YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME	

CG		INDEX NO	

MATHEMATICS 9758/02

Paper 2

15 SEPTEMBER 2021

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6
Marks						

Question	7	8	9	10	Total marks
Marks					100

This document consists of 22 printed pages and 2 blank pages.

Section A: Pure Mathematics [40 marks]

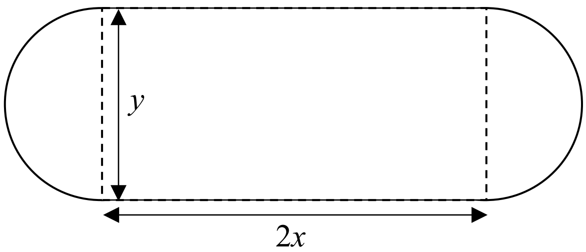
1 **(i)** Prove by the method of differences that $\sum_{r=3}^{n} \frac{1}{r^2 - 2r} = \frac{1}{2}\left(\frac{3}{2} - \frac{1}{n-1} - \frac{1}{n}\right)$. [4]

(ii) Explain why $\sum_{r=3}^{\infty} \frac{1}{r^2 - 2r}$ is a convergent series, and state the value of the sum to infinity. [2]

(iii) Hence show that $\sum_{r=4}^{\infty} \frac{1}{(r-1)^2} < \frac{5}{12}$. [2]

2 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$.]

A capsule is to be constructed using the curved surface of a cylinder and the curved surface of a hemisphere at each end. The length of the cylinder is $2x$ metres and the diameter of each hemisphere is y metres. The cross-sectional view of the capsule is shown in the diagram below.



It is given that the construction cost per square metre for the cylinder is \$$3k$ and that for the hemispheres is \$$5k$, where k is a constant.

Given that the volume of the capsule is π m^3, find, using differentiation, the exact values of x and y that give a minimum cost for its construction. [9]

3 **Do not use a calculator in answering this question.**

(a) The complex number z is given by $z = \frac{(1-\mathrm{i})^3}{\sqrt{2}(a+\mathrm{i})^2}$, where $a < 0$.

(i) Given that $|z| = \frac{1}{2}$, show that $\arg z = -\frac{5\pi}{12}$. [5]

(ii) Hence find the smallest positive integer n for which z^n has equal real and imaginary parts. [2]

(b) The complex number q is given by $\frac{\mathrm{e}^{-\mathrm{i}\theta}}{\mathrm{e}^{\mathrm{i}\theta} - \mathrm{i}}$, where $0 < \theta < \frac{\pi}{2}$. Show that

$$\mathrm{Re}(q) = \frac{1}{2}(1 + 2\sin\theta).$$ [4]

4 The line l has equation $\dfrac{x+1}{2}=\dfrac{y-a}{b}=\dfrac{z-4}{3}$. The plane p has equation $x+y-z-11=0$.

(i) If l and p do not intersect, what can be said about the values of a and b? [3]

For the rest of the question, use $a=-2$ and $b=1$.

(ii) Find the coordinates of the point B, the foot of perpendicular from the point $A(-1, a, 4)$ to p. [2]

(iii) Hence, or otherwise, find a vector equation of the line of reflection of l in p. [2]

(iv) Find the possible position vectors of the point C on l which is a distance of $\sqrt{164}$ from B. [3]

(v) Find the angle BCA. [2]

Section B: Probability and Statistics [60 marks]

5 12 students consisting of 4 from Class A, 5 from Class B and 3 from Class C participate in a team bonding activity where they are divided into 3 groups of 4.

(i) Find the number of ways such that each group has at most two students from Class A. [3]

During lunchtime, all the students from Class A and Class B are seated at a round table with 10 chairs.

(ii) How many different seating arrangements can be formed if no two students from Class A are seated next to each other? [2]

6 Events A, B and C are such that $\mathrm{P}(A)=0.5$, $\mathrm{P}(B)=0.6$ and $\mathrm{P}(C)=0.65$.
It is given that $\mathrm{P}(A|B)=0.45$, $\mathrm{P}(B\cap C)=0.4$ and $\mathrm{P}(A\cap B\cap C)=0.1$.

(i) Find $\mathrm{P}(A\cap B\cap C')$. [2]

(ii) It is also given that events A and C are independent. Find $\mathrm{P}(A\cup C')$. [2]

(iii) Given instead that events A and C are **not** independent, find the greatest and least possible values of $\mathrm{P}(A'\cap B'\cap C')$. [4]

7 A game is played with a set of 9 cards. Two of the cards are numbered 1, three of the cards are numbered 2 and four of the cards are numbered 3. A player randomly draws three cards without replacement. The random variable X denotes the sum of the numbers on the three cards drawn.

(i) Find the probability distribution of X. [4]

The player wins \$$w$ if the sum of the numbers on the three cards drawn is an odd number, and loses \$$w$ otherwise. The random variable Y denotes the player's profit in dollars.

(ii) Given that $\mathrm{E}(Y)=-0.8$, find $\mathrm{Var}(Y)$. [4]

(iii) Find the probability that the player wins \$$2w$ at the end of 10 rounds. [2]

8 In order to get a vaccination, people will go to either polyclinics or community clubs. The waiting time for vaccination in polyclinics follows a normal distribution with mean 25 minutes and standard deviation 6 minutes. The waiting time for vaccination in community clubs follows a normal distribution with mean 20 minutes and standard deviation 3 minutes.

(i) Two people waiting for their vaccinations in polyclinics are chosen at random. Find the probability that the waiting time for one of them is more than 30 minutes and the waiting time for the other is less than 30 minutes. [3]

(ii) Jim is waiting for his vaccination in a polyclinic and Tina is waiting for her vaccination in a community club. Find the probability that the difference between their waiting times is at least 3 minutes. [4]

A random sample of 5 people waiting for their vaccinations in polyclinics and 15 people waiting for their vaccinations in community clubs is taken.

(iii) Find the probability that their average waiting time is less than 21 minutes. State an assumption needed for your calculations to be valid. [5]

9 A chocolate manufacturer claims that the mean number of calories in a packet of chocolate is 300. A random sample of 50 packets of chocolate is selected and the number of calories, x, in each packet is measured. The results are summarised by:

$$\sum(x-300)=36, \quad \sum(x-300)^2=612.$$

(i) Find unbiased estimates of the population mean and variance. [2]

A nutritionist believes that the chocolate manufacturer has understated the mean number of calories in a packet of chocolate.

(ii) Test, at the 5% significance level, whether the nutritionist's belief is correct. [4]

(iii) Explain why there is no need for the nutritionist to know anything about the population distribution of the number of calories in the packets of chocolate. [1]

(iv) Explain, in the context of the question, the meaning of "at the 5% significance level". [1]

After receiving feedback from customers, the chocolate manufacturer improves the quality of the ingredients used. A random sample of 40 packets of the improved chocolate is taken. The mean number of calories of the 40 packets of chocolate is k and the variance is 10.

(v) Find the range of values of k such that there is sufficient evidence to conclude that the mean number of calories in a packet of the improved chocolate is not 300 at the 3% significance level. [4]

10 A factory produces a large number of bottles. Based on past records, 4% of the bottles are defective.

A departmental store manager wishes to purchase bottles from the factory. To decide whether to accept or reject a batch of bottles, the manager designs a sampling process. He takes a random sample of 30 bottles. The batch is accepted if there are no defective bottles and is rejected if there are more than 2 defective bottles. Otherwise, a second random sample of 30 bottles is taken. The batch is then accepted if there are fewer defective bottles in the second sample and is rejected otherwise.

(i) Find the probability of accepting a batch of bottles. [2]

(ii) If a batch is accepted, find the probability that there are at most 2 defective bottles found in the sampling process. [3]

Another departmental store manager purchases n randomly chosen boxes of 30 bottles each.

(iii) Find the probability that a box has exactly 20 fewer defective bottles than non-defective bottles. [2]

(iv) Find the greatest value of n if the probability that there are at least 2 boxes with exactly 20 fewer defective bottles than non-defective bottles is at most 0.05. [3]

(v) Given that $n = 50$, find the probability that the mean number of defective bottles in a box is less than 1. [3]