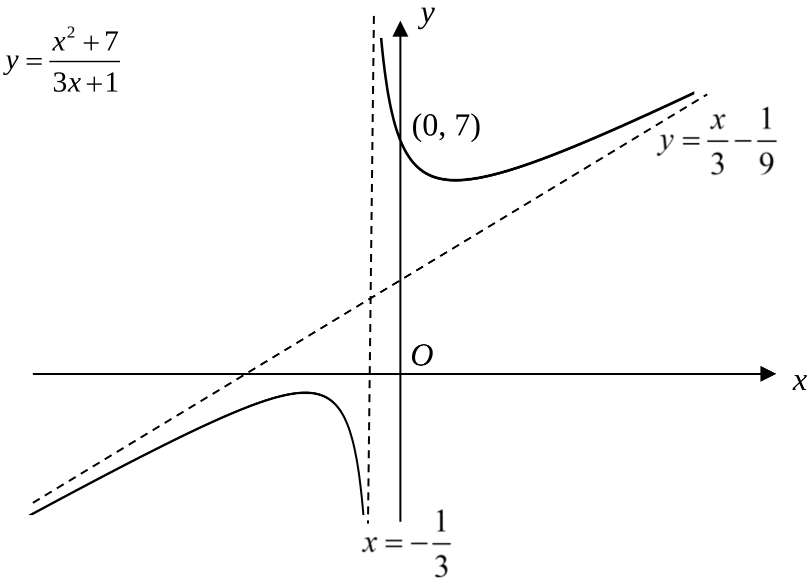
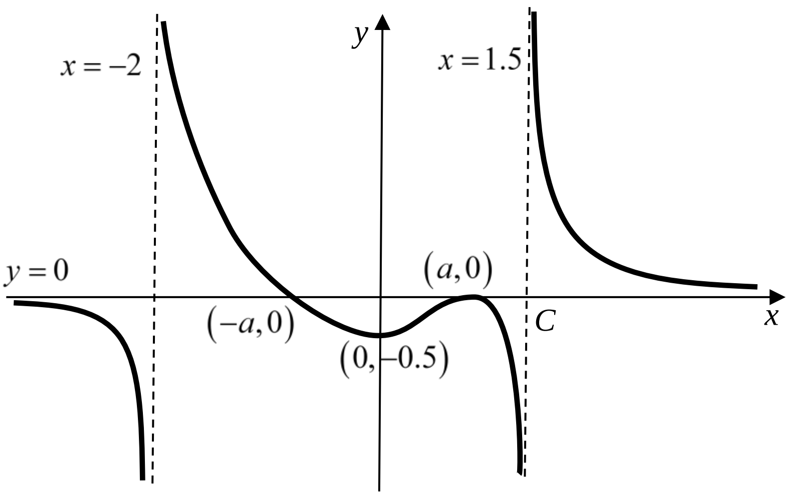
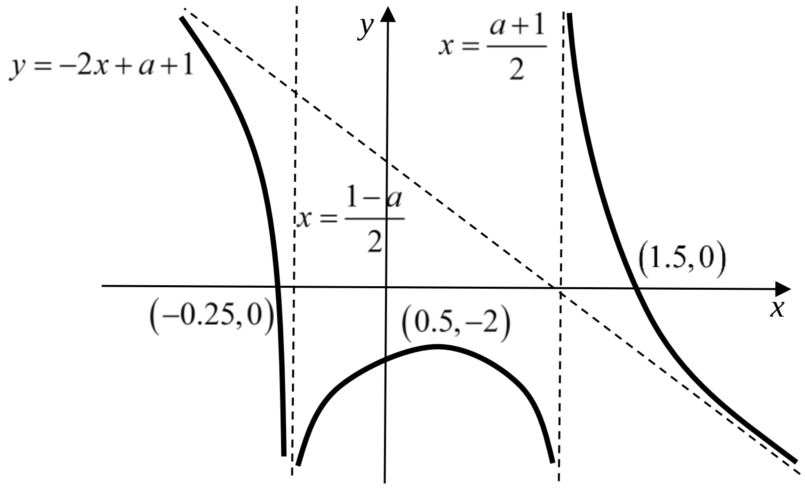
2022 ACJC H2 Math Promo Markers Report

Qn	Solution	Remarks
1(i)	$y=\ln\left(\dfrac{e^{\sqrt{x}}}{\cos^3 x}\right)$ $=\sqrt{x}\ln e-\ln\left(\cos^3 x\right)$ $=\sqrt{x}-3\ln\left(\cos x\right)$ $\dfrac{dy}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}-3\dfrac{-\sin x}{\cos x}$ $=\dfrac{1}{2\sqrt{x}}+3\tan x$	The ideal method is on the left, with log rules applied first. There were many solutions that applied quotient/product rule coupled with chain rule, leading to careless mistakes, especially with the derivative of $\cos^3 x$.
(ii)	$y^{\frac{1}{x}}=x^{\ln x}$ $\dfrac{1}{x}\ln y=\ln x\ln x$ $\ln y=x\left(\ln x\right)^2$ $\dfrac{1}{y}\dfrac{dy}{dx}=\left(\ln x\right)^2+2x\ln x\left(\dfrac{1}{x}\right)$ $\dfrac{dy}{dx}=y\ln x(\ln x+2)$ **Alternative:** $y^{\frac{1}{x}}=x^{\ln x}$ $\dfrac{1}{x}\ln y=\ln x\ln x$ $\dfrac{1}{x}\dfrac{1}{y}\dfrac{dy}{dx}-\dfrac{1}{x^2}\ln y=2\dfrac{1}{x}\ln x$ $\dfrac{1}{y}\dfrac{dy}{dx}-\dfrac{1}{x}\ln y=2\ln x$ $\dfrac{dy}{dx}=y\left(2\ln x+\dfrac{1}{x}\ln y\right)$	Generally those who applied 'ln' did well with the implicit differentiation that follows. A significant minority started differentiating straight away, leading to a lot of weird errors.

2(i)	$51-\frac{88}{x+2}\le 10x, \qquad x\ne -2$ $\frac{51(x+2)-88-10x(x+2)}{x+2}\le 0$ $\frac{51x+14-10x^2-20x}{x+2}\le 0$ $\frac{-10x^2+31x+14}{x+2}\le 0$ $\frac{10x^2-31x-14}{x+2}\ge 0$ $\frac{(2x-7)(5x+2)}{x+2}\ge 0$ $-2<x\le -\frac{2}{5}$ or $x\ge \frac{7}{2}$	About 15% of all scripts show reliance on GC to go from $\frac{-10x^2+31x+14}{x+2}\le 0$ to $\frac{(2x-7)(5x+2)}{x+2}\le 0$ without flipping the inequality sign or to $\frac{\left(x-\frac{7}{2}\right)\left(x+\frac{2}{5}\right)}{x+2}\ge 0$. Quite many students forgot to exclude "−2" from the final answer. Many do not have the "or" or do not draw their horizontal line for fraction long enough.
(ii)	$51\lvert x\rvert-\frac{88x^2}{1+2\lvert x\rvert}\le 10$ Dividing by $\lvert x\rvert$ and considering $x=0$ (which satisfies the above inequality) separately, $51-\frac{88\lvert x\rvert}{1+2\lvert x\rvert}\le\frac{10}{\lvert x\rvert}$ or $x=0$ $51-\frac{88}{\frac{1}{\lvert x\rvert}+2}\le\frac{10}{\lvert x\rvert}$ or $x=0$ Hence, "replacing x with $\frac{1}{\lvert x\rvert}$": $-2<\frac{1}{\lvert x\rvert}\le -\frac{2}{5}$ or $\frac{1}{\lvert x\rvert}\ge\frac{7}{2}$ or $x=0$ No solution. $\qquad -\frac{2}{7}\le x\le\frac{2}{7}$ $\therefore -\frac{2}{7}\le x\le\frac{2}{7}$	About 50% of all scripts saw or guessed the substitution correctly. Presentation was mostly very poor. E.g. starting from the inequality in part (i) or writing "$x=\frac{1}{\lvert x\rvert}$". Many who reached $\frac{1}{\lvert x\rvert}\ge\frac{7}{2}$ made the mistake of giving $\lvert x\rvert\ge\frac{2}{7}$.
3(i)	$y=\frac{ax^2+bx+c}{3x+1}$ Substitute $(-1,-4)$ and $(-3,-2)$ into equation, $a-b+c=8$ ----- (1) $9a-3b+c=16$ ----- (2) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(2ax+b)(3x+1)-3(ax^2+bx+c)}{(3x+1)^2}$ Substitute $x=-3$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$,	Many careless mistakes such as "−" becoming "+", not distributing the "−3" into the *c* in the bracket or (−4)(−2) = 2. About 15% of all scripts either did not use $(-3,-2)$ to obtain equation (2) or did not use

	$0=\left(-6a+b\right)\left(-8\right)-3\left(9a-3b+c\right)$ $21a+b-3c=0$ ----- (3) From GC, $a=1,\ b=0$ and $c=7$.	$x=-3$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ to obtain equation (3).
(ii)	$y=\frac{x^2+7}{3x+1}$ $=\frac{x}{3}-\frac{1}{9}+\frac{64}{9\left(3x+1\right)}$ $\begin{array}{r} \frac{x}{3}-\frac{1}{9} \\ 3x+1\overline{)x^2+0x+7} \\ \underline{x^2+\frac{x}{3}} \\ -\frac{x}{3}+7 \\ \underline{-\frac{x}{3}-\frac{1}{9}} \\ 7\frac{1}{9} \end{array}$  	About 20% of the scripts with correct answers in part (i) show mistakes in long-dividing. E.g. still having x in the remainder or $0x-\frac{x}{3}=\frac{x}{3}$ leading to $y=\frac{x}{3}+\frac{1}{9}$ as the oblique asymptote. Students who got wrong answers in part (i) and got a graph with no stationary points should have checked their part (i) since part (i) stated that the curve C has a turning point at $\left(-3,-2\right)$. Quite many students did not write the y-intercept in coordinates-form. A small number of students did not give the vertical asymptote as $x=\frac{1}{3}$ or the oblique asymptote as $y=-\frac{x}{3}-\frac{1}{9}$.
4(i)	 	About 20% of all scripts go up from $\left(a,0\right)$ instead of turning down. About 20% of all scripts do not have "$y=0$" for the horizontal asymptote.

(ii)		About 15% of all students could not get the correct coordinates for the intercepts and turning point. E.g. getting (−1, 0) and (6, 0) for the *x*-intercepts and (0, −2) for the turning point. Among students with correct coordinates for the intercepts and turning point, about 20% of them could not get the correct equations for the asymptotes. E.g. getting $y=-\frac{x}{2}+a-1$ for the oblique asymptote or $x=\frac{a-1}{2}$ or $x=2a+2$ for the vertical asymptote on the right.
5(i)	Since *N* lies on line *L*, $\overrightarrow{ON}=\begin{pmatrix}6-2\lambda\\ \lambda\\ 1-\lambda\end{pmatrix}$ for some $\lambda\in\mathbb{R}$. $\overrightarrow{OA}=\begin{pmatrix}3\\6\\1\end{pmatrix}$ $\therefore\ \overrightarrow{AN}=\begin{pmatrix}3-2\lambda\\ -6+\lambda\\ -\lambda\end{pmatrix}$ $\overrightarrow{AN}\bullet\begin{pmatrix}-2\\1\\-1\end{pmatrix}=0$ $\begin{pmatrix}3-2\lambda\\ -6+\lambda\\ -\lambda\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\-1\end{pmatrix}=0$ $-6+4\lambda-6+\lambda+\lambda=0$ $\lambda=2$ $\overrightarrow{ON}=\begin{pmatrix}2\\2\\-1\end{pmatrix}$	Common mistakes: 1. missing sign in the direction (transfer error). 2. calculation error. 3. Some students find the vector component parallel to the line as $\overrightarrow{AN}$.

(ii)	$\overrightarrow{OA'} = \overrightarrow{OA} + 2\overrightarrow{AN}$ $\overrightarrow{OA'} = \overrightarrow{OA} + 2\left(\overrightarrow{ON} - \overrightarrow{OA}\right)$ $\overrightarrow{OA'} = 2\overrightarrow{ON} - \overrightarrow{OA}$ $= 2\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$	Or use: Ratio theorem $\overrightarrow{ON} = \dfrac{\overrightarrow{OA} + \overrightarrow{OA'}}{2}$ $\overrightarrow{OA'} = 2\overrightarrow{ON} - \overrightarrow{OA}$
(iii)	Let D denote the point(s) that are $3\sqrt{3}$ units away from A on line L. Then $\overrightarrow{OD} = \begin{pmatrix} 6-2\lambda \\ \lambda \\ 1-\lambda \end{pmatrix}$ for some $\lambda \in \mathbb{R}$, And hence $\overrightarrow{AD} = \begin{pmatrix} 3-2\lambda \\ -6+\lambda \\ -\lambda \end{pmatrix}$. (can also be taken from (i)). $\left\| \begin{pmatrix} 3-2\lambda \\ -6+\lambda \\ -\lambda \end{pmatrix} \right\| = 3\sqrt{3}$ $(3-2\lambda)^2 + (-6+\lambda)^2 + (-\lambda)^2 = 27$ $6\lambda^2 - 24\lambda + 18 = 0$ $\lambda^2 - 4\lambda + 3 = 0$ $(\lambda-3)(\lambda-1) = 0$ $\lambda = 1$ or $\lambda = 3$ $\therefore D(4,1,0)$ or $D(0,3,-2)$	$\left\| \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right\| = \sqrt{(x)^2 + (y)^2 + (z)^2}$ Answer should be in coordinates form as stated in question. $\therefore D(4,1,0)$ or $D(0,3,-2)$
6(i)	$\mathrm{f}\left(2(r-1)\right) - \mathrm{f}\left(2r\right) = \cos(2r-2)\theta - \cos 2r\theta$ $= -2\sin(2r-1)\theta \sin(-\theta)$ $= 2\sin(2r-1)\theta \sin\theta$ $k = 2$ $\mathrm{f}\left(2(r-1)\right) - \mathrm{f}\left(2r\right) = \cos(2r-1-1)\theta - \cos(2r-1+1)\theta$ $= \cos(2r-1)\theta\cos\theta + \sin(2r-1)\theta\sin\theta$ $-\left[\cos(2r-1)\theta\cos\theta - \sin(2r-1)\theta\sin\theta\right]$ $= 2\sin(2r-1)\theta\sin\theta$	Common errors: $\cos 2(r-1)\theta = 2\cos(r-1)\theta$ $\cos 2(r-1)\theta = \cos 2r\theta - \cos 2\theta$ $\sin(-\theta) = \sin\theta$ so that $k = -2$ Those who did not use “sum to product” formula applied cos(A–B) or double angle. A few succeeded in proving using cos(A–B), but none did when using double angle. Added alternate solution.

<table>
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<td>(ii)</td>
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$$\sum_{r=1}^{n}\sin\left[(2r-1)\theta\right]=\frac{1}{2\sin\theta}\sum_{r=1}^{n}\left[\mathrm{f}\left(2(r-1)\right)-\mathrm{f}\left(2r\right)\right]$$

$$=\frac{1}{2\sin\theta}\left[\begin{array}{ll}\mathrm{f}(0) & \cancel{-\mathrm{f}(2)}\\ \cancel{\mathrm{f}(2)} & \cancel{-\mathrm{f}(4)}\\ \cancel{\mathrm{f}(4)} & \cancel{-\mathrm{f}(6)}\\ \ldots & \ldots\\ \cancel{\mathrm{f}(2(n-3))} & \cancel{-\mathrm{f}(2(n-2))}\\ \cancel{\mathrm{f}(2(n-2))} & \cancel{-\mathrm{f}(2(n-1))}\\ \cancel{\mathrm{f}(2(n-1))} & -\mathrm{f}(2n)\end{array}\right]$$

$$=\frac{1}{2\sin\theta}\left(\cos 0-\cos 2n\theta\right)$$

$$=\frac{1}{2\sin\theta}\left(1-\cos 2n\theta\right)$$

$$=\frac{1}{2\sin\theta}\left(2\sin^2 n\theta\right)$$

$$=\frac{\sin^2 n\theta}{\sin\theta}$$

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<td>MOD was mostly done correctly. Many students who did not get the value of k in (i) could work backwards to deduce k since final answer is given. No marks for giving k without correct working.</td>
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<td>(iii)</td>
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$$\sum_{r=1}^{n}\sin\left[(2r+1)\theta\right]$$

$$=\sum_{r-1=1}^{r-1=n}\sin\left[\left(2(r-1)+1\right)\theta\right] \qquad \text{("replace } r \text{ with } r-1\text{")}$$

$$=\sum_{r=2}^{n+1}\sin\left[(2r-1)\theta\right]$$

$$=\frac{\sin^2\left[(n+1)\theta\right]}{\sin\theta}-\sin\theta$$

</td>
<td>Many students changed from

$$\sum_{r=1}^{n}\sin\left[(2r-1)\theta\right]=\sum_{r=0}^{n-1}\sin\left[(2r+1)\theta\right]$$

but did not know how to proceed except a few. Some students did so after using earlier result (MOD),

$$\sum_{r=1}^{n}\mathrm{f}(2r)-\mathrm{f}(2(r+1))$$

to get this $\dfrac{\cos 2\theta-\cos 2(n+1)\theta}{2\sin\theta}$.</td>
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<td>7(a)</td>
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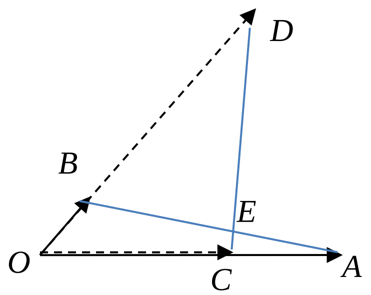
Since $\overrightarrow{AE}$ is parallel to $\overrightarrow{AB}$ and $\overrightarrow{CE}$ is parallel to $\overrightarrow{CD}$,

$$\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}$$

$$=\overrightarrow{OA}+\lambda\overrightarrow{AB}$$

$$=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$$

$$=(1-\lambda)\mathbf{a}+\lambda\mathbf{b}$$



$$\overrightarrow{OE}=\overrightarrow{OC}+\overrightarrow{CE}$$

$$=\overrightarrow{OC}+\mu\overrightarrow{CD}$$

$$=\frac{2}{3}\mathbf{a}+\mu\left(\mathbf{6b}-\frac{2}{3}\mathbf{a}\right)$$

$$=\frac{2}{3}(1-\mu)\mathbf{a}+6\mu\mathbf{b}$$

</td>
<td>This question on the whole is badly done.
(a) A large number of students who attempted this made the conceptual mistake of equating the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ in the hope of getting the point of intersection of l_{AB} and l_{CD}.
Of those who equated the line equations of l_{AB} and l_{CD}, about half did not know how to proceed (i.e. compare coefficients of a and b.)
There were also unnecessarily long methods to find $\overrightarrow{OC}$ and $\overrightarrow{OD}$,</td>
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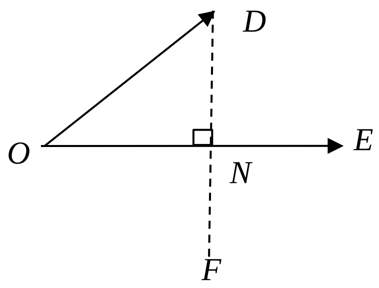
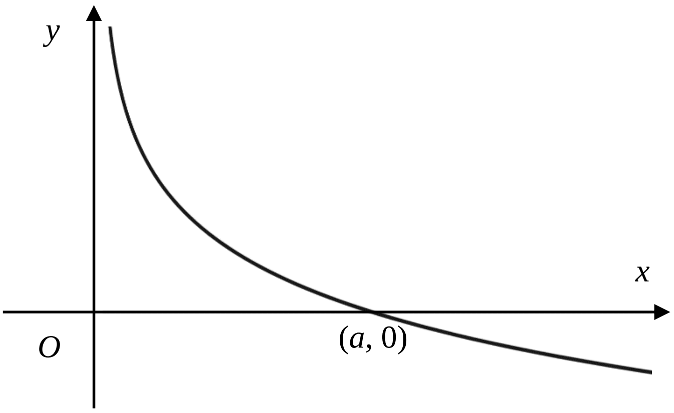
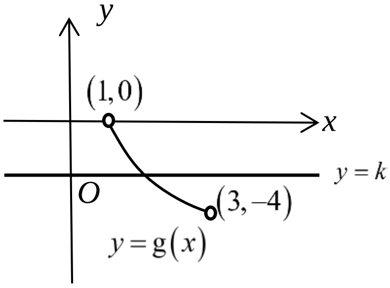
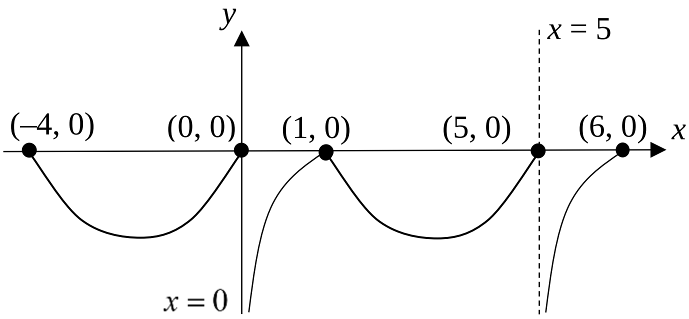
	Comparing the coefficients of **a** and **b**, $(1-\lambda)=\frac{2}{3}(1-\mu)$ $\lambda=6\mu$ $\lambda=\frac{6}{16},\ \mu=\frac{1}{16}$ $\overrightarrow{OE}=\left(1-\frac{6}{16}\right)\mathbf{a}+\frac{6}{16}\mathbf{b}=\frac{5}{8}\mathbf{a}+\frac{\mathbf{3}}{\mathbf{8}}\mathbf{b}$ Alternatively, consider the lines *AB* and *CD*: $l_{AB}:\mathbf{r}=\overrightarrow{OA}+\lambda\overrightarrow{AB}$ $\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$ $\mathbf{r}=(1-\lambda)\mathbf{a}+\lambda\mathbf{b}$ $l_{CD}:\mathbf{r}=\overrightarrow{OC}+\mu\overrightarrow{CD}$ $\mathbf{r}=\frac{2}{3}\mathbf{a}+\mu\left(\mathbf{6b}-\frac{2}{3}\mathbf{a}\right)$ $\mathbf{r}=\frac{2}{3}(1-\mu)\mathbf{a}+6\mu\mathbf{b}$ For the intersection point *E*, $(1-\lambda)\mathbf{a}+\lambda\mathbf{b}=\frac{2}{3}(1-\mu)\mathbf{a}+6\mu\mathbf{b}$ $(1-\lambda)=\frac{2}{3}(1-\mu)$ $\lambda=6\mu$ $\lambda=\frac{6}{16},\ \mu=\frac{1}{16}$ $\overrightarrow{OE}=\left(1-\frac{6}{16}\right)\mathbf{a}+\frac{6}{16}\mathbf{b}=\frac{5}{8}\mathbf{a}+\frac{\mathbf{3}}{\mathbf{8}}\mathbf{b}$	which can be easily obtained by using a simple diagram.
(b)(i)	Let the foot of perpendicular from point *D* to line *OE* be *N*, with position vector $\overrightarrow{ON}=\mathbf{n}$. Then $\mathbf{n}=\frac{(\mathbf{d}\cdot\mathbf{e})}{\lvert\mathbf{e}\rvert}\frac{\mathbf{e}}{\lvert\mathbf{e}\rvert}$.  $\overrightarrow{OF}=\overrightarrow{OD}+2\overrightarrow{DN}$ $\overrightarrow{OF}=\overrightarrow{OD}+2\left(\overrightarrow{ON}-\overrightarrow{OD}\right)$ $\overrightarrow{OF}=2\overrightarrow{ON}-\overrightarrow{OD}$ $\mathbf{f}=2\frac{(\mathbf{d}\cdot\mathbf{e})}{\lvert\mathbf{e}\rvert}\frac{\mathbf{e}}{\lvert\mathbf{e}\rvert}-\mathbf{d}$ $=2\frac{\pm3}{4}\mathbf{e}-\mathbf{d}$ $=\pm\frac{3}{2}\mathbf{e}-\mathbf{d}$	(b) This part is almost not done or done erroneously from the start. Common mistakes include assuming that *E* is the foot of perpendicular from *D* to the line *OE*, resulting in the wrong usage of midpoint theorem. Of those who used projection or dot product method, many did not take note that $\mathbf{d.e}=\pm3$.

(ii)	$\frac{1}{2}\left\|\overrightarrow{OD}\times\overrightarrow{OF}\right\| = \frac{1}{2}\left\|\mathbf{d}\times\left(\pm\frac{3}{2}\mathbf{e}-\mathbf{d}\right)\right\| = \frac{3}{4}\left\|\mathbf{d}\times\mathbf{e}\right\|$ **OR** $2\left(\frac{1}{2}\right)(ON)(DN) = 2\left(\frac{1}{2}\right)\left\|\frac{\mathbf{d}\cdot\mathbf{e}}{2}\right\|\left\|\frac{\mathbf{d}\times\mathbf{e}}{2}\right\| = \frac{3}{4}\left\|\mathbf{d}\times\mathbf{e}\right\|$	Again, many left this blank or just very badly done. Marks were given as long as student attempt to distribute the cross product and use the fact that $\mathbf{d}\times\mathbf{d}=\mathbf{0}$.
8(i)	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{a}{t^2}$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{a}{t}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{a}{t}\times\left(-\frac{t^2}{a}\right) = -t$ When $t = p$, $x = \frac{a}{p}$, $y = a\ln p$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -p$ Equation of tangent: $y - a\ln p = -p\left(x - \frac{a}{p}\right)$ $y = -px + a + a\ln p$ Equation of normal: $y - a\ln p = \frac{1}{p}\left(x - \frac{a}{p}\right)$ $y = \frac{1}{p}x - \frac{a}{p^2} + a\ln p$	Many careless mistakes when finding $\frac{\mathrm{d}y}{\mathrm{d}t}$ which can be avoided if they only have the decency to rewrite y as $y = a\ln t$. There were many careless mistakes when calculating $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$ such as missing negative signs and fumbling with reciprocals of reciprocals, or powers (of t). There were many cases when students sub in $t = p$ at the start even before differentiating and ended up differentiating in terms of p. Although marks were given, students should be warned that p here should be treated as a constant. Marks were given for such woeful working only to avoid a complete disaster. There were also a number who did not sub $t = p$ into $\frac{\mathrm{d}y}{\mathrm{d}x}$, resulting in equations with t and p appearing at the same time. The habit of substituting the parameter into x, y, $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the same time cannot be overemphasized as this is a recurring mistake made by the students (over the years). Finally, quite a number of students did not expand and simplify the equations for the tangents and normals. Once again marks are

		given, because calculations were needed anyway for the subsequent part. Note that this may not be the case in the actual 'A' level examinations.
(ii)	When $x=0,\ y=a+a\ln p$ When $x=0,\ y=-\dfrac{a}{p^2}+a\ln p$ Area of $APB=\dfrac{1}{2}\left(a+a\ln p+\dfrac{a}{p^2}-a\ln p\right)\left(\dfrac{a}{p}\right)$ $=\dfrac{1}{2}\left(a+\dfrac{a}{p^2}\right)\left(\dfrac{a}{p}\right)$ $=\dfrac{a^2(p^2+1)}{2p^3}$	Quite a good number of students misread the question and found *x*-intercepts instead for at least one of tangent or normal when the question clearly state the *y*-intercepts are needed. When finding area, some used Pythagoras to find the length *AP* and *BP* which is unnecessarily tedious and as a result, most did not go on to simplify their final answer.
(iii)		Quite a good number of students did not leave in coordinates form. Note that the *x*-intercept should be in terms of *a*, even though a particular value of *a* is used to obtained the shape of the graph in the G.C. There were many answers that, instead of cutting the x-axis, they show a minimum point at the x-axis. This could be due to students entering the G.C wrongly for $y=\ln t^a$. Students should err on the side of caution by entering with proper brackets, such as $y=\ln\left(t^a\right)$ instead of $y=\ln(t)^a$.
(iv)	Equation of tangent when $p=1$: $y=-x+a+a\ln 1$ $=-x+a$ Note that both the tangent and the line $y=mx+a$ pass through the point (0, *a*). Range of *m*: $-1<m<0$.	The first mark is a free gift. The marker is horrified by the number of students equating $y=-x+a$ and $y=mx+a$, hoping to find more than 2 intersections.

		How can two distinct straight lines cut more than once?
9(i)	$\mathrm{f}\left(x\right)=\left(x-1\right)\left(x-5\right)=\left(x-3\right)^2-4$ $\mathrm{R_f}=\left[-4,0\right]$	Careless in giving the range, lack precision when using brackets.
9(ii)	The line $y=k$ where $k\in\mathbb{R}$ cuts the graph of $y=\mathrm{g}\left(x\right)$ at most once. Hence g is one-one and g^{-1} exists. 	Students are reminded that the use of horizontal line test require sketch of graph. Note the domain of *g* in the sketch and the line drawn should be <u>within the range of *g*</u>. Many students gave only an example of horizontal line, which is a counter proof if inverse function does not exist. These students did not grasp the proof of one-one function correctly, that in the proof **no horizontal line should cut the graph more than once**.
(iii)	$y=\left(x-3\right)^2-4$ $\left(x-3\right)^2=y+4$ $x=3\pm\sqrt{y+4}$ $=3-\sqrt{y+4}\quad \because x<3$ $\mathrm{g}^{-1}(x)=3-\sqrt{x+4},\ \mathrm{D}_{\mathrm{g}^{-1}}=(-4,0)$	Mostly correct.
(iv)	$\mathrm{h}\left(\dfrac{11}{2}\right)+\mathrm{h}(-2)=\mathrm{h}\left(\dfrac{1}{2}\right)+\mathrm{h}(3)=\ln\left(\dfrac{1}{2}\right)-4$	
(v)		Missing asymptotes with many students who drew the "ln" graph that ended abruptly at a point, along the asymptotes.
10(i)	Equation of planes are: $\mathbf{r}\bullet\dfrac{\begin{pmatrix}-6\\-4\\2\end{pmatrix}}{\sqrt{56}}=\dfrac{4}{\sqrt{56}}\pm10$ $\mathbf{r}\bullet\dfrac{\begin{pmatrix}-3\\-2\\1\end{pmatrix}}{\sqrt{14}}=\dfrac{2}{\sqrt{14}}\pm10$	This part of Q10 was the most poorly done, with many students electing to skip it entirely. Of the minority who did attempt it, there were quite a few solutions which utilized $\left\lvert\dfrac{p-4}{\sqrt{56}}\right\rvert=10$ (distance between 2 planes), but went on to claim that:

		$\lvert p-4\rvert=10\sqrt{56}$ $p=\pm\left(4+10\sqrt{56}\right)$ This is a conceptual error with how the modulus function should be handled (or in which order). Some other errors include: (1) dividing the normal vector by 2, but kept the 4 on the RHS. (2) not converting answer to scalar product form (leaving it in cartesian form).
(ii)	$\theta=\cos^{-1}\dfrac{\left\lvert\begin{pmatrix}-6\\-4\\2\end{pmatrix}\bullet\begin{pmatrix}-1\\-1\\2\end{pmatrix}\right\rvert}{\left\lvert\begin{matrix}-6\\-4\\2\end{matrix}\right\rvert\left\lvert\begin{matrix}-1\\-1\\2\end{matrix}\right\rvert}=\cos^{-1}\dfrac{14}{\sqrt{56}\sqrt{6}}=40.2°$	Majority did this part. Errors include: (1) sine. (2) writing dot product of the 2 normals, but evaluating a cross product vector first. (3) usual careless mistakes. $\sqrt{4}$ instead of $\sqrt{6}$ was one of the common ones.
(iii)	$\begin{pmatrix}-6\\-4\\2\end{pmatrix}\times\begin{pmatrix}-1\\-1\\2\end{pmatrix}=\begin{pmatrix}-6\\10\\2\end{pmatrix}=2\begin{pmatrix}-3\\5\\1\end{pmatrix}$ Let $z=0$, $-6x-4y=4------(1)$ $-x-y=k\Rightarrow x=-k-y----(2)$ Subst. (2) into (1) $-6(-k-y)-4y=4$ $2y=4-6k\Rightarrow y=2-3k$ $\therefore x=-k-(2-3k)\Rightarrow x=2k-2$ $\therefore L:\mathbf{r}=\begin{pmatrix}2k-2\\2-3k\\0\end{pmatrix}+\lambda\begin{pmatrix}-3\\5\\1\end{pmatrix},\ \lambda\in\mathbb{R}$	As this is a "show" question, students are expected to show their steps clearly – writing "by using GC" usually will not serve as an adequate explanation as to how the given equation is obtained. There were a minority of solutions that approached this fully algebraically, which is fine, as long as the steps are clearly shown (i.e. not using cross product of normal to find direction vector).

(iv)	$P_3 : 5x+\beta y+5z=\mu$ $P_3 : \mathbf{r}\bullet\begin{pmatrix}5\\\beta\\5\end{pmatrix}=\mu$ $\begin{pmatrix}-3\\5\\1\end{pmatrix}\bullet\begin{pmatrix}5\\\beta\\5\end{pmatrix}=0$ $-15+5\beta+5=0$ $\beta=2$ Subst. $\mathbf{r}=\begin{pmatrix}2k-2\\2-3k\\0\end{pmatrix}$ into $P_3 : \mathbf{r}\bullet\begin{pmatrix}5\\2\\5\end{pmatrix}=\mu$ $\begin{pmatrix}2k-2\\2-3k\\0\end{pmatrix}\bullet\begin{pmatrix}5\\2\\5\end{pmatrix}=\mu$ $\mu=4k-6$	The most common "error" involves the students substituting the entire line into the plane, and not knowing what to do from there. Generally, this part was well-done though, with the appropriate dot products considered as shown. As usual, many careless mistakes, for example: $-15+5\beta+5=0$ $\beta=-2$ Surprisingly though, getting this part correct does not necessarily mean getting (v) correct, with a good percentage of those who got (iv) correct leaving (v) blank. The most common error with (v) was writing: $\beta\neq 2$.
(v)	$\beta=2$, $\mu\neq 4k-6$	
11(i)	$\$\left(7500(1.02)-x\right)$	In general, Question 11 was challenging for many students, with many of them leaving the questions fully or partially blank. The difficulties in understanding the problem are largely categorised into two groups: (a) misinterpretation and misuse of terms used in the question (e.g. debt, interest, repayment), and (b) misinterpretation of the mechanics of the repayment and interest scheme. Students need to have firm understanding in both aspects to approach this problem correctly. **(i)** The two most common errors were mixing up the sequence of interest and repayment (producing $1.02(7500-x)$ as the answer), and giving the amount after both the repayment and interest in November ($1.02\left(7500(1.02)-x\right)$).

(ii)

1st Nov 2022 (1st repayment)	$7500(1.02)-x$
1st Dec 2022 (2nd repayment)	$(7500(1.02)-x)(1.02)-x$ $=7500(1.02)^2-x(1.02)-x$ $=7500(1.02)^2-x(1+1.02)$
1st Jan 2023 (3rd repayment)	$(7500(1.02)^2-x(1+1.02))(1.02)-x$ $=7500(1.02)^3-x(1+1.02+1.02^2)$
*n*th repayment	$7500(1.02)^n-x(1+1.02+...+1.02^{n-1})$

$$7500(1.02)^n-x\left(1+1.02+...+1.02^{n-1}\right)$$
$$=7500(1.02)^n-x\left(\frac{1.02^n-1}{1.02-1}\right)$$
$$=7500(1.02)^n-50x\left(1.02^n\right)+50x$$
$$=(7500-50x)(1.02)^n+50x$$

(ii) Students had difficulty generating the correct pattern. Since the question requires the formula for the debt remaining after the *n*th repayment, students should focus on finding the series related to that quantity, and not on other quantities such as the debt amount after applying interest.

Students should be reminded to write down the full series that makes up the required quantity **before** applying any necessary formulas.

Most students were able to recognise that the formula for the sum of a GP needed to be applied, though the application is not always correct (e.g. the power corresponds to the number of terms in the series).

(iii)

$$(7500-50x)(1.02)^{60}+50x\le 0$$

By GC, $x\ge 215.75974\approx \$215.76$.

OR

$$(7500-50x)(1.02)^{60}+50x\le 0$$
$$\left(50-50(1.02)^{60}\right)x\le -7500(1.02)^{60}$$
$$x\ge\frac{-7500(1.02)^{60}}{50-50(1.02)^{60}}\approx \$215.76$$

$215.75974(60)-7500=\$5445.58$

OR

$215.76(60)-7500=\$5445.60$

(iii) Many students who could not do **(i)** and **(ii)** were still able to use the formula in **(ii)** to produce the correct answers here. Wrong answers often resulted from not understanding what the formula in **(ii)** represented, and hence forming the wrong equation or inequality. $n = 59$ and 61 were also commonly seen substituted in incorrect responses.

Students are reminded to give non-exact values in intermediate working to more significant figures or decimal places so that the final answer is more accurate. For final answers involving money, it is recommended to round off to the nearest **cent**, though answers rounded off to 3 significant figures were also accepted this round.

(iv)

Substituting in $x=500$, $n=12$, the amount Antonio still owes is $\left(7500-50(500)\right)(1.02)^{12}+50(500)=2805.768595$.

$$2805.77(1.01)^{n}-500\left(1+1.01+...+1.01^{n-1}\right)$$
$$=2805.77(1.01)^{n}-500\left(\frac{1.01^{n}-1}{1.01-1}\right)$$
$$=2805.77(1.01)^{n}-50000\left(1.01^{n}\right)+50000$$
$$=\left(2805.77-50000\right)(1.01)^{n}+50000$$

n	$\left(2805.77-50000\right)(1.01)^{n}+50000$
5	398.39
6	-97.63 √
7	-598.6

Antonio's last repayment will be on 1st April 2024, and the amount he will be repaying is \$(500 – 97.63) = \$402.37.

<u>**OR**</u>

$$\left(2805.77-50000\right)(1.01)^{n}+50000\le 0$$
$$(1.01)^{n}\ge\frac{-50000}{2805.77-50000}$$
$$n\ge\frac{\ln 1.059451547}{\ln 1.01}\approx 5.80$$

Least n is 6.

After the 5th repayment, Antonio will still owe $\left(2805.77-50000\right)(1.01)^{5}+50000=398.39$, and with the 1% interest charged on this amount, he will have to repay \$402.37 on 1st April 2024.

(iv) Similar to **(iii)**, the wrong value of *n* was often substituted to find the outstanding debt remaining. Once again, weaker responses resulted from not understanding what the formula represented.

Those who proceeded beyond the first step knew they had to reuse the formula from **(ii)** with the new interest rate of 1.01, but forgot that they had to recalculate and replace some terms in the formula to reflect the changes in the quantities (e.g. outstanding debt and new interest rate).

For those who were able to find the correct *n* at the end, some students miscounted the correct month or gave the wrong year for the date. A very common careless error was to give the amount of the last repayment as \$398.39, forgetting that this is the outstanding debt from 1 March 2024, and that interest still needs to be applied at the end of March 2024 before the remaining debt is fully paid on 1 April 2024.

(v)

Let the number of additional tables (on top of 10 tables) be *t*.

Gomez Hotel cost: $1500(t+10)$.

Grande Hotel cost: $20000+\frac{t}{2}\left(2(1950)+(t-1)(-50)\right)$.

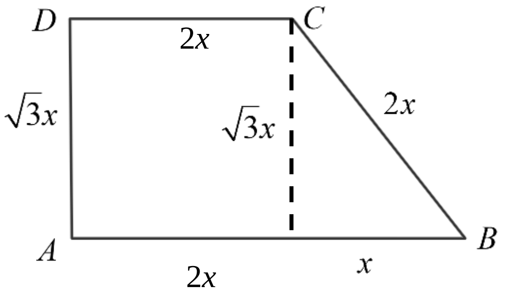
By GC,

t	$1500(t+10)$	$20000+\frac{t}{2}\left(2(1950)+(t-1)(-50)\right)$
26	54000	54450
27	55500	55100 √
28	57000	55700

The couple should need at least 37 tables.

(v) This question did not require any information from the earlier parts and could have been done independently from the rest. However, this question was also very challenging to students. The biggest oversight is for students to use the same variable *n* to represent both the total number of tables at Gomez Hotel and the number of tables at Grande Hotel after the first 10 tables.

Many students also could not apply the correct formula for the sum of an AP, or

		mistakenly used the formula for the term of an AP. A common difference of 50 instead of -50 was also commonly seen. Students are reminded to show sufficient evidence when solving inequalities, either using a quadratic curve or showing a table of values from the GC.
12(i)	 $AD = x\tan 60^\circ = \sqrt{3}x$ $BC = \dfrac{x}{\cos 60^\circ} = 2x$ $V = \dfrac{1}{2}(2x+3x)\times\sqrt{3}x\times y$ $5\sqrt{3}k = \dfrac{5\sqrt{3}x^2}{2}y$ $y = \dfrac{2k}{x^2}$ $A = \dfrac{1}{2}(2x+3x)\times\sqrt{3}x\times 2 + 3xy + 2xy + \sqrt{3}xy$ $= 5\sqrt{3}x^2 + \left(5+\sqrt{3}\right)xy$ $= 5\sqrt{3}x^2 + \left(5+\sqrt{3}\right)x\left(\dfrac{2k}{x^2}\right)$ $= 5\sqrt{3}x^2 + \dfrac{2\left(5+\sqrt{3}\right)k}{x}$ (Shown)	Generally well done. Careless mistake when finding *AD* and *BC*. Eg: $BC = \dfrac{x}{\cos 60^\circ} = \dfrac{x}{2}$ $BC = \dfrac{x}{\cos 60^\circ} = \dfrac{\sqrt{3}}{2}x$ $AD = \dfrac{x}{\tan 60^\circ} = \dfrac{x}{\sqrt{3}}$ Careless mistake when expressing *y* in terms of *x* and *k*. Eg: $y = 2kx^2$, $y = \dfrac{k}{x^2}$ When finding *A*, the area of *CDHG*, the top of the fish tank should not be counted.

(ii)	For minimum A, $\dfrac{\mathrm{d}A}{\mathrm{d}x}=0$ $10\sqrt{3}x-\dfrac{2\left(5+\sqrt{3}\right)k}{x^2}=0$ $5\sqrt{3}x=\dfrac{\left(5+\sqrt{3}\right)k}{x^2}$ $x^3=\dfrac{\left(5+\sqrt{3}\right)k}{5\sqrt{3}}$ $x=\left(\left(\dfrac{\sqrt{3}}{3}+\dfrac{1}{5}\right)k\right)^{\frac{1}{3}}$ $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}=10\sqrt{3}+\dfrac{4\left(5+\sqrt{3}\right)k}{x^3}$ At $x=\left(\left(\dfrac{\sqrt{3}}{3}+\dfrac{1}{5}\right)k\right)^{\frac{1}{3}}$, $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}=10\sqrt{3}+\dfrac{4\left(5+\sqrt{3}\right)k}{\dfrac{\left(5+\sqrt{3}\right)k}{5\sqrt{3}}}=30\sqrt{3}>0$ Hence, A is minimum when $x=\left(\left(\dfrac{\sqrt{3}}{3}+\dfrac{1}{5}\right)k\right)^{\frac{1}{3}}$.	Final answer should always simplify to the simplest form. Misconception in algebraic manipulation: Eg: 1. $5\sqrt{3}x=\dfrac{\left(5+\sqrt{3}\right)k}{x^2}$ $\Rightarrow 5\sqrt{3}=\dfrac{\left(5+\sqrt{3}\right)k}{x}$ 2. $x^3=\dfrac{k}{\sqrt{3}}+\dfrac{k}{5}$ $\Rightarrow x=\sqrt[3]{\dfrac{k}{\sqrt{3}}}+\sqrt[3]{\dfrac{k}{5}}$ Many forgot to check for minimum using 1st derivative or 2nd derivative test. When using 2nd derivative test, you need to explain why the expression for $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}$ is positive. Eg Sub the value of x you have found when $\dfrac{\mathrm{d}A}{\mathrm{d}x}=0$, show $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}$ is positive and thus conclude A is minimum.

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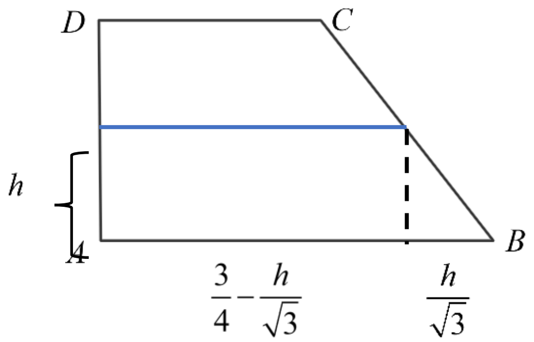
When $k=\dfrac{3}{160}$ and $y=0.6$, $0.6=\dfrac{2}{x^2}\times\dfrac{3}{160}$ => $x=0.25$

Hence, $AB=0.25\times 3=0.75=\dfrac{3}{4}$ and $AD=\sqrt{3}\times 0.25=\dfrac{\sqrt{3}}{4}$



V now denotes the volume of water in the fish tank.

$$V=\frac{1}{2}\left(\frac{3}{4}-\frac{h}{\sqrt{3}}+\frac{3}{4}\right)\times h\times 0.6$$
$$=0.3h\left(\frac{3}{2}-\frac{h}{\sqrt{3}}\right)$$
$$=0.45h-\frac{0.3}{\sqrt{3}}h^2$$

When the fish tank is half filled,

$$V=\frac{1}{2}\left(5\sqrt{3}\times\frac{3}{160}\right)=\frac{3\sqrt{3}}{64}$$
$$0.45h-\frac{0.3}{\sqrt{3}}h^2=\frac{3\sqrt{3}}{64}$$
$$\frac{0.3}{\sqrt{3}}h^2-0.45h+\frac{3\sqrt{3}}{64}=0$$

$h=0.19507$ *or* $h=2.40301$ (rejected since $h<AD\approx 0.433$)

$$\frac{dV}{dh}=0.45-\frac{0.6}{\sqrt{3}}h$$
$$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$$
$$0.015=\left(0.45-\frac{0.6}{\sqrt{3}}\times 0.19507\right)\times\frac{dh}{dt}$$
$$\frac{dh}{dt}=0.03922$$

Hence the rate of change of height is 0.03922 m per minute.

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This part is very badly done.

Note that length of $AB=3x$ is fixed and can be found given k and y now.

Many students mistaken x as the height of the water, h. Hence, they form V in terms of x and having difficulty to proceed from there.

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