中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

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PRELIMINARY EXAMINATION 2022
SECONDARY 4

MATHEMATICS   **4048/02**

Paper 2   **Wednesday 31 August 2022**

                                2 hours 30 minutes

Candidates answer on the Question Paper

Solutions

This document consists of **23** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

[Turn over for Question 1]

[Turn over

1 **(a)** Express as a single fraction in its simplest form $\dfrac{x}{6} - \dfrac{3(x-1)}{8}$.

$\dfrac{x}{6} - \dfrac{3(x-1)}{8}$

$= \dfrac{4x}{24} - \dfrac{9(x-1)}{24}$

$= \dfrac{4x - 9x + 9}{24}$

$= \dfrac{9 - 5x}{24}$

Answer ....$\dfrac{9-5x}{24}$................ [2]

(b) Simplify $\left(\dfrac{27b^6}{w^9}\right)^{-\frac{1}{3}}$.

$\left(\dfrac{27b^6}{w^9}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{w^9}{27b^6}\right)^{\frac{1}{3}}$

$= \dfrac{w^3}{27^{\frac{1}{3}} b^2}$

$= \dfrac{w^3}{3b^2}$

Answer ....$\dfrac{w^3}{3b^2}$................ [2]

(c) Simplify $\dfrac{5p^2 - 12p + 4}{25p^2 - 20p + 4}$.

$\dfrac{5p^2 - 12p + 4}{25p^2 - 20p + 4}$

$= \dfrac{(p-2)(5p-2)}{(5p-2)(5p-2)}$

$= \dfrac{p-2}{5p-2}$

Answer ....$\dfrac{p-2}{5p-2}$................ [3]

(d) Solve the equation $\dfrac{3}{x-2}+\dfrac{1}{2-x}=1$.

<u>Alternative</u>

$$\dfrac{3}{x-2}+\dfrac{1}{2-x}=1$$

$$\dfrac{3}{x-2}-\dfrac{1}{x-2}=1$$

$$\dfrac{2}{x-2}=1$$

$$x-2=2$$

$$x=4$$

$$\dfrac{3}{x-2}+\dfrac{1}{2-x}=1$$

$$\dfrac{3(2-x)+x-2}{(x-2)(2-x)}=1$$

$$6-3x+x-2=-x^2+4x-4$$

$$x^2-6x+8=0$$

$$(x-4)(x-2)=0$$

$$x=4 \ \text{ or } x=2 \ (\text{rej. } \because x \neq 2)$$

$x=4$

Answer   ……………………………………….. [3]

2 *A* is the point $(-5, -2)$ and *B* is the point $(-1, 3)$.

(a) Find the distance between point *A* and point *B*.

$$\text{distance } AB = \sqrt{\left(-5-(-1)\right)^2 + \left(-2-3\right)^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$
$$= 6.40 \text{ units (3 s.f.)}$$

Answer ……………6.40……………….. units [2]

(b) Line *p* passes through point *A* and point *B*.

Show that the equation of line *p* is $4y = 5x + 17$.

Answer

$$\text{gradient} = \frac{3-(-2)}{-1-(-5)}$$
$$= \frac{5}{4}$$

equation:

$$y - 3 = \frac{5}{4}\left(x-(-1)\right)$$
$$y - 3 = \frac{5}{4}(x+1)$$
$$4y - 12 = 5x + 5$$
$$4y = 5x + 17 \text{ (shown)}$$

Alternative

equation:

$$y = \frac{5}{4}x + c$$

sub. $(-1, 3)$

$$3 = \frac{5}{4}(-1) + c$$
$$c = \frac{17}{4}$$
$$y = \frac{5}{4}x + \frac{17}{4}$$
$$4y = 5x + 17 \text{ (shown)}$$

[3]

(c) The equation of line q is $8y - 7x = 43$.

Find the coordinates of the point of intersection of line p and line q.

$$4y = 5x + 17 \text{ --- (1)}$$

$$8y - 7x = 43 \text{ --- (2)}$$

(1) x 2: $8y = 10x + 34$ --- (3)

(3) - (2): $7x = 10x - 9$

$$3x = 9$$

$$x = 3$$

sub. $x = 3$ into (1):

$$4y = 15 + 17$$

$$y = 8$$

∴ the point of intersection is $(3, 8)$.

Answer (…………3………… , …………8…………) [3]

(d) The equation of another line r is $2y - 2.5x = k$, where k is a constant and $k \neq 8.5$. Without solving algebraically, explain why line p and line r will never meet.

Answer

Line r :

$$2y - 2.5x = k \text{ -- (1)}$$

(1) × 2: $4y = 5x + 2k$

gradient of line $r = \dfrac{5}{4}$

$2k \neq 17$

Since line r and line p have the same gradient, they are parallel lines or lines that overlap. As $2k \neq 17$, they do not have the same y-intercept and are parallel lines.

Therefore, line p and line r will never meet.t.

………………………………………………………………………………………… [2]

3 An empty pool has a capacity of 20 000 litres.

Hose *A* can fill the pool at a rate of *x* litres per minute.

Hose *B* can fill the pool at a rate of $(x - 8)$ litres per minute.

(a) Write down an expression, in terms of *x*, for the number of minutes it would take to fill the pool using hose *A*.

number of minutes to fill pool using hose A $= \dfrac{20000}{x}$ min

Answer ……………………$\dfrac{20000}{x}$………… minutes [1]

(b) Write down an expression, in terms of *x*, for the number of minutes it would take to fill the pool using hose *B*.

number of minutes to fill pool using hose A $= \dfrac{20000}{x-8}$ min

Answer ……………$\dfrac{20000}{x-8}$…………… minutes [1]

(c) It takes 2 hours longer to fill the pool using hose *B* than it does using hose *A*.

Write down an equation to represent this information and show that it reduces to

$$3x^2 - 24x - 4000 = 0.$$

$$\frac{20000}{x-8} - \frac{20000}{x} = 120$$

$$\frac{20000x - 20000(x-8)}{x(x-8)} = 120$$

$$\frac{160000}{x(x-8)} = 120$$

$$120x(x-8) = 160000$$

$$120x^2 - 960x - 160000 = 0$$

$$3x^2 - 24x - 4000 = 0 \text{ (shown)}$$

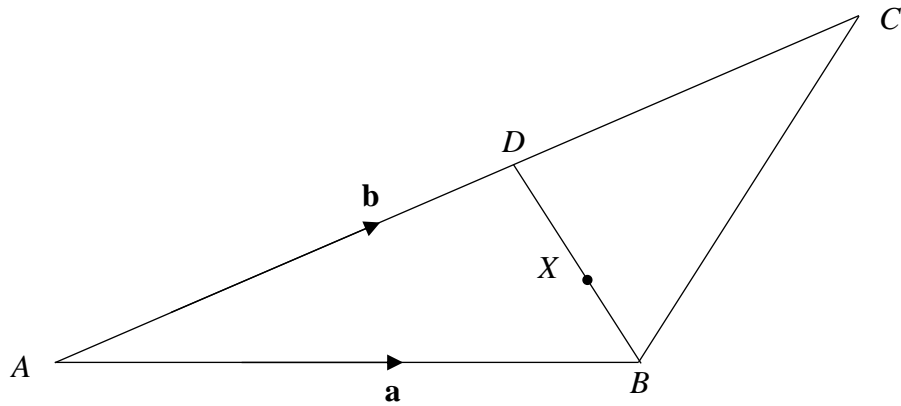**(d)** Solve the equation $3x^2 - 24x - 4000 = 0$, giving your solutions correct to two decimal places.

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(3)(-4000)}}{2(3)}$$

$$= \frac{24 \pm \sqrt{48576}}{6}$$

$= 40.73327...$ or $-32.73327...$

$= 40.73$ (2 d.p.) or -32.73 (2 d.p.)

Answer $x =$ ……40.73……… or ……−32.73……. [3]

(e) Calculate how long it would take to fill the empty pool using hose *A* and hose *B* together.

Give your answer in hours and minutes, correct to the nearest minute.

$$\text{time} = \frac{20000}{x + x - 8}$$

$$= \frac{20000}{2x - 8}$$

$$= \frac{20000}{2(40.73327...) - 8}$$

$= 272.23277...$ mins

$= 4$ h 32 mins (nearest minute)

Answer ……4……hours ……32……minutes [2]

4

ABC is a triangle where D is a point on AC.

$\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AC} = \mathbf{b}$ and $AD : DC = 3 : 2$

X is a point on BD such that $BX : XD = 1 : 2$.

(a) Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{CB}$

$$\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$$
$$= \mathbf{a} - \mathbf{b}$$

Answer ……………………$\mathbf{a} - \mathbf{b}$……………………….. [1]

(ii) $\overrightarrow{DB}$

$$\overrightarrow{DB} = \overrightarrow{AB} - \overrightarrow{AD}$$
$$= \mathbf{a} - \frac{3}{5}\mathbf{b}$$

Answer ……………………$\mathbf{a} - \dfrac{3}{5}\mathbf{b}$……………………….. [2]

(iii) $\overrightarrow{CX}$

$$\overrightarrow{CX} = \overrightarrow{CD} + \overrightarrow{DX}$$
$$= -\frac{2}{5}\mathbf{b} + \frac{2}{3}\overrightarrow{DB}$$
$$= -\frac{2}{5}\mathbf{b} + \frac{2}{3}\left(\mathbf{a} - \frac{3}{5}\mathbf{b}\right)$$
$$= -\frac{2}{5}\mathbf{b} + \frac{2}{3}\mathbf{a} - \frac{2}{5}\mathbf{b}$$
$$= \frac{2}{3}\mathbf{a} - \frac{4}{5}\mathbf{b}$$

Answer ……………………$\dfrac{2}{3}\mathbf{a} - \dfrac{4}{5}\mathbf{b}$……………………….. [2]

(b) *Y* is a point on *AB* such that *ADXY* is a trapezium.

(i) Explain why $\overrightarrow{XY}$ is not parallel to $\overrightarrow{CX}$.

Since *XY* is parallel to *AC*, $\overrightarrow{XY} = k\,\mathbf{b}$, where *k* is a constant.

Hence, $\overrightarrow{XY} \neq m\overrightarrow{CX}$, where *m* is a constant . $\overrightarrow{XY}$ is not parallel to $\overrightarrow{CX}$.

.... [1]

(ii) Prove that triangles *BXY* and *BDA* are similar.

Give a reason for each statement you make.

$\angle BXY = \angle BDA$ (corresponding angles, *XY*//*DA*)

$\angle YBX = \angle ABD$ (common angle)

By *AA* similarity test, triangles *BXY* and *BDA* are similar.

....

.... [3]

(iii) Find the ratio of the areas triangle *BXY* : trapezium *ADXY*.

$$\frac{\text{area of triangle } BXY}{\text{area of triangle } BDA} = \left(\frac{1}{3}\right)^2$$
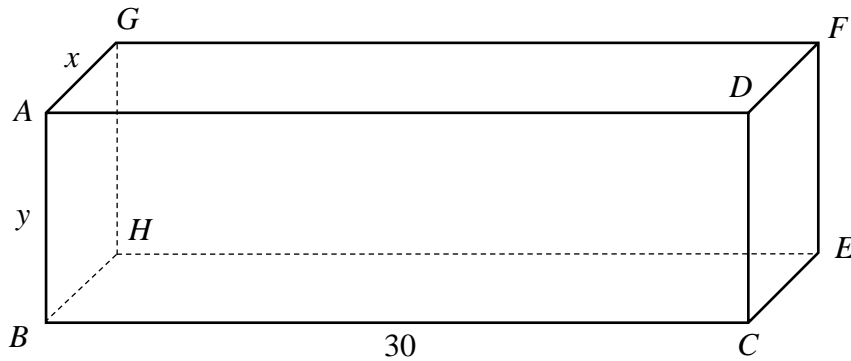
$$= \frac{1}{9}$$

$$\frac{\text{area of triangle } BXY}{\text{area of trapezium } ADXY} = \frac{1}{8}$$

Answer ....1:8.. [2]

5

The diagram shows a cuboid of length 30 cm.

The width of the cuboid is x cm.

The height of the cuboid is y cm.

The surface area of the cuboid is 1200 cm².

(a) Find an expression, in terms of x, for y.

$$\text{Surface area} = 1200$$

$$2xy + 2 \times 30x + 2 \times 30y = 1200$$
$$xy + 30x + 30y = 600$$
$$.\ y(x+30) = 600 - 30x$$
$$y = \frac{600 - 30x}{x + 30}$$

Answer $\qquad y = \dfrac{600 - 30x}{x + 30}$ .... [3]

(b) Show that the volume of the cuboid, V cm³, is given by $V = \dfrac{18000x - 900x^2}{x + 30}$.

Answer

$$\text{volume} = 30xy$$
$$= 30x\left(\frac{600 - 30x}{x + 30}\right)$$
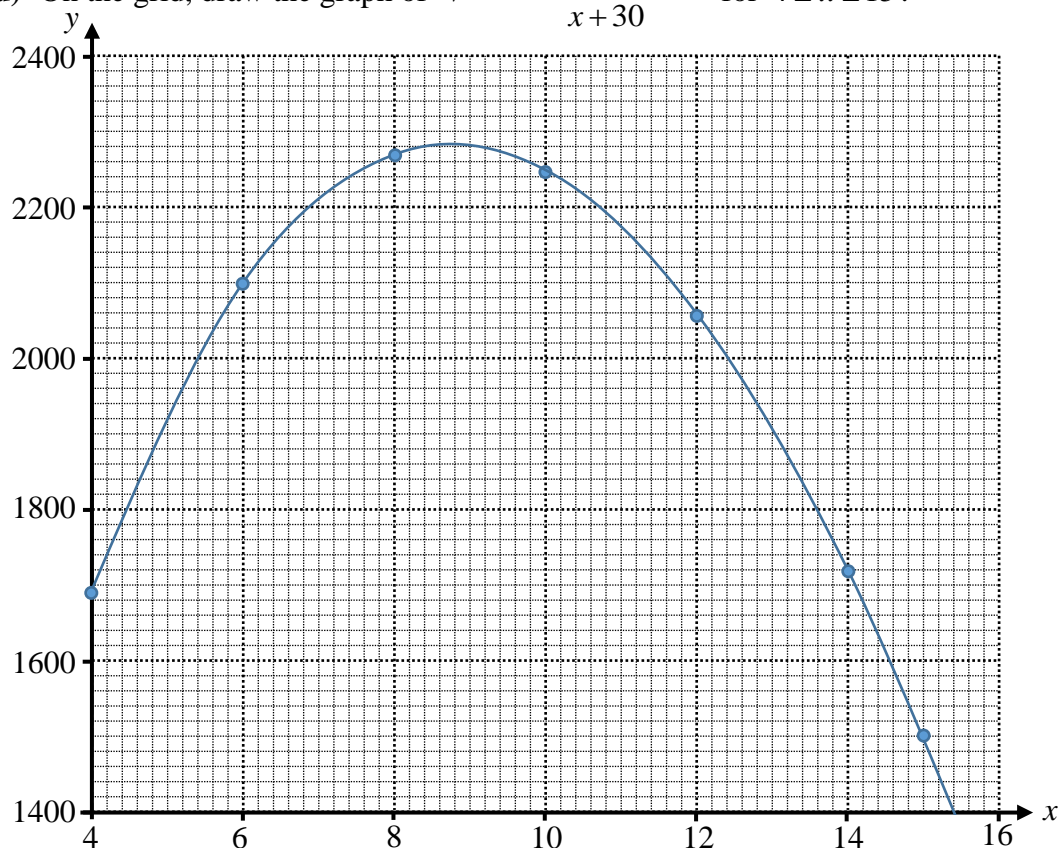$$= \frac{18000x - 900x^2}{x + 30} \text{ (shown)}$$

[2]

(c) Complete the table of values for $V = \dfrac{18000x - 900x^2}{x + 30}$.

Values are given to the nearest whole number.

x	4	6	8	10	12	14	15
V	1694	2100	2274	2250	2057	1718	1500

[1]

(d) On the grid, draw the graph of $V = \dfrac{18000x - 900x^2}{x + 30}$ for $4 \le x \le 15$.

[3]

(e) Use your graph to find the greatest value of x when the volume is equal to 2000 cm^3.

Answer $x = $ ………………………………… [1]
                                        12.4 ± 0.2

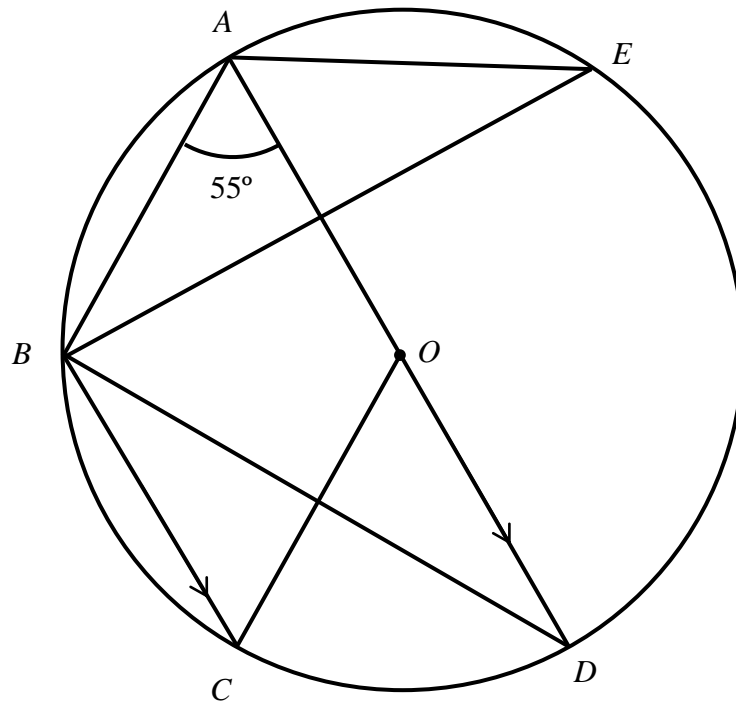(f) Explain how the graph shows that there is no solution to the equation
$$18000x - 900x^2 = 2400x + 72000 .$$

$$18000x - 900x^2 = 2400(x + 30)$$

$$\frac{18000x - 900x^2}{x + 30} = 2400$$

…………Since the maximum volume from the graph is about 2290 cm^3, the volume………… [2]

cannot be equal to 2400 cm^3. There is no solution to the equation.

6 **(a)**

In the diagram, *O* is the centre of the circle. Angle *BAO* = 55º. *BC* is parallel to *OD*. *AD* is a straight line passing through *O*.

Find, giving reasons for each answer,

(i) angle *ABD*,

$$\angle ABD = 90° \text{ (right angle in semicircle)}$$

Answer ....………………………………90…………….° [1]

(ii) angle *AEB*,

$$\angle ADB = 180° - 90° - 55° (\angle \text{ sum of triangle})$$
$$= 35°$$
$$\angle AEB = \angle ADB \ (\angle s \text{ in the same segment})$$
$$= 35°$$

Answer ....………………………………35…………….º [2]

(iii) angle *AOC*.

$$\angle CBD = \angle ODB \text{ (alternate } \angle s, OD // BC)$$
$$= 35°$$
$$\angle COD = 2 \times \angle CBD \text{ (}\angle \text{ at centre} = 2 \times \angle \text{ at circumference)}$$
$$= 70°$$
$$\angle AOC = 180° - 70° \text{(adjacent } \angle s \text{ on a straight line)}$$
$$= 110°$$

Answer ...……………………….…………… 110 .° [3]

(b)

The diagram shows a sector *DAC* of the circle, centre *D*. Angle *ADC* = *θ* radians . *ABC* is a semi-circular arc, centre *O*, with radius *r* cm. *ACD* is an equilateral triangle.

Calculate the percentage of the diagram that is shaded.

area of semicircle $= \dfrac{1}{2}\pi r^2$

area of sector $ACD = \dfrac{1}{2}(2r)^2\left(\dfrac{\pi}{3}\right) = \dfrac{2\pi}{3}r^2$

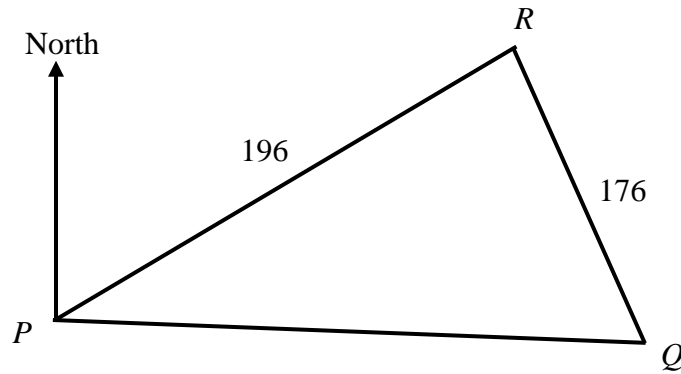area of triangle $ACD = \dfrac{1}{2}(2r)^2 \sin\left(\dfrac{\pi}{3}\right)$
$$= \sqrt{3}r^2$$

$\dfrac{\text{shaded area}}{\text{total area}} \times 100\% = \dfrac{\dfrac{1}{2}\pi r^2 - \left[\dfrac{2\pi}{3}r^2 - \sqrt{3}r^2\right]}{\dfrac{1}{2}\pi r^2 + \sqrt{3}r^2} \times 100\%$

$= \dfrac{\dfrac{1}{2}\pi - \left(\dfrac{2\pi}{3} - \sqrt{3}\right)}{\dfrac{1}{2}\pi + \sqrt{3}} \times 100\%$

Answer ...…………………………………… 36.6 ………% [5]

$= 36.6\% \text{ (3 s.f.)}$

7

North

R

196

176

P

Q

The diagram shows the positions of three buildings *P*, *Q* and *R* in a school.

R is 196 m from *P* on a bearing of 048º.

Q is 176 m from *R* on a bearing of 155º.

(a) Show that *PQ* = 222 m, correct to 3 significant figures.

Answer

$$\angle PRQ = 48° + (180° - 155°)$$
$$= 73°$$
$$PQ^2 = 196^2 + 176^2 - 2(196)(176)\cos 73°$$
$$= 49220.69135...$$
$$PQ = 221.85736...$$
$$= 222 \text{ m (3 s.f.) (shown)}$$

[4]

(b) Calculate the bearing of *P* from *Q*.

$$\frac{\sin \angle PQR}{196} = \frac{\sin \angle PRQ}{PQ}$$
$$\sin \angle PQR = \frac{196 \sin 73°}{221.85736...}$$
$$\angle PQR = 57.65564...°$$

Bearing of P from Q $= 360° - \angle PQR - \angle RQN$ (angles at a point)
$$= 360° - 57.65564...° - (180° - 155°)$$
$$= 277.34436...°$$
$$= 277.3° \text{ (1 d.p.)}$$

Answer ....277.3........................º [3]

(c) The diagram below shows a scale drawing of the positions of the three buildings *P*, *Q* and *R* in the same school.

Scale: 1 cm represents 20 m

(i) On the diagram, construct the bisector of angle *PQR*. [1]

(ii) A new building, *S*, will be added to the school.

S is equidistant from *Q* and from *R*. By making a suitable construction, show the possible positions of *S* on the diagram. [1]

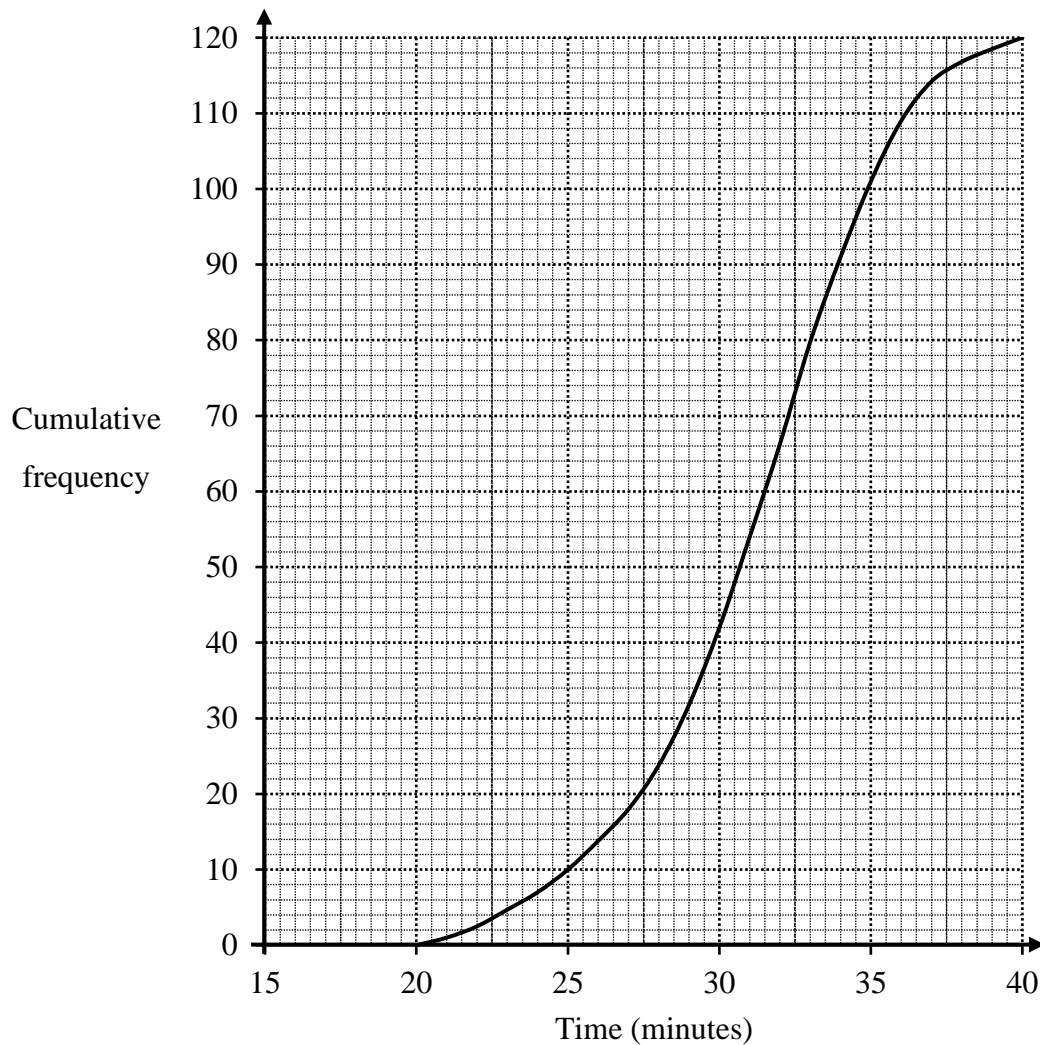**(iii)** Given further that *S* is equidistant from *PQ* and from *QR*, find the actual distance of *S* from *P*.

Distance on graph = 7.1 cm

Actual distance = 142 m

Answer ...…………………………………… m [1]

142

8 **(a)** Marc records his traveling time to school for 120 days.

The cumulative frequency curve summarises the results.

(i) Use the graph to find

(a) the median time,

Median time = 31.5 mins

Answer .... 31.5 .... minutes [1]

(b) the interquartile range of the times,

Q1 = 28.75 mins

Q3 = 34 mins

Interquartile range = 34 – 28.75

= 5.25 mins

Answer .... 5.25 .... minutes [2]

(c) the number of days his journey took more than half an hour.

At $t = 30$ mins, cumulative frequency = 42 days

Number of days it took more than half an hour = 120 − 42

$$= 78$$

Answer …………………………… 78 days [1]

(ii) Marc found a way to cut his traveling time by 5 minutes each day. Describe the effect this change would have on the cumulative frequency curve.

The cumulative frequency curve will shift to the left by 5 minutes.

………………………………………………………………………………………………...

………………………………………………………………………………………….. [1]

(b) There are $(2n + 6)$ blue balls, $(27 − n)$ yellow balls and $(n − 3)$ red balls in a bag.

A ball is drawn at random.

(i) Find, in terms of n, the probability that a blue ball is drawn.

probability of drawing a blue ball $= \dfrac{2n+6}{2n+6+27-n+n-3}$

$$= \dfrac{2n+6}{2n+30}$$

$$= \dfrac{n+3}{n+15}$$

Answer …………………………$\dfrac{n+3}{n+15}$………………….. [1]

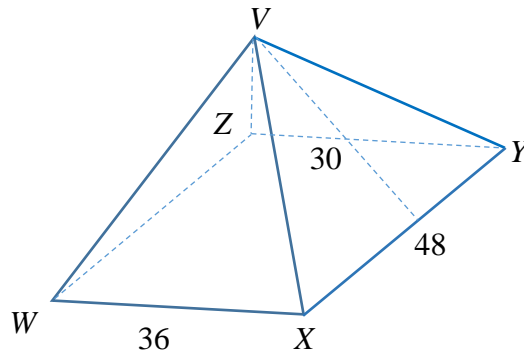**(ii)** The probability that a yellow ball or a red ball is drawn is $\dfrac{6}{13}$.

Find, as a fraction in its simplest form, the probability that a red ball is drawn.

$$1 - \dfrac{n+3}{n+15} = \dfrac{6}{13}$$

$$\dfrac{12}{n+15} = \dfrac{6}{13}$$

$$6n + 90 = 156$$

$$n = 11$$

probability of drawing a red ball $= \dfrac{11-3}{2(11)+30}$

$$= \dfrac{2}{13}$$

Answer …………………………$\dfrac{2}{13}$……………….. [3]

9

The diagram shows a pyramid *VWXYZ*.

The base of the pyramid is a rectangle of sides 36 cm by 48 cm.

V is vertically above the centre of the rectangular base.

The slant height of the pyramid is 30 cm.

(a) Find the vertical height of the pyramid.

$$h^2 + 18^2 = 30^2$$
$$h^2 = 576$$
$$h = 24 \text{ cm}$$

Answer ...……………………24………………… cm [2]

(b) Find the largest angle of elevation of *V* from any point on the sides of the rectangle *WXYZ*.

For the largest angle of elevation, we need the shortest distance from *WXYZ* to the point directly below *V*.

Shortest distance = 18 cm from midpoint of *WZ* or *XY*

$$\tan x = \frac{24}{18}$$
$$x = 53.1301...°$$
$$= 53.1° \text{ (1 d.p.)}$$

Answer ...……………………53.1………………° [2]

(c) A smaller, geometrically similar pyramid of 64 cm³ is removed from the top of the original pyramid. Find the height of the smaller pyramid.

$$\text{volume of pyramid} = \frac{1}{3} \times 36 \times 48 \times 24$$
$$= 13824 \text{ cm}^3$$

$$\frac{\text{volume of smaller pyramid}}{\text{volume of original pyramid}} = \frac{64}{13824}$$

$$\frac{\text{height of smaller pyramid}}{\text{height of original pyramid}} = \sqrt[3]{\frac{64}{13824}}$$
$$= \frac{1}{6}$$

$$\text{height of smaller pyramid} = \frac{1}{6} \times 24$$
$$= 4 \text{ cm}$$

Answer ...…………………………………… 4 cm [3]
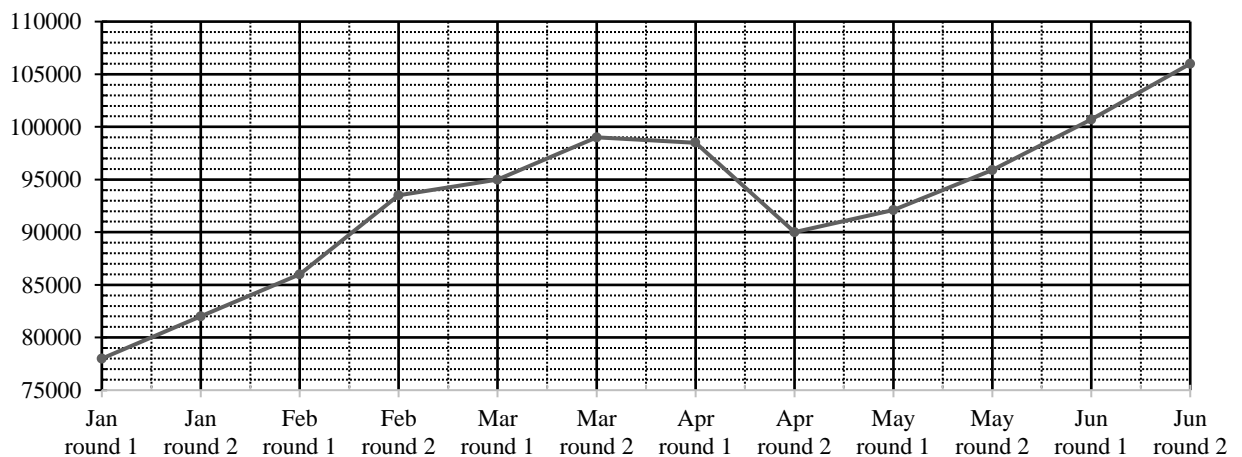
[Turn over

10 Paul intends to buy a car.

He wants to work out how much he needs to pay for his car monthly.

He finds the information below to help him work out the cost of his desired car.

Cost of Desired Car

Components	Cost
Certificate of Entitlement (COE)	Depending on the COE open bidding results
Open Market Value	$16 356
Excise Duty	$3 313
GST of 7% on the combined value of Open Market Value + Excise Duty	$1376.83
Registration Fee	$250
Other Charges	$7 604

COE Open Bidding Results in First Half of Year 2022

(a) Write down the price of the Certificate of Entitlement (COE) for March round 2.

Answer $ ......99 000.......................... [1]

(b) Use the graph to find the percentage increase in price of the COE from the lowest price to the highest price in the first half of year 2022.

Lowest = 78 000, highest = 10 6000

$$\text{percentage increase} = \frac{106000 - 78000}{78000} \times 100\%$$
$$= 35.8974...\%$$
$$= 35.9\% \ (3 \text{ s.f.})$$

Answer ......35.9......................% [3]

(c) Paul wishes to buy the car on hire purchase in July.

He will make a down-payment of one fifth of the cost of the car and then take out a loan on the remaining sum over 7 years at a flat (simple) interest rate of 3% per annum.

Paul predicts that the price of the COE is unlikely to go down in the next round of bidding. He also finds this information about other car expenses that he may incur.

Other car expenses	Cost per month
Road tax	$48.67
Insurance	$300
Petrol	$380
Parking & Electronic Road Pricing (ERP)	$360

Suggest a suitable amount of money he should set aside monthly for a new car.

Justify any decisions you make and show your calculations clearly.

Take the most recent price of COE = $10 6000

Total cost on car = 10 6000 + 16 356 + 3 313 + 1 376.83 + 250 + 7604

$\qquad$ = $13 4899.83

total interest on loan = $\frac{4}{5} \times 134899.83 \times 3\% \times 7$

$\qquad$ = $22 663.17144

Monthly payment for car loan = $\left(22\ 663.17144 + \frac{4}{5} \times 134899.83 \right) \div (7 \times 12)$

$\qquad$ = $1554.559946

Other monthly expenses for car = 48.67 + 300 + 380 +360

$\qquad$ = $1088.67

Total monthly expenditure on car = 1554.559946 + 1088.67

$\qquad$ = $2643. 229946

$\qquad$ = $2643.23 (nearest cent)

Suggested amount to set aside is $2800 as the car expenses such as parking and petrol may be higher on certain months.

[6]

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