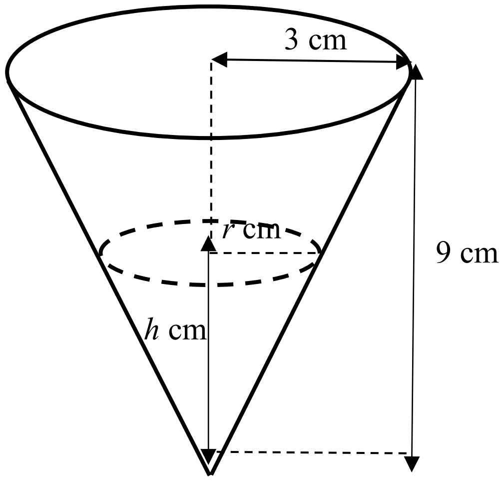
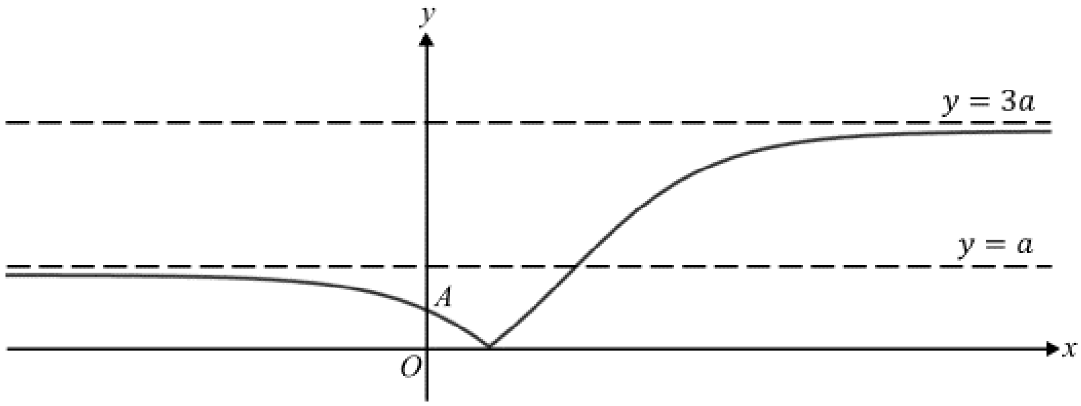
Promos Practice Paper 3 [YIJC 2022] 98marks

1 [It is given that the volume of a circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.]



A cone-shaped paper drinking cup with height 9 cm and radius 3 cm is shown above. Water is poured at a constant rate of 5 cm^3 per second into the cup. When the depth of water is h cm, the surface of the water has radius r cm (see diagram). Find the rate of increase of the depth of water when $r = 2$. **[4]**

2 The diagram below shows the graph of $y = |\mathrm{f}(x)|$ with asymptotes $y = a$ and $y = 3a$, where $a > 0$. The curve crosses the y-axis at the point $A\left(0, \frac{a}{2}\right)$.



Given that f is an increasing function, sketch the graphs of

(a) $y = \mathrm{f}(x) + a$, **[2]**

(b) $y = \mathrm{f}(-|x|)$, **[2]**

stating the equations of any asymptotes and the coordinates of the point corresponding to A after the transformation.

3 **MI Promo 9758/2021/PU2/01/Q2**

(i) Given that θ is sufficiently small, show that

$$\sin\left(\frac{\pi}{3}-\theta\right)\approx a+b\theta+c\theta^2,$$

where a, b and c are constants to be determined. **[3]**

(ii) By using the substitution $\theta=\frac{\pi}{10}$, find an approximate value for $\cos\left(\frac{4\pi}{15}\right)$, giving your answer correct to 5 decimal places. **[2]**

(iii) By using a calculator to evaluate $\cos\left(\frac{4\pi}{15}\right)$, correct to 5 decimal places, explain why the approximation in part **(ii)** is not good. **[1]**

4 **MI PU2 Promo 9758/2019/01/Q4**

Do not use a calculator in answering this question.

(i) Given that $z=2+\mathrm{i}$ is a root of the equation $z^3+2z^2-19z+30=0$, find the other roots. [4]

(ii) Hence find in cartesian form the roots of the equation $\mathrm{i}w^3-2w^2+19\mathrm{i}w+30=0$. [2]

5 A curve C has equation $xy^2-3x^2y+144=0$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the coordinates of the turning point of C. **[5]**

The point P on C has coordinates (4, k) for some constant k.

(ii) Find the equation of the tangent at P. **[3]**

6 An arithmetic series has first term 3 and common difference d, where d is non-zero. A geometric series has first term a and common ratio r. Given that the 37th, 7th and 1st terms of the arithmetic series are consecutive terms of the geometric series, find d. **[3]**

Deduce that the geometric series is convergent. Find, in terms of a, the sum to infinity of the odd-numbered terms (i.e. the 1st, 3rd, 5th, …. terms) of the geometric series. **[3]**

Given further that a = 3072, find the least value of n such that the sum of the first n terms of the arithmetic series exceeds the sum to infinity of the odd-numbered terms of the geometric series. **[2]**

7 A curve C has parametric equations

$$x = 2t, \qquad y = \frac{1}{1-t}, \quad t \neq 1.$$

Show that the normal to C at the point with parameter t has equation

$$(1-t)y + 2(1-t)^3 x = 1 + 4t(1-t)^3. \qquad \textbf{[4]}$$

State the equation of the normal at the point P where $t = 2$. This normal cuts C again at the point Q. Find the exact coordinates of Q. **[4]**

8 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 1 + \frac{2}{x^2 - 4}, \quad x \in \mathbb{R}, \ 0 \leq x < 2,$$

$$\mathrm{g} : x \mapsto \frac{1}{x}, \quad x \in \mathbb{R}, \ x \geq 1.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[3]**

(ii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the x- and y-axes. **[4]**

(iii) Solve the equation $\mathrm{fg}(x) = \frac{2}{5}$. **[2]**

9 On 1 January 2022, Jerald puts \$500 into a savings account which pays interest at a rate of 0.1% per month on the last day of each month. On the first day of each subsequent month from February 2022, he puts another \$$x$ into the account.

(i) Write down how much Jerald's initial deposit of \$500 will become on 30 November 2022 after adding compound interest earned. **[1]**

(ii) Taking January 2022 as the first month, show that the amount of money in Jerald's account on the last day of the nth month is

$$500(1.001)^n + 1001x\left[(1.001)^{n-1} - 1\right]. \qquad \textbf{[3]}$$

(iii) Find the least integer value of x so that the interest earned for the first six months of the year 2022 exceeds \$30. **[2]**

(iv) Given that $x = 300$, how much will Jerald have in his account on 31 December 2025? Hence state the date on which the amount will first exceed \$15 000. **[3]**

10 The curves C and D have equations $y=\frac{5}{3}-x-\frac{4}{x-3}$ and $\frac{(x-1)^2}{6^2}+\frac{y^2}{k^2}=1$ respectively, where k is a positive constant.

(i) Using an algebraic method, find the exact range of values of y that C can take. **[4]**

(ii) On the same axes, sketch

(a) the graph of C, stating the equations of any asymptotes and the coordinates of the turning points, **[2]**

(b) the graph of D for the case where $k=2$, stating the coordinates of the centre, the turning points and the points of intersection with the x-axis. **[2]**

(iii) State the exact range of values of k such that C and D intersect at more than one point. **[1]**

(iv) State the range of values of m such that the line with equation $y+\frac{4}{3}=m(x-3)$ does not intersect C. **[1]**

11 **(a)** Mr Tan buys 5 bottles of cooking oil, 4 packets of biscuits and 2 packets of rice. Based on the usual retail price in the supermarket, the total amount paid is $73.45.

Currently, there is a "Buy 6 Get 1 Free" promotion for the biscuits. Under this promotion, Mr Suresh pays $53.30 and receives 2 bottles of cooking oil, 14 packets of biscuits and a packet of rice.

Ms Siti receives a 5% membership discount on the total bill and pays $103.93 for 4 bottles of cooking oil, 2 packets of biscuits and 5 packets of rice.

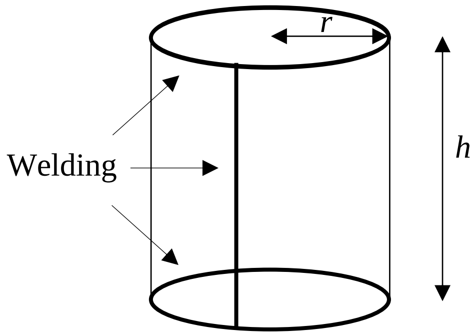
Write down and solve equations to find the usual retail price of a bottle of cooking oil, a packet of biscuits and a packet of rice. **[4]**

(b) Without using a calculator, solve

(i) $\frac{x^2}{6-x}\geq 1$, **[3]**

(ii) $x^2-4\geq (x+2)\left(4x^2-3x-1\right)$. **[3]**

12 A closed cylindrical can with radius r cm and height h cm has fixed volume 20π cm^3. The material for the top and bottom faces costs \$0.50 per cm^2 and the material for the curved surface costs \$0.30 per cm^2. It also costs \$0.80 per cm to weld the top and bottom faces onto the cylinder and \$0.60 per cm to weld the seam up the curved surface of the cylinder (see diagram).



(i) The total cost of the can is \$$C$. Show that $C = \pi r^2 + 3.2\pi r + \dfrac{12\pi}{r} + \dfrac{12}{r^2}$. **[3]**

(ii) Use differentiation to find the values of r and h which give a minimum value of C, proving that C is a minimum. State this value of C. **[7]**

(iii) It is given instead that $0.5 \le r \le 2$. Find the corresponding range of values of C. **[2]**

Answers

1	0.398 (or $\frac{5}{4\pi}$) cm/s
2	**Graph**
3	**(i)** $\therefore a=\frac{\sqrt{3}}{2}, b=-\frac{1}{2}, c=-\frac{\sqrt{3}}{4}$ **(ii)** (ii) $0.66621\ (5\text{ d.p})$
4	$z=2-\mathrm{i}$ and $z=-6$
5	(i) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y(6x-y)}{x(2y-3x)}$, $(-2,-12)$ $x=4$
6	$d=2$, Sum to infinity of odd-numbered terms $=\frac{25}{24}a$, least $n=56$
7	$y+2x=7$, $Q\left(\frac{3}{2},4\right)$.
8	(i) $\mathrm{f}^{-1}(x)=\sqrt{4+\frac{2}{x-1}}$, $\mathrm{D}_{\mathrm{f}^{-1}}=\left(-\infty,\frac{1}{2}\right]$ $x\approx 1.22\ (3\text{ s.f.})$
9	(i) \$505.53 (iii) 1798 \$14968.22 , 1st Jan 2026
10	(i) $y\le -\frac{16}{3}$ or $y\ge \frac{8}{3}$ (iii) $k>\frac{8}{3}$ $m\ge -1$
11	(a) Let \$*x*, \$*y* and \$*z* be the usual retail price of a bottle of cooking oil, a packet of biscuits and rice respectively. $x=6.85$, $y=2$ and $z=15.6$ (i) $x\le -3$ or $2\le x<6$, (ii) $x\le -2$ or $x=\frac{1}{2}$
12	$r=1.61$ (3 s.f.) and $h=7.68$ (3 s.f.). The least cost is \$52.37. $52.37\le C\le 129.21$