



**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION 2022**

S4

SOLUTIONS

MATHEMATICS

4048/01

Paper 1

25 August 2022

Candidates answer on the Question Paper.

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the space at the top of this page.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiners' Use

For Examiners Use									
Question	1	2	3	4	5	6	7	8	9
Marks									
Question	10	11	12	13	14	15	16	17	18
Marks									
Question	19	20	21	22	23	24	25		
Marks									
Table of Penalties			Units				80		
			Clarity / Logic						
			Accuracy / Precision						
Parent's Name and Signature:									
Date:									

This document consists of **17** printed pages.

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 Calculate $\frac{-12.2^2}{3.15 - \sqrt{17}}$ and correct your answer to 4 significant figures.

1	$= 152.95359$ $= 153.0$ (4s.f)	B1
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Answer [1]

- 2 Rearrange the formula $y = \frac{x^2 - 2}{(x - 2)(x + 2)}$ to make x the subject.

2	$y = \frac{x^2 - 2}{(x - 2)(x + 2)}$ $y(x^2 - 4) = x^2 - 2$ $yx^2 - x^2 = 4y - 2$ $x^2 = \frac{4y - 2}{y - 1}$ $x = \pm \sqrt{\frac{2(2y - 1)}{y - 1}}$	<p>M1: for expansion</p> <p>M1: for grouping</p> <p>A1: not given without the plus/minus Accept if never factorise out 2.</p>
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Answer [3]

- 3 Solve the equation $3 - \frac{2x - 3}{5} = \frac{2x + 15}{7}$.

3	$3 - \frac{2x - 3}{5} = \frac{2x + 15}{7}$ $\frac{15 - (2x - 3)}{5} = \frac{2x + 15}{7}$ $\frac{18 - 2x}{5} = \frac{2x + 15}{7}$ $126 - 14x = 10x + 75$ $24x = 51$ $x = \frac{51}{24}$ $x = \frac{17}{8} \text{ or } 2\frac{1}{8} \text{ or } 2.125$	<p>M1 for forming algebraic fraction on either side</p> <p>M1 for cross multiply and form the linear equation</p> <p>A1</p>
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Answer $x =$ [3]

- 4 The area of triangle ABC is 36.7cm^2 . The length of AB is 14.2cm and BC is 6.1cm . Find the possible values of angle ABC .

4	$\frac{1}{2}(14.2)(6.1)\sin \hat{ABC} = 36.7$ $\sin \hat{ABC} = 0.847379$ $\hat{ABC} = \sin^{-1} 0.847379$ $= 57.9277^\circ \text{ or } 180^\circ - 57.9277^\circ$ $= 57.9^\circ \text{ or } 122.1^\circ$	B2: 1 mark for each answer
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Answer Angle $ABC = \dots\dots\dots^\circ$ or $\dots\dots\dots^\circ$ [2]

- 5 Simplify $\left(\frac{8p^6}{27m^3}\right)^{-\frac{1}{3}}$.

5	$\left(\frac{8p^6}{27m^3}\right)^{-\frac{1}{3}}$ $= \left(\frac{27m^3}{8p^6}\right)^{\frac{1}{3}}$ $= \frac{27^{\frac{1}{3}}m}{8^{\frac{1}{3}}p^2}$ $= \frac{3m}{2p^2}$	<p>M1 for making index form positive</p> <p>A1</p>
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Answer $\dots\dots\dots$ [2]

- 6 (a) Express 1134 as a product of its prime factors.

6(a)	$1134 = 2 \times 3^4 \times 7$	B1
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Answer $\dots\dots\dots$ [1]

- (b) Find the smallest integer value of k such that $\frac{1143}{k}$ is a perfect cube.

6(b)	$\frac{1134}{k} = \frac{2 \times 3^4 \times 7}{2 \times 3 \times 7} \text{ is a perfect cube}$ <p>Therefore $k = 2 \times 3 \times 7$</p> $= 42$	B1
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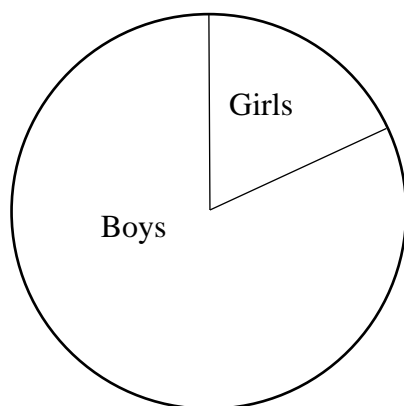
Answer $k = \dots\dots\dots$ [1]

- (c) A perfect square number less than 150 has exactly 7 factors. List the factors.

6(c)	<p>The number is 64</p> <p>Factors of 64 are 1, 2, 4, 8, 16, 32, 64</p>	<p>B1 if 64 is listed.</p> <p>B1 for correct list</p>
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Answer $\dots\dots\dots$ [2]

- 7 Based on the pie chart drawn, Audrey concluded that boys did better than the girls in the Mathematics Common Test. Do you agree? Justify your answer.

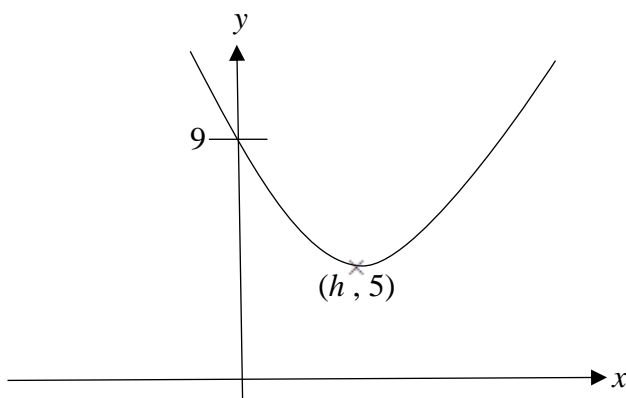


Students who scored Distinction in Mathematics Common Test

7	<p>I do not agree.</p> <ol style="list-style-type: none"> 1) Although the number of girls who scored distinction may be lower than the number of boys, the percentage of passing may be higher. 2) There is no actual number of boys and girls given taking the test, hence it is not conclusive of the statement. 3) This data does not show the central tendency of distribution (mean, mode, median) 	<p>B1</p> <p>B1 accept any valid reason</p>
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Answer [2]

- 8 The diagram below shows the graph of $y = (x - h)^2 + k$.
The graph cuts the y-axis at the point (0, 9) and has a minimum point at (h, 5).



(a) State the value of k .

8(a)	$k = 5$	B1
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Answer $k =$ [1]

(b) Find the value of h .

8(b)	Using the point $(0, 9)$, $9 = (0 - h)^2 + 5$ $4 = h^2$ $\therefore h = \pm 2$ Since $h > 0$, $h = 2$	M1 A1. No marks awarded if students never state the condition of h .
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Answer $h = \dots\dots\dots$ [2](c) State the number of solutions when $y < 5$.

8(c)	0	B1
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Answer $\dots\dots\dots$ [1]

9 Factorise the following completely.

(a) $2a^2 - 6b^2 + ab$.

9(a)	$2a^2 - 6b^2 + ab = (a + 2b)(2a - 3b)$	B1
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Answer $\dots\dots\dots$ [1](b) $2 - 8x^2 + 4x^2y - y$.

9(b)	$2 - 8x^2 + 4x^2y - y = 2(1 - 4x^2) + y(4x^2 - 1)$ $= (2 - y)(1 - 4x^2)$ $= (2 - y)(1 - 2x)(1 + 2x)$	M1: for grouping M1 A1 o.e
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Answer $\dots\dots\dots$ [3]10 Express as a single fraction in the simplest form $\frac{2x}{(2x-1)^2} - \frac{2}{2x-1}$.

10	$\frac{2x}{(2x-1)^2} - \frac{2}{2x-1} = \frac{2x - 2(2x-1)}{(2x-1)^2}$ $= \frac{-2x + 2}{(2x-1)^2}$ $= \frac{2(1-x)}{(2x-1)^2}$	M1: for correct denominator A1 either answer accepted
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Answer $\dots\dots\dots$ [2]

- 11** A fruit seller purchased 100 mangoes at \$2 each. Four were spoilt. After selling all the remaining mangoes, his total profit was 20%. Calculate the selling price of each mango.

11	Profit = 20% of \$200 = \$40 96 mangoes are sold for \$240 Each mango is sold for $\$240 \div 96 = \2.50	M1 obtaining \$40 or \$240 A1
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Answer \$. [2]

- 12 (a)** Given that T is directly proportional to \sqrt{x} , find the percentage increase in T when the value of x is increased by 8 times.

12(a)	$T = k\sqrt{x}$ When x was increased by 8 times, New $T = k\sqrt{9x}$ $= 3k\sqrt{x}$ Percentage increase in $T = \frac{3k\sqrt{x} - k\sqrt{x}}{k\sqrt{x}} \times 100\%$ $= 200\%$	M1 A1
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Answer % [2]

- (b)** Jane can prepare 4 full cups of bubble tea in 3 minutes. Danny can prepare 9 full cups in 7 minutes. Find the total number of full cups prepared by them in 20 minutes.

12(b)	In 1 minute, Jane can prepare $\frac{4}{3}$ cups Danny can prepare $\frac{9}{7}$ cups In 20 minutes, Jane can prepare $\frac{80}{3}$ cups $= 26\frac{2}{3}$ cups Danny can prepare $\frac{180}{7}$ cups $= 25\frac{5}{7}$ cups Together they prepare $26 + 25 = 51$ cups or Together they prepare $26\frac{2}{3} + 25\frac{5}{7} = 52\frac{8}{21}$ ≈ 52	M1 M1 A1 Accept either answers
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Answer [3]

- 13 (a)** Given
 $\xi = \{x : x \text{ is an integer and } 0 < x \leq 14\}$.
 $A = \{x : x \text{ is a factor of } 12\}$.
 $B = \{x : x \text{ is an integer such that } 4x \geq 23\}$.

(i) List the elements in $A' \cap B'$.

13(ai)	$A' \cap B' = \{5\}$	B1. Accept 5 if students never write the set.
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Answer [1]

(ii) Write down the number of elements in the set $A \cup B$.

13a(ii)	13	B1
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Answer [1]

- 14** A map has a scale of 1: 4 000 000. The length of a river running through a national park is 1.4 cm.

(a) Calculate the length of the river in km.

14a)	$1:4000000$ $1\text{cm} : \frac{4000000}{100000} \text{ km}$ $1\text{cm} : 40\text{km}$ $\text{Therefore } 1.4\text{cm} : 1.4 \times 40\text{km}$ $\quad \quad \quad : 56\text{km}$	M1 correct conversion to km. A1
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Answer km [2]

(b) The area of the park is 52 000 km². Calculate, the area of the park in the map.

14b)	$1\text{cm} : 40\text{km}$ Area scale $(1\text{cm})^2 : (40\text{km})^2$ $1\text{cm}^2 : 1600\text{km}^2$ $\text{Therefore } 52000\text{km}^2 = \frac{52000}{1600} \text{ cm}^2$ $\quad \quad \quad = 32.5 \text{ cm}^2$	M1 A1
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Answer cm² [2]

- 15 $ABCD$ is a parallelogram. $\overrightarrow{AB} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$ and $\overrightarrow{BD} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$. The coordinates of D is $(6, 2)$. Find

(a) the coordinates of B ,

15a	$\overrightarrow{BO} + \overrightarrow{OD} = \overrightarrow{BD}$ $\overrightarrow{OB} = \overrightarrow{OD} - \overrightarrow{BD}$ $= \begin{pmatrix} 6 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -6 \end{pmatrix}$ $\therefore B(2, -6)$	B1
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Answer (.....,) [1]

(b) \overrightarrow{BC} ,

15b	$\overrightarrow{BA} + \overrightarrow{BC} = \overrightarrow{BD}$ $\overrightarrow{BC} = \overrightarrow{BD} - \overrightarrow{BA}$ $= \begin{pmatrix} 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$	B1
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Answer $\overrightarrow{BC} = \begin{pmatrix} \\ \end{pmatrix}$ [1]

(c) the magnitude of \overrightarrow{CD} .

15c	$ \overrightarrow{CD} = \overrightarrow{AB} $ $= \sqrt{(-1)^2 + (-5)^2}$ $= \sqrt{26}$ $= 5.09902$ $= 5.10$	M1 A1
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Answerunits[1]

- 16 (a)** Find the interior angle of a regular 18-sided polygon.

16a)	Each interior angle = $\frac{(18-2)}{18} \times 180^\circ$ $= 160^\circ$	B1
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Answer° [1]

- (b)** An n -sided polygon has 3 exterior angles measuring 70° each and the remaining exterior angles measuring w° each. Find w in terms of n .

16b)	Sum of exterior of polygon is always = 360° $3 \times 70^\circ + (n-3)w = 360^\circ$ $(n-3)w = 360^\circ - 210^\circ$ $w = \frac{150^\circ}{n-3}$	M1 A1
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Answer $w = \dots\dots\dots$ [2]

- 17 (a)** The thickness of Mary's hair is 0.036 mm. Express the thickness of the hair, in metres, in standard form.

17a)	$1\text{m} = 1000\text{mm}$ $1\text{mm} = \frac{1}{1000}\text{m}$ $1\text{mm} = 1 \times 10^{-3}\text{m}$ $0.036\text{mm} = 0.036 \times 10^{-3}$ $= 3.6 \times 10^{-5}$	B1
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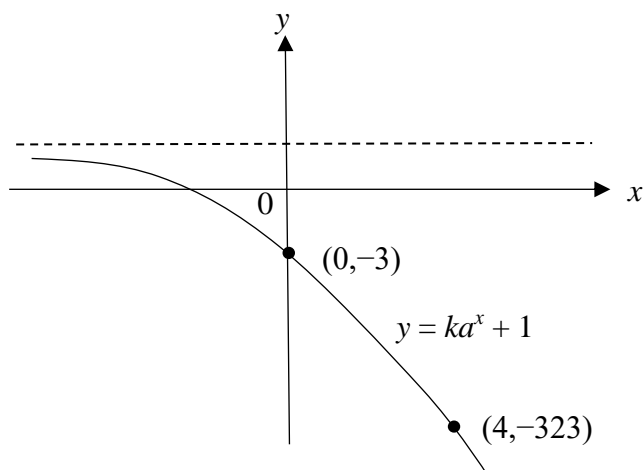
Answerm [1]

- (b)** After a hair bonding treatment, Mary's hair length is 4 cm longer than Betty. Given that the rate of their hair growth is 1.2 cm and 1.6 cm per month respectively, calculate the time taken, in months, for their hair to be of the same length.

17b)	Let t be the time taken $1.2t + 4 = 1.6t$ $0.4t = 4$ $t = 10\text{months}$	M1 A1
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Answermonths [2]

- 18 The sketch shows the graph of $y = ka^x + 1$ where $a > 0$.
The points $(0, -3)$ and $(4, -323)$ lie on the graph.



Find the value of

(a) k and

18(a)	Using the point $(0, -3)$, substitute into the equation of the curve, $-3 = ka^0 + 1$ $-4 = k$	B1
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Answer $k = \dots\dots\dots$ [1]

(b) a .

18(b)	Using the point $(4, -323)$, substitute into the equation of the curve, $-323 = -4a^4 + 1$ $-324 = -4a^4$ $a^4 = 81$ $\therefore a = 3$ or -3 Since $a > 0$, hence, $a = 3$ (reject $a = -3$)	B1 Penalise from overall (under clarity) if students did not reject -3
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Answer $a = \dots\dots\dots$ [2]

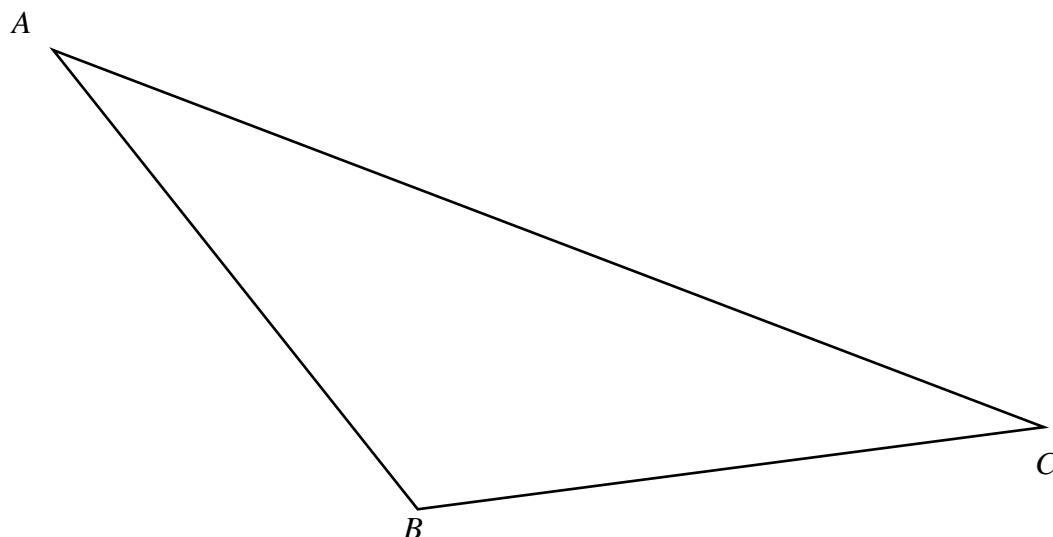
19 In the diagram below, construct

(a) a perpendicular bisector of AB ,

[1]

(b) an angle bisector of angle ABC .

[1]



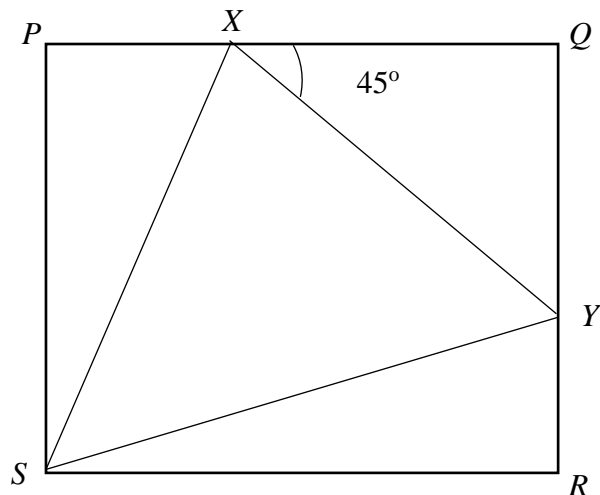
(c) These 2 lines from (a) and (b) meet at point P . Complete the sentence below.

Answer The point P isfrom point A and point B . It is also
from the lines and

[2]

19a,b		
19c	<p>1 mark for each construction</p> <p>The point P is <u>equidistant</u> from point A and point B. It is also <u>equidistant</u> from lines <u>AB</u> and <u>BC</u>.</p>	<p>2 marks for all 4 answers. Accept 'same distance', 'same length'. Do not accept 'equal'</p>

- 20 In the diagram, $PQRS$ is a square and $\angle QXY = 45^\circ$. Giving reasons clearly, prove that triangle PXS is congruent to triangle RYS .



20	<p>$PS = SR$ (property of square: all sides are equal in length)</p> <p>$\angle SPX = \angle SRY$ (property of square: all angles are equal)</p> <p>Triangle QXY is an isosceles triangle, thus $QX = QY$.</p> <p>Hence, $PX = RY$</p> <p>By SAS test of congruency, triangle PXS is congruent to triangle RYS.</p>	<p>M1 for stating the property of square.</p> <p>M1 for stating isosceles triangle.</p> <p>A1. Do not award this mark if students never state the test.</p> <p>No marks if students state the test as RHS.</p>
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Answer

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..... [3]

21 Solve the equation $\frac{4m+7}{m^2+1} - \frac{5}{2m-1} = \frac{3}{m^2+1}$

$\frac{4m+7}{m^2+1} - \frac{5}{2m-1} = \frac{3}{m^2+1}$ $\frac{4m+7}{m^2+1} - \frac{3}{m^2+1} = \frac{5}{2m-1}$ $\frac{4m+4}{m^2+1} = \frac{5}{2m-1}$ $(4m+4)(2m-1) = 5(m^2+1)$ $8m^2 + 4m - 4 = 5m^2 + 5$ $3m^2 + 4m - 9 = 0$ $m = \frac{-4 \pm \sqrt{4^2 - 4(3)(-9)}}{2(3)}$ $m = \frac{-4 \pm \sqrt{124}}{6}$ $m = 1.1892 \text{ or } -2.5225$ $m = 1.19 \text{ or } -2.52 \text{ (3s.f.)}$ <p>or</p> $\frac{4m+7}{m^2+1} - \frac{5}{2m-1} = \frac{3}{m^2+1}$ $\frac{(4m+7)(2m-1)}{(m^2+1)(2m-1)} - \frac{5(m^2+1)}{(2m-1)(m^2+1)} = \frac{3}{m^2+1}$ $\frac{8m^2 - 4m + 14m - 7 - 5m^2 - 5}{(m^2+1)(2m-1)} = \frac{3}{m^2+1}$ $3m^2 + 10m - 12 = 3(2m-1)$ $3m^2 + 10m - 12 = 6m - 3$ $3m^2 + 4m - 9 = 0$ $m = \frac{-4 \pm \sqrt{4^2 - 4(3)(-9)}}{2(3)}$ $m = \frac{-4 \pm \sqrt{124}}{6}$ $m = 1.1892 \text{ or } -2.5225$ $m = 1.19 \text{ or } -2.52 \text{ (3s.f.)}$	<p>M1 for forming the algebraic fraction</p> <p>M1 for forming the quadratic equation M1 for using the formula or any method to solve x</p> <p>A1</p> <p>M1 LCM approach on LHS (Do not expand denominator)</p> <p>M1 Simplify. Cross multiply. Cancel $(m^2 + 1)$ on RHS. Obtain the quadratic equation.</p> <p>M1 for using formula to solve m</p> <p>A1</p>
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Answer $m = \dots\dots\dots$ or $m = \dots\dots\dots$ [4]

- 22 Given that $B = \{\text{black grapes}\}$, $G = \{\text{green grapes}\}$, $S = \{\text{sweet grapes}\}$ and $L = \{\text{seedless grapes}\}$.

(a) Use set notation to express each of the following statements:

(i) all seedless grapes are sweet but not all sweet grapes are seedless,

(ai)	$L \subset S$. Accept $S \cap L = L$	B1
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Answer [1]

(a) no green grapes are seedless.

(aii)	$G \cap L = \emptyset$ or $\{ \}$	B1.
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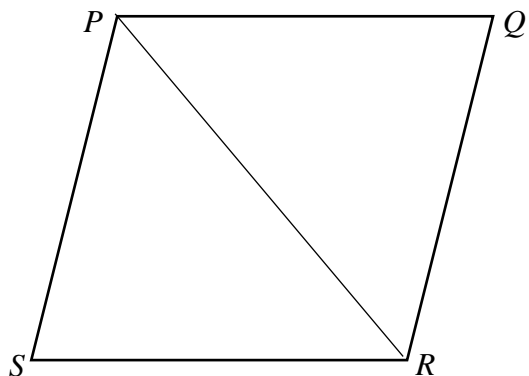
Answer [1]

(b) Write a statement to explain what does $(G \cup B) \cap S \neq \emptyset$ means.

(b)	There are <u>black</u> <u>or</u> green grapes that are sweet.	B1. 1 st underlined, accept 'some', 'many', 'most' Do not accept 'All' (1 st underlined), 'and' (2 nd underlined)
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Answer [1]

- 23 The figure below is made up of 2 equilateral triangles SPR and PQR . PQ is $(x + 2y)$ cm, QR is $(3x + y)$ cm and PS is $(2x + 4y - 3)$ cm.



(a) Form, without simplifying, 2 simultaneous equations in terms of x and y .

Answer

(a)	$x + 2y = 3x + y$ -----(1) $x + 2y = 2x + 4y - 3$ -----(2) or $3x + y = 2x + 4y - 3$ -----(2)	B1
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[1]

(b) Hence, find the value of x and of y .

(b) Giving reasons clearly, find

(i) angle AEB ,

(bi)	Angle $AEB = 180^\circ - 58^\circ - 66^\circ$ (sum of angle in triangle ABE) $= 56^\circ$	B1 Minus 1 from this qn if reasons not stated
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Answer Angle $AEB = \dots\dots\dots^\circ$ [1](ii) angle DCB .

(bii)	Angle $DCB = 360^\circ - 116^\circ - 66^\circ - 132^\circ$ (sum of angle in quadrilateral $ABCD$) $= 46^\circ$ or Angle $DCB = 360^\circ - 132^\circ - 58^\circ - (180^\circ - 56^\circ)$ (sum of angle in quadrilateral $BEDC$) $= 46^\circ$	B1 As above
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Answer Angle $DCB = \dots\dots\dots^\circ$ [1](c) Show that BC is not parallel to AD .

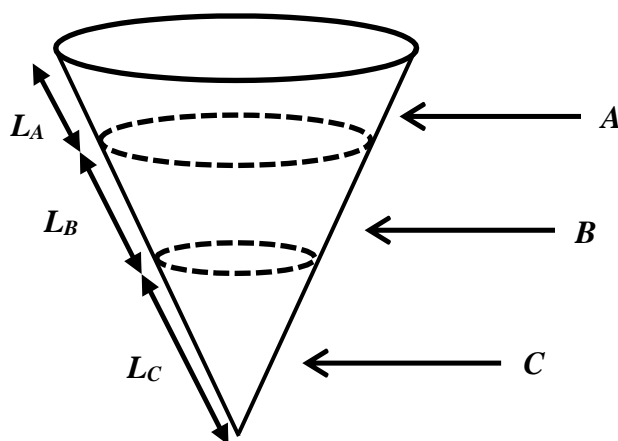
(c)	For two lines to be parallel, the sum of interior angles is 180° . Angle $ABC = 2 \times 58^\circ$ Angle $BAE = 66^\circ$ Since angle $ABC + \text{angle } BAE = 182^\circ$ and not 180° , $\therefore BC$ is not parallel to AD . or Since angle AEB is not equal to angle EBC , hence BC is not parallel to AD (converse of alternate angle) or For two lines to be parallel, the sum of interior angles is 180° . Angle $ADC = 132^\circ$ Angle $BCD = 46^\circ$ Since angle $ADC + \text{angle } BCD = 178^\circ$ and not 180° , $\therefore BC$ is not parallel to AD .	B1 when sum of angles = 182° and $\neq 180^\circ$ is sighted. or B1 or B1 when sum of angles = 178° and $\neq 180^\circ$ is sighted.
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Answer

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..... [1]

- 25 A right solid cone is made up of frustrum A , frustrum B and cone C .
The ratio of the slant height of A , B and C is given by $L_A:L_B:L_C=2:3:5$.



- (a) Find the ratio of volume of A : volume of whole cone.

24(a)	Using similar figures, Volume of A : Volume of whole cone $(10^3 - 8^3) : 10^3$ 488 : 1000 61 : 125	M1 for able to find the volume part of A A1
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Answer : [2]

- (b) Given that the volume of A is 122 cm^3 , calculate the volume of B .

(b)	By similar figures, Volume of A : Volume of B 488 : $(8^3 - 5^3)$ 488 : 387 By proportion, Volume of $B = \frac{387}{488} \times 122$ $= 96.75 \text{ cm}^3$ (exact) Or Volume of whole cone $= \frac{125}{61} \times 122$ $= 250 \text{ cm}^3$ Volume of cone $C = \left(\frac{5}{10}\right)^3 \times 250$ $= 31.25 \text{ cm}^3$ Volume of $B = 250 - 122 - 31.25 \text{ cm}^3$ $= 96.75 \text{ cm}^3$	M1 for finding the volume part of B A1. Students can leave in improper or mixed number. If students correct to 3s.f, no marks awarded. M1 for finding both whole cone and cone C . A1. As above
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Answer cm^3 [2]

End of Paper