

NAME: _____ ()

CLASS: 4 ()



SOLUTIONS

S4

MATHEMATICS

Paper 2

4048/02

31 August 2022

2 hours 30 minutes

Candidates answer on the Question Paper and Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

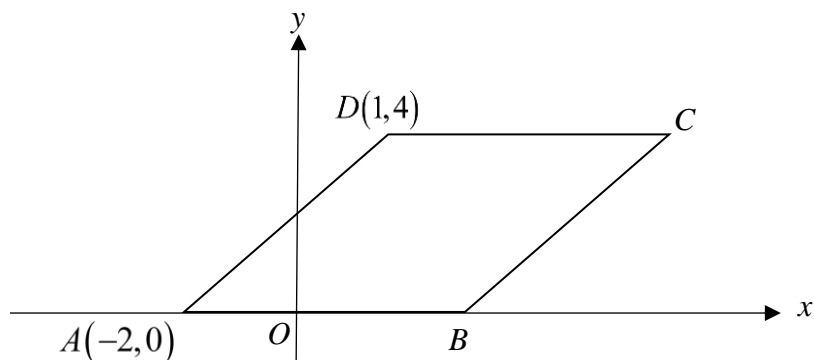
The total number of marks for this paper is 100.

For Examiners' Use

For Examiners' Use							
Questions	1	2	3	4	5	6	7
Marks							
Questions	8	9	10	11	12	13	14
Marks							
Table of Penalties	Units					100	
	Clarity/Logic						
	Accuracy/Precision						
Parent's Name and Signature:							
Date:							

This document consists of 23 printed pages and 1 blank page.

2



The diagram shows a rhombus $ABCD$, with sides 5 units, and coordinates $A(-2, 0)$ and $D(1, 4)$.

(a) Show that the equation of line BC is $3y = 4x - 12$.

(a)	$B(-2 + 5, 0) = B(3, 0)$ $\text{Gradient} = \frac{4 - 0}{1 - (-2)}$ $= \frac{4}{3}$ $y = \frac{4}{3}x + c$ $0 = \frac{4}{3}(3) + c$ $c = -4$ $\text{Equation of } BC: y = \frac{4}{3}x - 4$ $3y = 4x - 12$	<p>M1: coordinates of B or C</p> <p>M1</p> <p>M1</p>
-----	--	--

E is a point on BC , such that DE is perpendicular to AD . The gradient of the line DE is $-\frac{3}{4}$.

(b) Find the coordinates of E .

(b)	$y = -\frac{3}{4}x + c$ $4 = -\frac{3}{4}(1) + c$ $c = \frac{19}{4}$ <p>Equation of DE: $y = -\frac{3}{4}x + \frac{19}{4}$</p> $-\frac{3}{4}x + \frac{19}{4} = \frac{4}{3}x - 4$ $x = \frac{21}{5}$ $y = \frac{8}{5}$ <p>Coordinates of E: $\left(\frac{21}{5}, \frac{8}{5}\right)$ or $\left(4\frac{1}{5}, 1\frac{3}{5}\right)$</p>	<p>M1</p> <p>M1</p> <p>A1</p>
-----	---	-------------------------------

(c) Given that the length of $BE = 2$ units, hence, find the area of $ABED$.

(c)	$DE = \sqrt{\left(\frac{21}{5} - 1\right)^2 + \left(\frac{8}{5} - 4\right)^2}$ $= \sqrt{16}$ $= 4 \text{ units}$ <p>Area of $ABED = \frac{1}{2}(2+5)(4)$</p> $= 14 \text{ units}^2$	<p>M1, accept shoelace method</p> <p>A1</p>
-----	--	---

3 (a) A group of 24 tourists visited the National Museum.

One of the 24 tourists is selected at random. The probability that it is a Korean male tourist is $\frac{1}{3}$. By showing clear workings, complete the table of information about the 24 tourists.

	Male	Female
Korean		5
Japanese		2

(a)	<p>Number of Korean male tourists</p> $= \frac{1}{3} \times 24$ $= 8$	<p>M1</p>
-----	---	-----------

- | | | |
|-----|---|---------------------|
| (b) | $P(\text{both tourists are males})$ $= \left(\frac{8+7}{24} \right) \left(\frac{8+7-1}{24-1} \right)$ $= \frac{35}{92}$ | <p>M1</p> <p>A1</p> |
|-----|---|---------------------|

- | | | |
|----------|--|----|
| 4 | $50 + 8x < 15x$ | M1 |
| | $50 < 7x$ | M1 |
| | $x > 7\frac{1}{7}$ | A1 |
| | \therefore James will have to pay for a minimum of
8 months for his scheme to be cheaper than Terry. | B1 |

(a) Express angle ABC in terms of x .

(a)	Angle $ABO = 2x^\circ$ (isocles triangle, $OA = OB$ same radii)	M1
	Angle $OBC = 3x^\circ$ (isocles triangle, $OB = OC$ same radii)	
	Angle $ABC = 2x^\circ + 3x^\circ$ $= 5x^\circ$	A1
	OR $\angle ABC + \angle ADC = 180^\circ$ (angles in opposite segments are supplementary) $\angle ABC + 2.5x^\circ = 180^\circ$ $\angle ABC = (180 - 2.5x)^\circ$	M1 A1

(b) Find the value of x .

(b)	$\angle ABC + \angle ADC = 180^\circ$ (angles in opposite segments are supplementary)	M1
	$5x + 2.5x = 180^\circ$	
	$7.5x = 180^\circ$	
	$x = 24$	A1

6 (a) Given that $\frac{5x-3a}{8} - \frac{3x-4a}{6} = 1$, express x in terms of a .

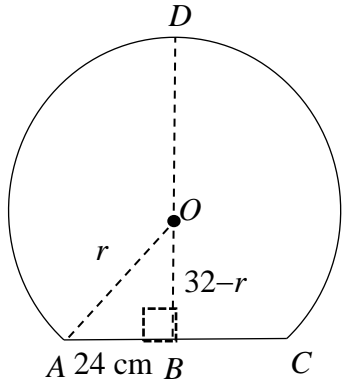
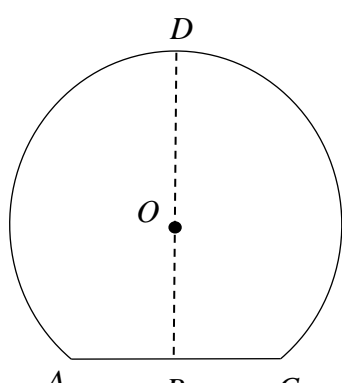
(a)	$\frac{5x-3a}{8} - \frac{3x-4a}{6} = 1$	M1 for combining into a single algebraic fraction
	$\frac{3(5x-3a) - 4(3x-4a)}{24} = 1$	
	$15x - 9a - 12x + 16a = 24$	
	$3x = 24 - 7a$	A1 for expressing x in terms of a
	$x = \frac{24-7a}{3}$	

(b) Find the largest positive integer value of a if $x \geq -29$.

(a)	Given that $x \geq -29$,	M1 for forming this inequality
	$\frac{24-7a}{3} \geq -29$	
	$24 - 7a \geq -87$	M1 for solving this inequality
	$-7a \geq -87 - 24$	
	$a \leq \frac{-111}{-7}$	
	$a \leq 15\frac{6}{7}$	A1
	\therefore The largest integer value of $a = 15$	

- 7 (a) $ABCD$ is a major segment of a circle, centre O . BD is 32 cm, AC is 48 cm and angle DBA is $\frac{\pi}{2}$.

(i) Show that the radius of the circle is 25 cm.

(ai)	<p>Let the radius of the circle be r cm.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> $r^2 = \left(\frac{48}{2}\right)^2 + (32-r)^2$ $r^2 - (32-r)^2 = 24^2$ $(r-32+r)(r+32-r) = 576$ $(2r-32)32 = 576$ $2r-32 = \frac{576}{32}$ $2r-32 = 18$ $2r = 50$ $r = 25$ <p>The radius of the circle is 25 cm. (shown)</p>	<p>M1 for forming an equation to solve for radius.</p> <p>M1 for solving the equation to find the radius</p> <p>A1</p>
------	---	--

(ii) Calculate the area of the major segment $ABCD$.

(aii)	$\sin \angle AOB = \frac{24}{25}$ $\angle AOB = 1.2870$ $\angle AOC = 1.2870 \times 2$ ≈ 2.574	<p>M1 for finding angle AOC.</p>
-------	--	---

	<p>Area of major segment $AODC$</p> <p>$=$ Area of major sector $OADC$ + Area of triangle $OABC$</p> <p>$= \frac{1}{2} r^2 (\text{reflex } \angle AOC) + \frac{1}{2} r^2 \sin(\text{acute } \angle AOC)$</p> <p>$= \frac{1}{2} (25)^2 (2\pi - 2.574) + \frac{1}{2} (25)^2 \sin(2.574)$</p> <p>$\approx 1327.12$</p> <p>$\approx 1330 \text{ cm}^2$</p>	<p>M2 (one mark for each part)</p> <p>A1</p>
--	--	--

(b) The sector $OADC$ was cut from the above to form a cone where the OA is glued to OC .

(i) Calculate the circumference of the base circle of the cone.

(bi)	<p>Circumference of the base of the cone</p> <p>$=$ arc length of the major sector $OADC$</p> <p>$= r(\text{reflex } \angle AOC)$</p> <p>$= 25(2\pi - 2.574)$</p> <p>$\approx 92.730$</p> <p>$\approx 92.7 \text{ cm}$</p>	<p>M1</p> <p>A1</p>
------	--	---------------------

(ii) Hence, calculate the vertical height of the cone.

(bii)	<p>Radius of base circle</p> <p>$= \frac{92.730}{2\pi}$</p> <p>$\approx 14.758 \text{ cm}$</p> <p>Let the vertical height be $h \text{ cm}$.</p> <p>$25^2 = h^2 + (14.758)^2$</p> <p>$h^2 = 25^2 - (14.758)^2$</p> <p>$h = \sqrt{25^2 - (14.758)^2}$</p> <p>$h \approx 20.179$</p> <p>$h \approx 20.2$</p> <p>The vertical height is about 20.2 cm</p>	<p>M1</p> <p>M1</p> <p>A1</p>
-------	--	-------------------------------

8 (a) The heights of a group of 30 students were measured and the results are shown in the stem-and-leave diagram.

14	4 5 5 7 7 7 8 8 8 9 9 9 9
15	1 1 2 3 3 3 3 5 5 6 9 9 9
16	0 3 8
17	
18	9

<p>(ai)</p>	<p>Position of median height</p> $= \frac{30+1}{2}$ $= 15.5$ <p>= between 15th and 16th position</p> <p>Median height = $\frac{151+152}{2}$</p> $= 151.5 \text{ cm}$	<p>B1</p>
--------------------	--	-----------

(a)	Mean is not an appropriate measure as there is an outlier of 189 cm	B1
-----	---	----

Height (h cm)	$140 \leq h < 150$	$150 \leq h < 160$	$160 \leq h < 170$	$170 \leq h < 180$
Frequency	p	9	11	q

[illegible]

	$p = 10 - 6$ $p = 4$	A1 for both answers
--	-------------------------	---------------------

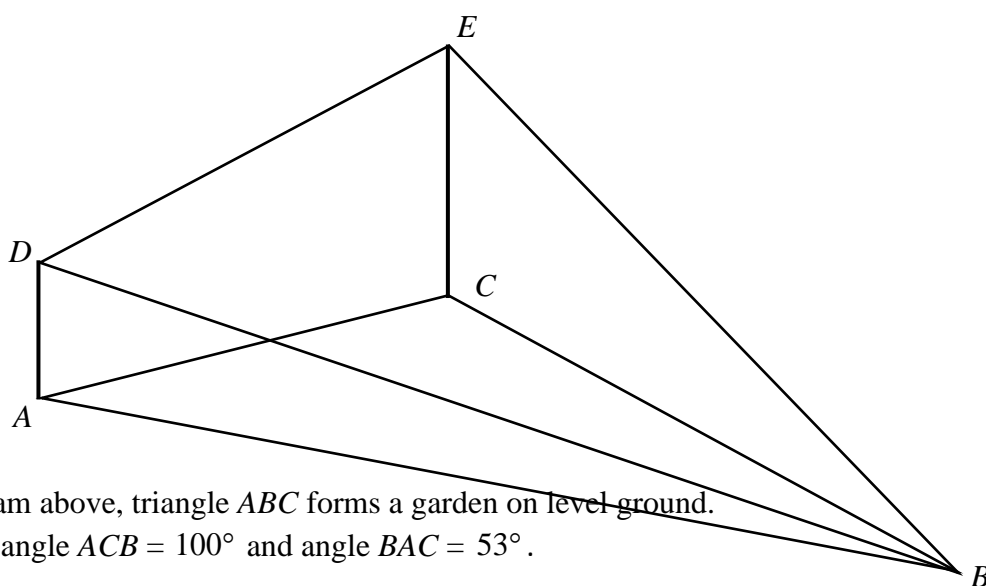
(ii) Calculate an estimate of the standard deviation.

(bii)	Estimated standard deviation ≈ 9.2912 cm ≈ 9.29 cm	B1
-------	--	----

(iii) Given that the estimated standard deviation of the first group of students is 8.83 cm, make one comparison between the heights of the two groups of students.

(biii)	Since the first group of students have a lower standard deviation of 8.83 cm as compared to the second group which is 9.29 cm, the heights of the first group of students are generally more consistent.	B1
--------	--	----

9



In the diagram above, triangle ABC forms a garden on level ground.
 $BC = 32$ m, angle $ACB = 100^\circ$ and angle $BAC = 53^\circ$.

(a) Show that $AC = 18.19$ m, correct to 4 significant figures.

(a)	$\angle ABC = 180^\circ - 100^\circ - 53^\circ$ $= 27^\circ$ (adj. \angle s on a str. line) $\frac{AC}{\sin 27^\circ} = \frac{BC}{\sin 53^\circ}$ $AC = \frac{32 \sin 27^\circ}{\sin 53^\circ}$ $= 18.19065$ $= 18.19$ m	M1 M1: for correct use of sine rule.
-----	---	---

(b) Triangle BDE is a roof designed for the garden. Two vertical poles, $AD = 2$ m and $CE = 5$ m, were built to hold up the roof. Find

(i) the length of DE ,

(bii)	$\frac{AB}{\sin 100^\circ} = \frac{BC}{\sin 53^\circ}$	
	$AB = \frac{32 \sin 100^\circ}{\sin 53^\circ}$	
	$= 39.4596 \text{ m}$	M1
	$BD = \sqrt{3^2 + 39.4596^2}$	
	$= 39.5735 \text{ m}$	M1
	$BE = \sqrt{5^2 + 32^2}$	
	$= 32.3883 \text{ m}$	M1
	$DE^2 = BD^2 + BE^2 - 2(BD)(BE)\cos \angle DBE$	
	$18.4357^2 = 39.5735^2 + 32.3883^2 - 2(39.5735)(32.3883)\cos \angle DBE$	
	$\cos \angle DBE = 0.887554$	M1
$\angle DBE = \cos^{-1} 0.887554$		
$= 27.4325^\circ$		
$= 27.4^\circ$	A1	

- Level 1 Level 2

$$C = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix} \begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix}$$

(a) Represent the payment per session with a 2×1 matrix, \mathbf{N} .

(a)	$\begin{pmatrix} 80 \\ 120 \end{pmatrix}$	B1
-----	---	----

(b) Evaluate $\mathbf{Q} = \mathbf{CN}$.

(b)	$\mathbf{Q} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 80 \\ 120 \end{pmatrix}$ $= \begin{pmatrix} 3 \times 80 + 0 \times 120 \\ 2 \times 80 + 1 \times 120 \end{pmatrix}$ $= \begin{pmatrix} 240 \\ 280 \end{pmatrix}$	B1
-----	---	----

(c) Describe the elements in \mathbf{Q} .

(c)	\mathbf{Q} represent the total amount of course fee collect on weekday and weekend respectively.	B1
-----	--	----

(d) Using matrix multiplication, calculate his total weekly income.

(d)	<p>Total weekly collection</p> $= (1 \ 1) \begin{pmatrix} 240 \\ 280 \end{pmatrix}$ $= (1 \times 240 + 1 \times 280)$ $= (520)$ <p>His weekly income is \$520.</p>	B1
-----	--	----

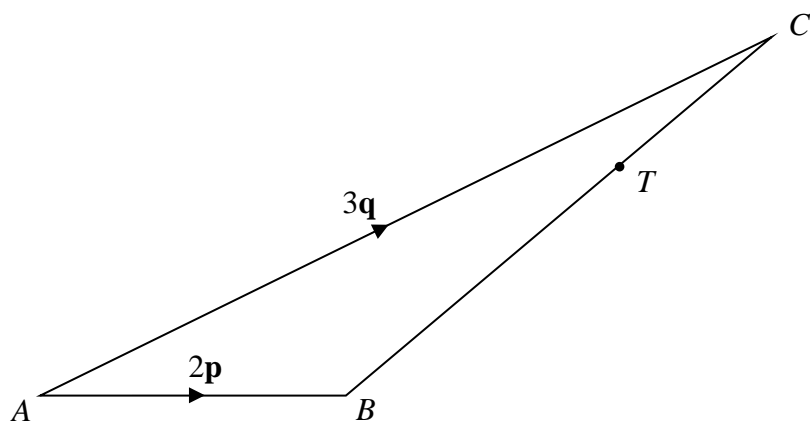
(e) Due to the popularity of the courses, Billy is paid 5% more for each session of Level 1 and Level 2 course he conducts. The matrix \mathbf{D} shows the number of sessions he teaches each week.

$$D = \begin{matrix} & \begin{matrix} \text{Level 1} & \text{Level 2} \end{matrix} \\ \begin{pmatrix} 4 & x \\ 3 & y \end{pmatrix} & \begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix} \end{matrix}$$

Given that his total income on weekdays and weekends are \$462 and \$504 respectively, find the value of x and of y .

(e)	<p>New fee = $1.05 \begin{pmatrix} 80 \\ 120 \end{pmatrix} = \begin{pmatrix} 84 \\ 126 \end{pmatrix}$</p> $\begin{pmatrix} 4 & x \\ 3 & y \end{pmatrix} \begin{pmatrix} 84 \\ 126 \end{pmatrix} = \begin{pmatrix} 462 \\ 504 \end{pmatrix}$ $\begin{pmatrix} 4 \times 84 + 126x \\ 3 \times 84 + 126y \end{pmatrix} = \begin{pmatrix} 462 \\ 504 \end{pmatrix}$ $\therefore 336 + 126x = 462 \quad \text{and} \quad 252 + 126y = 504$ $x = 1 \quad \text{and} \quad y = 2$	B2 – 1 mark each for answer
-----	---	-----------------------------------

11



ABC is a triangle. $\overrightarrow{AB} = 2\mathbf{p}$ and $\overrightarrow{AC} = 3\mathbf{q}$.

T is a point on BC such that $CT : TB = 2 : 3$.

(a) Express, as simply as possible, in terms of \mathbf{p} and/or \mathbf{q} ,

(i) \overrightarrow{BC} ,

(ai)	$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ $= -2\mathbf{p} + 3\mathbf{q}$	B1
------	--	----

(ii) \overrightarrow{TC} .

(aii)	$\overrightarrow{TC} = \frac{2}{5} \overrightarrow{BC}$ $= \frac{2}{5} (-2\mathbf{p} + 3\mathbf{q})$	B1 accepted if expanded
-------	--	-------------------------

(b) R is a point on AC such that triangle ABC is similar to triangle RTC .

Express \overrightarrow{RC} , as simply as possible, in terms of \mathbf{p} and/or \mathbf{q} .

(b)	$\overrightarrow{RC} = \frac{2}{5} \overrightarrow{AC}$ $= \frac{2}{5} (3\mathbf{q})$ $= \frac{6}{5} \mathbf{q}$	B1
-----	--	----

(c) Find the ratio of

(i) $\frac{\text{area of triangle } RTC}{\text{area of triangle } ABC},$

(ci)	$\frac{\text{area of triangle } RTC}{\text{area of triangle } ABC} = \left(\frac{2}{5}\right)^2$ $= \frac{4}{25}$	B1
------	---	----

(ii) $\frac{\text{area of triangle } RBC}{\text{area of triangle } ABC}.$

(cii)	$\frac{\text{area of triangle } RBC}{\text{area of triangle } ABC} = \frac{RC}{AC}$ $= \frac{2}{5}$	B1
-------	---	----

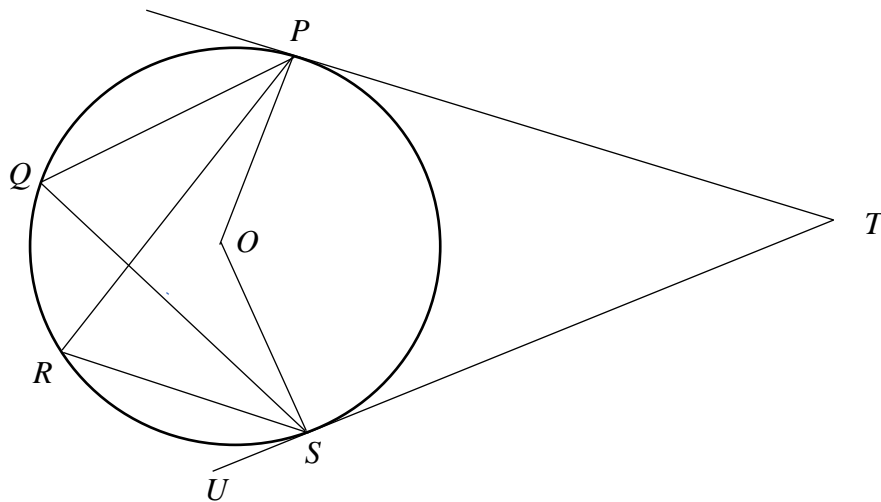
(d) S is a point on AC .

Triangle ASB has the same area as triangle SBC .

Express \overrightarrow{SB} , as simply as possible, in terms of \mathbf{p} and/or \mathbf{q} .

(d)	S is a midpoint of AC . $\overrightarrow{SB} = \overrightarrow{SA} + \overrightarrow{AB}$ $= \frac{1}{2}(-\overrightarrow{AC}) + 2\mathbf{p}$ $= 2\mathbf{p} - \frac{3}{2}\mathbf{q}$	M1: for recognising midpoint A1
-----	--	---

12



In the diagram, P , Q , R and S are four points on a circle with centre O . PT and ST are tangents to the circle at P and S respectively. Angle $PRS = 68^\circ$ and angle $QSU = 70^\circ$. Find, giving reasons for each answer,

(a) angle PQS ,

(a)	Angle $PQS = 68^\circ$ (angle in the same segment)	B1
-----	--	----

(b) angle OSU ,

(b)	Angle $OSU = 90^\circ$ (radius perpendicular to tangent)	B1
-----	--	----

(c) angle PTS .

(c)	Angle $POS = 2 \times 68^\circ$ (angle at centre is 2 times angle at circumference) $= 136^\circ$	M1
	Angle $PTS = 360^\circ - 90^\circ - 90^\circ - 136^\circ$ (sum of angle in quadrilateral) $= 44^\circ$	A1

13 (a) Complete the table of values for $y = \frac{x^3}{5} - x + 4$ below.

x	-3.5	-3	-2	-1	0	1	2	3
y	-1.1	1.6	4.4 (B1)	4.8	4	3.2	3.6	6.4

(b) Using a scale of 2 cm to represent 1 unit on both axes, plot the points given in the table and join them with a smooth curve for $-3.5 \leq x \leq 3$.

(b)		B1 for points plotted correctly B1 for smoothness B1 for labelling of axes and curve
-----	--	--

(c) By drawing a tangent, find the gradient of the curve at $x = 2$.

(c)	Gradient $= 1.38 \pm 0.1$	M1 for showing the working A1
-----	---------------------------	----------------------------------

(d) Use your graph to find the solutions of the equation $\frac{x^3}{5} - \frac{1}{2}x = 0$ for $-4 \leq x \leq 3$.

(d)	$\frac{x^3}{5} - \frac{1}{2}x = 0$ $\frac{x^3}{5} = \frac{1}{2}x$ $\frac{x^3}{5} - x + 4 = \frac{1}{2}x - x + 4$ $y = -\frac{1}{2}x + 4$ $x = -1.45, 0, 1.45$	<p>M1 for drawing this line to intersect curve</p> <p>A1</p>
-----	---	--

- (e) The solutions in part (d) are also the solutions for the equation $2x^3 + Ax^2 + Bx = 0$. Find the value of A and the value of B .

(e)	$\frac{x^3}{5} - x + 4 = -\frac{1}{2}x + 4$ <p>Multiply by 10,</p> $2x^3 - 10x + 40 = -5x + 40$ $2x^3 - 5x = 0$ $\therefore A = 0, B = -5$	<p>M1</p> <p>A1 for both values</p>
-----	--	-------------------------------------

- 15 Mrs Lee intends to buy a 7-seater car so that she can bring her family out for outings. Typically a 7-seater car has an engine capacity that is more than 1600 cc. After doing some research, she realizes that the total cost of purchasing a car includes the following components which are summarised in Table 1.

Table 1: Costs Incurred in Car Purchase

Category	A	B
Engine Capacity (in cc)	≤ 1600	> 1600
Certificate of Entitlement, COE Prices	\$74 989	\$106 001
Open Market Value, OMV	\$15,602 – \$41,124	\$26,211 – \$74,924
Road Tax, RT	\$371.45 per 6 months	\$606.05 per 6 months
Insurance	\$1,684 per annum	
Additional Registration Fee, (ARF)	Refer to Table 2 for the guide to calculating ARF	
Goods and Services Tax, GST	7% of the OMV	

Table 2: Guide to Calculating ARF

Additional Registration Fee, (ARF)	Chargeable ARF	Rate	ARF Payable
	First \$20,000:	100% of OMV	\$20,000
	Next \$30,000:	140% of OMV	\$42,000
	Next \$30,000:	180% of OMV	\$54,000

	In excess of \$80,000:	220% of OMV	\$176,000
--	------------------------	-------------	-----------

- (a) How much is the maximum amount of the total Additional Registration Fee that Mrs Lee will have to pay for a 7-seater car.

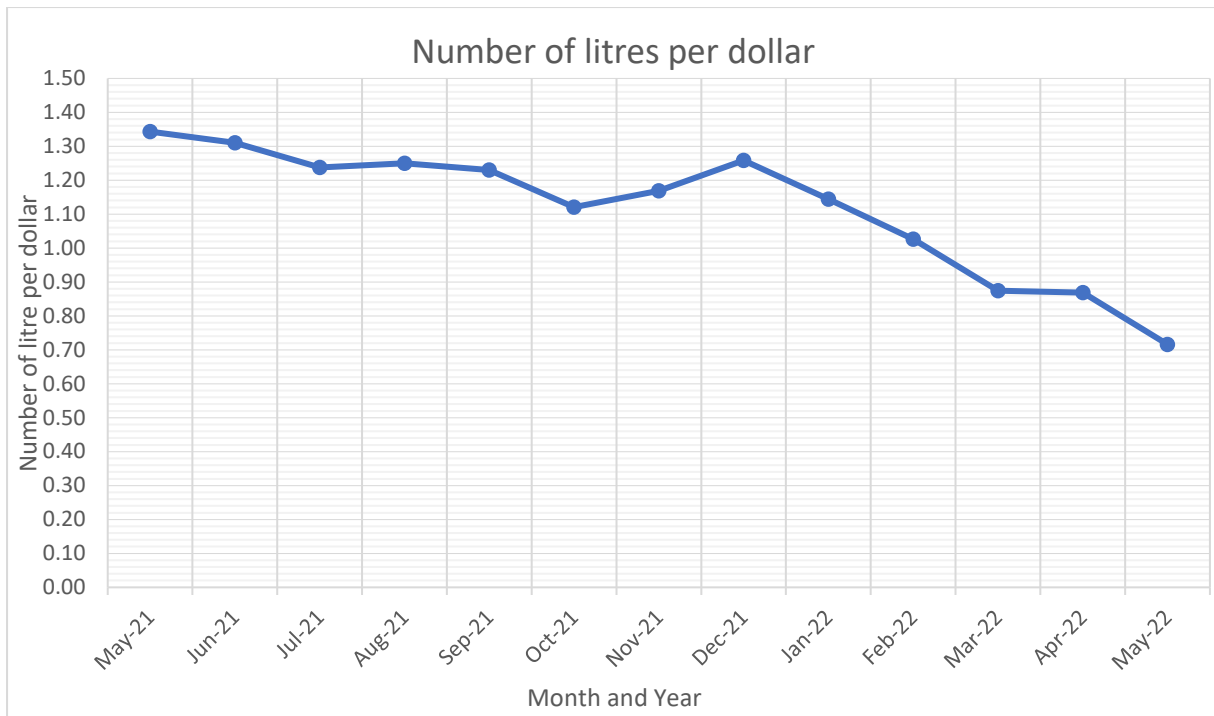
(a)	<p>For an OMV of \$74 924,</p> $\text{ARF of the last } \$24\,924 = \frac{180}{100} \times \$24\,924$ $= \$44863.20$ <p>Total max amount of ARF to be paid</p> $= \$20\,000 + \$42\,000 + \$44863.20$ $= \$10\,6863.20$ <p>If they choose Category B but value is not the max, we give method marks and not A1.</p> <p>If they choose Category A, zero marks.</p>	<p>M1</p> <p>M1</p> <p>A1</p>
-----	---	-------------------------------

- (b) What is the total cost of the car, excluding the road tax and the insurance.

(b)	<p>GST to be paid = $\frac{7}{100} \times \\$74\,924$</p> $= \$5244.68$ <p>Total cost of a car excluding insurance and road tax</p> $= \text{COE} + \text{OMV} + \text{ARF} + \text{GST}$ $= \$106\,001 + \$74\,924 + \$106\,863.20 + \$5\,244.68$ $= \$293\,032.88$	<p>M1</p> <p>A1</p>
-----	---	---------------------

- (c) The car company allows Mrs Lee to pay 30% of the total cost as down-payment and the subsequent amount at a simple interest rate of 1.5% per annum over 7 years. To ensure that her monthly salary is enough to pay for the monthly instalment of the car and the petrol, she will need to calculate how much money she needs to set aside every month for the car.

Mrs Lee also found the following information about the petrol prices over the last one year and the national average distance travelled is 1000 km per month with one litre of petrol being able to cover an average of 12 km in city area or 15 km on highway.



Suggest a suitable amount of money that Mrs Lee need to set aside every month in her bank to pay for **all** expenses of the car during the first seven years.

Justify any decisions you made and show your calculations clearly.

(c)	<p>Amount owed before interest</p> $= \frac{70}{100} \times \$293\,032.88$ $= \$205\,123.016$ <p>Total amount owed including interest</p> $= \frac{1.5}{100} \times \$205\,123.016 \times 7 + \$205\,123.016$ $= \$226\,660.9327$ <p>Monthly cost of a car excluding petrol cost, insurance and road tax</p> $= \frac{\$226\,660.9327}{7 \times 12}$ $= \$2\,698.3444$ <p>Cost of petrol for a month = $\frac{1000}{12} \div 0.72$</p> $= \$115.7407$ <p>12km is chosen to calculate for the worst case scenario where Mrs Lee has to purchase the maximum amount of petrol.</p> <p>0.72 litres per dollar is chosen to calculate for the worst case scenario where petrol price is the highest.</p>	<p>T1</p> <p>I1</p> <p>P1</p> <p>J₁-1 mark</p> <p>J₂ – 1 mark</p>
-----	---	---

	<p> Total monthly cost of a car = $\\$2\,698.3444 + \\$115.7407 + \frac{\\$606.05}{6} + \frac{\\$1684}{12}$ $= \\$3\,055.4268$ $\approx \\$3\,055.43$ </p> <p>Therefore, Mrs Lee should have at least about \$3100 in the bank per month to pay for the costs of the car.</p>	<p>RT + I = 1mark</p> <p>C1 (award this mark as long as student suggests at least \$10 more. in multiples of \$10, \$100 or \$1000)</p>
--	--	---