1 By finding the expansion of $(1+3x)^{-1}$ or otherwise, find the expansion of $\frac{\sqrt{1+2x}}{1+3x}$ in ascending powers of $x$, up to and including the term in $x^2$. Find the range of values of $x$ for which the expansion is valid. [4]

Solution

$$(1+3x)^{-1}=1+(-1)(3x)+\frac{(-1)(-2)}{2!}(3x)^2+\ldots$$

Commented [NCM1]: Some apply Maclurins Theorem instead of Binomial Theorem. Some wrongly applied the Binomial expansion for positive integer n.

$$\frac{\sqrt{1+2x}}{1+3x}=(1+2x)^{\frac{1}{2}}(1+3x)^{-1}$$

$$=\left(1+\left(\frac{1}{2}\right)(2x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(2x)^2+\ldots\right)\left(1+(-1)(3x)+\frac{(-1)(-2)}{2!}(3x)^2\ldots\right)$$

$$=\left(1+x-\frac{1}{2}x^2+\ldots\right)\left(1-3x+9x^2\ldots\right)$$

$$=1-3x+x-\frac{1}{2}x^2+9x^2-3x^2+\ldots$$

$$=1-2x+\frac{11}{2}x^2+\ldots$$

Commented [NCM2]: The expansion should be in ascending powers of x, and not quadratic expression.

Valid for $|2x|<1$ and $|3x|<1$

$-\frac{1}{2}<x<\frac{1}{2}$ and $-\frac{1}{3}<x<\frac{1}{3}$

Commented [NCM3]: Final validity range of x should be the intersection of the 2 sets.

$\therefore -\frac{1}{3}<x<\frac{1}{3}$

2 Using an algebraic method, solve $\dfrac{x^2+2x+3}{x^2+x-2} \geq 1$. [4]

Commented [LDL4]: Question reading
Pay attention to the requirement of the question. Some students solve by graphical method.

Solution

$$\frac{x^2+2x+3}{x^2+x-2} - 1 \geq 0$$

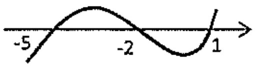
$$\frac{x^2+2x+3-x^2-x+2}{x^2+x-2} \geq 0$$

$$\frac{(x+5)}{(x-1)(x+2)} \geq 0, \quad x \neq -2, 1$$

Commented [LDL5]: Algebraic manipulation
Many were careless with the negative sign and got the wrong expression for the numerator.

$$(x+5)(x-1)(x+2) \geq 0$$

Commented [LDL6]: Concept / Method
A number of students wasted precious time in expanding out all the terms instead of the factorised form.




From the graph, $\therefore -5 \leq x < -2$ or $x > 1$

Commented [LDL7]: Carelessness
Make it a good practice to exclude the values of x which cannot be included at the start of the question.

3 It is given that $(y+x)^3=3x+1$.

Commented [NCY8]: This question generally is well answered.

**(a)** Show that $2\left(\frac{\mathrm{d}y}{\mathrm{d}x}+1\right)^2+(y+x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$. [2]

Solution

$(y+x)^3=3x+1$

$3(y+x)^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}+1\right)=3$

$2(y+x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}+1\right)^2+(y+x)^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$

$2\left(\frac{\mathrm{d}y}{\mathrm{d}x}+1\right)^2+(y+x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$

**(b)** Hence find the Maclaurin series of $y$ up to and including the term in $x^3$. [3]

Commented [NCY9]: Concept
The expectation is to use the 2nd derivative expression in (a) and do implicit differentiation to get 3rd derivative and NOT attempt to make y the subject then differentiate

Solution

$4\left(\frac{\mathrm{d}y}{\mathrm{d}x}+1\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}+1\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(y+x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=0$

$5\left(\frac{\mathrm{d}y}{\mathrm{d}x}+1\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(y+x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=0$

When $x=0$,

$$y=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=0,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2,\ \frac{\mathrm{d}^3y}{\mathrm{d}x^3}=10.$$

So, $y=1+0x+\frac{-2}{2}x^2+\frac{10}{3!}x^3+\cdots$

$=1-x^2+\frac{5}{3}x^3+\cdots$

4 **Do not use a calculator in answering this question.**

It is given that $f(z)=z^4-2z^3+Az^2+Bz+6$, where $A$ and $B$ are real constants. The equation $f(z)=0$ has a root $z=1+i$. Find the values of $A$ and $B$, and the other roots of the equation $f(z)=0$. [5]

**Commented [TLR10]:** Question reading
Do show all your workings clearly and give them in exact form.

Solution

Since all the coefficients of $f(z)$ are all real and $1+i$, then $1-i$ is also a root.

Quadratic factor: $(z-1-i)(z-1+i)=z^2-2z+2$

**Commented [TLR11]:** Presentation
Many students did not explain why $1-i$ is also a root.

**Commented [TLR12]:** Calculation
The correct way is
$[(z-1)-i][(z-1)+i]=(z-1)^2-i^2$
Many incorrectly wrote
$[z-(1+i)][z+(1+i)]=z^2+(1+i)^2$

Let $f(z)=z^4-2z^3+Az^2+Bz+6=(z^2-2z+2)(z^2+pz+q)$

Compare coefficients of $z^3$: $-2=p-2 \quad \Rightarrow p=0$

constants: $6=2q \quad \Rightarrow q=3$

of $z^2$: $A=q+2-2p \quad \Rightarrow A=5$

of $z$: $B=-2q+2p \quad \Rightarrow B=-6$

**Commented [TLR13]:** Presentation
Many attempt to compare coefficients, however unclear in what they are actually comparing.

Therefore $f(z)=(z^2-2z+2)(z^2+3)=(z-1-i)(z-1+i)(z-i\sqrt{3})(z+i\sqrt{3})$

The other roots are $1-i$, $i\sqrt{3}$ and $-i\sqrt{3}$.

**Commented [TLR14]:** Concept
The roots for this quadratic factor $(z^2+3)$ can simply be obtained by
$z^2=-3 \Rightarrow z=\pm\sqrt{-3}=\pm i\sqrt{3}$
Some explained that "$z^2 \neq -3$ hence no roots" which is incorrect.

Alternative method

$(1+i)^2=1+2i-1=2i$

$(1+i)^3=2i(1+i)=2i-2$

$(1+i)^4=(2i)^2=-4$

Since $(1+i)$ is a root of $f(z)=0$, $f(1+i)=0$.

$$(1+i)^4-2(1+i)^3+A(1+i)^2+B(1+i)+6=0$$
$$-4-2(2i-2)+A(2i)+B(1+i)+6=0$$

Compare real terms: $-4+4+B+6=0 \quad \Rightarrow B=-6$

Compare imaginary terms: $-4+2A+B=0 \quad \Rightarrow A=5$

**Commented [TLR15]:** Note that this method is not encouraged since you are not supposed to use calculator.

5 Solve the following simultaneous equations

$$iz + w = 1$$

$$(1+i)w^* - 2z = 0,$$

giving $z$ and $w$ in the form $a + b$i where $a$ and $b$ are real numbers to be determined. [5]

Solution

Commented [FYJ16]: Conceptual understanding
Many students think that $w = 1 - iz, w^* = 1 + iz$. That is a wrong concept as Z is a complex number which contains both real and imaginary parts.

Given $iz + w = 1$ --- (1)

$(1+i)w^* - 2z = 0$ --- (2)

$2i \times (1) - (2)$:

$2iw - (1+i)w^* = 2i$

Let $w = x + yi$

$2i(x+yi) - (1+i)(x-yi) = 2i$

$2xi - 2y - x + (y-x)i - y = 2i$

Comparing real parts: $-2y - x - y = 0$

$x = -3y$ --- (3)

Comparing imaginary parts: $2x + y - x = 2$

$x + y = 2$ --- (4)

Substitute (3) into (4): $-3y + y = 2$

$y = -1$

Substitute $y = -1$ into (3): $x = 3$

Therefore $w = 3 - i$

Substitute $w = 3 - i$ into (1): $iz + 3 - i = 1$

Multiply by i: $-z + 3i + 1 = i$

$z = 1 + 2i$

Commented [FYJ17]: Careless
Students who did not make the fatal wrong conceptual of $w = 1 - iz, w^* = 1 + iz$ will manage to solve and get most of the method marks. However, students are very careless with their manipulation, which resulted in wrong answers.

**6 (a)** Find $\int x\tan^{-1}(2x)\,dx$. [3]

Solution

$$\int x\tan^{-1}(2x)\,dx = \frac{x^2}{2}\tan^{-1}(2x) - \int \frac{x^2}{2}\frac{2}{1+(2x)^2}\,dx$$

$$= \frac{1}{2}x^2\tan^{-1}(2x) - \frac{1}{4}\int 1 - \frac{1}{1+4x^2}\,dx$$

$$= \frac{1}{2}x^2\tan^{-1}(2x) - \frac{1}{4}x + \frac{1}{16}\int \frac{1}{x^2+\left(\frac{1}{2}\right)^2}\,dx$$

$$= \frac{1}{2}x^2\tan^{-1}(2x) - \frac{1}{4}x + \frac{1}{16}\left(\frac{1}{\frac{1}{2}}\right)\tan^{-1}\left(\frac{x}{\frac{1}{2}}\right) + c$$

$$= \frac{1}{2}x^2\tan^{-1}(2x) - \frac{1}{4}x + \frac{1}{8}\tan^{-1}(2x) + c$$

$\int \frac{1}{1+4x^2}$

$= \int \frac{1}{1^2+(2x)^2}$

$= \left[\frac{1}{1}\tan^{-1}\frac{2x}{1}\left(\frac{1}{2}\right)\right]$

**Commented [FYJ18]: Manipulation**
Many students are not able to apply the integration by parts formula correctly.
Many students do not know what to do when they see $\int \frac{x^2}{1+(2x)^2}\,dx$. Many tried to do by parts again and end up not being able to solve the question.

**Commented [FYJ19]:** $\int \frac{1}{1+(2x)^2}\,dx$ will give you $\tan^{-1}$, NOT ln. Students need to memorise/practice more to improve on their integration techniques.

**(b)** Use the substitution $u = 2-3x$ to find $\int \frac{x}{\sqrt{2-3x}}\,dx$. [3]

Solution

Let $u = 2-3x \Rightarrow x = \frac{2-u}{3}$

$\frac{du}{dx} = -3 \quad \Rightarrow \frac{dx}{du} = -\frac{1}{3}$

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \int \frac{2-u}{3}\frac{1}{\sqrt{u}}\left(-\frac{1}{3}\right)du$$

$$= \frac{1}{9}\int u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{9}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2u^{\frac{1}{2}}}{\frac{1}{2}}\right) + c$$

$$= \frac{2}{27}(2-3x)^{\frac{3}{2}} - \frac{4}{9}(2-3x)^{\frac{1}{2}} + c$$

$$= -\frac{2}{27}(2-3x)^{\frac{1}{2}}(3x+4) + c$$

**Commented [FYJ20]:** Many students did not know how to solve $\int \frac{2-u}{\sqrt{u}}\,du$, and attempt to do integration by parts. Also, some students were confused and did differentiation instead of integration.

**Commented [FYJ21]: Presentation**
Many students forgot to change back into $x$.

7 Referred to the origin, the position vectors of two points $A$ and $B$ are $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-parallel and non-zero. The point $C$, with position vector $4\mathbf{b}-\lambda\mathbf{a}$, lies on $AB$ produced such that $AB : BC = \alpha : 1$, where $\alpha$ and $\lambda \in \mathbb{R}$.

**(i)** Find the values of $\alpha$ and $\lambda$. [3]

Solution

$\overrightarrow{AB} = \alpha\overrightarrow{BC}$

i.e. $\mathbf{b}-\mathbf{a} = \alpha(4\mathbf{b}-\lambda\mathbf{a}-\mathbf{b})$

Since $\mathbf{a}$ is not parallel to $\mathbf{b}$ and $\mathbf{a}$ and $\mathbf{b}$ are non-zero,

$1 = 3\alpha$ and $1 = \alpha\lambda$

$\therefore \alpha = \frac{1}{3}$ and $\lambda = 3$

Alternative method

$\mathbf{b} = \dfrac{\mathbf{a}+\alpha\mathbf{c}}{\alpha+1}$

$(\alpha+1)\mathbf{b} = \mathbf{a}+\alpha(4\mathbf{b}-\lambda\mathbf{a})$

$1 = 3\alpha$ and $0 = -\alpha\lambda + 1$

$\therefore \alpha = \frac{1}{3}$ and $\lambda = 3$

comparing

$\alpha b + b = a + 4b\alpha - \lambda\alpha a$

$(m>1)$

The point $D$ lies on $OB$ produced such that $OB : OD = 1 : m$, where $m \in \mathbb{R}$.

**(ii)** Show that the area of triangle $ABD$ can be written as $k|\mathbf{a}\times\mathbf{b}|$ where $k$ is an expression in terms of $m$ to be determined. [2]

Solution

**Method 1**

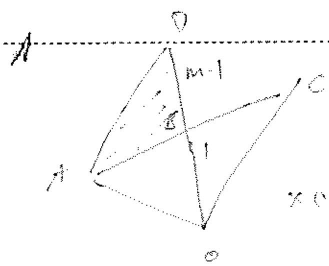
Area of triangle $ABD$

$= \left|\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{BD}\right|\right|$

$= \frac{1}{2}|(\mathbf{b}-\mathbf{a})\times(m-1)\mathbf{b}|$

$= \frac{m-1}{2}|(\mathbf{b}\times\mathbf{b})-(\mathbf{a}\times\mathbf{b})|$ since $\mathbf{b}\times\mathbf{b}=0$

$= \frac{m-1}{2}|\mathbf{a}\times\mathbf{b}|$ where $k = \frac{m-1}{2}$



$\times$ OA/OD as values

**Commented [LDL22]: Wrong concept**
There is no division of vectors. Vector has both direction and magnitude. It is WRONG to write $\frac{\overline{AB}}{\overline{BC}} = \frac{\alpha}{1}$. This is a common misconception which was evident in part (ii) too.

**Commented [LDL23]: Presentation**
This is an important condition which many omitted in their presentation before you can compare the coefficients.

**Commented [LDL24]: Concept / Method**
This is the more commonly used method by most students. Some applied ratio theorem wrongly – wrong formula or wrong ratio.

**Commented [LDL25]: Application of formula**
1. Wrong formula without ½ or modulus sign
2. Wrong choice of vectors e.g. ½ |a x d|
3. Wrong interpretation of ratio for vector BD

**Commented [LDL26]: Careless mistake**
Many students gave the answer $k = \frac{1-m}{2}$. Based on the question, D lies on OB produced which implies that m>1. Be careful as you bring out this constant and remove the modulus sign.

**Method 2:**

Area of triangle $OAB = \frac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OB}\right| = \frac{1}{2}|\mathbf{a}\times\mathbf{b}|$

Area of triangle $OAD = \frac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OD}\right| = \frac{1}{2}|\mathbf{a}\times m\mathbf{b}| = \frac{m}{2}|\mathbf{a}\times\mathbf{b}|$

Area of triangle $ABD = \frac{m}{2}|\mathbf{a}\times\mathbf{b}| - \frac{1}{2}|\mathbf{a}\times\mathbf{b}|$

$= \frac{m-1}{2}|\mathbf{a}\times\mathbf{b}|$

**(iii)** Show that $OADC$ cannot be a parallelogram. [2]

Solution

**Method 1**

If $OADC$ is a parallelogram, then

$$\overrightarrow{OA} = \overrightarrow{CD} \qquad \underset{\sim}{a} = k\underset{\sim}{b}$$

$$\mathbf{a} = m\mathbf{b} - (4\mathbf{b} - 3\mathbf{a})$$

$$2\mathbf{a} = (4-m)\,\mathbf{b} \qquad \underset{\sim}{a} = \frac{4-m}{2}\underset{\sim}{b}$$

$\Rightarrow$ $\mathbf{a}$ parallel to $\mathbf{b}$ since $\mathbf{a}$ and $\mathbf{b}$ are non-zero

But $\mathbf{a}$ is not parallel to $\mathbf{b}$. Thus, $OADC$ cannot be a parallelogram.

**Commented [LDL27]:** Concept
The critical condition for OADC to be parallelogram is $\overrightarrow{OA}$ and $\overrightarrow{CD}$ must be equal vectors. Most students wrote $\overrightarrow{OA} = k\overrightarrow{CD}$.

**Commented [LDL28]:** Presentation
This is a show question which requires clear explanation in your argument. Most students omitted this critical information in their explanation or attempt to show that $\overrightarrow{OA}$ is not // $\overrightarrow{CD}$.

Many students have this misconception that by observing and comparing
$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{CD} = 3\mathbf{a} + (m-4)\mathbf{b}$
and concluded that these pair of vectors are not parallel. If a and b are parallel vectors, this will not be true!

**Method 2**

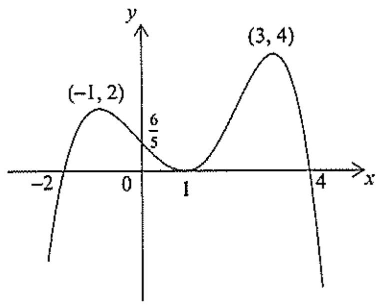
$B$ is the point of intersection between 2 diagonals of $OADC$.

For $OADC$ to be a parallelogram, $\overline{AB} = \overline{BC}$ .

However, $AB : BC = \frac{1}{3} : 1 \Rightarrow BC = 3AB$

$OADC$ is not a parallelogram (shown)

**8 (a)** The diagram below shows the graph of $y = \mathrm{f}(x)$. The curve passes through the points $(-2,0)$, $\left(0,\frac{6}{5}\right)$ and $(4, 0)$, and has turning points at $(-1, 2)$, $(1, 0)$ and $(3, 4)$.




**(i)** Sketch the graph of $y = \mathrm{f}(2x + 3)$, indicating clearly the axial intercepts, stationary points and asymptotes (if any). [3]

[Solution]

(i). $y = \mathrm{f}(x) \to \mathrm{f}(x+3) \to y = \mathrm{f}((2x)+3)$

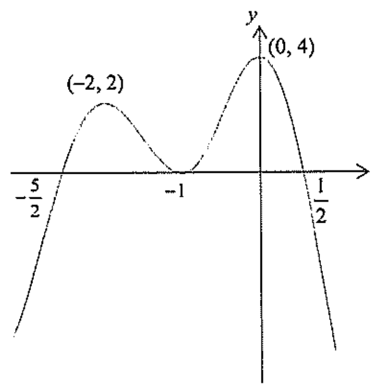
1. All $x$ – values minus 3

2. All $x$ – values divide by 2

**Commented [TLR29]:** Concept/method
Take the original coordinates, and let it under go the 2 steps change. Example $(-1,2) \rightarrow (2,2) \rightarrow (1,2)$

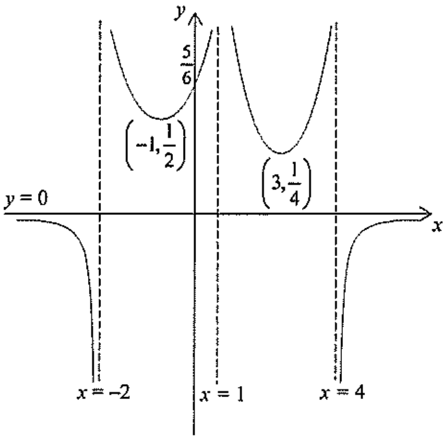
There should be no change in the $y$ values.

Given that this is a simple linear transformation, it was not as well done as it should be.



**(ii)** Sketch the graph of $y = \left|\dfrac{1}{\mathrm{f}(x)}\right|$, indicating clearly the axial intercepts, stationary points and asymptotes (if any). [3]

[Solution]




Commented [TLR30]: Good effort in attempting.
1. All y values should be reciprocated, including the turning points.
2. Some students reciprocated AND added negative sign, resulted in error.

**(b)** Describe the sequence of transformations that will transform the graph of $y = x^3$ to the graph of $y = 1 - 8x^3$. [3]

[Solution]

$y = x^3 \rightarrow y = 8x^3$ or $(2x)^3$ — Scaling of scale factor of 8 parallel to the $y$-axis

OR: Scaling of scale factor of $\dfrac{1}{2}$ parallel to the $x$-axis

$\rightarrow y = -8x^3$ — Reflection about the $x$-axis/$y$-axis

$\rightarrow y = 1 - 8x^3$ — Translation of 1 unit in the positive $y$-direction

Commented [TLR31]: Student should describe the transformation rather than showing the algebraic manipulations.

Students should use words like scale, reflect and translate. Students should NOT use words like stretch, expand, flip, invert, move, shift.

Commented [TLR32]: Concept
Common mistakes

1. "Scaling of scale factor of $1/8$ parallel to the $x$-axis" Student did not realise that this will result in the following algebraic change

$$y = x^3 \xrightarrow{x \to 8x} y = (8x)^3 = 512x^3$$

*Note: you will never replace $x^3$ by $8x^3$, you can only replace $x$ by something.

2. "Translation of 1 unit in the negative x-direction" Student did not realise that this will result in the following algebraic change

$$y = x^3 \xrightarrow{x \to x+1} y = (x+1)^3$$

9 The line $l_1$ passes through the points $A$ and $B$ with position vectors $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\-1\\3\end{pmatrix}$ respectively. The line $l_2$ has equation $\mathbf{r}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\0\\-3\end{pmatrix}$, $\mu\in\mathbb{R}$ and passes through the point $C$. It is given that $BC$ is perpendicular to line $l_2$.

**(i)** Find the coordinates of $C$. [4]

Commented [NCY33]: Concept
This part is well answered generally. It is useful to visualise via a diagram & realised that C is just the foot of perpendicular of B to $l_2$, hence $\overrightarrow{BC}\cdot\mathbf{d}_2=0$

Commented [NCY34]: Question Reading
Required the answer to be presented as coordinates and not vector

Solution

$$l_2:\mathbf{r}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\0\\-3\end{pmatrix},\ \mu\in\mathbb{R}$$

Since $C$ lies on line $l_2$, $\overrightarrow{OC}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\0\\-3\end{pmatrix}$, for some $\mu\in\mathbb{R}$.

$$\overrightarrow{BC}=\begin{pmatrix}1+\mu\\2\\-1-3\mu\end{pmatrix}-\begin{pmatrix}2\\-1\\3\end{pmatrix}=\begin{pmatrix}\mu-1\\3\\-4-3\mu\end{pmatrix}$$

Since $\overrightarrow{BC}$ is perpendicular to line $l_2$, $\begin{pmatrix}\mu-1\\3\\-4-3\mu\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-3\end{pmatrix}=0$

$\mu-1+12+9\mu=0$

$\Rightarrow\mu=-\dfrac{11}{10}$ [A1]

$$\overrightarrow{OC}=\begin{pmatrix}1\\2\\-1\end{pmatrix}-\frac{11}{10}\begin{pmatrix}1\\0\\-3\end{pmatrix}=\frac{1}{10}\begin{pmatrix}-1\\20\\23\end{pmatrix}$$

Hence, $C$ has coordinates $\left(-\dfrac{1}{10},2,\dfrac{23}{10}\right)$.

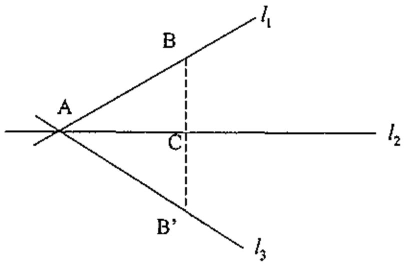
**(ii)** The line $l_3$ is a reflection of the line $l_1$ in the line $l_2$. Find a vector equation of the line $l_3$. [3]

Commented [NCY35]: Concept
Important to visualise via diagram and realised by observation, pt A is a common pt on $l_1$ and $l_2$.

Solution

Method 1: find vector AB then vector AB'
Method 2 : find vector OB' then vector AB'

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$$




Let $B'$ be the image of $B$.

$$\overrightarrow{AC} = \frac{\overrightarrow{AB} + \overrightarrow{AB'}}{2}$$

$$\overrightarrow{AB'} = 2\overrightarrow{AC} - \overrightarrow{AB} = 2\left[\frac{1}{10}\begin{pmatrix} -1 \\ 20 \\ 23 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}\right] - \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -16 \\ 15 \\ 13 \end{pmatrix}$$

Hence, a vector equation of the line $l_3$ is,

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} -16 \\ 15 \\ 13 \end{pmatrix}, s \in \mathbb{R}$$

**Commented [NCY36]:** Presentation
Students should write the equation of line in vector form like this.

**(iii)** The point $P$ has position vector $6\mathbf{j} + \alpha\mathbf{k}$, where $\alpha \in \mathbb{R}$. The length of projection of $\overrightarrow{AP}$ onto the $l_2$ is given by $\sqrt{\frac{5}{2}}$. Find the possible values of $\alpha$. [2]

Solution

$$\overrightarrow{AP} = \begin{pmatrix} 0 \\ 6 \\ \alpha \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ \alpha + 1 \end{pmatrix}$$

$$\left| \frac{\begin{pmatrix} -1 \\ 4 \\ \alpha+1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{1^2 + (-3)^2}} \right| = \sqrt{\frac{5}{2}}$$

**Commented [NCY37]:** Concept
Important to put the modulus when looking at length of projection, especially so when there's unknown $\alpha$

$$|-1 - 3\alpha - 3| = \sqrt{\frac{5}{2}}\left(\sqrt{10}\right)$$

$$-4 - 3\alpha = \pm 5$$

**Commented [NCY38]:** Concept
Need to consider 2 cases when removing modulus

$\alpha = -3$ or $\alpha = \frac{1}{3}$

10 A curve $C$ has parametric equations

$$x = 2\sin^2 t \ , \ y = 2t - \sin(2t) \ \ , \ 0 < t < \frac{\pi}{2}.$$

(i) Show that $\frac{dy}{dx} = \tan t$. [3]

[Solutions]

$\frac{dy}{dt} = 2 - 2\cos 2t$ $\qquad$ $\frac{dx}{dt} = 4\sin t \cos t$

Commented [NCM39]: Concepts
Students need to polish on their Trigonometry skills especially on identities and calculus.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 - 2\cos 2t}{4\sin t \cos t}$$

$$= \frac{2 - 2\left(1 - 2\sin^2 t\right)}{4\sin t \cos t}$$

$$= \frac{4\sin^2 t}{4\sin t \cos t}$$

$$= \tan t \quad \text{(Shown)}$$

(ii) The normal to the curve at the point where $t = a$ meets the $y$-axis at the point $A$, where $a \in \mathbb{R}$. Show that the $y$-coordinate of $A$ is $\lambda a$, where $\lambda$ is a constant to be found. [3]

[Solutions]

Equation of normal at $t = a$: $y - (2a - \sin(2a)) = -\cot a\left(x - 2\sin^2 a\right)$

When $x = 0$, $y - (2a - \sin(2a)) = -\frac{\cos a}{\sin a}\left(-2\sin^2 a\right)$

$$\begin{aligned} y &= 2\sin a \cos a + 2a - \sin(2a) \\ &= \sin 2a + 2a - \sin 2a \\ &= 2a \end{aligned}$$

Therefore $\lambda = 2$.

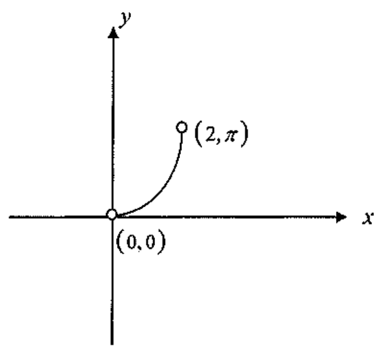
**(iii)** What can be said about the values of the gradients to the curve $C$ as $t \to 0$ and as $t \to \frac{\pi}{2}$. Hence sketch $C$, clearly showing the features of the curve when $t \to 0$ and $t \to \frac{\pi}{2}$. [4]

**Commented [NCM40]: Concept**
(1) Gradient is a value. Tangent is a straight line. Therefore, gradient is not equivalent to tangent. It is meaningless to say 'gradient is a horizontal line', or 'horizontal gradient'.
(2) Undefined is not equivalent to infinity.
(3) Gradient tends to infinity does not imply existence of asymptote.

[Solutions]

As $t \to 0$, $\cos t \to 1, \sin t \to 0$ $\therefore \frac{dy}{dx} \to 0$.

As $t \to \frac{\pi}{2}$, $\cos t \to 0, \sin t \to 1$ $\therefore \frac{dy}{dx} \to \infty$.

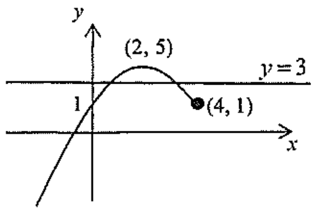



**Commented [NCM41]: Graphing Techniques**
(1) the (x- and y-) axes must be clearly labelled, especially when there is a 3rd parameter t.
(2) there is no asymptotes for this curve C.
(3) must give the coordinates of the end points, and the end points must be excluded.
(4) must show clearly the slope of the curve near the end points, need not draw the tangents.

11 The function f is defined by $f : x \mapsto 5-(x-2)^2,\ \ x \le 4$.

**(a)** Sketch the graph of $y = f(x)$. Hence state the range of f. [2]

[Solution]




From the graph, $R_f = (-\infty, 5]$.

Commented [KKL42]: Question Reading
Question reading remains an issue for some students as they did not draw the function in the given domain, i.e. $x \le 4$. Others drew the graph and did not give the range of f.

Commented [KKL43]: Presentation/Misconception
A number of students wrote their answers incorrectly as $[5, -\infty)$. It should always be smaller value in front and larger number behind.

**(b)** Explain why $f^{-1}$ does not exist. [1]

[Solution]

The horizontal line $y = 3$ cuts the graph twice. Therefore, the function f is not a one – one function and thus $f^{-1}$ does not exist.

Commented [KKL44]: Misconception
(a) To show that it is not 1-1, you need to choose a suitable value of k, e.g. 3 in the solution, to show a counter-example. Only when we want to show that it is 1-1, we indicate that it is for all real values.
i.e. y = k for all $k \in \mathbb{R}$.
(b) Some students left out the critical line that f is not 1-1. Students should note that the reason why the function has no inverse is not directly because there are more than 1 point of intersection between the line y = 3 and y = f(x). It is because f is not 1-1.

The function g is such that $g : x \mapsto f(x),\ x \le k$, where $k$ is a real constant. It is given that given that $g^{-1}$ exists.

**(c)** State the greatest possible value of $k$. [1]

[Solution]

The largest possible value of $k$ is 2.

Use the largest possible value of $k$ found in part **(c)** to answer the remaining parts of this question.

**(d)** Define $g^{-1}$ in similar form. [3]

[Solution]

$y = 5-(x-2)^2,\ x \le 2$

$(x-2) = \pm\sqrt{5-y}$

$x = 2 \pm \sqrt{5-y}$

$x = 2-\sqrt{5-y}\ \because x \le 2$

$\therefore g^{-1}: x \mapsto 2-\sqrt{5-x},\quad x \in \mathbb{R},\ x \le 5$

**Commented [KKL45]:** Carelessness
A number of students forgot to insert a "±" sign when taking square root.

**(e)** The function h is defined by $h: x \mapsto \dfrac{\alpha x+1}{x-2},\ x \in \mathbb{R},\ x \ne 2$ where $\alpha$ is a positive constant.

**(i)** By finding the range of h in terms of $\alpha$, determine the range of values of $\alpha$ such that $h^{-1}g$ exist. [3]

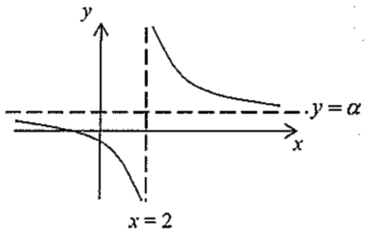
**Commented [KKL46]:** Concept
A number of students did not know how to start this question. Finding range usually involves sketching the graph of the function. The function is a rectangular hyperbola and the SOP for this is to write it in its proper fraction form first. The SOP remains the same even if there is an unknown constant in the expression.

**Commented [KKL47]:** Notation
Please note that $\alpha \ne \alpha$

[Solutions]

For function $h^{-1}g$ to exist, $R_g \subseteq D_{h^{-1}} = R_h$

$h(x) = \dfrac{\alpha x+1}{x-2} = \alpha + \dfrac{2\alpha+1}{x-2}$




$R_h = (-\infty, \alpha) \cup (\alpha, \infty)$.

For $R_g = (-\infty, 5] \subseteq R_h = (-\infty, \alpha) \cup (\alpha, \infty)$

This means $\alpha > 5$

**(ii)** Solve $h^{-1}g(x) = -11$ for the case where $\alpha = 6$. [2]

[Solution]

$h^{-1}g(x) = -11$

$\Rightarrow g(x) = h(-11)$

$\Rightarrow 5-(x-2)^2 = \dfrac{6(-11)+1}{-11-2}$

$\Rightarrow 5-(x-2)^2 = 5$

$\Rightarrow x = 2$

**Commented [KKL48]:** Approach
A large number of students approached the question using a long method despite this method being highlighted in tutorial.
i.e. there is no necessity to work out $h^{-1}$ at all.

End of First month | 2nd
1500 (1.11) | 1500(1.11)(1.11)
| = 1500 (1.11)^2



12 Betty received an SMS from a friend regarding a Bitcoin (BTC) investment that guarantees a fixed percentage of interest per month. Betty clicked on the link in the SMS and found out that the BTC investment has an account for each investor. An interest is added to the account at the end of each month at a fixed rate of 11% of the amount in the account at the beginning of the month. (1500 × 1.11) → end of the month

(i) Betty decides to start an investment account with \$1500 on 1 Jan 2022. She invests with the intention of not withdrawing any money out of the account but just leave it for the interest to build up. Find the total profit at the end of one full year. [2]

[Solution]

The amount of money in the investment account after 1 month is 111% of the investment at the beginning of the month, i.e. GP with $a = 1500$ and $r = 1.11$

Hence the projected profit at the end of 1 full year = $1500(1.11)^{12} - 1500$

= \$3747.68 (correct to nearest cent)

$T = ar^{n-1}$

**Commented [TGY49]:** Carelessness
This question is generally well attempted by most students, with a minority forgetting to subtract the principal sum of \$1500 to get the correct answer.

**Commented [TGY50]:** Miscellaneous
This is a contextual question with a mathematical model for real-life scenario in finance, students are expected to give the final answer to 2 decimal places, not 3 significant places.

Betty forwards the SMS to her friend, Carl who decides to invest \$200 on the first day of each month starting from 1 Jan 2022.

(ii) Show that the projected amount of money in Carl's investment account at the end of $n$ months of investment is given by

$$\frac{22200}{11}\left[\left(\frac{111}{100}\right)^n - 1\right].$$ [3]

must show this

**Commented [TGY51]:** Presentation
This question requires students to give a prove. As such, candidates showing little or insufficient working will not be given full credit.

[Solution]

Draw Table (see pattern)

| End of $r^{th}$ month | Amt in account |
|---|---|
| 1 | $200(1.11)$ |
| 2 | $[200 + 200(1.11)](1.11) = 200(1.11) + 200(1.11)^2$ |
| 3 | $[200 + 200(1.11) + 200(1.11)^2](1.11)$ $= 200(1.11) + 200(1.11)^2 + 200(1.11)^3$ |

Do not simplify exp

$r = 1.11$

$\therefore$ the projected amount of money $= 200(1.11) + 100(1.11)^2 + \ldots + 100(1.11)^n$

$$= \frac{200(1.11)(1.11^n - 1)}{1.11 - 1}$$

Formula: $a\left(\frac{r^n-1}{r-1}\right)$

$$= \frac{22200}{11}\left[\left(\frac{111}{100}\right)^n - 1\right]$$

**(iii)** Find the projected amount of money in Carl's investment account at the end of one year. [1]

[Solution]

Projected amount of money in Carl's investment account $= \frac{22200}{11}\left[\left(\frac{111}{100}\right)^{12} - 1\right]$

$= \$5042.33$

Commented [TGY52]: Miscellaneous
Almost all students answered this question correctly, with a handful of students forgetting to round off the final answer to 2 decimal places.

**(iv)** Determine with clear reasons, the date on which the projected amount of money in Carl's investment account first exceeds the projected amount of money in Betty's investment account. [4]

[Solution]

Let $n$ be the number of months after Carl started his investment.

$$\frac{22200}{11}\left[\left(\frac{111^n}{100}\right) - 1\right] > 1500(1.11)^n$$

$$(1.11)^n - 1 > \frac{55}{74}(1.11)^n$$

$$(1.11)^n > \frac{74}{19}$$

$$n\ln(1.11) > \ln\left(\frac{74}{19}\right)$$

$$n > 13.028$$

Commented [TGY53]: Carelessness
Many students were able to set up this correct inequality. A handful of students make careless mistakes which include writing $1500(1.11)^n - 1$ or writing \$3747.68 for the right-hand side of the inequality.

On 31 Jan 2023, Carl's projected amount in his account $= \frac{22200}{11}\left[\left(\frac{111^{13}}{100}\right) - 1\right] = \$5818.98$

Betty's projected amount in her account $= 1500(1.11^{13}) = \$5824.92$

Commented [TGY54]: Misconceptions
Many students use the inequality method to solve this question, correcting arriving at $n > 13.028$. However, almost all of them do not know how to interpret this result and assume that it takes 13.028 months (13 months and 0.85 days) for the Carl's account to exceed Betty's. Although these students may successfully arrive at the correct answer, no marks is credited for the erroneous concept.

On 31 Jan 2023, Betty has \$5.94 more in her account than Carl.

On 1 Feb 2023, Carl's projected amount will increase to \$6018.98 > \$5824.92.

Hence, Carl will have more money in his account than Betty has in hers on 1 Feb 2023 after he deposited \$200 into his account.

Commented [TGY55]: Question reading
The question requires candidates to justify the answer with clear reasons. Failure in doing so results in loss of marks.

Alternative Method

$$\frac{22200}{11}\left[\left(\frac{111^n}{100}\right)-1\right]>1500(1.11)^n$$

$$\text{Let } T_n=\frac{22200}{11}\left[\left(\frac{111^n}{100}\right)-1\right]-1500(1.11)^n$$

From GC, $T_{13}=-5.937<0$

$T_{14}=215.41>0$

$T_{15}=461.11>0$

On 31 Jan 2023, Carl's projected amount in his account $=\frac{22200}{11}\left[\left(\frac{111^{13}}{100}\right)-1\right]=\$5818.98$

Betty's projected amount in her account $=1500(1.11^{13})=\$5824.92$

On 31 Jan 2023, Betty has \$5.94 more in her account than Carl.

On 1 Feb 2023, Carl's projected amount will increase to \$6018.98 > \$5824.92.

Hence, Carl will have more money in his account than Betty has in hers on 1 Feb 2023 after he deposited \$200 into his account.

Unfortunately, the BTC investment is a scam. According to the Police, victims in Singapore lost \$633.3 million to scams in 2021. With public education campaigns, a researcher predicts that the amount lost to scams each year will drop by 5% of the amount lost in the preceding year.

(v) If the researcher is correct in his predictions, what would be the theoretical total amount of money lost to scams from 2021 (inclusive) onwards? [2]

[Solution]

The amount of money lost due to scam cases each year forms a GP with first term \$633.3 million and common ratio 0.95.

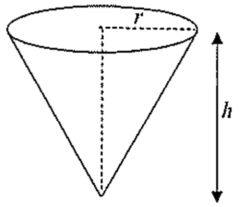
$\therefore$ The estimated total amount of money lost due to scam cases $=\frac{633.3}{1-0.95}$ million

$=\$12666$ million

**Commented [TGY56]:** Question reading
Quite a significant proportion of the cohort do not know that this is asking for the sum to infinity. For those who knew, some students thought that the common ratio is 0.05, where the correct common ratio is 0.95.

**Commented [TGY57]:** Miscellaneous
The answer \$12666 million is an exact answer, and there is no need to round this to 3 sf.
Few students omit the word "million" from the final answer.

**13** The diagram below shows a hanging plant pot with negligible thickness, in the shape of an inverted cone with radius $r$ cm and height $h$ cm.




**(a)** The plant pot has fixed external curved surface area $a\pi$ cm$^2$, where $a > 0$.

**(i)** Show $9V^2 = \pi^2 r^2 \left(a^2 - r^4\right)$, where $V$ is the volume of the pot. [2]

[Curved surface area of a cone = $\pi r l$]

[Solution]

$A = \pi r l$

$= \pi r\sqrt{h^2 + r^2}$

$h = \sqrt{\left(\frac{A}{\pi r}\right)^2 - r^2}$

$\therefore V = \frac{1}{3}\pi r^2 \sqrt{\left(\frac{a\pi}{\pi r}\right)^2 - r^2}$

$= \frac{1}{3}\pi r\sqrt{a^2 - r^4}$

$9V^2 = \pi^2 r^2 \left(a^2 - r^4\right)$ (shown)

**(ii)** Use differentiation to find, in terms of $a$, the exact maximum volume of the pot, proving that it is indeed a maximum. [5]

[Solution]

$9V^2 = \pi^2 r^2 a^2 - \pi^2 r^6$

$18V\frac{\mathrm{dV}}{\mathrm{dr}} = 2\pi^2 a^2 r - 6\pi^2 r^5$

When $\frac{\mathrm{dV}}{\mathrm{dr}} = 0$,

$\pi^2 a^2 r - 3\pi^2 r^5 = 0$

$r\left(a^2 - 3r^4\right) = 0$

$r = \sqrt[4]{\frac{a^2}{3}}$ *or* $r = 0$ (rejected)

**Commented [LHMD58]:** Question Reading
Some thought that the external curved surface area consists of the circular base as well i.e. $\pi rl + \pi r^2$.

**Commented [LHMD59]:** Formula
Some do not know the formula for the volume of a cone.

Algebraic error
Many thought that $h = \sqrt{\left(\frac{a}{r}\right)^2 - r^2} = \sqrt{a^2 - r^4}$. It seems like they completely ignored the denominator.

**Commented [LHMD60]:** Careless mistake
Some copied the equation wrongly i.e. $\pi$ instead of $\pi^2$ and $r^2$ instead of $r^4$ and ended up with a wrong derivative.

**Commented [LHMD61]:** Technique
Some who attempted implicit differentiation left out $V$.

Careless mistake
Some were very careless with the constant terms like $\pi^2, a^2$ and left them out after differentiation.

Conceptual error
Some differentiated $a^2 - r^4$ to $2a - 4r^3$, without realising that $a$ is a constant.

**Commented [LHMD62]:** Conceptual error
Many thought a is a variable and found $\frac{\mathrm{d}V}{\mathrm{d}a}$ instead.

**Commented [LHMD63]:** Algebraic error
Quite a number struggled with the algebraic manipulation with the most common error being $V = \frac{1}{3}\pi r\sqrt{a^2 - r^4} = \frac{1}{3}\pi r\left(a - r^2\right)$ leading to a wrong derivative. Some others had problems manipulating the algebraic terms to arrive at the correct value for $r$.

$$\frac{\mathrm{dV}}{\mathrm{dr}}=\frac{\pi^2 r\left(a^2-3r^4\right)}{9V}=\frac{-\pi^2 r}{3V}\left(r^2+\frac{a}{\sqrt{3}}\right)\left(r+\sqrt{\frac{a}{\sqrt{3}}}\right)\left(r-\sqrt{\frac{a}{\sqrt{3}}}\right)$$

$$=-\left[\frac{\pi^2 r}{3V}\left(r^2+\frac{a}{\sqrt{3}}\right)\left(r+\sqrt{\frac{a}{\sqrt{3}}}\right)\right]\left(r-\sqrt{\frac{a}{\sqrt{3}}}\right)$$

Since $V>0,\ r>0$ ,

| $r$ | $\left(\sqrt{\frac{a}{\sqrt{3}}}\right)^-$ | $\sqrt{\frac{a}{\sqrt{3}}}$ | $\left(\sqrt{\frac{a}{\sqrt{3}}}\right)^+$ |
|---|---|---|---|
| $\frac{\mathrm{d}V}{\mathrm{d}r}$ | $-[+](-)=+ve$ | 0 | $-[+](+)=-ve$ |
| Tangent | / | — | \ |

$\therefore V$ is maximum at $r=\sqrt[4]{\frac{a^2}{3}}$ (cannot be −ve)

$$\therefore Max.\ V=\frac{1}{3}\pi r\sqrt{a^2-r^4}$$

$$=\frac{1}{3}\pi\left(\sqrt[4]{\frac{a^2}{3}}\right)\sqrt{a^2-\left(\sqrt[4]{\frac{a^2}{3}}\right)^4}=\frac{1}{3}\pi\left(\sqrt[4]{\frac{a^2}{3}}\right)\sqrt{\frac{2a^2}{3}}$$

$$=\frac{\sqrt{2}\pi}{3}\left(\frac{a^2}{3}\right)^{\frac{3}{4}}\quad or\quad =\pi\sqrt{2}\,a^{3/2}\,3^{-7/4}$$

Commented [LHMD64]: **Presentation**
Many did not factorise to $\frac{\pi^2 r\left(a^2-3r^4\right)}{9V}$ (at the very least) and instead just left the 1st derivative as $\frac{\pi^2\left(a^2 r-3r^5\right)}{9V}$. Students have to learn to rewrite the expression to an extent where it is pretty obvious what the sign is.

Alternative Method (Second Derivative Test)

$$9V\frac{\mathrm{dV}}{\mathrm{dr}}=\pi^2a^2r-3\pi^2r^5$$

$$9\left(\frac{\mathrm{d}V}{\mathrm{d}r}\right)^2+9V\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=\pi^2a^2-15\pi^2r^4 \qquad \text{--- (*)}$$

Substitute $r=\sqrt[4]{\frac{a^2}{3}}$ and $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ into (*):

$$9(0)^2+9V\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=\pi^2a^2-15\pi^2\left(\frac{a^2}{3}\right)$$

$$\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=-\frac{4\pi^2a^2}{9V}$$

Since $V>0$, therefore $\frac{\mathrm{d}^2V}{\mathrm{d}r^2}<0$

$\therefore V$ is maximum when $r=\sqrt[4]{\frac{a^2}{3}}$ .

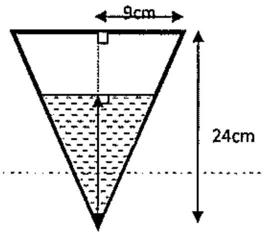
Commented [LHMD65]: **Technique**
Quite a number differentiated $9V\frac{\mathrm{dV}}{\mathrm{dr}}$ wrongly i.e. $9V\frac{\mathrm{d}^2V}{\mathrm{d}r^2}+9\frac{\mathrm{d}V}{\mathrm{d}r}$ or $9\frac{\mathrm{d}^2V}{\mathrm{d}r^2}+9\left(\frac{\mathrm{d}V}{\mathrm{d}r}\right)^2$.

**Presentation**
Some did not actually substitute $r=\sqrt[4]{\frac{a^2}{3}}$ into the expression for the 2nd derivative and immediately concluded that $\frac{\mathrm{d}^2V}{\mathrm{d}r^2}<0$.

**(b)** It is now given that $r = 9$ and $h = 24$. The pot was initially filled completely with water. Water is leaking from the tip of the pot at a constant rate of 0.5 cm$^3$ per second. At time $t$ (in seconds), the depth of water in the pot and the radius of the circular water surface is given by $y$ cm and $x$ cm respectively. Find the rate of change in the depth of water when the depth of water is 1 cm. [5]

[Solution]

Using similar triangles,




$$\frac{x}{9} = \frac{y}{24}$$

$$x = \frac{3y}{8}$$

Let $W$ be the volume of the water in the pot at time $t$

$$W = \frac{1}{3}\pi\left(\frac{3}{8}y\right)^2 y$$

$$= \frac{3}{64}\pi y^3$$

$$\frac{dW}{dy} = \pi\left(\frac{9}{64}\right)y^2$$

When $y = 1$,

$$\frac{dW}{dy} = \pi\left(\frac{9}{64}\right)y^2 = \frac{9}{64}\pi$$

Using chain rule,

$$\frac{dy}{dt} = \frac{dy}{dW} \times \frac{dW}{dt}$$

$$= \frac{64}{9\pi} \times \left(-\frac{1}{2}\right)$$

$$= -\frac{32}{9\pi} \text{ cms}^{-1}$$

decrease in rate

$\therefore$ the rate of decrease in the depth of water is $\dfrac{32}{9\pi}$ (or 1.13) cm per second.

**Commented [LHMD66]:** Method
Many attempted to find the initial volume of the water when it is actually not needed.
Many also tried to find the value of a given the values of $r$ and $h$. However, they did not realise that this was not required as the question is asking about the rate of change for the depth of water and not the pot, which is already fixed.

**Commented [LHMD67]:** Question Reading
Many used $r,h$ instead of $x,y$ as stated in the question. They also conveniently used V which was used in (a) to represent the volume of the pot instead of the volume of water in the pot.
There was a large group of students who used the expression for V in (a) and this is obviously wrong as they are using the volume of the pot (which is now fixed) to find the rate of change for the height of the water in the pot.

**Commented [LHMD68]:** Formula
Some thought that the volume of cone is $\pi r^2 h$ or $\frac{1}{3}\pi rh$.

Conceptual error
- Many differentiated $\frac{1}{3}\pi r^2 h$ or $\frac{1}{3}\pi x^2 y$ to $\frac{1}{3}\pi r^2$ or $\frac{1}{3}\pi x^2$ without realising that both $x,y$ are variables.
- Many students substituted $y = 1$ into the expression for the volume of water before they differentiated.

**Commented [LHMD69]:** Question Reading
Many failed to realise that it is a negative rate.

**Commented [LHMD70]:** Presentation
Many did not write the final statement to answer the question but they were not penalised.