\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$$

$$= 3\mathbf{a} - 2\mathbf{b} - 3\mathbf{a} + 4\mathbf{b}$$

$$= 2\mathbf{b}$$

M1 for correct working

$$\overrightarrow{SR} = \overrightarrow{SO} + \overrightarrow{OR}$$

$$= 9\mathbf{a} - 6\mathbf{b}$$

$$= 3(3\mathbf{a} - 2\mathbf{b})$$

A1 for correct answer

- (iv) Show that triangles  $OPQ$  and  $OSR$  are similar.  
Give a reason for each statement you make.

[2]

$$\overrightarrow{SR} = 3\overrightarrow{PQ}$$

$\therefore SR$  is parallel to  $PQ$

M1 for correct working

$$\angle OPQ = \angle OSR \text{ (corresponding angles)}$$

Any 2

$$\angle OQP = \angle ORS \text{ (corresponding angles)}$$

$$\angle POQ = \angle SOR \text{ (common angle)}$$

A1 for correct answer

$\therefore \triangle OPQ$  is similar to  $\triangle OSR$

Or any reasonable similar triangles test

- (v) Find the ratio of the area of triangle  $OPQ$  to the area of triangle  $TSR$ .

[2]

$$\frac{\text{Area of } \triangle OPQ}{\text{Area of } \triangle TPQ} = \frac{1}{2}$$

$$\frac{\text{Area of } \triangle OQT}{\text{Area of } \triangle ORT} = \frac{1}{2}$$

$$\frac{\text{Area of } \triangle OTR}{\text{Area of } \triangle TRS} = \frac{2}{1}$$

$$\frac{\text{Area of } \triangle OPQ}{\text{Area of } \triangle TSR} = \frac{1}{3}$$

M1 for any correct ratio of areas

$$\therefore \text{ratio is } 1:3$$

A1 for correct answer

(b) It is given that  $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ,  $\overrightarrow{QR} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$  and  $\overrightarrow{RS} = \begin{pmatrix} k \\ 7.5 \end{pmatrix}$ .

(i) The coordinates of  $R$  are  $(-3, 2)$ . Find the coordinates of  $P$ .

[2]

Method 1

Method 2

$$\overrightarrow{OR} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$$

$$= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$$

$$\begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \overrightarrow{OP}$$

$$\overrightarrow{OP} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$$

$$\begin{pmatrix} -3 \\ 2 \end{pmatrix} - \overrightarrow{OP} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

M1 for correct working

M1 for correct working

Coordinates of  $P = (-1, 3)$  A1 for correct answer

(ii) Given that  $\overrightarrow{RS}$  is parallel to  $\overrightarrow{PQ}$ , find the value of  $k$ .

[2]

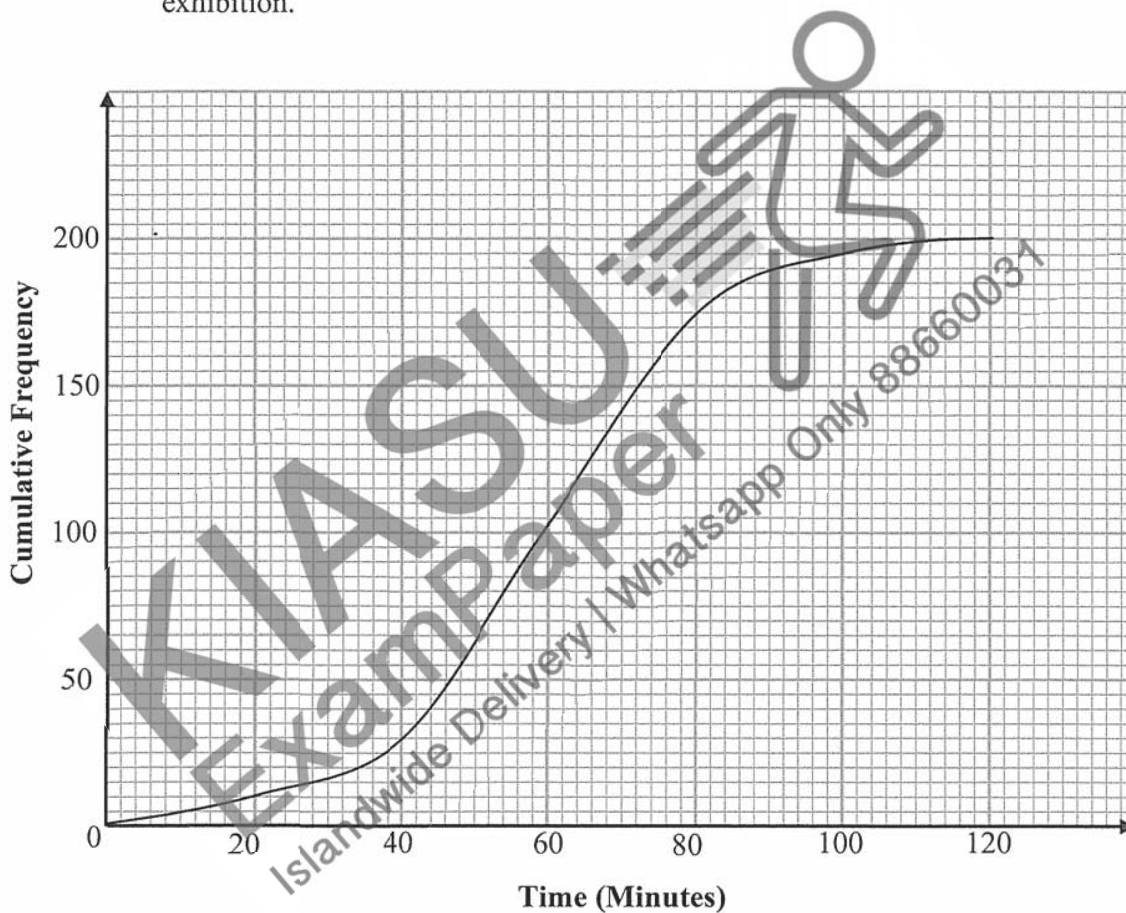
$$\frac{-3}{2} = \frac{7.5}{k}$$

$$k = -5$$

M1 for correct working

A1 for correct answer

- 9 (a) The cumulative graph shows the time (in minutes) spent by 200 visitors at an exhibition.



Use the graph to estimate

- (i) the median time, [1]

59 minutes B1 for correct answer

- (ii) the interquartile range, [2]

Interquartile range =  $72 - 47$  M1 for correct working  
 $= 25$  mins A1 for correct answer

- (iii) the value of  $T$  if 45% of the visitors spent more than  $T$  minutes at the exhibition. [2]

No. of visitors who spent  $T$  mins or less =  $\frac{55}{100} \times 200$   
 $= 110$  M1 for correct working  
 $T = 62$  A1 for correct answer

- (b) The organiser of the exhibition set up two boxes at the lucky draw booth. All the balls in the boxes are identical except colour. Box *A* contains 15 coloured balls of which 8 are yellow, 5 are red and remaining balls are blue. Box *B* contains 8 yellow and 4 red balls. Visitors first draw a ball randomly from Box *A* and place in Box *B*. A second ball is then drawn from Box *B*. Below is the table for the prizes.

Two blue balls	Prize X
Two red balls	Prize Y
Two yellow balls	Prize Z
Two different coloured balls	Consolation prize

Find, as a fraction in its simplest form, the probability that a visitor wins

- (i) prize X, Y or Z. [2]

$$P(X, Y \text{ or } Z) = \frac{2}{15} \times \frac{1}{13} + \frac{5}{15} \times \frac{5}{13} + \frac{8}{15} \times \frac{9}{13}$$

M1 for correct working

$$= \frac{33}{65}$$

A1 for correct answer

- (ii) a consolation prize. [2]

$$P(\text{consolation}) = 1 - \frac{33}{65}$$

M1 for correct working

$$= \frac{32}{65}$$

A1 for correct answer

- (iii) The organiser wants most of the visitors to win the consolation prize. Do you agree with the current setup? Explain. [2]

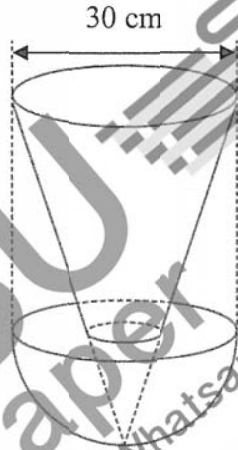
$$P(\text{consolation}) < P(X, Y \text{ or } Z)$$

The setup is wrong as more people will win one of the top prizes instead of consolation prizes.

M1 for correct comparison of probability

A1 for any acceptable conclusion

- 10 Stephen came across beautiful concrete garden ornaments during his overseas trip recently. He decided to make his own concrete ornaments based on the dimensions below. The base diameter of a solid cone is 30 cm. A horizontal cut is made such that a smaller cone is removed. The remaining part of the cone is then attached to a hemisphere.



- (a) The curved surface area of the entire solid cone is 12.25 times of the curved surface area of the smaller cone that is removed. Show that the height of the remaining part of the cone is 37.5 cm. [2]

Let the height of the remaining part of the cone be  $h$  cm.

$$\frac{\text{Curved surface area of big cone}}{\text{Curved surface area of small cone}} = 12.25$$

$$\left(\frac{h+15}{15}\right)^2 = 12.25 \quad (\text{M1})$$

$$\frac{h+15}{15} = 3.5$$

$$h+15 = 52.5$$

$$h = 37.5 \quad (\text{A1})$$

- (b) Show that the volume of the concrete ornament is  $19\,200\text{ cm}^3$ , correct to the nearest 3 significant figures. [2]

$$\frac{r}{15} = \frac{15}{15 + 37.5}$$

$$r = \frac{30}{7}$$

$$= 4\frac{2}{7}\text{ cm}$$

$$\begin{aligned}\text{Volume} &= \frac{2}{3}\pi(15)^3 + \left(\frac{1}{3}\right)\pi(15)^2(37.5 + 15) - \left(\frac{1}{3}\right)\pi\left(\frac{30}{7}\right)^2(15) \\ &= 19150.1 \\ &\approx 19200\text{ cm}^3\end{aligned}$$

M1 correct working

A1 for correct answer



Through his research, he found the following information.

- 1 m<sup>3</sup> of concrete weight around 2.4 metric ton (2400 kg)
- Typically, 1 m<sup>3</sup> of concrete is made up of about 350 kg cement, 700 kg sand, 1 200 kg aggregate\* and about 150 litres of water

The mass and cost (without 7% GST) per packet are shown in the table below.

Materials are sold in the following quantities only.

Item	Mass (kg) per packet	Price without 7% GST per packet
Cement	20	\$5.40
	50	\$7.80
Sand	4	\$5.86
Aggregate	4.5	\$16.80

Item	Volume (litres)**	Price without 7% GST
Water	1 000	\$1.20

\* Aggregates are composed of geological materials such as gravel, sand and crushed rock

\*\* 1 litre of water is equivalent to 1 kg

- (c) He would like to produce four of the concrete ornaments showed. He has a budget of \$500. The GST is 7%. Assuming there is no other cost incurred, comment if he can achieve his plan. Show all essential workings. [6]

$$\text{Volume of 1 ornament} = 19150.1 \text{ cm}^3 = \frac{19150.1}{1000000} = 0.0191501 \text{ m}^3$$

$$\text{Volume of 4 ornaments} = 0.0191501 \times 4 = 0.0766004 \text{ m}^3 \quad \begin{array}{l} \text{1 mark for correct} \\ \text{amount of concrete} \end{array}$$

	Concrete	Sand	Aggregate
Mass	$0.0766004 \times 350$ $= 26.81 \text{ kg}$	$0.0766004 \times 700$ $= 53.62 \text{ kg}$	$0.0766004 \times 1200$ $= 91.92 \text{ kg}$
Number of packets	1 pack of 50 kg	$\frac{53.62}{4} = 13.405$ $= 14 \text{ packets}$	$\frac{91.92}{4.5} = 20.427$ $= 21 \text{ packets}$
Cost with GST	$\$7.80 \times 1.07$ $= \$8.35$	$\$5.86 \times 14 \times 1.07$ $= \$87.78$	$\$16.80 \times 21 \times 1.07$ $= \$377.50$

1 mark for correct masses

1 mark for correct packets

1 mark for correct calculation with GST

$$\begin{aligned} \text{Total cost} &= \$8.35 + \$87.78 + \$377.50 + (\$1.20 \times 1.07) \\ &= \$474.91 \end{aligned} \quad \begin{array}{l} \text{1 mark for correct} \\ \text{cost} \end{array}$$

Since  $\$474.91 < \$500$ , he can achieve his plan. 1 mark for correct conclusion

Alternative answer

$$\begin{aligned}\text{Amount of concrete needed} &= 4 \times \frac{19200}{1000000} \times 2400 \\ &= 184.32 \text{ kg}\end{aligned}$$

1 mark for correct amount of concrete

$$\begin{aligned}\text{Amount of cement needed} &= \frac{350}{2400} \times 184.32 \\ &= 26.88 \text{ kg}\end{aligned}$$

$$\begin{aligned}\text{Amount of sand needed} &= \frac{700}{2400} \times 184.32 \\ &= 53.76 \text{ kg}\end{aligned}$$

1 out of 2 marks for 1 correct item

$$\begin{aligned}\text{Amount of aggregate needed} &= \frac{1200}{2400} \times 184.32 \\ &= 92.16 \text{ kg}\end{aligned}$$

2 out of 2 marks for all correct items

$$\begin{aligned}\text{Total cost} &= [\$7.80 + (14 \times \$5.86) + (21 \times \$16.80) + (\$1.20)] \times 1.07 \\ &= [\$7.80 + \$82.04 + \$352.80 + \$1.20] \times 1.07 \\ &= \$474.91\end{aligned}$$

1 mark for correct quantity needed

1 mark for correct cost

Since  $\$474.91 < \$500$ , he can achieve his plan.

1 mark for correct final answer

**End Of Paper**