

NAME: \_\_\_\_\_ ( )

CLASS: 4 ( )



**ANGLICAN HIGH SCHOOL  
SECONDARY FOUR  
PRELIMINARY EXAMINATIONS 2022**

**S4**

**MATHEMATICS**

Paper 2

**4048/02**

**31 August 2022**

**2 hours 30 minutes**

Candidates answer on the Question Paper and Graph Paper

**READ THESE INSTRUCTIONS FIRST**

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

**For Examiners' Use**

For Examiners Use							
Questions	1	2	3	4	5	6	7
Marks							
Questions	8	9	10	11	12	13	14
Marks							
Table of Penalties	Units					100	
	Clarity/Logic						
	Accuracy/Precision						
Parent's Name and Signature:							
Date:							

This document consists of 23 printed pages and 1 blank page.

***Mathematical Formulae****Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left( \frac{\Sigma fx}{\Sigma f} \right)^2}$$

- 1 (a) Below are the first five terms of a sequence.

$$\frac{1}{17} \quad \frac{4}{25} \quad \frac{9}{33} \quad \frac{16}{41} \quad \frac{25}{49}$$

- (i) Find the sixth term of the sequence.

Answer ..... [1]

- (ii)  $T_n$  is the  $n$ th term of the sequence.  
Find an expression, in terms of  $n$ , for  $T_n$ .

Answer  $T_n =$  ..... [2]

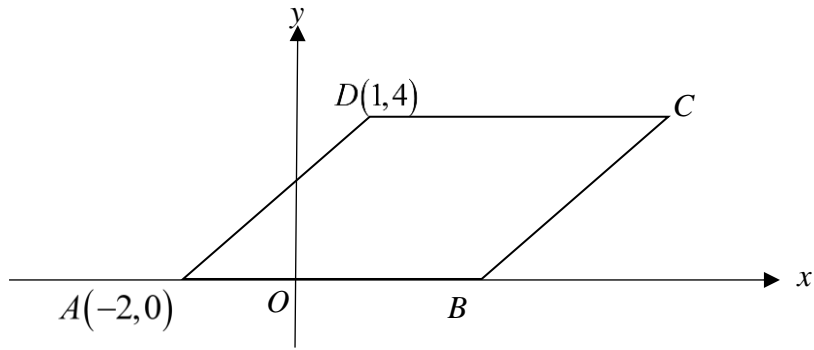
- (iii) The  $k$ th term is 1. Find the value of  $k$ .

Answer  $k =$  ..... [2]

- (b) Ali invested \$7500 in a saving account for 3 years. The rate of compound interest was fixed at  $r\%$  per annum. At the end of 3 years, there was \$8436.48 in his account. Find the value of  $r$ .

Answer  $r =$  ..... [3]

2



The diagram shows a rhombus  $ABCD$ , with sides 5 units, and coordinates  $A(-2, 0)$  and  $D(1, 4)$ .

- (a) Show that the equation of line  $BC$  is  $3y = 4x - 12$ .

*Answer*

[3]

$E$  is a point on  $BC$ , such that  $DE$  is perpendicular to  $AD$ . The gradient of the line  $DE$  is  $-\frac{3}{4}$ .

- (b) Find the coordinates of  $E$ .

*Answer* (..... , ..... ) [3]

- (c) Given that the length of  $BE = 2$  units, hence, find the area of  $ABED$ .

*Answer* ..... units<sup>2</sup> [2]

- 3 (a) A group of 24 tourists visited the National Museum.

One of the 24 tourists is selected at random. The probability that it is a Korean male tourist is  $\frac{1}{3}$ . By showing clear workings, complete the table of information about the 24 tourists.

	Male	Female
Korean		5
Japanese		2

[2]

- (b) Two of the 24 tourists are selected at random. Find the probability that they are both male tourists.

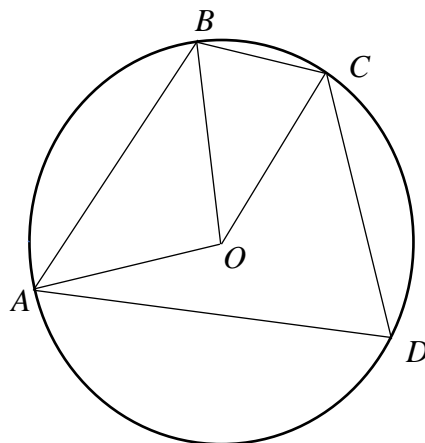
*Answer* ..... [2]

- 4 Terry pays \$15 a month for a season parking of his motorcycle in his workplace. James, also a motorist, uses a different scheme in which he pays a deposit of \$50 and then a monthly payment of \$8.

Let the number of months that both Terry and James have been paying for their motorcycle be  $x$ . By forming an inequality, find the minimum number of months James will have to pay to ensure that his scheme is cheaper than Terry.

Answer ..... [4]

5



In the diagram,  $A$ ,  $B$ ,  $C$  and  $D$  are four points on a circle with centre  $O$ . Angle  $OAB = 2x^\circ$ , angle  $OCB = 3x^\circ$ , and angle  $ADC = 2.5x^\circ$ .

- (a) Express angle  $ABC$  in terms of  $x$ .

Answer angle  $ABC = \dots\dots\dots^\circ$  [2]

- (b) Find the value of  $x$ .

Answer  $x = \dots\dots\dots$  [2]

- 6 (a) Given that  $\frac{5x-3a}{8} - \frac{3x-4a}{6} = 1$ , express  $x$  in terms of  $a$ .

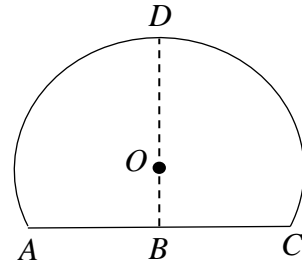
*Answer*  $x = \dots\dots\dots$  [2]

- (b) Find the largest positive integer value of  $a$  if  $x \geq -29$ .

*Answer*  $a = \dots\dots\dots$  [3]

- 7 (a)  $ABCD$  is a major segment of a circle, centre  $O$ .  
 $BD$  is 32 cm,  $AC$  is 48 cm and angle  $DBA$  is  $\frac{\pi}{2}$ .  
 (i) Show that the radius of the circle is 25 cm.

*Answer*



[3]

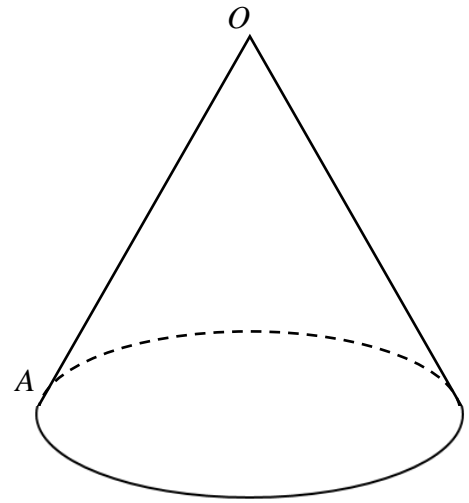
- (ii) Calculate the area of the major segment  $ABCD$ .

*Answer* .....  $\text{cm}^2$  [4]



(b) The sector  $OADC$  was cut from the above to form a cone where the  $OA$  is glued to  $OC$ .

(i) Calculate the circumference of the base circle of the cone.



Answer ..... cm [2]

(ii) Hence, calculate the vertical height of the cone.

Answer ..... cm [3]

- 8 (a) The heights of a group of 30 students were measured and the results are shown in the stem-and-leave diagram.

14	4 5 5 7 7 7 8 8 8 9 9 9 9
15	1 1 2 3 3 3 3 5 5 6 9 9 9
16	0 3 8
17	
18	9
	Key: 14   4 means 144 cm

- (i) Find the median height.

Answer ..... cm [1]

- (ii) Explain why mean may not be an appropriate average to use to summarise the heights of this group of students?

Answer .....

.....

..... [1]

- (b) The table shows the height of another group of 30 students.

Height ( $h$ cm)	$140 \leq h < 150$	$150 \leq h < 160$	$160 \leq h < 170$	$170 \leq h < 180$
Frequency	$p$	9	11	$q$

- (i) The estimated mean height is  $161\frac{1}{3}$  cm. Find the value of  $p$  and of  $q$ .

*Answer*  $p = \dots\dots\dots$

$q = \dots\dots\dots$  [3]

- (ii) Calculate an estimate of the standard deviation.

*Answer*  $\dots\dots\dots$  cm [1]

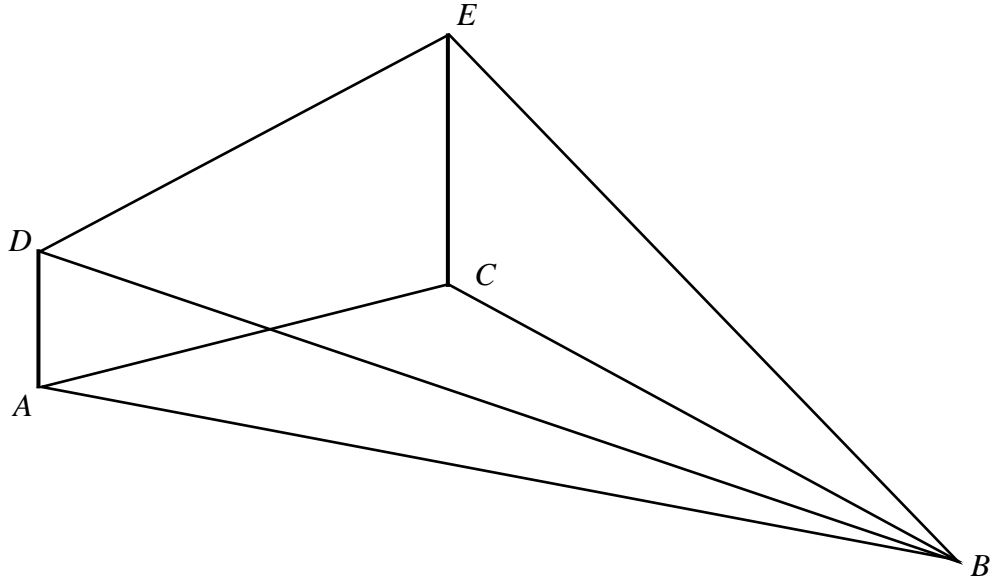
- (iii) Given that the estimated standard deviation of the first group of students is 8.83 cm, make one comparison between the heights of the two groups of students.

*Answer*  $\dots\dots\dots$

$\dots\dots\dots$

$\dots\dots\dots$  [1]

9



In the diagram above, triangle  $ABC$  forms a garden on level ground.  
 $BC = 32$  m, angle  $ACB = 100^\circ$  and angle  $BAC = 53^\circ$ .

- (a) Show that  $AC = 18.19$  m, correct to 4 significant figures.

*Answer*

[2]

- (b) Triangle  $BDE$  is a roof designed for the garden. Two vertical poles,  $AD = 2$  m and  $CE = 5$  m, were built to hold up the roof. Find

- (i) the length of  $DE$ ,

*Answer* ..... m [2]

(ii) angle  $DBE$ .

*Answer* angle  $DBE = \dots\dots\dots^\circ$  [5]

- 10** Billy is engaged as a coding instructor to conduct Level 1 and Level 2 course on weekdays and weekends. The matrix **C** shows the number of sessions he teaches each week.

$$\mathbf{C} = \begin{matrix} & \begin{matrix} \text{Level 1} & \text{Level 2} \end{matrix} \\ \begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix} & \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix} \end{matrix}$$

He is paid \$80 per session for conducting Level 1 course and \$120 per session for conducting Level 2 course.

- (a) Represent the payment per session with a  $2 \times 1$  matrix, **N**.

$$\text{Answer } \mathbf{N} = \begin{pmatrix} & \\ & \end{pmatrix} \quad [1]$$

- (b) Evaluate  $\mathbf{Q} = \mathbf{CN}$ .

$$\text{Answer } \mathbf{Q} = \begin{pmatrix} & \\ & \end{pmatrix} \quad [1]$$

- (c) Describe the elements in **Q**.

Answer.....  
 .....  
 ..... [1]

- (d) Using matrix multiplication, calculate his total weekly income.

*Answer* \$ ..... [1]

- (e) Due to the popularity of the courses, Billy is paid 5% more for each session of Level 1 and Level 2 course he conducts. The matrix **D** shows the number of sessions he teaches each week.

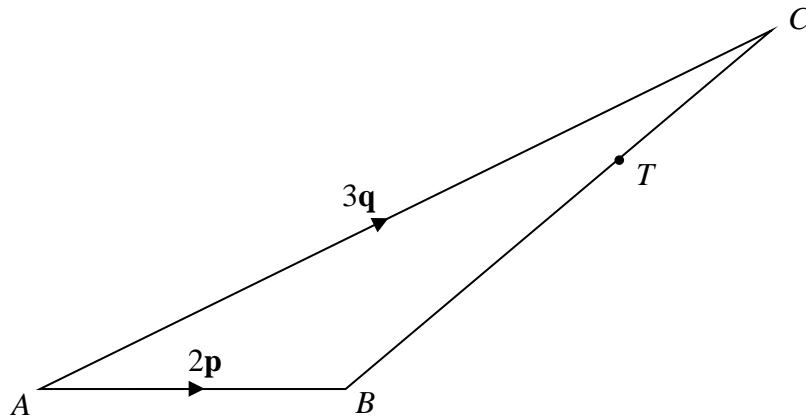
$$\mathbf{D} = \begin{matrix} & \begin{matrix} \text{Level 1} & \text{Level 2} \end{matrix} \\ \begin{pmatrix} 4 & x \\ 3 & y \end{pmatrix} & \begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix} \end{matrix}$$

Given that his total income on weekdays and weekends are \$462 and \$504 respectively, find the value of  $x$  and of  $y$ .

*Answer*  $x =$  .....

$y =$  ..... [2]

11



$ABC$  is a triangle.  $\overrightarrow{AB} = 2\mathbf{p}$  and  $\overrightarrow{AC} = 3\mathbf{q}$ .  
 $T$  is a point on  $BC$  such that  $CT : TB = 2 : 3$ .

(a) Express, as simply as possible, in terms of  $\mathbf{p}$  and/or  $\mathbf{q}$ ,

(i)  $\overrightarrow{BC}$ ,

Answer  $\overrightarrow{BC} = \dots\dots\dots$  [1]

(ii)  $\overrightarrow{TC}$ .

Answer  $\overrightarrow{TC} = \dots\dots\dots$  [1]

(b)  $R$  is a point on  $AC$  such that triangle  $ABC$  is similar to triangle  $RTC$ .

Express  $\overrightarrow{RC}$ , as simply as possible, in terms of  $\mathbf{p}$  and/or  $\mathbf{q}$ .

Answer  $\overrightarrow{RC} = \dots\dots\dots$  [1]



(c) Find the value of

(i)  $\frac{\text{area of triangle } RTC}{\text{area of triangle } ABC},$

Answer ..... [1]

(ii)  $\frac{\text{area of triangle } RBC}{\text{area of triangle } ABC}.$

Answer ..... [1]

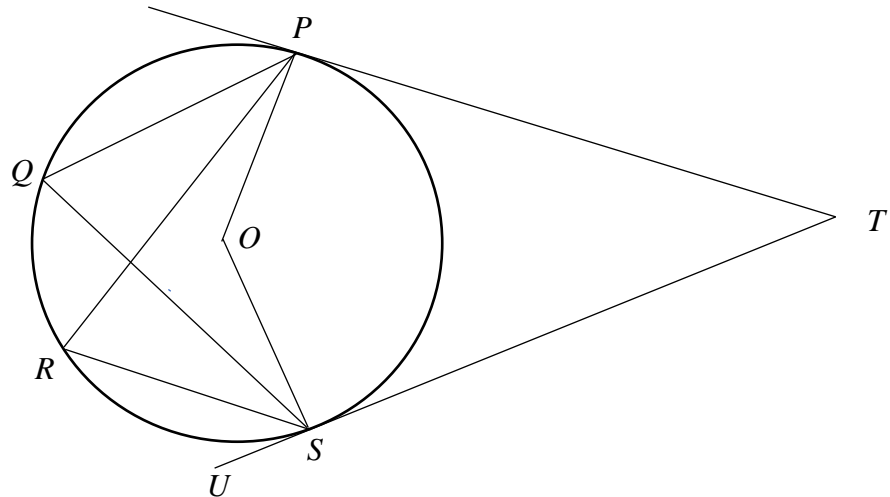
(d)  $S$  is a point on  $AC$ .

Triangle  $ASB$  has the same area as triangle  $SBC$ .

Express  $\overrightarrow{SB}$ , as simply as possible, in terms of  $\mathbf{p}$  and/or  $\mathbf{q}$ .

Answer  $\overrightarrow{SB} = \dots\dots\dots$  [2]

12



In the diagram,  $P$ ,  $Q$ ,  $R$  and  $S$  are four points on a circle with centre  $O$ .  $PT$  and  $ST$  are tangents to the circle at  $P$  and  $S$  respectively. Angle  $PRS = 68^\circ$  and angle  $QSU = 70^\circ$ . Find, giving reasons for each answer,

(a) angle  $PQS$ ,

Answer angle  $PQS = \dots\dots\dots^\circ$  [1]

(b) angle  $OSU$ ,

Answer angle  $OSU = \dots\dots\dots^\circ$  [1]

(c) angle  $PTS$ .

Answer angle  $PTS = \dots\dots\dots^\circ$  [2]

- 13 (a) Complete the table of values for  $y = \frac{x^3}{5} - x + 4$  below.

$x$	-3.5	-3	-2	-1	0	1	2	3
$y$	-1.1	1.6		4.8	4	3.2	3.6	6.4

[1]

- (b) Using a scale of 2 cm to represent 1 unit on both axes, plot the points given in the table and join them with a smooth curve for  $-3.5 \leq x \leq 3$ .

[3]

- (c) By drawing a tangent, find the gradient of the curve at  $x = 2$ .

Answer ..... [2]

- (d) Use your graph to find the solutions of the equation  $\frac{x^3}{5} - \frac{1}{2}x = 0$  for  $-4 \leq x \leq 3$ .

Answer  $x =$  ..... or ..... or ..... [2]

- (e) The solutions in part (d) above are also the solutions for the equation  $2x^3 + Ax^2 + Bx = 0$ . Find the value of  $A$  and of  $B$ .

Answer  $A =$  .....

$B =$  ..... [2]

- 14** Mrs Lee intends to buy a 7-seater car so that she can bring her family out for outings. Typically a 7-seater car has an engine capacity that is more than 1600 cc. After doing some research, she realizes that the total cost of purchasing a car includes the following components which are summarised in Table 1.

Table 1: Costs Incurred in Car Purchase

Category	A	B
Engine Capacity (in cc)	$\leq 1600$	$> 1600$
Certificate of Entitlement, COE Prices	\$74 989	\$106 001
Open Market Value, OMV	\$15,602 – \$41,124	\$26,211 – \$74,924
Road Tax, RT	\$371.45 per 6 months	\$606.05 per 6 months
Insurance	\$1,684 per annum	
Additional Registration Fee, (ARF)	Refer to Table 2 for the guide to calculating ARF	
Goods and Services Tax, GST	7% of the OMV	

Table 2: Guide to Calculating ARF

Additional Registration Fee, (ARF)	Chargeable ARF	Rate	ARF Payable
	First \$20,000:	100% of OMV	\$20,000
	Next \$30,000:	140% of OMV	\$42,000
	Next \$30,000:	180% of OMV	\$54,000
	In excess of \$80,000:	220% of OMV	\$176,000

- (a) How much is the maximum amount of the total Additional Registration Fee that Mrs Lee will have to pay for a 7-seater car.

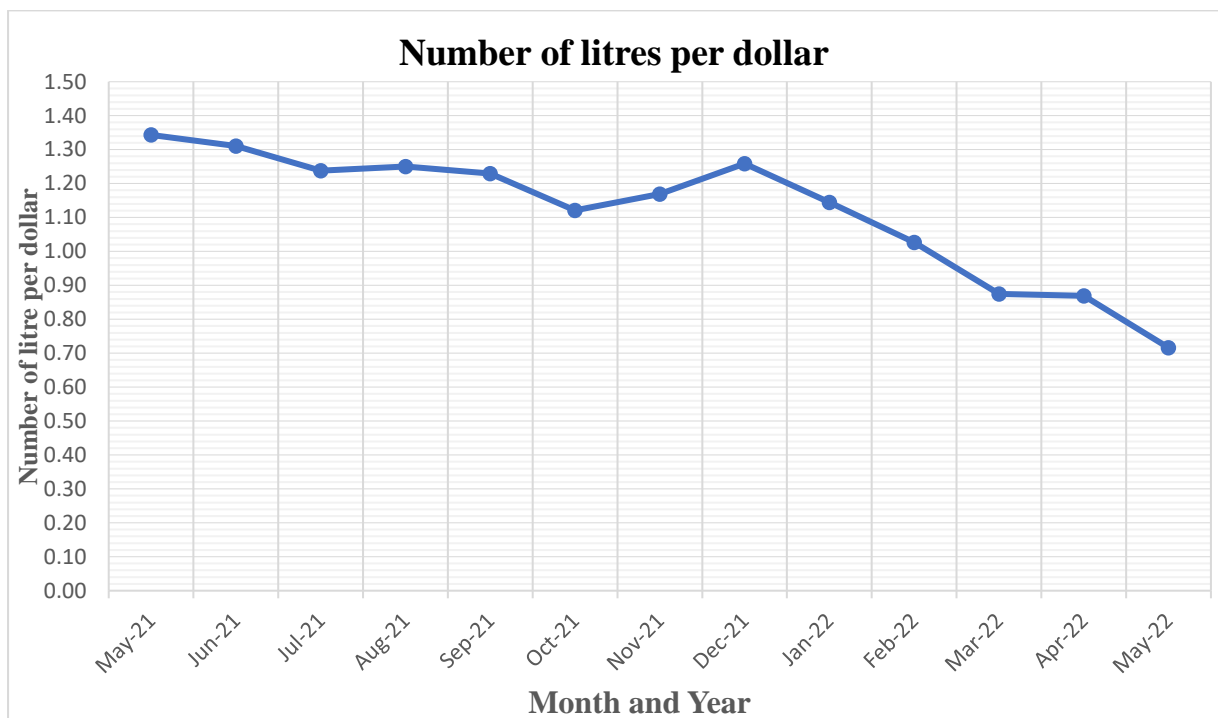
*Answer* \$ ..... [3]

- (b) What is the total cost of the car, excluding the road tax and the insurance.

*Answer* \$ ..... [2]

- (c) The car company allows Mrs Lee to pay 30% of the total cost as down-payment and the subsequent amount at a simple interest rate of 1.5% per annum over 7 years in monthly instalment. To ensure that her monthly salary is enough to pay for the monthly instalment of the car and the petrol, she will need to calculate how much money she needs to set aside every month for the car.

Mrs Lee also found the following information about the petrol prices over the last one year and the national average distance travelled is 1000 km per month with one litre of petrol being able to cover an average of 12 km in city area and 15 km on highway.



Suggest a suitable amount of money that Mrs Lee need to set aside every month in her bank to pay for **all** the expenses of the car during the first seven years. Justify any decisions you made and show your calculations clearly.

*Answer*

-----  
----- [7]

**End of Paper**

## Answer Key

<b>1ai</b>	$\frac{36}{57}$ or $\frac{12}{19}$				<b>10a</b>	$\begin{pmatrix} 80 \\ 120 \end{pmatrix}$
<b>1aii</b>	$T_n = \frac{n^2}{8n+9}$				<b>10b</b>	$\begin{pmatrix} 240 \\ 280 \end{pmatrix}$
<b>1aiii</b>	$k = 9$				<b>10c</b>	Q represent the total amount of course fee collect on weekday and weekend respectively.
<b>1b</b>	4				<b>10d</b>	\$520
<b>2b</b>	$\left(\frac{21}{5}, \frac{8}{5}\right)$ or $\left(4\frac{1}{5}, 1\frac{3}{5}\right)$				<b>10e</b>	$x = 1, y = 2$
<b>2c</b>	14 units <sup>2</sup>				<b>11ai</b>	$-2\mathbf{p} + 3\mathbf{q}$
<b>3a</b>			<b>Male</b>	<b>Female</b>	<b>11aii</b>	$\frac{2}{5}(-2\mathbf{p} + 3\mathbf{q})$
		<b>Korean</b>	8	5		
		<b>Japanese</b>	7	2		
<b>3b</b>	$\frac{35}{92}$				<b>11b</b>	$\frac{6}{5}\mathbf{q}$
<b>4</b>	8 months				<b>11ci</b>	$\frac{4}{25}$
<b>5a</b>	$5x^\circ$ or $(180 - 2.5x)^\circ$				<b>11cii</b>	$\frac{2}{5}$
<b>5b</b>	24				<b>11d</b>	$2\mathbf{p} - \frac{3}{2}\mathbf{q}$
<b>6a</b>	$x = \frac{24-7a}{3}$				<b>12a</b>	$68^\circ$
<b>6b</b>	15				<b>12b</b>	$90^\circ$
<b>7aii</b>	1330 cm <sup>2</sup>				<b>12c</b>	$44^\circ$
<b>7bi</b>	92.7 cm				<b>13a</b>	4.4
<b>7bii</b>	20.2 cm				<b>13b</b>	Refer to next page
<b>8ai</b>	151.5 cm				<b>13c</b>	$1.38 \pm 0.1$
<b>8aii</b>	There is an outlier.				<b>13d</b>	$x = -1.45, 0, 1.45$
<b>8bi</b>	$p = 4, q = 6$				<b>13e</b>	$A = 0, B = -5$
<b>8bii</b>	9.29 cm				<b>14a</b>	\$10 6863.20
<b>8biii</b>	The height of first group of students are more consistent.				<b>14b</b>	\$293 032.88
<b>9bi</b>	18.4 m				<b>14c</b>	\$3100
<b>9bii</b>	27.4°					