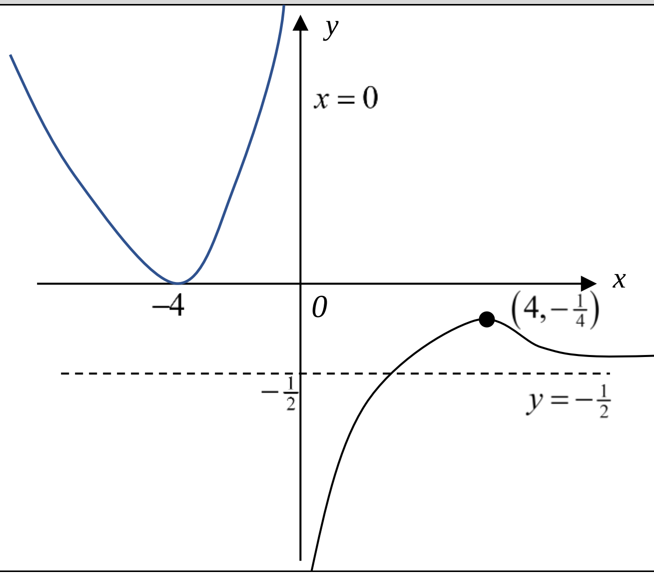
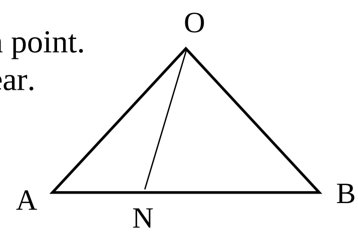
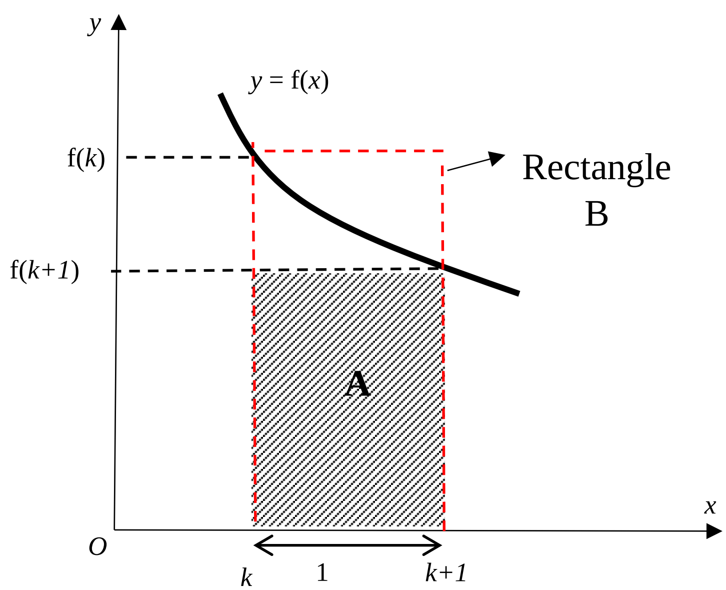
2022 H2 Maths Prelim Paper 1 Markers Report

<table>
<tr><th>Qn</th><th>Solutions</th><th>Comments</th></tr>
<tr><td>1</td><td>
</td><td>Many students misread the question and were not aware that the given graph was $y=\mathrm{f}(2x)$. As a result, the x-coordinates remained as −2 and 2, or worse −1 and 1, instead of −4 and 4.

It is also good to note that $\left(4,-\frac{1}{4}\right)$ should lie exactly between the x-axis and the asymptote $y=-\frac{1}{2}$.</td></tr>
<tr><td>2</td><td>$\int x\tan^{-1}3x\,\mathrm{d}x = \left(\tan^{-1}3x\right)\frac{x^2}{2}-\int\left(\frac{x^2}{2}\frac{3}{1+9x^2}\right)\mathrm{d}x$
$=\frac{x^2}{2}\tan^{-1}3x-\frac{3}{2}\int\frac{x^2}{1+9x^2}\,\mathrm{d}x$
$=\frac{x^2}{2}\tan^{-1}3x-\frac{1}{6}\int\left(1-\frac{1}{1+9x^2}\right)\mathrm{d}x$
$=\frac{x^2}{2}\tan^{-1}3x-\frac{1}{6}\left[x-\frac{1}{3}\tan^{-1}3x\right]+c$
$=\left(\frac{x^2}{2}+\frac{1}{18}\right)\tan^{-1}3x-\frac{1}{6}x+c$</td><td>Common mistakes:
1) $\frac{\mathrm{d}}{\mathrm{d}x}(\tan^{-1}3x)=\frac{1}{1+(3x)^2}$
2) Problem with long division $\frac{x^2}{1+9x^2}$ interpret as $x^2\,\overline{)9x^2+1}$
3) $\int\frac{1}{1+9x^2}=\tan^{-1}3x+C$
4) $\int u\mathrm{d}v=uv+\int v\mathrm{d}u$</td></tr>
<tr><td>3
(i)</td><td>Area of triangle ONB =
$\frac{1}{2}\left|\overrightarrow{\boldsymbol{ON}}\times\overrightarrow{\boldsymbol{OB}}\right|$
$=\frac{1}{2}\left|\left(\frac{5\boldsymbol{a}\times 3\boldsymbol{b}}{8}\right)\times\boldsymbol{b}\right|$
$=\frac{1}{2}\left|\left(\frac{5\boldsymbol{a}\times\boldsymbol{b}+3\boldsymbol{b}\times\boldsymbol{b}}{8}\right)\right|$
$=\frac{5}{16}\left|\boldsymbol{a}\times\boldsymbol{b}\right|$
$\boldsymbol{a}.\boldsymbol{b}=40=\left|\boldsymbol{a}\right|\left|\boldsymbol{b}\right|\cos\theta$ where angle $AOB=\theta$
$\cos\theta=\frac{40}{20\sqrt{10}}=\frac{2}{\sqrt{10}}$ Note that : $\sin^2\theta=1-\cos^2\theta$</td><td>Common mistakes & Presentation errors:
1) wrong notation vector $\underset{\sim}{a}$ written as "a"
2) $\boldsymbol{a}\times\boldsymbol{b}=\left|\boldsymbol{a}\right|\left|\boldsymbol{b}\right|\sin\theta$
3) Area of triangle ONB = $\frac{1}{2}(\overrightarrow{\boldsymbol{ON}}\times\overrightarrow{\boldsymbol{OB}})$
4) Did not include $\mid\;\mid$ when finding the area of the triangle.
5) Did not simplify the answer $\frac{5}{16}(20\sqrt{10})\sin[\cos^{-1}(\frac{2}{\sqrt{10}})]$</td></tr>
</table>

	$\sin\theta = \dfrac{\sqrt{6}}{\sqrt{10}}$ $\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = (20\sqrt{10})\times\dfrac{\sqrt{6}}{\sqrt{10}} = 20\sqrt{6}$ Area of triangle ONB = $\dfrac{5}{16}\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = \dfrac{5}{16}(20\sqrt{6}) = \dfrac{25\sqrt{6}}{4}$	6) Left the answer as 1.53 instead of the exact form
	Method 2 $\overrightarrow{NB} = \overrightarrow{OB} - \overrightarrow{ON} = \boldsymbol{b} - \dfrac{5\boldsymbol{a}+3\boldsymbol{b}}{8} = \dfrac{8\boldsymbol{b}-5\boldsymbol{a}-3\boldsymbol{b}}{8}$ $= \dfrac{5\boldsymbol{b}-5\boldsymbol{a}}{8} = \dfrac{5(\boldsymbol{b}-\boldsymbol{a})}{8} = \dfrac{5}{8}\overrightarrow{AB}$ $\overrightarrow{NB} // \overrightarrow{AB}$ with B as common point. Hence A, B and N are collinear. Area triangle ONB = $\dfrac{5}{8}$ area triangle OAB=  $(\dfrac{5}{8})\dfrac{1}{2}\lvert \overrightarrow{\boldsymbol{OA}}\times\overrightarrow{\boldsymbol{OB}}\rvert = \dfrac{5}{16}\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert$ $\boldsymbol{a}.\boldsymbol{b} = 40 = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos\theta$ where angle AOB = θ $\cos\theta = \dfrac{40}{20\sqrt{10}} = \dfrac{2}{\sqrt{10}}$ $\sin\theta = \dfrac{\sqrt{6}}{\sqrt{10}}$ $\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = (20\sqrt{10})\times\dfrac{\sqrt{6}}{\sqrt{10}} = 20\sqrt{6}$ Area of triangle ONB = $\dfrac{5}{16}\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = \dfrac{5}{16}(20\sqrt{6}) = \dfrac{25\sqrt{6}}{4}$	For Method 2 Many students did not explain why A, N and B are collinear.
(ii)	$\overrightarrow{AB}\times\overrightarrow{AC}$ $=(\boldsymbol{b}-\boldsymbol{a})\times(\boldsymbol{c}-\boldsymbol{a})$ $=\boldsymbol{b}\times\boldsymbol{c}-\boldsymbol{b}\times\boldsymbol{a}-\boldsymbol{a}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{a}$ $=\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{c}\times\boldsymbol{a}+\boldsymbol{0}$ $=\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{c}\times\boldsymbol{a}$ Hence the angle between $\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{c}\times\boldsymbol{a}$ and plane containing points A, B and C is $\dfrac{\pi}{2}$.	Either no attempt or not well done. In this question O is not on the plane π containing A, B and C. Hence π is not parallel to vectors $\boldsymbol{a},\boldsymbol{b}$ and $\boldsymbol{c}$ but students tend to assume that π is parallel to vectors $\boldsymbol{a},\boldsymbol{b}$ and $\boldsymbol{c}$

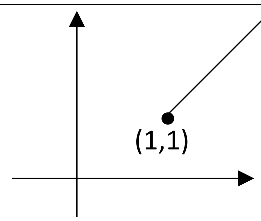
		Hence common errors like $\boldsymbol{a}\times\boldsymbol{b}$, $\boldsymbol{b}\times\boldsymbol{c}$, $\boldsymbol{c}\times\boldsymbol{a}$ are all normals to the plane π containing A, B and C commonly seen. No marks awarded for the answer $\frac{\pi}{2}$ if the method is incorrect.
4 (a)	 Area under curve = $\int_k^{k+1} \mathrm{f}\left(x\right)\mathrm{d}x$ Area of rectangle A= $\mathrm{f}\left(k+1\right)$ x 1 Area of rectangle B= $\mathrm{f}\left(k\right)$ x 1 As seen from diagram: $\mathrm{f}\left(k+1\right) < \int_k^{k+1} \mathrm{f}\left(x\right)\mathrm{d}x < \mathrm{f}\left(k\right)$	Q4 was badly done. The information given in the question was not followed: 1) $\mathrm{f}(x) > 0$ means the graph should not be drawn below the x-axis. 2) <u>curve</u> $y = \mathrm{f}(x)$ means a straight line should not be drawn. 3) for $k < x < k+1$ and $k \geq 1$ means k and $k+1$ should be clearly indicated on the x-axis. There should be only 2 rectangles involved and these should be clearly labelled as *ABCD* and *ABEF* or appropriately shaded.
(b) (i)	$\int_1^{10} \frac{1}{x}\mathrm{d}x = \int_1^{2} \frac{1}{x}\mathrm{d}x + \int_2^{3} \frac{1}{x}\mathrm{d}x + \int_3^{4} \frac{1}{x}\mathrm{d}x + ... + \int_9^{10} \frac{1}{x}\mathrm{d}x$ $< \mathrm{f}(1) + \mathrm{f}(2) + \mathrm{f}(3) + ... + \mathrm{f}(9)$ $= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9}$ $= \sum_{k=1}^{9} \frac{1}{k}$	Many of the students were totally clueless as to how to use part (a) to do part (b). For the very few who were able to write a few lines of $\int_k^{k+1} \frac{1}{x}\mathrm{d}x < \frac{1}{k}$ as

		$\int_1^2 \frac{1}{x}\,dx < \frac{1}{1}$, $\int_2^3 \frac{1}{x}\,dx < \frac{1}{2}$, $\int_3^4 \frac{1}{x}\,dx < \frac{1}{3}$, etc, they were able to appreciate the upper and lower of limits of an integral and connect all the integrals on LHS to $\int_1^{10} \frac{1}{x}\,dx$.
(ii)	$\int_1^9 \frac{1}{x}dx = \int_1^2 \frac{1}{x}dx + \int_2^3 \frac{1}{x}dx + \int_3^4 \frac{1}{x}dx + ... + \int_8^9 \frac{1}{x}dx$ > f(2) + f(3) + f(4) + ... + f(9) $= \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9} = \sum_{k=1}^{9} \frac{1}{k} - 1$ $1 + \int_1^9 \frac{1}{x}dx > \sum_{k=1}^{9} \frac{1}{k}$ $\therefore \sum_{k=1}^{9} \frac{1}{k} < 1 + \int_1^9 \frac{1}{x}dx$	An elegant solution in a script: $1 + \int_1^9 \frac{1}{x}dx$ $= 1 + \int_1^2 \frac{1}{x}dx + \int_2^3 \frac{1}{x}dx + ... + \int_8^9 \frac{1}{x}dx$ $> 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9}$ $= \sum_{k=1}^{9} \frac{1}{k}$
	$\int_1^{10} \frac{1}{x}dx < \sum_{k=1}^{9} \frac{1}{k} < 1 + \int_1^9 \frac{1}{x}dx$ $[\ln x]_1^{10} < 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9} < 1 + [\ln x]_1^9$ $\ln 10 < 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9} < 1 + \ln 9$	Again, majority of the students were not able to combine (b) (i) & (ii) results to write out the inequalities properly. There were bad presentations with no inequalities seen at all but random workings of $\int_1^{10} \frac{1}{x}dx$ and $\int_1^9 \frac{1}{x}dx$ to ln10 and ln9 directly without showing the proper integration process.

5(i)	$z_1 = 2\left(\cos\frac{\pi}{18} - \mathrm{i}\sin\frac{\pi}{18}\right) = 2e^{-\mathrm{i}\frac{\pi}{18}}$	It is important to note that the properties of modulus and argument can only be applied for $\frac{z_1^{\,2}}{z_1^{\,*}}$ and <u>not</u> for $\left(\frac{z_1^{\,2}}{z_1^{\,*}} + z_2\right)$.

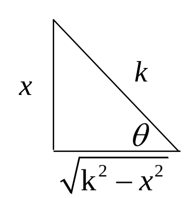
	$\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2$ $=\frac{4e^{-i\frac{\pi}{9}}}{2e^{i\frac{\pi}{18}}}+2i$ $=2e^{-i\frac{\pi}{6}}+2i$ $=2\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)+2i$ $=2\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)+2i$ $=\sqrt{3}+i$	Conversion of $2e^{-i\frac{\pi}{6}}$ from exponential form to cartesian form would be via the trigonometric form. It was inefficient to solve for a and b in $\lvert a+bi\rvert=2$ and $\arg(a+bi)=-\frac{\pi}{6}$. There was not a need to convert 2i to exponential form $2e^{i\frac{\pi}{2}}$.
(ii)	$\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3$ is real and $\left\lvert\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3\right\rvert=\frac{2}{3}$ $\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3=\frac{2}{3}$ or $-\frac{2}{3}$ $\left(\sqrt{3}+i\right)z_3=\frac{2}{3}$ or $-\frac{2}{3}$ $z_3=\frac{2}{3\left(\sqrt{3}+i\right)}$ or $-\frac{2}{3\left(\sqrt{3}+i\right)}$ $=\frac{2}{3\left(2e^{i\frac{\pi}{6}}\right)}$ or $e^{i\pi}\frac{2}{3\left(2e^{i\frac{\pi}{6}}\right)}$ $=\frac{1}{3}e^{-i\frac{\pi}{6}}$ or $\frac{1}{3}e^{i\frac{5\pi}{6}}$ $=\frac{1}{3}\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$ or $\frac{1}{3}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$	Many students failed to see that z_3 could be found from $\left(\sqrt{3}+i\right)z_3=\pm\frac{2}{3}$ or $\left(2e^{i\frac{\pi}{6}}\right)z_3=\pm\frac{2}{3}$ easily, instead of letting $z_3=a+bi$ and making careless mistakes along the way. Some students thought $\lvert z_3\rvert=\frac{2}{3}$, either reading error or calculation error.
	<u>Method 2</u> $\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2=\sqrt{3}+i=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$	Those students who used method 2 were usually successful in their solutions. Some students did not express the answers in the form

	$\left\|\left(\frac{z_1^{\,2}}{z_1^{\,*}}+z_2\right)z_3\right\|=\frac{2}{3}$ $\left\|\frac{z_1^{\,2}}{z_1^{\,*}}+z_2\right\|\left\|z_3\right\|=\frac{2}{3}$ $2\left\|z_3\right\|=\frac{2}{3}$ $\left\|z_3\right\|=\frac{1}{3}$ $\left(\frac{z_1^{\,2}}{z_1^{\,*}}+z_2\right)z_3$ is real $\Rightarrow \arg\left(\frac{z_1^{\,2}}{z_1^{\,*}}+z_2\right)z_3=0 \quad \text{or} \quad \pi$ $\Rightarrow \arg\left(\frac{z_1^{\,2}}{z_1^{\,*}}+z_2\right)+\arg z_3=0 \quad \text{or} \quad \pi$ $\Rightarrow \frac{\pi}{6}+\arg z_3=0 \quad \text{or} \quad \pi$ $\Rightarrow \arg z_3=-\frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}$ $z_3=\frac{1}{3}\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(-\frac{\pi}{6}\right)\right) \quad \text{or} \quad \frac{1}{3}\left(\cos\frac{5\pi}{6}+\mathrm{i}\sin\frac{5\pi}{6}\right)$	$r\left(\cos\theta+\mathrm{i}\sin\theta\right)$, but gave their final answers as $z_3=\frac{1}{3}\left(\cos\frac{\pi}{6}-\mathrm{i}\sin\frac{\pi}{6}\right)$, were not given the accuracy mark.
(6)(i)	Maximum value of λ for which the inverse function h exist.is $\frac{1}{2}$	This is well-done.
(ii) (a)	When $0<x\le\frac{1}{2}$, ln(2x) < 0 $\Rightarrow \left\|\ln(2x)\right\|=-\ln(2x)$ $y=-\ln(2x)+1$ $\ln(2x)=1-y$ $2x=\mathrm{e}^{1-y}$ $x=\frac{1}{2}\mathrm{e}^{1-y}$ $\mathrm{h}^{-1}(x)=\frac{1}{2}\mathrm{e}^{1-x},\ x\ge 1$	Most common mistake is writing $\left\|\ln(2x)\right\|$ as $\ln(2x)$ or $\pm\ln(2x)$. Domain is sometimes wrongly written as $(1,\infty)$ or $(\infty,1]$.

(b)		This was poorly done. Many students did not realise that $y = \mathrm{h}\,\mathrm{h}^{-1}(x) = x$ and wasted time finding $\mathrm{h}\,\mathrm{h}^{-1}(x)$. Many wrote $y = \mathrm{h}\,\mathrm{h}^{-1}(x)$ $= \left\|\ln\left(\mathrm{e}^{1-x}\right)\right\| + 1$ $= \|1-x\| + 1$ and sketched a modulus graph instead. $D_{\mathrm{h}^{-1}} = [1, \infty)$ must be considered to further interpret $\|1-x\|$ as $-(1-x)$. Those who correctly wrote $y = \mathrm{h}\,\mathrm{h}^{-1}(x) = x$ often did not consider $D_{\mathrm{h}^{-1}} = [1, \infty)$ in sketching the graph.
(c)	**<u>Method 1</u>** $\mathrm{R_h} = [1, \infty)$ and $\mathrm{R}_{\mathrm{h}^{-1}} = (0, \frac{1}{2}]$ $\mathrm{R_h} \cap \mathrm{R}_{\mathrm{h}^{-1}} = \varnothing$ No solution **<u>Method 2</u>** From GC When h(x) = x $\|\ln(2\boldsymbol{x})\| + 1 = \boldsymbol{x}$ From GC x = 2.678 > 0.5 (no solution) **<u>Method 3</u>** From GC When h(x) = x $-\ln(2\boldsymbol{x}) + 1 = \boldsymbol{x}$ From GC x = 0.685 > 0.5 (no solution)	This was poorly done. Many do not understand that solution set refers to the values of x such that $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$. Those who wrote the solution as 2.678 or 0.685 failed to consider that solution set is such that $0 < \boldsymbol{x} \le \dfrac{1}{2}$ since $D_{\mathrm{h}} = \left(0, \frac{1}{2}\right]$.

7(i)	$y = \ln(2 + \sin 2x)$ $\mathrm{e}^{y} \dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\cos 2x$	Those who showed by implicit differentiation showed in 2 steps.

	$\mathrm{e}^y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-4\sin 2x$ (shown)	However, a significant number of students did direct differentiation and wasted time by showing the result in at least 7-8 steps.
(ii)	$\mathrm{e}^y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+\mathrm{e}^y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}+2\mathrm{e}^y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{e}^y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3=-2\cos 2x$ $\mathrm{e}^y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+3\mathrm{e}^y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{e}^y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3=-2\cos 2x$ When $x=0, y=\ln 2, \dfrac{\mathrm{d}y}{\mathrm{d}x}=1, \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-1, \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=-2$ $\therefore\ y=\ln 2+x+\dfrac{(-1)}{2!}x^2+\dfrac{(-2)}{3!}x^3+\ldots$ $=\ln 2+x-\dfrac{1}{2}x^2-\dfrac{1}{3}x^3+\ldots$	Common mistakes: (a) Writing $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^y\right)=\mathrm{e}^y$ (b) Writing $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$ Some students are not able to simplify $\mathrm{e}^{\ln 2}$.
(iii)	$y=\ln\left(2+\sin 2x\right)$ $\approx\ln\left(2+2x-\dfrac{(2x)^3}{6}\right)$ $=\ln\left(2\left(1+x-\dfrac{2}{3}x^3\right)\right)$ $=\ln 2+\ln\left(1+x-\dfrac{2}{3}x^3\right)$ $=\ln 2+\left(x-\dfrac{2}{3}x^3\right)-\dfrac{\left(x-\frac{2}{3}x^3\right)^2}{2}+\dfrac{\left(x-\frac{2}{3}x^3\right)^3}{3}+\ldots$ $\approx\ln 2+x-\dfrac{2}{3}x^3-\dfrac{1}{2}x^2+\dfrac{1}{3}x^3$ $=\ln 2+x-\dfrac{1}{2}x^2-\dfrac{1}{3}x^3+\ldots$ (verified)	A significant number of students do not know how to start on the question. Common mistakes: (a) Writing $=\ln(2+\sin 2x)$ $=\ln\left[1+(1+\sin 2x)\right]$ $=(1+\sin 2x)+\dfrac{(1+\sin 2x)^2}{2}$ $+\dfrac{(1+\sin 2x)^3}{3}\ldots$ (b) Writing $\sin 2x\approx 2x$, omitting the x^3 term.
(iv)	$\dfrac{\ln(2+\sin 2x)}{\sqrt{1-x}}$	This is not well-done. Many students did not know that they should consider

	$\approx \dfrac{\ln 2 + x - \frac{1}{2}x^2 - \frac{1}{3}x^3}{\sqrt{1-x}}$ $= \left(\ln 2 + x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right)(1-x)^{-\frac{1}{2}}$ $\approx \left(\ln 2 + x - \frac{1}{2}x^2\right)\left(1 + \frac{1}{2}x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x)^2\right)$ $= \left(\ln 2 + x - \frac{1}{2}x^2\right)\left(1 + \frac{1}{2}x + \frac{3}{8}x^2\right)$ $\approx \ln 2 + x - \frac{1}{2}x^2 + \left(\frac{1}{2}\ln 2\right)x + \frac{1}{2}x^2 + \left(\frac{3}{8}\ln 2\right)x^2$ $= \ln 2 + \left(1 + \frac{1}{2}\ln 2\right)x + \left(\frac{3}{8}\ln 2\right)x^2$	binomial expansion of $(1-x)^{-\frac{1}{2}}$. Some students wrote $\frac{1}{\sqrt{1-x}}$ wrongly as $(1-x)^{\frac{1}{2}}$ or $(1-x)^{-1}$. A significant number of students who considered binomial expansion of $(1-x)^{-\frac{1}{2}}$ made careless mistakes in evaluating the coefficients of x and/or x^2 e.g. $1 - \frac{1}{2}x + \frac{1}{8}x^2$.
8(i)	$x = k\sin\theta$ $\frac{dx}{d\theta} = k\cos\theta$ $\int \sqrt{k^2 - x^2}\,dx$ $= \int \sqrt{k^2 - k^2\sin^2\theta}\;k\cos\theta\,d\theta$ $= \int k^2\cos^2\theta\,d\theta$ $= \frac{k^2}{2}\int \cos 2\theta + 1\,d\theta$ $= \frac{k^2}{2}\left[\frac{1}{2}\sin 2\theta + \theta\right] + c$ $= \frac{k^2}{2}\left[\sin\theta\cos\theta + \theta\right] + c$ $= \frac{k^2}{2}\left[\frac{x}{k}\sqrt{1 - \left(\frac{x}{k}\right)^2} + \sin^{-1}\left(\frac{x}{k}\right)\right] + c$ since $\cos\theta = \sqrt{1 - \sin^2\theta}$ $= \frac{x}{2}\sqrt{k^2 - x^2} + \frac{k^2}{2}\sin^{-1}\frac{x}{k} + c$	Generally, well done. Since the result to be proved is given, the working to show that $\frac{k^2}{2}\left[\sin\theta\cos\theta + \theta\right] + c$ leads to $\frac{x}{2}\sqrt{k^2 - x^2} + \frac{k^2}{2}\sin^{-1}\frac{x}{k} + c$ should be written out clearly as shown. OR Alternatively, using a right-angled triangle to get expression for $\cos\theta$ given $\sin\theta = \frac{x}{k}$.  $\cos\theta = \frac{\sqrt{k^2 - x^2}}{k}$

(ii) (a)	Point of intersection of $\frac{x^2}{4}+\frac{y^2}{16}=1$ and $x^2+y^2=7$ Substitute $y^2=7-x^2$ into $4x^2+y^2=16$ $4x^2+7-x^2=16,\ \therefore x=\pm\sqrt{3}$ and $y=\pm 2$	To find the area enclosed between two curves, the points of intersection between the two curves must be found first to get the correct the limits for integration.
	Area $=4\int_0^{\sqrt{3}}\sqrt{16-4x^2}-\sqrt{7-x^2}\,dx$ $=4\int_0^{\sqrt{3}}2\sqrt{4-x^2}-\sqrt{7-x^2}\,dx$ $=8\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^{\sqrt{3}}$ $-4\left[\frac{x}{2}\sqrt{7-x^2}+\frac{7}{2}\sin^{-1}\frac{x}{\sqrt{7}}\right]_0^{\sqrt{3}}$ $=8\left[\frac{\sqrt{3}}{2}+2\sin^{-1}\frac{\sqrt{3}}{2}-0\right]-4\left[\sqrt{3}+\frac{7}{2}\sin^{-1}\sqrt{\frac{3}{7}}\right]$ $=4\sqrt{3}+16\left(\frac{\pi}{3}\right)-4\sqrt{3}-14\sin^{-1}\sqrt{\frac{3}{7}}$ $=\frac{16\pi}{3}-14\sin^{-1}\sqrt{\frac{3}{7}}$ $A=\frac{16}{3},\ \ B=-14,\ \ C=\sqrt{\frac{3}{7}}$ OR For area taken with respect to y-axis	Many students used the wrong values for the limits of the integral. Note the correct method to use the answer from part (i) to evaluate $\int_0^{\sqrt{3}}\sqrt{16-4x^2}\,dx$. The coefficient of x^2 must be one before the result can be applied directly as shown. OR Alternatively $\int_0^{\sqrt{3}}\sqrt{16-4x^2}\,dx$ $=\frac{1}{2}\int_0^{\sqrt{3}}2\sqrt{4^2-(2x)^2}\,dx$ $=\frac{1}{2}\left[\frac{2x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\frac{2x}{4}\right]$ $+c$

	Area $=4\left[\int_2^4 \frac{1}{2}\sqrt{16-y^2}\,\mathrm{d}y-\int_2^{\sqrt{7}}\sqrt{7-y^2}\,\mathrm{d}y\right]$ $=2\left[\frac{y}{2}\sqrt{16-y^2}+8\sin^{-1}\frac{y}{4}\right]_2^4$ $-4\left[\frac{y}{2}\sqrt{7-y^2}+\frac{7}{2}\sin^{-1}\frac{y}{\sqrt{7}}\right]_2^{\sqrt{7}}$ $=2\left[8\sin^{-1}1-2\sqrt{3}-8\sin^{-1}\frac{1}{2}\right]$ $-4\left[\frac{7}{2}\sin^{-1}1-\sqrt{3}-\frac{7}{2}\sin^{-1}\frac{2}{\sqrt{7}}\right]$ $=16\left(\frac{\pi}{2}\right)-4\sqrt{3}-16\left(\frac{\pi}{6}\right)-14\left(\frac{\pi}{2}\right)+4\sqrt{3}+14\sin^{-1}\frac{2}{\sqrt{7}}$ $=14\sin^{-1}\frac{2}{\sqrt{7}}-\frac{5\pi}{3}$ $A=\frac{-5}{3},\ B=14,\ C=\frac{2}{\sqrt{7}}$	
(b)	Volume $=2\pi\left[\int_2^4 \frac{1}{4}\left(16-y^2\right)\mathrm{d}y-\int_2^{\sqrt{7}} 7-y^2\,\mathrm{d}y\right]$ $=14.58$ (to 2 dp)	Many students used the wrong expressions for volume. Since the rotation is about y axis the basic formula should be Volume $=\pi\int_{y_1}^{y_2} x^2\,\mathrm{d}y$ The limits are y coordinates of the region enclosed and the volume generated using this formula is multiplied by 2 and not 4.
9(a) (i)	<table><tr><td>Quarter</td><td>Increment</td><td>Monthly pay for the quarter</td></tr><tr><td>1</td><td>-</td><td>3000</td></tr><tr><td>2</td><td>0.6(3000)</td><td>3000+0.6(3000)</td></tr><tr><td>3</td><td>0.6^2(3000)</td><td>3000+0.6(3000)+ 0.6^2(3000)</td></tr><tr><td></td><td></td><td></td></tr></table>$Q_n=3000(1+0.6+0.6^2+0.6^3+....0.6^{n-1})$ $=3000\frac{1-0.6^n}{1-0.6}=7500(1-0.6^n)$	Correct reading and understanding of the question are key here. Many students attempted the question using previously studied standard methods, which will not apply to this question. Students who took the time to read and write out the monthly pay for at least the first three quarters, manged to proceed well with the question.

		It is good to label the columns of the table you are forming, as it also clarifies your own thinking. **It is necessary to show the geometric series before summing it using the sum of terms of GP formula.** Since the result to be proved is given, merely working backwards a few steps will not be sufficient to prove the result.
(ii)	17^{th} month is in the 6^{th} quarter $Q_6 = 7500(1-0.6^6) = 7{,}150.08$ His salary for the 17^{th} month of work is \$7,150(to nearest dollar)	Reading the question correctly would have allowed students to use the answer from (i) above, to answer the rest of the question very easily. Many students did this part without understanding what Q_n stands for.
(iii)	Total salary for first two years of work $= 3\sum_{n=1}^{8} 7500(1-0.6^n) = 146816.87$ (from GC) Total salary for two years of work is \$146,817 (to nearest dollar) Alternatively, Total salary for first two years of work $= 3\sum_{n=1}^{8} 7500(1-0.6^n)$ $= 22500\sum_{n=1}^{8}(1-0.6^n)$ $= 22500\left[8-\sum_{1}^{8}0.6^n\right]$ $= 22500\left[8-\frac{0.6(1-0.6^8)}{1-0.6}\right] = 146816.87$ Total salary for two years of work is \$146,817 (to nearest dollar)	Several students resorted to adding up individual terms and wasting their time, instead of using the GC or the sum of GP formula.
(iv)	Increment in the n^{th} quarter if he stays in the job, $3000\left(0.6^{n-1}\right) \geq 80$ Use GC table: $n \leq 8$ He will stay in his job for 24 months.	The general expression for the increment can be written down easily. Many students used the difference between the monthly pay in two consecutive quarters

<table>
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<td>to get the expression which is a very long-winded method.

Many also did not use tables from GC to get answer.

When forming the inequality, you should be clear which case you are considering.
Ie if he quits the job then $3000\left(0.6^{n-1}\right)<80$ will be used, which leads to the answer $n>8.091$. Therefore, he quits in the 9th quarter. So, he stays on the job for 8 quarters or 24months.</td>
</tr>
<tr>
<td>(b)
(i)

(ii)</td>
<td>Total amount required for 10 workers over n years
$10\times12\times\frac{n}{2}\left[2(90)+(n-1)40\right]\leq200000$
$24n^2+84n-2000\leq0$
$-11.045<n\leq7.5449$

Or using GC table
<table>
<tr><td>n</td><td>Total sum</td></tr>
<tr><td>7</td><td>176,400</td></tr>
<tr><td>8</td><td>202,800</td></tr>
</table>
$n\leq7$
The budget will last for a maximum of 7 years.
Highest incentive amount will be in the 7th year
i.e. 90 +(7-1)40=$330</td>
<td>Again, reading the question accurately will help.

The loyalty incentive is increasing by a fixed amount each year, meaning it is an arithmetic progression. i.e
90, 90+40, 90+2(40), 90+3(40), ……for n years.
Sum of AP can be used to get the total.
Since it is a monthly incentive and it is for 10 persons, the sum must be multiplied by 12 x10 to get the total amount required.</td>
</tr>
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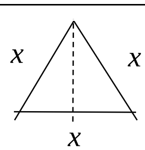
<tr>
<td>10(i)</td>
<td>

By Pythagoras' Theorem,

height of triangle $=\sqrt{x^2-\left(\frac{1}{2}x\right)^2}=\sqrt{\frac{3}{4}x^2}=\frac{\sqrt{3}}{2}x$</td>
<td>Well done</td>
</tr>
</table>

	$x+2y+2\left(\frac{\sqrt{3}}{2}x\right)=a$ $y=\frac{1}{2}\left(a-\left(1+\sqrt{3}\right)x\right)$ (shown)	
(ii)	height of pyramid $=\sqrt{\left(\frac{\sqrt{3}}{2}x\right)^2-\left(\frac{1}{2}x\right)^2}=\sqrt{\frac{1}{2}x^2}=\frac{1}{\sqrt{2}}x$ $V=x^2y+\frac{1}{3}x^2\left(\frac{1}{\sqrt{2}}x\right)$ $=\frac{1}{2}ax^2-\frac{1}{2}\left(1+\sqrt{3}\right)x^3+\frac{1}{3\sqrt{2}}x^3$ $=\frac{1}{2}ax^2+\left(\frac{1}{3\sqrt{2}}-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x^3$ $\frac{\mathrm{d}V}{\mathrm{d}x}=ax+3\left(\frac{1}{3\sqrt{2}}-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x^2$ $=ax+\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)x^2$ When $\frac{\mathrm{d}V}{\mathrm{d}x}=0$ $\Rightarrow ax+\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)x^2=0$ $\Rightarrow x\left(a+\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)x\right)=0$ $\Rightarrow x=0$ (rejected since $x>0$) or $x=\frac{a}{-\frac{1}{\sqrt{2}}+\frac{3}{2}+\frac{3\sqrt{3}}{2}}=0.295a$ Using second derivative test,	Most students could find the height of pyramid. For V, most students found the volume of the pyramid only. Students need to add the volume of the cuboid: x^2y. Most students went on to differentiate V but did not attempt to solve $\frac{\mathrm{d}V}{\mathrm{d}x}=0$. Some have forgotten to prove maximum. For the first derivative test, students must indicate the actual values in the table. For the second derivative test, students must substitute the x value found and get a negative value for $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$.

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$$\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=a+2\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)x$$

$$=a+2\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)\left(\frac{a}{-\frac{1}{\sqrt{2}}+\frac{3}{2}+\frac{3\sqrt{3}}{2}}\right)$$

$$=a-2a$$

$$=-a<0$$

Maximum V.

</td>
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OR Using first derivative test,

<table>
<tr><td>x</td><td>$0.290a$</td><td>$\dfrac{a}{-\frac{1}{\sqrt{2}}+\frac{3}{2}+\frac{3\sqrt{3}}{2}}$</td><td>$0.3a$</td></tr>
<tr><td>$\dfrac{\mathrm{d}V}{\mathrm{d}x}$</td><td>$0.00482a^2$
>0</td><td>0</td><td>$-0.00519a^2$
<0</td></tr>
<tr><td>Slope</td><td>/</td><td>-</td><td>\</td></tr>
</table>

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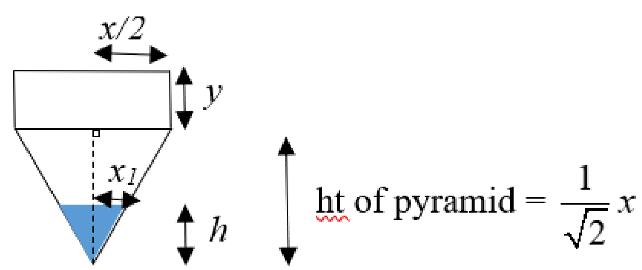
<td></td>
</tr>
<tr>
<td>**(iii)**</td>
<td>



By similar triangles,

$$\frac{x_1}{\frac{x}{2}}=\frac{h}{\frac{1}{\sqrt{2}}x}$$

$$x_1=\frac{\sqrt{2}}{2}h$$

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<td>

Many students cannot distinguish between height of the pyramid, $\frac{1}{\sqrt{2}}x$ in (ii), and the height of water, h in this case.

Many students used the volume, V in (ii) instead of finding the volume of water, V_1 in this case.

Students should note the following:

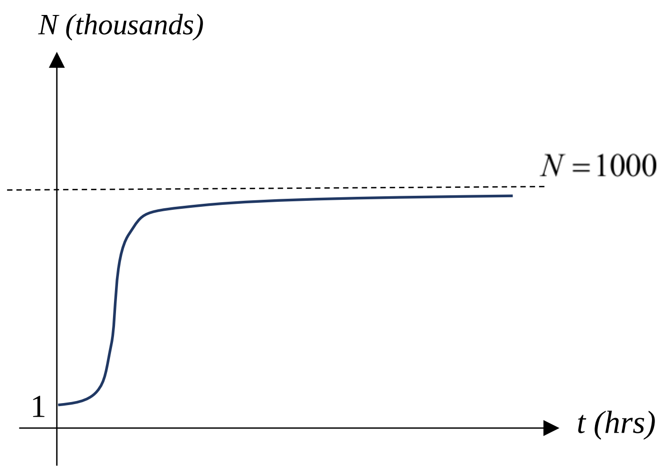
They can then express x_1 in terms of h and hence V_1 in terms of h in order to find $\frac{dV_1}{dh}$ before applying the chain rule.

</td>
</tr>
</table>

	Since it is a square base pyramid, its base area is $(2x_1)^2$. Volume of water, V_1 $=\frac{1}{3}(2x_1)^2 h=\frac{1}{3}\left(2\left(\frac{\sqrt{2}}{2}h\right)\right)^2 h=\frac{2}{3}h^3$ $\frac{\mathrm{d}V_1}{\mathrm{d}h}=2h^2$ $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V_1}{\mathrm{d}t}\cdot\frac{\mathrm{d}h}{\mathrm{d}V_1}$ $=(0.1)\left(\frac{1}{2h^2}\right)$ $=(0.1)\left(\frac{1}{2(0.05a)^2}\right)$ $=\frac{20}{a^2}$ The depth of the water is increasing at a rate of $\frac{20}{a^2}$ m/mins.	To find $\frac{dV_1}{dh}$, students must attempt to express V_1 in terms of h first. It is WRONG to differentiate V_1 when it is in terms of x_1 and h.

11 (i)	$\frac{\mathrm{d}N}{\mathrm{d}t}=kN(P-N)$	Well done
11 (ii)	Using $N=1$ and $\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{P-1}{10}$ when $t=0$, $\frac{P-1}{10}=k(1)(P-1)$ $\therefore k=\frac{1}{10}$	Many did not use the conditions given to find k.

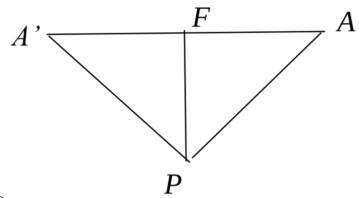
	$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{N}{10}(P-N)$ $\int \frac{1}{N(P-N)}\,\mathrm{d}N=\int \frac{1}{10}\,\mathrm{d}t$ $\int \frac{\frac{1}{P}}{N}+\frac{\frac{1}{P}}{(P-N)}\,\mathrm{d}N=\int \frac{1}{10}\,\mathrm{d}t$ $\frac{1}{P}\ln(N)-\frac{1}{P}\ln(P-N)=\frac{t}{10}+C$ (Note: $N>0$ and $P>N$) $\frac{1}{P}\ln\left(\frac{N}{P-N}\right)=\frac{t}{10}+C$ $\ln\left(\frac{N}{P-N}\right)=0.1Pt+PC$ $\frac{N}{P-N}=\mathrm{e}^{0.1Pt+PC}$ $\frac{N}{P-N}=A\mathrm{e}^{0.1Pt}$ whereby $A=\mathrm{e}^{PC}$ $N=A\mathrm{e}^{0.1Pt}(P-N)$ $N=AP\mathrm{e}^{0.1Pt}-AN\mathrm{e}^{0.1Pt}$ $N+AN\mathrm{e}^{0.1Pt}=AP\mathrm{e}^{0.1Pt}$ $N\left(1+A\mathrm{e}^{0.1Pt}\right)=AP\mathrm{e}^{0.1Pt}$ $N=\frac{AP\mathrm{e}^{0.1Pt}}{1+A\mathrm{e}^{0.1Pt}}$ $N=\frac{AP\mathrm{e}^{0.1Pt}}{\mathrm{e}^{0.1Pt}\left(\mathrm{e}^{-0.1Pt}+A\right)}$ $N=\frac{AP}{A+\mathrm{e}^{-0.1Pt}}$ (Shown)	Students separated the variables correctly, but they should show this step before they start integrating: $\int \frac{1}{N(P-N)}\,\mathrm{d}N=\int \frac{1}{10}\,\mathrm{d}t$ The integration techniques used were correct, but many careless mistakes were seen. Students should introduce the constant A and attempt to make N the subject. Working MUST be shown to illustrate how students can proceed from this $N=\frac{AP\mathrm{e}^{0.1Pt}}{1+A\mathrm{e}^{0.1Pt}}$ to the expression given.

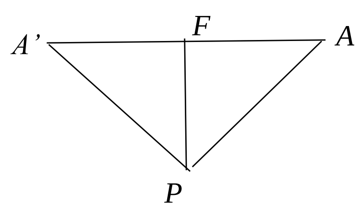
11 (iii)	Since the population size is 1 million, $\therefore P=1000$. Using $N=1$ when $t=0$, $1=\dfrac{1000A}{A+e^{0}}$ $A+1=1000A$ $999A=1$ $A=\dfrac{1}{999}$ $N=\dfrac{\frac{1000}{999}}{\frac{1}{999}+e^{-100t}}$ 	Many students attempted to find A, but they did not write down the value of P in the expression for N. For the sketch, the horizontal asymptote and the x-intercept MUST be shown. The curve can only be drawn in the first quadrant as t, $N > 0$. Students must learn how to get the curvature (near the origin) from their GC.
11 (iv)	In the long term, the number of people who will receive a piece of information will <u>tend towards</u> the population size of 1 million.	Students must explain in CONTEXT and use key words such as tends to or approaches 1 million. It is *incorrect* to say the entire population *will get* the piece of information.

11 (v)	When $N = 0.99\times 1000 = 990,$ using GC to solve $990 = \dfrac{\frac{1000}{999}}{\frac{1}{999}+\mathrm{e}^{-100t}}$ $t = 0.1150187 \text{ hr} = 6.901122 \text{ min} \approx 7 \text{ min}$ (nearest minute) It takes about 7 minutes for a piece of information to reach 99% of the population of one million.	Well done for those with the correct A and P stated in the earlier parts. However, students must READ the question carefully to correct their answer to the nearest minute.

12(i)	$\boldsymbol{l}_{AP} : \boldsymbol{r} = \begin{pmatrix}4\\-5\\10\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}$ ……..(1) $\pi_1 : \boldsymbol{r}.\begin{pmatrix}0\\1\\3\end{pmatrix}=\begin{pmatrix}0\\-1\\2\end{pmatrix}.\begin{pmatrix}0\\1\\3\end{pmatrix}=5$ …….(2) Subst (1) into (2) $\begin{pmatrix}4-2\lambda\\-5+2\lambda\\10+\lambda\end{pmatrix}.\begin{pmatrix}0\\1\\3\end{pmatrix}=5$ $-5+2\lambda+30+3\lambda=5$ $\lambda=-4$ $\overrightarrow{OP}=\begin{pmatrix}4+8\\-5-8\\10-4\end{pmatrix}=\begin{pmatrix}12\\-13\\6\end{pmatrix}$ $P(12,-13,6)$	**Most students can score at least 2 marks** **Common mistakes:** 1) Some just verify (12,-13,6) falls on π_1 by substituting $\begin{pmatrix}12\\-13\\6\end{pmatrix}$ into equation of π_1 i.e $\begin{pmatrix}12\\-13\\6\end{pmatrix}.\begin{pmatrix}0\\1\\3\end{pmatrix}=0-13+18=5$ 2) Some wrote $P = \begin{pmatrix}12\\-13\\6\end{pmatrix}$ = (12,-13,6) 3) Some did not conclude that coordinates of P is (12,-13,6) 4) Many write vector $\boldsymbol{a}$ as "a" and not "$\underset{\sim}{a}$"
(ii)	**Method 1** Equation of normal to π_1 $L_N : \boldsymbol{r} = \begin{pmatrix}12\\-13\\6\end{pmatrix}+\alpha\begin{pmatrix}0\\1\\3\end{pmatrix}=\begin{pmatrix}12\\-13+\alpha\\6+3\alpha\end{pmatrix}$	**Not well done especially students who used method 2.** **Common mistakes:** 1)Many assume that

	Let F be the foot of perpendicular from A to normal to plane $\overline{OF}=\begin{pmatrix}12\\-13+\alpha\\6+3\alpha\end{pmatrix}$ ………………(1) $\overrightarrow{AF}=\begin{pmatrix}12\\-13+\alpha\\6+3\alpha\end{pmatrix}-\begin{pmatrix}4\\-5\\10\end{pmatrix}=\begin{pmatrix}8\\-8+\alpha\\-4+3\alpha\end{pmatrix}$ $\overrightarrow{AF}\perp L_N$ $\overrightarrow{AF}\bullet\begin{pmatrix}0\\1\\3\end{pmatrix}=0$ $\begin{pmatrix}8\\-8+\alpha\\-4+3\alpha\end{pmatrix}\bullet\begin{pmatrix}0\\1\\3\end{pmatrix}=0$ $-8+\alpha-12+9\alpha=0$ $\alpha=2$ $\overrightarrow{AF}=\begin{pmatrix}8\\-8+\alpha\\-4+3\alpha\end{pmatrix}=\begin{pmatrix}8\\-6\\2\end{pmatrix}$	$\mathbf{r}=\begin{pmatrix}4\\-5\\10\end{pmatrix}+\alpha\begin{pmatrix}0\\1\\3\end{pmatrix}$ is vector equation of line AF and found the position vector of the foot of perpendicular from A to plane π_1. 2) Some assume that $\overline{OF}=\begin{pmatrix}4\\-5+\alpha\\10+3\alpha\end{pmatrix}$

(ii)	**Method 2** Let *A*’ be the mirror image of *A* in the normal to the plane and *F* be the foot of perpendicular from *A* to the normal  $\overrightarrow{AP} = \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \\ -4 \end{pmatrix}$ $\overrightarrow{FP} = \left[\overrightarrow{AP} \bullet \frac{1}{\sqrt{1+9}} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right] \frac{1}{\sqrt{1+9}} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ $= \frac{1}{10} \left[\begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right] \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \frac{1}{10}(-8-12) \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$ $\overrightarrow{AF} = \overrightarrow{AP} + \overrightarrow{PF} = \begin{pmatrix} 8 \\ -8 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 2 \end{pmatrix}$	Students who used method 2 (vector component) to find $\overrightarrow{FP}$ did badly. Common mistakes are 1) $\overrightarrow{FP} = \left\| \overrightarrow{AP}.\hat{\boldsymbol{u}} \right\| \hat{\boldsymbol{u}}$ 2) $\overrightarrow{FP} = (\overrightarrow{PA}.\hat{\boldsymbol{u}})\hat{\boldsymbol{u}}$ 3) $\overrightarrow{AF} = (\overrightarrow{AP} \times \hat{\boldsymbol{u}})\hat{\boldsymbol{u}}$ where $\hat{\boldsymbol{u}} = \frac{1}{\sqrt{1+9}} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

(iii)	Let L_N be the normal to π_1 at P **Method 1** From (1) in (ii) $\overrightarrow{OF}=\begin{pmatrix}12\\-13+2\\6+6\end{pmatrix}=\begin{pmatrix}12\\-11\\12\end{pmatrix}$ Let A' be the mirror image of A in L_N $\dfrac{\overrightarrow{OA}+\overrightarrow{OA'}}{2}=\overrightarrow{OF}$ $\overrightarrow{OA'}=2\overrightarrow{OF}-\overrightarrow{OA}=2\begin{pmatrix}12\\-11\\12\end{pmatrix}-\begin{pmatrix}4\\-5\\10\end{pmatrix}=\begin{pmatrix}20\\-17\\14\end{pmatrix}$ $\overrightarrow{PA'}=\begin{pmatrix}20\\-17\\14\end{pmatrix}-\begin{pmatrix}12\\-13\\6\end{pmatrix}=\begin{pmatrix}8\\-4\\8\end{pmatrix}=4\begin{pmatrix}2\\-1\\2\end{pmatrix}$ *A'* falls on line *PC* Hence equation of l_{PC} $: \boldsymbol{r}=\begin{pmatrix}12\\-13\\6\end{pmatrix}+\beta\begin{pmatrix}2\\-1\\2\end{pmatrix}$ $\mathbf{l_{PC}}: \left(\boldsymbol{r}-\begin{pmatrix}12\\-13\\6\end{pmatrix}\right)\times\begin{pmatrix}2\\-1\\2\end{pmatrix}=\mathbf{0}$	**Not well Done** Many assume that C is the mirror image of A in L_N. i.e $\dfrac{\overrightarrow{OA}+\overrightarrow{OC}}{2}=\overrightarrow{OF}$ commonly seen which is incorrect. The mirror image A' falls on the line PC . Hence although they manage to obtain the equation of line PC because P, A' and C are collinear, the working is incorrect. Expressing the equation in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$ is badly done
(iii)	**Method 2** Let *A'* be the mirror image of A in the normal to the plane and F be the foot of perpendicular from A to the normal  $\overrightarrow{AA'}=2\overrightarrow{AF}=\begin{pmatrix}16\\-12\\4\end{pmatrix}$	Students are careless regarding the directions of the vectors 1)**Method 2** $\overrightarrow{FA'}=\overrightarrow{FA}$

(iii)	$\overrightarrow{PA'} = \overrightarrow{PA} + \overrightarrow{AA'} = \begin{pmatrix}-8\\8\\4\end{pmatrix} + \begin{pmatrix}16\\12\\4\end{pmatrix} = \begin{pmatrix}8\\-4\\8\end{pmatrix} = 4\begin{pmatrix}2\\-1\\2\end{pmatrix}$ *A'* falls on line *PC* Hence equation of l_{PC} **:** $\boldsymbol{r} = \begin{pmatrix}12\\-13\\6\end{pmatrix} + \beta\begin{pmatrix}2\\-1\\2\end{pmatrix}$ $\mathbf{l_{PC}}: \left(\boldsymbol{r} - \begin{pmatrix}12\\-13\\6\end{pmatrix}\right) \times \begin{pmatrix}2\\-1\\2\end{pmatrix} = \mathbf{0}$ **Method 3** $\overrightarrow{PF} = \dfrac{\overrightarrow{PA'} + \overrightarrow{PA}}{2}$ $\overrightarrow{PA'} = 2\overrightarrow{PF} - \overrightarrow{PA} = 2\begin{pmatrix}0\\2\\6\end{pmatrix} - \begin{pmatrix}-8\\8\\4\end{pmatrix} = \begin{pmatrix}8\\-4\\8\end{pmatrix} = 4\begin{pmatrix}2\\-1\\2\end{pmatrix}$ *A*' falls on line *PC* Hence equation of l_{PC} **:** $\boldsymbol{r} = \begin{pmatrix}12\\-13\\6\end{pmatrix} + \beta\begin{pmatrix}2\\-1\\2\end{pmatrix}$ $\mathbf{l_{PC}}: \left(\boldsymbol{r} - \begin{pmatrix}12\\-13\\6\end{pmatrix}\right) \times \begin{pmatrix}2\\-1\\2\end{pmatrix} = \mathbf{0}$	2) **Method 3 (Ratio theorem)** $\overrightarrow{FP} = \dfrac{\overrightarrow{PA'} + \overrightarrow{PA}}{2}$ instead of $\overrightarrow{PF} = \dfrac{\overrightarrow{PA'} + \overrightarrow{PA}}{2}$

<table>
<tr>
<td>(iv)</td>
<td>
<u>Method 1</u>
$\boldsymbol{l_{AP}}:\boldsymbol{r}=\begin{pmatrix}4\\-5\\10\end{pmatrix}+\lambda\begin{pmatrix}-2\\2\\1\end{pmatrix}$
$\pi_2:\boldsymbol{r}.\begin{pmatrix}2\\1\\2\end{pmatrix}=1$
$\begin{pmatrix}-2\\2\\1\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}=-4+2+2=0$
$\therefore \boldsymbol{n}\perp\pi_2$ and $\boldsymbol{n}\perp l_{AP}$
$l_{AP}\ //\ \pi_2$
</td>
<td>
This is a “show” question but many just wrote
$\begin{pmatrix}-2\\2\\1\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}=0$
and did not show the scalar product.

Although no marks were deducted, students should explain why when the scalar product is zero, this implies $l_{AP}\ //\ \pi_2$.
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<u>Method 2</u>
$\boldsymbol{l_{AP}}:\boldsymbol{r}=\begin{pmatrix}4\\-5\\10\end{pmatrix}+\lambda\begin{pmatrix}-2\\2\\1\end{pmatrix}$ …………(1)
$\pi_2:\boldsymbol{r}.\begin{pmatrix}2\\1\\2\end{pmatrix}=1$…………(2)
Substitute (1) into (2)

$\begin{pmatrix}4-2\lambda\\-5+2\lambda\\10+\lambda\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}=1$
LHS= $8-4\lambda-5+2\lambda+20+2\lambda=23\neq 1$
No solution
None of the points on the line passing through AP falls on π_2. Hence $l_{AP}\ //\ \pi_2$
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Insufficient working like method 1 was also seen for method 2.
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Method 1
Pick a point Q (0,1,0) on π_2.
$A(4,-5,10)$ is a point on l_{AP}.
$\overrightarrow{QA}=\begin{pmatrix}4\\-5\\10\end{pmatrix}-\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}4\\-6\\10\end{pmatrix}$
Shortest distance between the ray AP and π_2
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Common mistake
Shortest distance =
$\dfrac{\left\|\begin{pmatrix}4\\-6\\10\end{pmatrix}\times\begin{pmatrix}2\\1\\2\end{pmatrix}\right\|}{\sqrt{2^2+1^2+2^2}}$
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	$= \dfrac{\left\|\begin{pmatrix}4\\-6\\10\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix}\right\|}{\sqrt{2^2+1^2+2^2}} = \dfrac{\|8-6+20\|}{3} = \dfrac{22}{3}$	
	Method 2 Let N be the foot of perpendicular from A to plane π_2 $\boldsymbol{l_{AN}} : \boldsymbol{r} = \begin{pmatrix}4\\-5\\10\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\2\end{pmatrix} = \begin{pmatrix}4+2\lambda\\-5+\lambda\\10+2\lambda\end{pmatrix}$ $\begin{pmatrix}4+2\lambda\\-5+\lambda\\10+2\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix} = 1$ $8-4\lambda-5+\lambda+20+4\lambda = 1$ $\lambda = -\dfrac{22}{9}$ $\overrightarrow{ON} = \begin{pmatrix}4-\frac{44}{9}\\-5-\frac{22}{9}\\10-\frac{44}{9}\end{pmatrix} = \begin{pmatrix}-\frac{8}{9}\\-\frac{67}{9}\\\frac{46}{9}\end{pmatrix}$ $\overrightarrow{AN} = \begin{pmatrix}4+\frac{8}{9}\\-5+\frac{67}{9}\\10-\frac{46}{9}\end{pmatrix} = \dfrac{1}{9}\begin{pmatrix}44\\22\\44\end{pmatrix}$ $\left\|\overrightarrow{AN}\right\| = \dfrac{1}{9}\sqrt{44^2+22^2+44^2} = \dfrac{22}{3}$	**Not well Done** Some thought that the shortest distance is $\left\|\overrightarrow{ON}\right\|$