

	Register No.	Class
Name :		



**BENDEMEER SECONDARY SCHOOL**  
**2022 PRELIMINARY EXAMINATION**  
**SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)**  
**Elementary Mathematics Paper 1**  
**4048/01**

**DATE : 23 August 2022**  
**DURATION : 2 hours**  
**TOTAL : 80 marks**

<p><b>MARKING SCHEME</b></p>
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**READ THESE INSTRUCTIONS FIRST**

Write your name, class and register number on all the work you hand in.  
 Write in dark blue or black pen on both sides of the paper.  
 You may use a 2B pencil for any diagrams or graphs.  
 Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions on the question booklet unless otherwise stated by the question.  
 All the diagrams in this paper are **not** drawn to scale.  
 If working is needed for any question, it must be shown with the answer.  
 Omission of essential working will result in loss of marks.  
 The use of an approved scientific calculator is expected, where appropriate.  
 If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.  
 For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.  
 The number of marks is given in brackets [ ] at the end of each question or part question.  
 The total number of marks for this paper is 80.

<p><b>FOR EXAMINER'S USE</b></p>
<p><b>80</b></p>

## MATHEMATICAL FORMULAE

### *Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

### *Mensuration*

$$\text{Curved surface area of cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

### *Trigonometry*

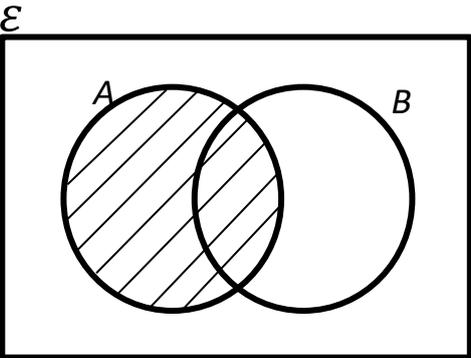
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

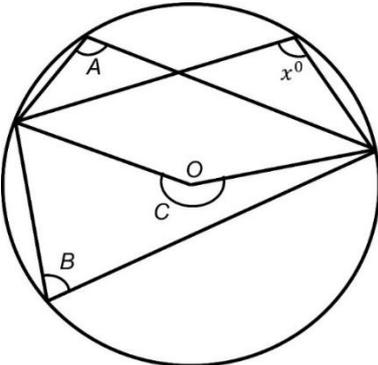
$$a^2 = b^2 + c^2 - 2bc \cos A$$

### *Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

1.	<p>The mass of tracing paper is 45 grams per square metre. What is the mass of an A4-size tracing paper in kg. Give your answer in standard form. [Dimension of A4-size paper is 210 mm by 297 mm]</p> <p>Area of A4  <math>= 0.21 \times 0.297</math>  <math>= 0.06237 \text{ m}^2</math> ..... M1</p> <p>Mass  <math>= 0.06237 \times 0.045</math> ..... M1  <math>= 2.81 \times 10^{-3} \text{ kg}</math> ..... A1</p> <p style="text-align: right;"><i>Answer</i> ..... kg [3]</p>
2.	<p>Two geometrically similar containers have volume 250 ml and 54 ml respectively. Find the ratio of the base area of the bigger container to the base area of the smaller container.</p> <p><math>\frac{L_1}{L_2} = \frac{\sqrt[3]{250}}{\sqrt[3]{54}}</math>  <math>= \frac{5}{3}</math> ..... M1</p> <p><math>\frac{BA_1}{BA_2} = \left(\frac{5}{3}\right)^2</math>  <math>= 25 : 9</math> ..... A1</p> <p style="text-align: right;"><i>Answer</i> ..... [2]</p>
3.	<p>(a) In the Venn diagram below, shade the region(s) represented by the set notation <math>A \cup (B' \cap A)</math>. [1]</p> <div style="text-align: center;">  </div> <p style="text-align: right;">B1</p>
	<p>(b) Given that <math>\varepsilon = \{x: x \text{ is an integer such that } 1 \leq x &lt; 20\}</math>,  <math>G = \{x: x \text{ is an odd number}\}</math> and  <math>T = \{x: x \text{ is a square number}\}</math>.</p> <p>List the elements of <math>(G \cup T)'</math>.</p> <p><math>\{2, 6, 8, 10, 12, 14, 18\}</math> ..... B1</p> <p style="text-align: right;"><i>Answer (b)</i> ..... [1]</p>

4.	A bag contains 8 white balls and 12 black balls. Two balls are drawn from the bag with replacement. Find the probability of drawing	
	<p>(a) 2 white balls,  <math>\frac{8}{20} \times \frac{8}{20} = \frac{4}{25}</math> ..... B1</p>	Answer (a)..... [1]
	<p>(b) 2 balls of different colours,  <math>2 \left( \frac{8}{20} \times \frac{12}{20} \right) = \frac{12}{25}</math> ..... B1</p>	Answer (b)..... [1]
	<p>(c) at least one black ball.  <math>1 - \frac{4}{25} = \frac{21}{25}</math> ..... B1</p>	Answer (c)..... [1]
5.	<p>A bag of sweets is shared among a group of children. If each child gets 7 sweets, there will be 3 sweets left. If each child gets 8 sweets, there is a shortage of 4 sweets.          How many sweets are in the bag and how many children are there in the group?</p> <p>If 3 sweets are left: <math>3+7 = 10, 17, 24, 31, 38, 44, 51</math>          If shortage of 4 sweets: <math>-4+8 = 4, 12, 20, 28, 36, 44, 52</math> } <span style="border: 1px solid black; padding: 2px;">M1 (or other appropriate working)</span></p> <p>Therefore, there are 52 sweets and 7 children ..... A1, A1</p> <p style="text-align: right;">Answer ..... sweets ; ..... children [3]</p>	
6.	<p>In the diagram below, <math>O</math> is the centre of the circle.</p> <div style="text-align: center;">  </div> <p>Stating the properties of circles, write down the values of angles <math>A</math>, <math>B</math> and <math>C</math> in terms of <math>x</math>.</p> <p><math>\angle A = x^\circ</math> ; Property: Angles in the same segment ..... B1</p> <p><math>\angle B = 180^\circ - x^\circ</math> ; Property: Angles in opposite segments ..... B1</p> <p><math>\angle C = 2x^\circ</math> ; Property: Angle at centre = 2 x Angle at circumference ..... B1</p>	

7.	<p>A firm is offering a 10-year investment plan.  For the first seven years, investors are offered a simple interest of 2.5% per annum of their principal amount.  At the end of the seventh year, the total amount is invested for 3 years at 1.8% per annum, compounded half-yearly.  If an investor invested \$200 000, how much would he receive at the end of 10 years?  Give your answer to the nearest dollar.</p> <p>Total amount at the end of 7 years  <math>= 200000 + \left(200000 \times \frac{2.5}{100} \times 7\right)</math> ..... M1  <math>= 235\ 000</math></p> <p>Total amount at the end of 10 years  <math>= 235000 \left(1 + \frac{0.9}{100}\right)^6</math> ..... M1  <math>= 247\ 979</math> (nearest whole number) ..... A1</p> <p style="text-align: right;"><i>Answer</i> \$..... [3]</p>
8.	<p>Solve <math>125 \times \sqrt{5} = 5^{2n} 5^{2n}</math></p> <p><math>5^3 \times 5^{\frac{1}{2}} = 5^{2n}</math> ..... M1  <math>3 + \frac{1}{2} = 2n</math> ..... M1  <math>n = 1\frac{3}{4}</math> ..... A1</p> <p style="text-align: right;"><i>Answer</i> <math>n =</math> ..... [3]</p>
9.	<p><math>R</math> is inversely proportional to the cube of <math>p</math>. When the value of <math>p</math> is halved, the value of <math>R</math> changes by the factor, <math>u</math>. Find <math>u</math>.</p> <p><math>R = \frac{k}{p^3}</math>, where <math>k</math> is a constant ..... M1  <math>k = Rp^3</math></p> <p><math>R_1 = \frac{k}{\left(\frac{p}{2}\right)^3}</math> ..... M1</p> <p><math>R_1 = \frac{Rp^3}{\frac{p^3}{8}}</math></p> <p><math>R_1 = 8R</math></p> <p>Therefore, <math>u = 8</math> ..... A1</p> <p style="text-align: right;"><i>Answer</i> <math>u =</math> ..... [3]</p>

10.

Given  $\begin{pmatrix} 3 & x & 4 \\ -1 & -5 & 0 \end{pmatrix} \begin{pmatrix} y & 2 \\ -2 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 14 & 30 \\ -7 & -2 \end{pmatrix}$ , find the value of  $x$  and  $y$ .

$$\begin{pmatrix} 3y - 2x & 6 + 24 \\ -y + 10 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 30 \\ -7 & -2 \end{pmatrix} \dots\dots\dots \text{M1}$$

$$-y + 10 = -7$$

$$y = 17 \dots\dots\dots \text{A1}$$

$$3y - 2x = 14$$

$$3(17) - 2x = 14$$

$$x = 18\frac{1}{2} \dots\dots\dots \text{A1}$$

*Answer*  $x = \dots\dots\dots$ ,  $y = \dots\dots\dots$  [3]

11.

Solve the inequality  $\frac{3-7x}{2} \leq \frac{4x+5}{3}$  and hence write down the smallest rational number that satisfies the inequality.

$$\frac{3 - 7x}{2} \leq \frac{4x + 5}{3}$$

$$3(3 - 7x) \leq 2(4x + 5) \dots\dots\dots \text{M1}$$

$$9 - 21x \leq 8x + 10$$

$$-1 \leq 29x \dots\dots\dots \text{M1}$$

$$\frac{-1}{29} \leq x \dots\dots\dots \text{A1}$$

$$\text{smallest rational number} = \frac{-1}{29} \dots\dots\dots \text{B1}$$

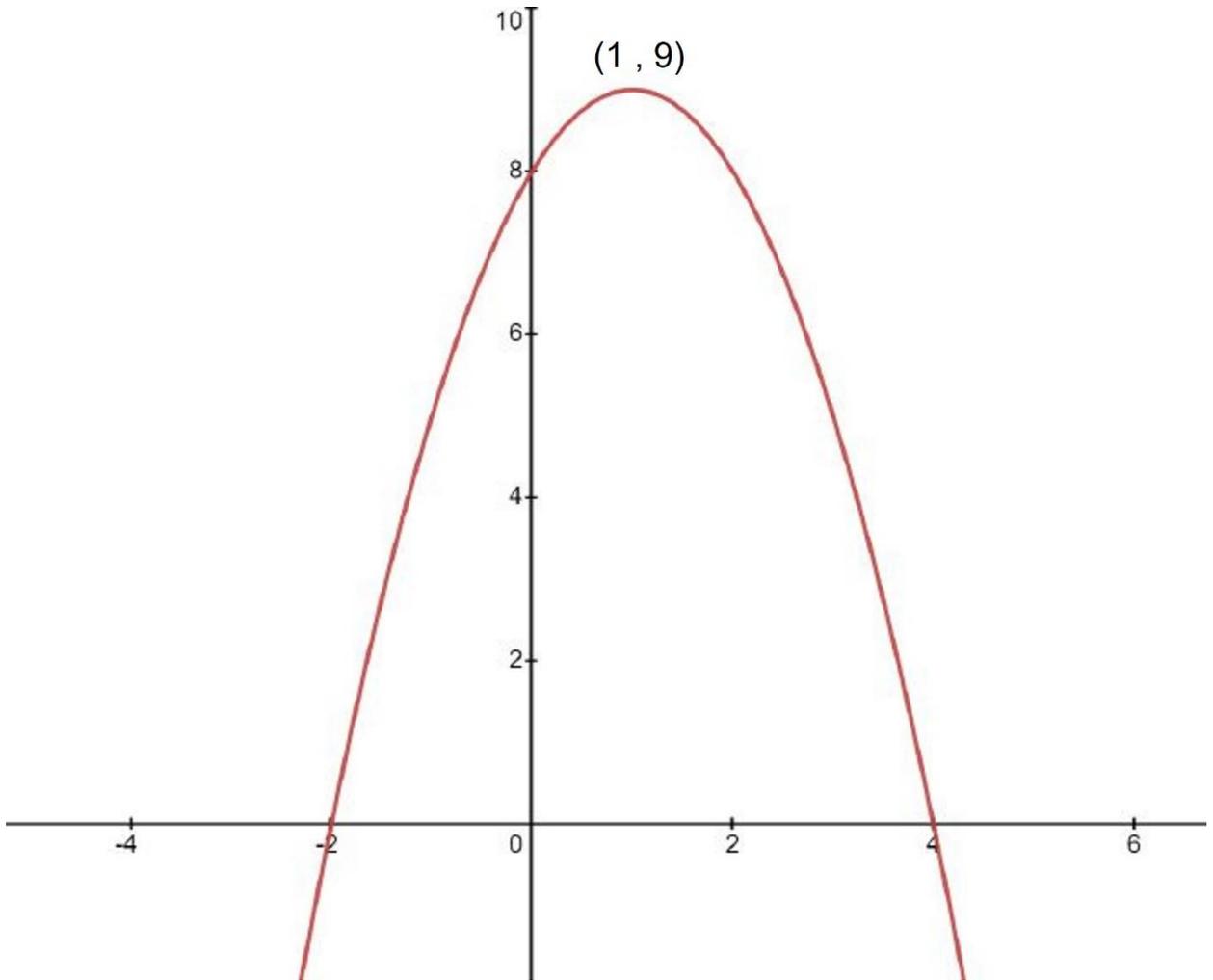
*Answer*  $\dots\dots\dots$  [3]

Smallest rational number =  $\dots\dots\dots$  [1]

<b>12.</b>	Factorise each of the following completely.
	<p>(a) <math>3kh - h - 3k^2 + k</math>  <math>= h(3k - 1) - k(3k - 1)</math> ..... M1  <math>= (3k - 1)(h - k)</math> ..... A1</p> <p style="text-align: right;"><i>Answer (a)</i> ..... [2]</p>
	<p>(b) <math>288x^3 - 8x</math>  <math>= 8x(36x^2 - 1)</math> ..... M1  <math>= 8x(6x - 1)(6x + 1)</math> ..... A1</p> <p style="text-align: right;"><i>Answer (b)</i> ..... [2]</p>
<b>13.</b>	<p>Solve <math>\frac{1}{x+1} - \frac{2x-1}{2} = \frac{3}{2}</math>.</p> <p><math>2 - (2x - 1)(x + 1) = 3(x + 1)</math></p> <p><math>2 - 2x^2 - x + 1 = 3x + 3</math> ..... M1</p> <p><math>2x^2 + 4x = 0</math> ..... M1</p> <p><math>2x(x + 2) = 0</math> ..... M1</p> <p><math>x = 0</math> or <math>x = -2</math> ..... A1</p> <p style="text-align: right;"><i>Answer x =</i> ..... [4]</p>

14. Sketch the graph of  $y = (4 - x)(2 + x)$ .  
State clearly the horizontal and vertical intercepts, and coordinates of the turning point. [3]

Write down the equation of the tangent to the curve where gradient is zero.



Shape ..... B1

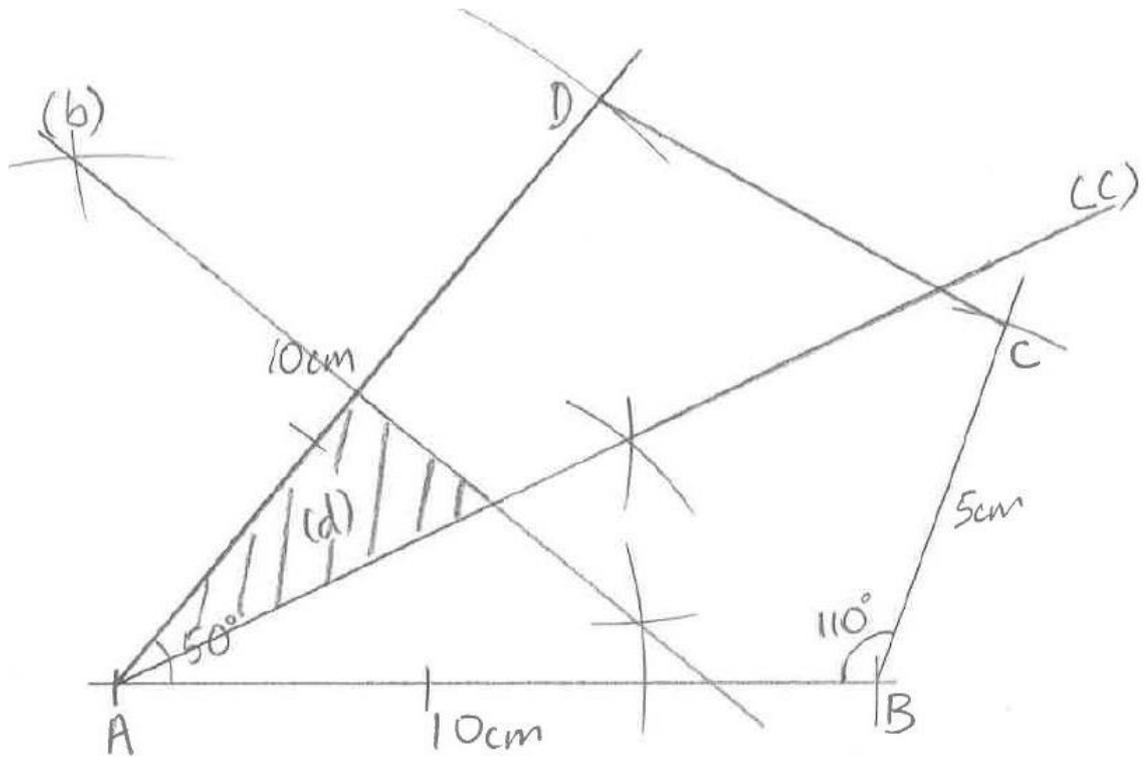
$x$  and  $y$  intercepts ..... B1

Coordinates of turning point ..... B1

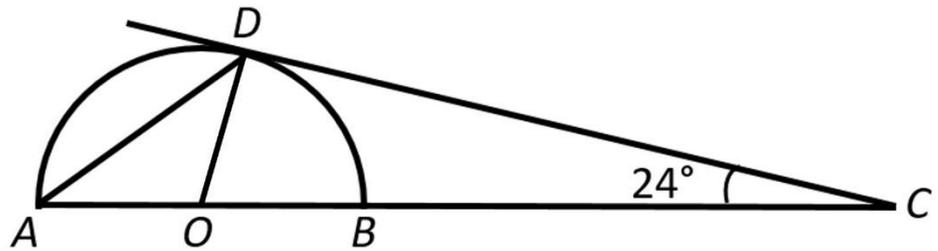
Equation of the tangent to the curve where gradient is zero:  $y = 9$  ..... B1

*Answer* Equation of tangent ..... [1]

15. (a) Construct a quadrilateral such that  $AB = AD = 10$  cm,  $BC = 5$  cm,  $\angle DAB = 50^\circ$  and  $\angle ABC = 110^\circ$ . Measure  $DC$ .
- Construction ..... B1
- $DC = 6.1 \pm 0.1$  cm ..... B1
- Answer (a)  $DC = \dots\dots\dots$  cm [2]
- (b) Construct the perpendicular bisector of  $AD$ . [1]
- (c) Construct the angle bisector of  $\angle DAB$ . [1]
- (d) Hence, shade the region that is nearer to  $A$  than to  $D$ , and nearer to  $AD$  than to  $AB$ . [1]



16. The diagram shows a semicircle with centre,  $O$ .  $DC$  is the tangent to the semicircle at  $D$ .  $BC = 16$  m.



- (a) Find  $\angle DAB$ .

$$\begin{aligned} \angle DOB &= 180^\circ - 90^\circ - 24^\circ \text{ (radius perpendicular to tangent)} \\ &= 66^\circ \dots\dots\dots \text{M1} \end{aligned}$$

$$\begin{aligned} \angle DAB &= 66^\circ \div 2 \text{ (angle at centre = 2 x angle at circumference)} \\ &= 33^\circ \dots\dots\dots \text{A1} \end{aligned}$$

*Answer (a)*  $\angle DAB = \dots\dots\dots^\circ$  [2]

- (b) Find the radius of the semicircle.

$$\sin 24^\circ = \frac{r}{r+16} \dots\dots\dots \text{M1}$$

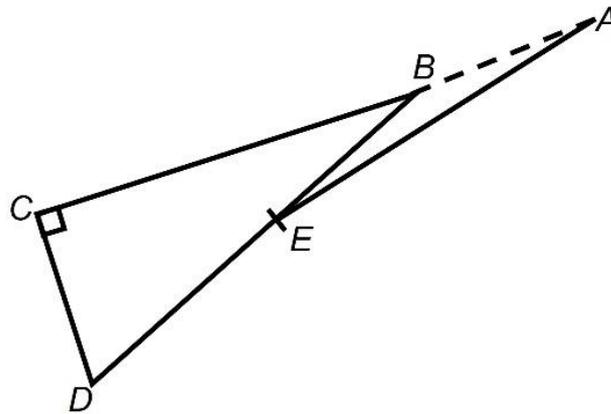
$$\sin 24^\circ(r + 16) = r \dots\dots\dots \text{M1}$$

$$r(1 - \sin 24^\circ) = 16 \sin 24^\circ$$

$$r = 11.0 \text{ m (3sf)} \dots\dots\dots \text{A1}$$

*Answer (b)* radius =  $\dots\dots\dots$  m [3]

17. (a) The diagram shows a right-angled triangle  $BCD$ .  $A$  is a point on  $CB$  produced such that  $BA = BE = 6$  cm.  $CD = 5$  cm and  $CB = 12$  cm.



Giving your answer in fraction, find the value of

(i)  $\sin \angle CBD$ ,

(ii)  $\cos \angle ABD$ .

(i)

$$BD = \sqrt{5^2 + 12^2} = 13 \dots\dots\dots \text{M1}$$

$$\sin \angle CBD = \frac{5}{13} \dots\dots\dots \text{A1}$$

$$\text{(ii) } \cos \angle ABD = -\frac{12}{13} \dots\dots\dots \text{A1}$$

Answer (a)(i) ..... [2]

(b)(ii) ..... [1]

(b) Hence, find the area of  $\triangle BEA$ .

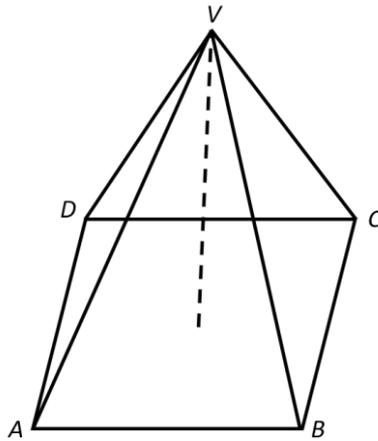
$$\frac{1}{2} \times 6 \times 6 \times \frac{5}{13} \dots\dots\dots \text{M1}$$

$$= 6.92 \text{ cm}^2 \text{ (3sf)} \dots\dots\dots \text{A1}$$

(b) .....  $\text{cm}^2$  [2]

18.	<p><math>P</math> is the point <math>(7, -6)</math> and <math>\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}</math>.</p>
	<p>(a) Find the coordinates of point <math>Q</math>.</p> $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$ $\overrightarrow{OQ} = \begin{pmatrix} 7 \\ -6 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix} \dots\dots\dots \text{M1}$ $\overrightarrow{OQ} = \begin{pmatrix} 12 \\ -7 \end{pmatrix}$ <p>Therefore <math>Q</math> has coordinates <math>(12, -7)</math> ..... A1</p> <p style="text-align: right;"><i>Answer (a)</i> ..... [2]</p>
	<p>(b) Find the magnitude of <math>\overrightarrow{PQ}</math>.</p> $\sqrt{5^2 + (-1)^2}$ $= 5.10 \text{ units (3sf)} \dots\dots\dots \text{B1}$ <p style="text-align: right;"><i>Answer (b)</i> ..... units [1]</p>
	<p>(c) Given that <math>\overrightarrow{QP} = 3\overrightarrow{PR}</math>, find the coordinates of point <math>R</math>.</p> $\begin{pmatrix} -5 \\ 1 \end{pmatrix} = 3(\overrightarrow{OR} - \overrightarrow{OP}) \dots\dots\dots \text{M1}$ $\begin{pmatrix} -5 \\ 1 \end{pmatrix} = 3\overrightarrow{OR} - 3\begin{pmatrix} 7 \\ -6 \end{pmatrix}$ $\begin{pmatrix} -5 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 7 \\ -6 \end{pmatrix} = 3\overrightarrow{OR} \dots\dots\dots \text{M1}$ $3\overrightarrow{OR} = \begin{pmatrix} 16 \\ -17 \end{pmatrix}$ $\overrightarrow{OR} = \begin{pmatrix} \frac{16}{3} \\ -\frac{17}{3} \end{pmatrix}$ <p>Therefore <math>R</math> has coordinates <math>\left(\frac{16}{3}, -\frac{17}{3}\right)</math> ..... A1</p> <p style="text-align: right;"><i>Answer (c)</i> ..... [3]</p>

19.  $VABCD$  is a pyramid with a square base of sides 6cm.  $\Delta VAB, \Delta VBC, \Delta VCD$  and  $\Delta VDA$  are equilateral triangles.



(a) Show that the height of the pyramid is 4.24 cm. [2]

$$DB = \sqrt{6^2 + 6^2} \dots\dots\dots \text{M1}$$

$$\frac{1}{2}DB = 3\sqrt{2} \text{ or } 4.24264 \text{ cm}$$

$$\text{Height} = \sqrt{6^2 - 4.24264^2} \dots\dots\dots \text{M1}$$

$$= 4.24 \text{ cm (shown)}$$

(b) Find the volume of the pyramid.

$$\begin{aligned} \text{Volume} \\ = \frac{1}{3} \times 6^2 \times 4.24 \dots\dots\dots \text{M1} \end{aligned}$$

$$= 50.88 \text{ cm}^3 \text{ (3sf) } \dots\dots\dots \text{A1}$$

*Answer (b)*.....  $\text{cm}^3$  [2]

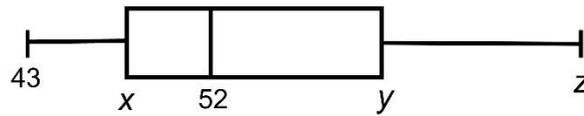
(c) Find the total surface area of the pyramid.

$$\begin{aligned} \text{Total surface area} \\ = 4 \left( \frac{1}{2} \times 6 \times 6 \times \sin 60^\circ \right) + (6 \times 6) \dots\dots\dots \text{M1, M1} \end{aligned}$$

$$= 98.4 \text{ cm}^2 \text{ (3sf) } \dots\dots\dots \text{A1}$$

*Answer (c)*.....  $\text{cm}^2$  [3]

20. (a) The box-and-whisker plot shows the distribution of the mass of 40 students in Sec 3.



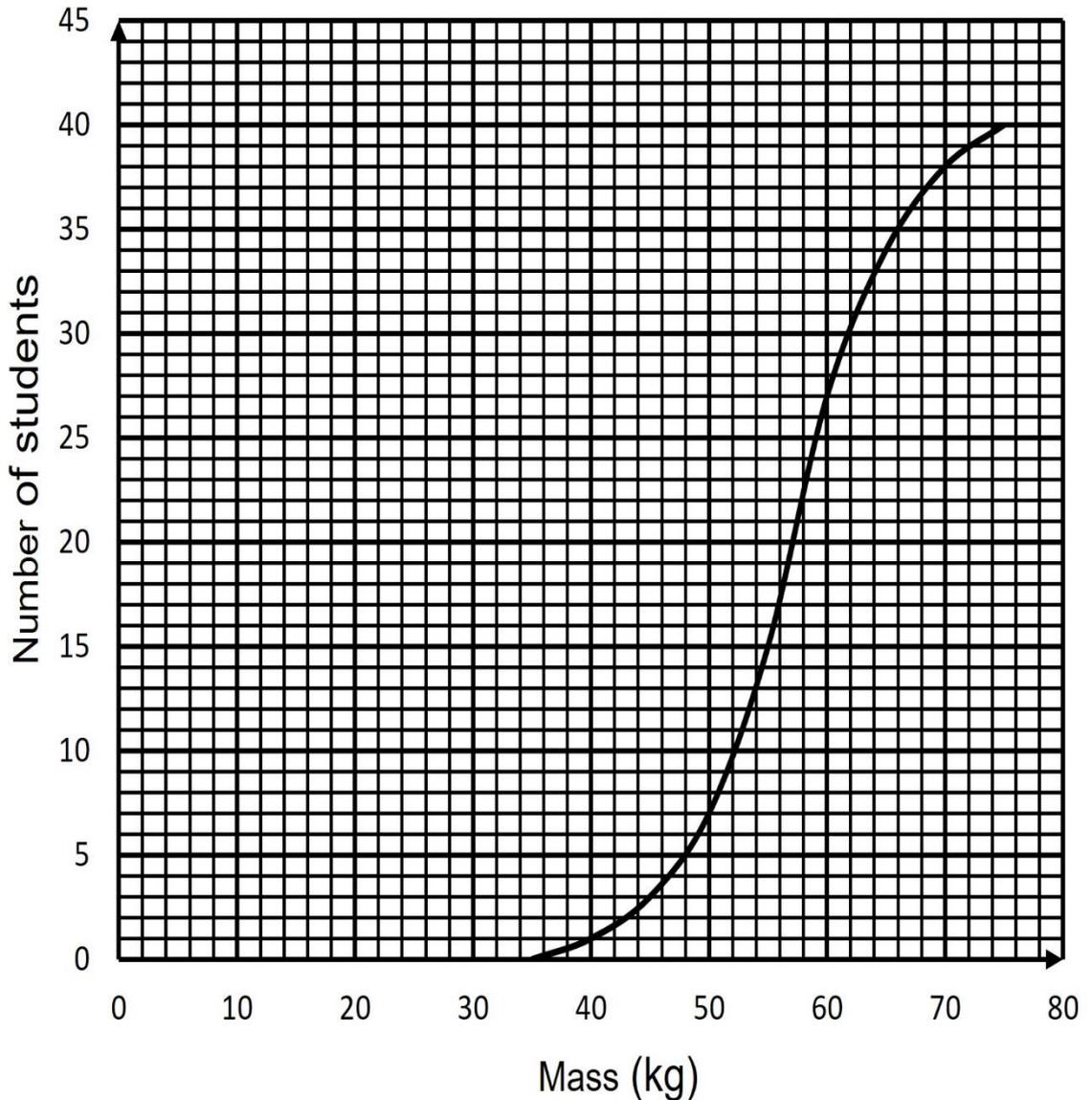
The range of the mass is 33 kg. 25% of the students weigh 47 kg or less and the interquartile range is 18 kg. Find the value of  $x$ ,  $y$  and  $z$ .

$x = 47$  ..... B1  
 $y = 65$  ..... B1  
 $z = 76$  ..... B1

Answer (a)  $x = \dots\dots\dots$ ,  $y = \dots\dots\dots$ ,  $z = \dots\dots\dots$  [3]

(b) The cumulative frequency curve shows the distribution of the mass of 40 students in Sec 4.

Mass of students in Sec 4



		<p>Find the (i) median, 57 kg ..... B1  <i>Answer (b)(i)</i> ..... kg [1]</p> <p>(ii) interquartile range, and 62 – 52 = 10 kg ..... B1  <i>Answer (b)(ii)</i> ..... kg [1]</p> <p>(iii) percentage of students who are above 55kg <math>\frac{40-15}{40} \times 100\% = 62.5\%</math> ..... B1  <i>Answer (b)(ii)</i> ..... % [1]</p>
	(c)	<p>Make two comparisons between the mass of the students in Sec 3 and those in Sec 4. [2]</p> <p><i>Answer (c)</i></p> <p>The <b>median mass</b> of the students in Sec 4 is <b>higher</b> than the median mass of the students in Sec 3. Hence, the students in Sec 4 are generally heavier than the students in Sec 3 ..... B1</p> <p>The <b>spread</b> of the mass of students in Sec 4 is <b>smaller</b> than the spread of the mass of students in Sec 3. Hence, the mass of the students in Sec 4 is more consistent than the mass of the students in Sec 3..... B1</p>

*End of Paper*