2021 JC2 H2MA Preliminary Examination Paper 2 Solution

1 (i)

$$\frac{1}{r^2-2r}=\frac{A}{r}+\frac{B}{r-2}$$

$$1=A(r-2)+Br$$

$$-2A=1\Rightarrow A=-\frac{1}{2}$$

$$A+B=0\Rightarrow B=\frac{1}{2}$$

$$\therefore \frac{1}{r^2-2r}=-\frac{1}{2r}+\frac{1}{2(r-2)}$$

$$\sum_{r=3}^{n}\frac{1}{r^2-2r}$$

$$=\frac{1}{2}\sum_{r=3}^{n}\left(\frac{1}{r-2}-\frac{1}{r}\right)$$

$$=\frac{1}{2}\left\{\begin{array}{ll} 1 & -\frac{1}{3} \\ +\frac{1}{2} & -\frac{1}{4} \\ +\frac{1}{3} & -\frac{1}{5} \\ +\frac{1}{4} & -\frac{1}{6} \\ +\ldots & \\ +\frac{1}{n-4} & -\frac{1}{n-2} \\ +\frac{1}{n-3} & -\frac{1}{n-1} \\ +\frac{1}{n-2} & -\frac{1}{n} \end{array}\right\}$$

$$=\frac{1}{2}\left(\frac{3}{2}-\frac{1}{n-1}-\frac{1}{n}\right)\text{ (shown)}$$

(ii)

As $n\to\infty$, $\frac{1}{n-1}\to 0$ and $\frac{1}{n}\to 0$.

Thus $\sum_{r=3}^{\infty}\frac{1}{r^2-2r}$ is a convergent series.

$$\sum_{r=3}^{\infty}\frac{1}{r^2-2r}=\frac{3}{4}$$

(iii)	$(r-1)^2 = r^2 - 2r + 1 > r^2 - 2r$ $\dfrac{1}{(r-1)^2} < \dfrac{1}{r^2 - 2r}$ for $r > 2$ $\displaystyle\sum_{r=4}^{\infty} \frac{1}{(r-1)^2} < \sum_{r=4}^{\infty} \frac{1}{r^2-2r}$ $\displaystyle = \sum_{r=3}^{\infty} \frac{1}{r^2-2r} - \frac{1}{3^2-2(3)}$ $= \dfrac{3}{4} - \dfrac{1}{3}$ $= \dfrac{5}{12}$ (shown)

2	Surface area of cylindrical piece $= \pi y(2x)$ $= 2\pi xy$ Surface area of 2 hemispherical pieces $= 4\pi\left(\dfrac{y}{2}\right)^2$ $= \pi y^2$ $C = 2\pi xy(3k) + \pi y^2(5k)$ $= \pi ky(6x + 5y)$ Since $V = \pi$, $\pi = \pi\left(\dfrac{y}{2}\right)^2(2x) + \dfrac{4}{3}\pi\left(\dfrac{y}{2}\right)^3$ $6 = 3y^2x + y^3$ $x = \dfrac{6-y^3}{3y^2}$ $C = \pi ky\left(6\left(\dfrac{6-y^3}{3y^2}\right) + 5y\right)$ $= \pi ky\left(\dfrac{12}{y^2} - 2y + 5y\right)$ $= \dfrac{12\pi k}{y} + 3\pi ky^2$

	$\frac{\mathrm{d}C}{\mathrm{d}y}=-\frac{12\pi k}{y^2}+6\pi ky=0$ $\frac{12\pi k}{y^2}=6\pi ky$ $y^3=2 \quad \Rightarrow y=\sqrt[3]{2}$ When $y=\sqrt[3]{2}$, $x=\frac{6-2}{3\left(\sqrt[3]{2}\right)^2}=\frac{4}{3\left(\sqrt[3]{4}\right)}=\frac{\sqrt[3]{16}}{3}$ $\frac{\mathrm{d}^2C}{\mathrm{d}y^2}=\frac{24\pi k}{y^3}+6\pi k=\frac{24\pi k}{2}+6\pi k=18\pi k>0$ Hence, C is minimum when $y=\sqrt[3]{2}$ and $x=\frac{\sqrt[3]{16}}{3}$.

3(a) (i)	$z=\frac{(1-\mathrm{i})^3}{\sqrt{2}(a+\mathrm{i})^2}$ $\Rightarrow \lvert z\rvert=\frac{\lvert 1-\mathrm{i}\rvert^3}{\sqrt{2}\lvert a+\mathrm{i}\rvert^2}$ $\Rightarrow \frac{1}{2}=\frac{\left(\sqrt{2}\right)^3}{\sqrt{2}\lvert a+\mathrm{i}\rvert^2}$ $\Rightarrow \lvert a+\mathrm{i}\rvert^2=4$ $\Rightarrow \lvert a+\mathrm{i}\rvert=2$ $\Rightarrow \sqrt{a^2+1}=2$ $\Rightarrow a=-\sqrt{3}$ (since $a<0$) $3\arg(1-\mathrm{i})-\left[\arg\sqrt{2}+2\arg(-\sqrt{3}+\mathrm{i})\right]$ $=3\left(-\frac{\pi}{4}\right)-2\left(\frac{5\pi}{6}\right)=-\frac{29\pi}{12}$ $\therefore\ \arg z=-\frac{29\pi}{12}+2\pi$ $=-\frac{5\pi}{12}$ (shown)
(a) (ii)	For z^n to have equal real and imaginary parts, $n\arg z=\frac{\pi}{4}+k\pi \Rightarrow -\frac{5n\pi}{12}=\pi\left(\frac{1}{4}+k\right)$, where $k\in\mathbb{Z}$ $n=-\frac{3+12k}{5}$ For smallest integer n, let $k=-4$ $\therefore$ smallest integer $n=9$

(b)	Method 1: $q=\left(\dfrac{\mathrm{e}^{-\mathrm{i}\theta}}{\mathrm{e}^{\mathrm{i}\theta}-\mathrm{i}}\right)\left(\dfrac{\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{i}}{\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{i}}\right)$ $=\dfrac{\mathrm{e}^{-2\mathrm{i}\theta}+\mathrm{i}\mathrm{e}^{-\mathrm{i}\theta}}{2+\mathrm{i}\left(\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}\right)}$ $\mathrm{Re}(q)=\dfrac{\cos 2\theta+\sin\theta}{2-2\sin\theta}$ $=\dfrac{1-2\sin^2\theta+\sin\theta}{2-2\sin\theta}$ $=\dfrac{(1+2\sin\theta)(1-\sin\theta)}{2(1-\sin\theta)}$ $=\dfrac{1}{2}(1+2\sin\theta)$ (shown) Method 2: $q=\dfrac{\mathrm{e}^{-\mathrm{i}\theta}}{\mathrm{e}^{\mathrm{i}\theta}-\mathrm{i}}$ $=\dfrac{\cos\theta-\mathrm{i}\sin\theta}{\cos\theta-\mathrm{i}(1-\sin\theta)}$ $=\dfrac{\cos\theta-\mathrm{i}\sin\theta}{\cos\theta-\mathrm{i}(1-\sin\theta)}\left(\dfrac{\cos\theta+\mathrm{i}(1-\sin\theta)}{\cos\theta+\mathrm{i}(1-\sin\theta)}\right)$ $\mathrm{Re}(q)=\dfrac{\cos^2\theta+\sin\theta(1-\sin\theta)}{\cos^2\theta+(1-\sin\theta)^2}$ $=\dfrac{\cos^2\theta+\sin\theta-\sin^2\theta}{\cos^2\theta+1-2\sin\theta+\sin^2\theta}$ $=\dfrac{1-2\sin^2\theta+\sin\theta}{2(1-\sin\theta)}$ $=\dfrac{(1+2\sin\theta)(1-\sin\theta)}{2(1-\sin\theta)}$ $=\dfrac{1}{2}(1+2\sin\theta)$ (shown)

4 (i)	If l and p do not intersect, 2 conditions have to be satisfied. **Condition 1:** l is parallel to p Direction vector of l is perpendicular to normal vector of p $\begin{pmatrix}2\\b\\3\end{pmatrix}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}=0$ $2+b-3=0$ $b=1$

	Condition 2: point on l is not on p Point on l does not satisfy plane equation $\begin{pmatrix}-1\\a\\4\end{pmatrix}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}\neq 11$ $-1+a-4\neq 11$ $a\neq 16$
(ii)	Since B lies on p, $l_{AB}:\underset{\sim}{r}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}+s\begin{pmatrix}1\\1\\-1\end{pmatrix}$, $s\in\mathbb{R}$ $p:\underset{\sim}{r}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}=11$ To find point B, $\left[\begin{pmatrix}-1\\-2\\4\end{pmatrix}+s\begin{pmatrix}1\\1\\-1\end{pmatrix}\right]\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}=11$ $-1+s-2+s-4+s=11$ $3s=18$ $s=6$ $\overrightarrow{OB}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}+6\begin{pmatrix}1\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\4\\-2\end{pmatrix}$ Coordinates of B are $(5,4,-2)$.
(iii)	Let A' be the reflected point of A in p. $\overrightarrow{OB}=\dfrac{\overrightarrow{OA'}+\overrightarrow{OA}}{2}$ $\overrightarrow{OA'}=2\begin{pmatrix}5\\4\\-2\end{pmatrix}-\begin{pmatrix}-1\\-2\\4\end{pmatrix}=\begin{pmatrix}11\\10\\-8\end{pmatrix}$ Line of reflection of l in p: $\underset{\sim}{r}=\begin{pmatrix}11\\10\\-8\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\3\end{pmatrix}$, $\lambda\in\mathbb{R}$
(iv)	Since C is on l, $\overrightarrow{OC}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\3\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\overrightarrow{BC}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\3\end{pmatrix}-\begin{pmatrix}5\\4\\-2\end{pmatrix}=\begin{pmatrix}-6\\-6\\6\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\3\end{pmatrix}$

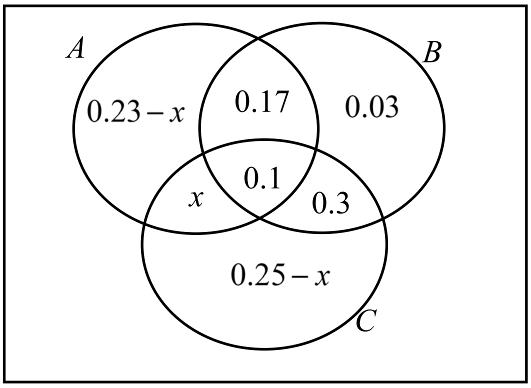
	$\left\|\overrightarrow{BC}\right\| = \sqrt{164}$ $\sqrt{(-6+2\lambda)^2+(-6+\lambda)^2+(6+3\lambda)^2} = \sqrt{164}$ $36-24\lambda+4\lambda^2+36-12\lambda+\lambda^2+36+36\lambda+9\lambda^2=164$ $14\lambda^2-56=0$ $\lambda=-2$ or $\lambda=2$ $\overrightarrow{OC}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}-2\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}-5\\-4\\-2\end{pmatrix}$ or $\overrightarrow{OC}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}+2\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}3\\0\\10\end{pmatrix}$
(v)	Method I $\overrightarrow{AB}=\begin{pmatrix}5\\4\\-2\end{pmatrix}-\begin{pmatrix}-1\\-2\\4\end{pmatrix}=\begin{pmatrix}6\\6\\-6\end{pmatrix}$ $\left\|\overrightarrow{AB}\right\|=\sqrt{3\left(6^2\right)}=\sqrt{108}$ $\sin\measuredangle BCA=\frac{\sqrt{108}}{\sqrt{164}}$ $\measuredangle BCA=54.2°$ (1 d.p.) Method 2 B and the 2 points of C form an isosceles triangle. $\therefore \measuredangle BCA=$ acute angle between $\overrightarrow{BC}$ and l $\lambda=2\Rightarrow\overrightarrow{BC}=\begin{pmatrix}-6\\-6\\6\end{pmatrix}+2\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}-2\\-4\\12\end{pmatrix}$ $\cos\measuredangle BCA=\frac{\left\|\begin{pmatrix}-2\\-4\\12\end{pmatrix}\cdot\begin{pmatrix}2\\1\\3\end{pmatrix}\right\|}{\sqrt{164}\sqrt{2^2+1^2+3^2}}$ $=\frac{28}{\sqrt{164}\sqrt{14}}$ $\measuredangle BCA=54.2°$ (1 d.p.) Method 3 Consider vectors $\overrightarrow{BC}$ & $\overrightarrow{AC}$. $\lambda=2\Rightarrow\overrightarrow{BC}=\begin{pmatrix}-6\\-6\\6\end{pmatrix}+2\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}-2\\-4\\12\end{pmatrix},\overrightarrow{AC}=\begin{pmatrix}3\\0\\10\end{pmatrix}-\begin{pmatrix}-1\\-2\\4\end{pmatrix}=\begin{pmatrix}4\\2\\6\end{pmatrix}$ $\cos\measuredangle BCA=\frac{\begin{pmatrix}-2\\-4\\12\end{pmatrix}\cdot\begin{pmatrix}4\\2\\6\end{pmatrix}}{\sqrt{164}\sqrt{4^2+2^2+6^2}}$ $=\frac{56}{\sqrt{164}\sqrt{56}}$

	$\measuredangle BCA = 54.2°$ (1 d.p.)

5 (i)	Method 1 (Direct Method) Case 1: Students from Class A are grouped as 2,2,0 Number of ways = $\dfrac{{}^4C_2\,{}^8C_2 \times {}^6C_2}{2!} = 1260$ Case 2: Students from Class A are grouped as 2,1,1 Number of ways = $\dfrac{{}^4C_2\,{}^8C_2 \times {}^2C_1\,{}^6C_3}{2!} = 3360$ Total number of ways = 4620 -- Method 2 (Using complement) Case 1: Students from Class A are grouped as 3,1,0 Number of ways = ${}^4C_3\,{}^8C_1 \times {}^7C_3 = 1120$ Case 2: Students from Class A are grouped in 4,0,0 Number of ways = $\dfrac{{}^4C_4 \times {}^8C_4}{2!} = 35$ Number of ways = $\dfrac{{}^{12}C_4\,{}^8C_4\,{}^4C_4}{3!} - (1120 + 35) = 4620$
(ii)	No of ways = $(6-1)! \times {}^6C_4 \times 4!$ $= 43200$

6 (i)	$\mathrm{P}(A\|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ $0.45 = \dfrac{\mathrm{P}(A \cap B)}{0.6}$ $\mathrm{P}(A \cap B) = 0.27$ $\mathrm{P}(A \cap B \cap C') = 0.27 - 0.1 = 0.17$
(ii)	Since A and C are independent, A and C' are independent, $\mathrm{P}(A \cap C') = \mathrm{P}(A) \times \mathrm{P}(C') = 0.5 \times (1 - 0.65) = 0.175$ $\mathrm{P}(A \cup C') = \mathrm{P}(A) + \mathrm{P}(C') - \mathrm{P}(A \cap C')$ $= 0.5 + (1 - 0.65) - 0.175$ $= 0.675$

	Let $\mathrm{P}(A \cap B' \cap C)$ be x.



$\mathrm{P}(A' \cap B' \cap C')$
$= 1 - (0.23 - x) - 0.17 - 0.1 - 0.3 - 0.03 - x - (0.25 - x)$
$= x - 0.08$

Consider:
$x - 0.08 \geq 0$ and $0.23 - x \geq 0$ and $x \geq 0$ and $0.25 - x \geq 0$
$x \geq 0.08$ and $x \leq 0.23$ and $x \geq 0$ and $x \leq 0.25$

Therefore, $0.08 \leq x \leq 0.23$.

When $x = 0.08$, $\mathrm{P}(A' \cap B' \cap C') = 0.08 - 0.08 = 0$ (least)
When $x = 0.23$, $\mathrm{P}(A' \cap B' \cap C') = 0.23 - 0.08 = 0.15$ (greatest)

Alternative
Least $\mathrm{P}(A' \cap B' \cap C')$ occurs when x is minimised:
Suppose $x = 0$, then $\mathrm{P}(A \cup B \cup C) = 0.6 + 0.23 + 0.25 = 1.08$ which is impossible as $\mathrm{P}(A \cup B \cup C) \leq 1$.
Greatest $\mathrm{P}(A \cup B \cup C) = 1$ and least $\mathrm{P}(A' \cap B' \cap C') = 0$

Greatest $\mathrm{P}(A' \cap B' \cap C')$ occurs when x is maximised:
$0.23 - x = 0$
$x = 0.23$
$\mathrm{P}(A' \cap B' \cap C') = 1 - 0.23 - 0.02 - 0.6 = 0.15$
Greatest $\mathrm{P}(A' \cap B' \cap C') = 0.15$

7 (i)	$\mathrm{P}(X = 9) = \mathrm{P}(3,3,3) = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{21}$ $\mathrm{P}(X = 8) = \mathrm{P}(3,3,2) = \frac{4}{9} \times \frac{3}{8} \times \frac{3}{7} \times 3 = \frac{3}{14}$ $\mathrm{P}(X = 7) = \mathrm{P}(3,3,1) + \mathrm{P}(3,2,2) = \left(\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times 3\right) + \left(\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times 3\right) = \frac{2}{7}$

	$\mathrm{P}(X=6)=\mathrm{P}(3,2,1)+\mathrm{P}(2,2,2)=\left(\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\times 3!\right)+\left(\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\right)=\frac{25}{84}$ $\mathrm{P}(X=5)=\mathrm{P}(3,1,1)+\mathrm{P}(2,2,1)=\left(\frac{4}{9}\times\frac{2}{8}\times\frac{1}{7}\times 3\right)+\left(\frac{3}{9}\times\frac{2}{8}\times\frac{2}{7}\times 3\right)=\frac{5}{42}$ $\mathrm{P}(X=4)=\mathrm{P}(2,1,1)=\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\times 3=\frac{1}{28}$ **OR** $\mathrm{P}(X=9)=\frac{{}^4C_3}{{}^9C_3}=\frac{1}{21}$ $\mathrm{P}(X=8)=\frac{{}^4C_2\times{}^3C_1}{{}^9C_3}=\frac{3}{14}$ $\mathrm{P}(X=7)=\frac{{}^4C_2\times{}^2C_1}{{}^9C_3}+\frac{{}^4C_1\times{}^3C_2}{{}^9C_3}=\frac{2}{7}$ $\mathrm{P}(X=6)=\frac{{}^4C_1\times{}^3C_1\times{}^2C_1}{{}^9C_3}+\frac{{}^3C_3}{{}^9C_3}=\frac{25}{84}$ $\mathrm{P}(X=5)=\frac{{}^4C_1\times{}^2C_2}{{}^9C_3}+\frac{{}^3C_2\times{}^2C_1}{{}^9C_3}=\frac{5}{42}$ $\mathrm{P}(X=4)=\frac{{}^2C_2\times{}^3C_1}{{}^9C_3}=\frac{1}{28}$
(ii)	$\mathrm{P}(X \text{ is odd})=\frac{1}{21}+\frac{2}{7}+\frac{5}{42}=\frac{19}{42}$ $\mathrm{E}(Y)=\left(1-\frac{19}{42}\right)(-w)+\frac{19}{42}(w)$ $-0.8=-\frac{23}{42}w+\frac{19}{42}w$ $w=8.4$ $\mathrm{Var}(Y)=\mathrm{E}(Y^2)-[\mathrm{E}(Y)]^2$ $=\left[\left(1-\frac{19}{42}\right)(-8.4)^2+\left(\frac{19}{42}\right)(8.4)^2\right]-(-0.8)^2$ $=69.92$
(iii)	Player wins \$$2w$ $\Rightarrow$ wins 6 out of 10 rounds Let R be the number of rounds that the player wins, out of 10 rounds. $R\sim\mathrm{B}\left(10,\frac{19}{42}\right)$

x	4	5	6	7	8	9
$\mathrm{P}(X=x)$	$\frac{1}{28}$	$\frac{5}{42}$	$\frac{25}{84}$	$\frac{2}{7}$	$\frac{3}{14}$	$\frac{1}{21}$

	$\text{P}(R=6)=0.162$ (3 s.f.) **Alternative** Required probability $=\left(\frac{19}{42}\right)^6\left(1-\frac{19}{42}\right)^4\times\frac{10!}{6!4!}$ $=0.162$ (3 s.f.)

8 (i)	Let X denote the waiting time for vaccination in Polyclinics. $X\sim \text{N}\left(25,6^2\right)$ $\text{P}(X>30)=0.20233$ $\text{P}(X<30)=1-0.20233=0.79767$ Required probability $=\text{P}(X>30)\times\text{P}(X<30)\times 2!$ $=0.323$ (3 s.f.)
(ii)	Let Y denote the waiting time for vaccination in Community Clubs. $Y\sim \text{N}\left(20,3^2\right)$ $X-Y\sim \text{N}\left(25-20,6^2+3^2\right)$ $\therefore X-Y\sim \text{N}(5,45)$ $\text{P}(\lvert X-Y\rvert\geq 3)$ $=\text{P}(X-Y\geq 3)+\text{P}(X-Y\leq -3)$ $=0.61720+0.11652$ $=0.734$ (3 s.f.)
(iii)	Let $A=\frac{X_1+\cdots+X_5+Y_1+\cdots+Y_{15}}{20}$ $\text{E}(A)=\text{E}\left(\frac{X_1+\cdots+X_5+Y_1+\cdots+Y_{15}}{20}\right)$ $=\frac{1}{20}\text{E}\left(X_1+\cdots+X_5+Y_1+\cdots+Y_{15}\right)$ $=\frac{1}{20}\left(5(25)+15(20)\right)$ $=21.25$

	$\mathrm{Var}(A)=\mathrm{Var}\left(\dfrac{X_1+\cdots+X_5+Y_1+\cdots+Y_{15}}{20}\right)$ $=\left(\dfrac{1}{20}\right)^2\mathrm{Var}\left(X_1+\cdots+X_5+Y_1+\cdots+Y_{15}\right)$ $=\left(\dfrac{1}{20}\right)^2\left[\mathrm{Var}\left(X_1\right)+...+\mathrm{Var}\left(X_5\right)+\mathrm{Var}\left(Y_1\right)+...+\mathrm{Var}\left(Y_{15}\right)\right]$ $=\dfrac{1}{20^2}\left(5\left(6^2\right)+15\left(3^2\right)\right)$ $=0.7875$ $\therefore A\sim \mathrm{N}\left(21.25, 0.7875\right)$ $\mathrm{P}(A<21)=0.389$ (3 s.f.) Required assumption: The waiting times for vaccinations of the people in Polyclinics and Community Clubs are all mutually independent with one another.

9 (i)	An unbiased estimate of μ is: $\overline{x}=\dfrac{36}{50}+300=300.72$ An unbiased estimate of σ^2 is: $s^2=\dfrac{1}{49}\left(612-\dfrac{36^2}{50}\right)$ $=11.961$ $=12.0$ (3 s.f.)
(ii)	$\mathrm{H_0}$: $\mu=300$ $\mathrm{H_1}$: $\mu>300$ Under $\mathrm{H_0}$, since $n=50$ is large, by Central Limit Theorem, $\overline{X}\sim \mathrm{N}\left(300,\dfrac{11.961}{50}\right)$ approximately. The test statistic is $Z=\dfrac{\overline{X}-300}{\sqrt{11.961}/\sqrt{50}}\sim \mathrm{N}(0,1)$ approximately. From GC, p-value = 0.0705 (3 s.f.) Since the p-value = 0.0705 > 0.05, we **do not reject $\mathrm{H_0}$** and conclude that there is **insufficient** evidence at the 5% level that the mean number of calories is more than 300 (*or* the nutritionist's belief is correct *or* the mean number of calories has been understated).
(iii)	Since sample size is large, by Central Limit Theorem, the sample mean number of calories for a packet of chocolate will be approximately normal.
(iv)	There is a probability of 0.05 that the test will show that the mean number of calories for a packet of chocolate is more than 300, when it is in fact 300.

(v)	$s^2=\frac{40}{39}(10)=10.256$ H_0 : $\mu=300$ H_1 : $\mu\neq 300$ Under H_0, since $n=40$ is large, by Central Limit Theorem, $\overline{X}\sim \text{N}\left(300,\frac{10.256}{40}\right)$ approximately. The test statistic is $Z=\frac{\overline{X}-300}{\sqrt{10.256}/\sqrt{40}}\sim \text{N}(0,1)$ approximately. Critical Region: $z\leq -2.1701$ or $z\geq 2.1701$ Given that H_0 is rejected, $\frac{k-300}{\sqrt{10.256}/\sqrt{40}}\leq -2.1701$ or $\frac{k-300}{\sqrt{10.256}/\sqrt{40}}\geq 2.1701$ $k-300\leq -2.1701\left(\frac{\sqrt{10.256}}{\sqrt{40}}\right)$ or $k-300\geq 2.1701\left(\frac{\sqrt{10.256}}{\sqrt{40}}\right)$ $k\leq 299$ (3 s.f.) or $k\geq 301$ (3 s.f.)

10 (i)	Let X be the number of defective bottles, out of 30 bottles. Then, $X\sim \text{B}(30, 0.04)$ P(accepting a batch of bottles) $=\text{P}(X_1=0)+\text{P}(X_1=1)\text{P}(X_2=0)+\text{P}(X_1=2)\text{P}(X_2\leq 1)$ $=0.54853$ $=0.549$ (3 s.f.)
(ii)	Required probability = P(at most 2 defective bottles | batch is accepted) $=\frac{0.54853-\text{P}(X_1=2)\text{P}(X_2=1)}{0.54853}$ $=\frac{0.46701}{0.54853}$ $=0.851$ (3 s.f.)
(iii)	Exactly 20 fewer defective bottles than non-defective bottles means $X=5$ $\text{P}(X=5)=0.0052591$ $=0.00526$ (3 s.f.)
(iv)	Let Y be the number of boxes, out of n, with exactly 20 fewer defective bottles than non-defective bottles. $Y\sim \text{B}(n, \text{P}(X=5))$ ie. $Y\sim \text{B}(n, 0.0052591)$

	$P(Y \geq 2) \leq 0.05$ $1 - P(Y \leq 1) \leq 0.05$ From GC, <table><tr><td>n</td><td>$1 - P(Y \leq 1)$</td></tr><tr><td>67</td><td>0.0488</td></tr><tr><td>68</td><td>0.0501</td></tr></table> Greatest possible value of $n = 67$
(v)	$X \sim B(30, 0.04)$ $E(X) = 30(0.04) = 1.2$ $Var(X) = 30(0.04)(0.96) = 1.152$ Since sample size = 50 is large, by Central Limit Theorem, $\bar{X} \sim N\left(1.2, \frac{1.152}{50}\right)$ approximately $\bar{X} \sim N(1.2, 0.02304)$ Required probability = $P(\bar{X} < 1) = 0.0938$ (3 s.f.)