YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME

CG | INDEX NO

MATHEMATICS 9758/01

Paper 1 **25 AUGUST 2021**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

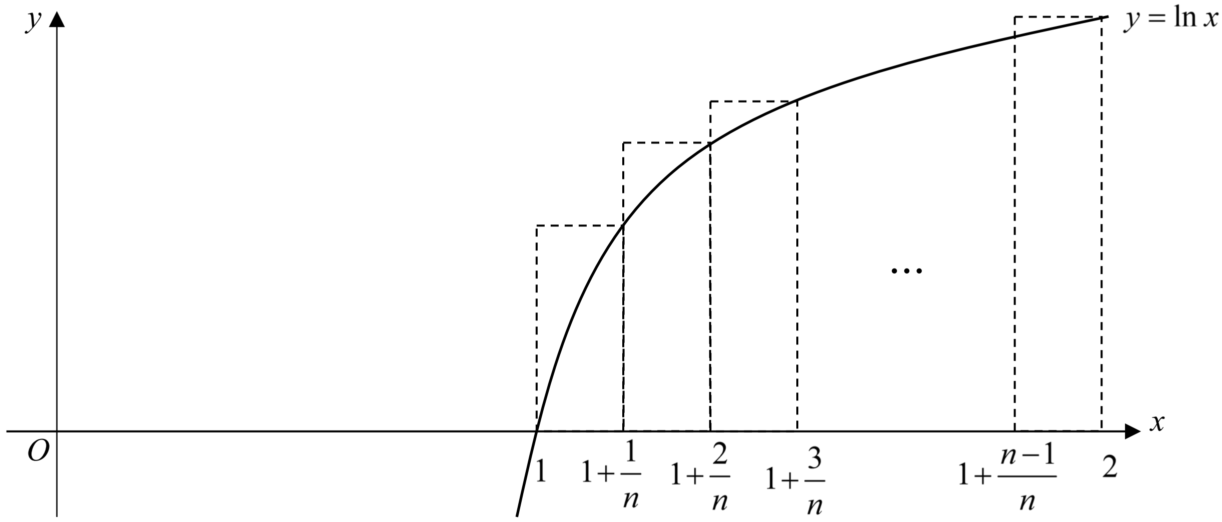
Write your CG, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6	7
Marks							

Question	8	9	10	11	12	Total marks	/100
Marks							

1 **(a)** The diagram shows part of the graph of $y = \ln x$, with n rectangles of equal width, where n is a positive integer.



(i) Show that the total area of the n rectangles, A, is

$$A = \frac{1}{n}\sum_{r=1}^{n}\left[\ln(n+r)\right] - \ln n.$$ [2]

(ii) Evaluate $\lim_{n\to\infty} A$, giving your answer correct to 4 decimal places. [2]

(b) It is given that $\mathrm{f}(x) = \frac{a}{x^2} + bx + c$, where a, b and c are constants. The curve with equation $y = \mathrm{f}(x)$ has a minimum point with coordinates $(1, 2)$ and $\int_1^3 \mathrm{f}(x)\,\mathrm{d}x = \frac{20}{3}$.

Find the equation of the curve. [4]

2 A curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \begin{cases} \frac{1}{2}(x+5) & \text{for } x < -3, \\ 1 & \text{for } -3 \le x \le -1, \\ x^2 & \text{for } x > -1. \end{cases}$

(i) Sketch the curve for $-4 \le x \le 2$. [3]

(ii) On a separate diagram, sketch the curve with equation $y = \mathrm{f}(2x-1)$, for $-2 \le x \le \frac{1}{2}$. [2]

3 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The point P with position vector $\mathbf{p}$ lies on AB such that $\mathbf{a} \bullet \mathbf{p} = \mathbf{b} \bullet \mathbf{p}$.

(i) Show that AB is perpendicular to OP. [2]

(ii) It is now given that $\mathbf{a}$ is perpendicular to $\mathbf{b}$. Show that

$$\mathbf{p} = \mathbf{a} + \frac{|\mathbf{a}|^2}{|\mathbf{a}|^2 + |\mathbf{b}|^2}(\mathbf{b} - \mathbf{a}).$$ [4]

4 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2 + \frac{3}{1 - x^2}, \qquad x \in \mathbb{R}, x < -1,$$
$$\mathrm{g} : x \mapsto -x^2 + 6x + a, \quad x \in \mathbb{R}, x \geq 0,$$

where a is a positive integer.

(i) Show that f has an inverse. [1]

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) State whether the composite function fg exists, justifying your answer. [3]

5 It is given that O is the origin and A is the point on the curve $y = x\mathrm{e}^x$ where $x = 3$. The region bounded by the curve $y = x\mathrm{e}^x$ and the line OA is rotated through 360° about the x-axis. Find the exact volume of the solid formed. [7]

6 **Do not use a calculator in answering this question.**

(i) One of the roots of the equation $2z^3 - 9z^2 + 30z + b = 0$ is $1 + a\mathrm{i}$, where a and b are non-zero real numbers. Find the values of a and b and the roots of this equation. [5]

(ii) Hence solve the equation $bz^3 + 30z^2 - 9z + 2 = 0$. [2]

7 **(i)** Sketch the curve with equation $y=\left|\dfrac{5-4x}{x+5}\right|$, stating the equations of the asymptotes. On the same diagram, sketch the line with equation $y=-2x-2$. [3]

(ii) Solve exactly the inequality $\left|\dfrac{5-4x}{x+5}\right|>-2x-2$. [4]

(iii) Hence, or otherwise, solve exactly the inequality $\left|\dfrac{10-4x}{x+10}\right|>-x-2$. [2]

8 It is given that $y=\sin\left(\ln(1+\mathrm{e}x)\right)$.

(i) Show that $\left(1+\mathrm{e}x\right)^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}\left(1+\mathrm{e}x\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\mathrm{e}^2y$. [2]

(ii) By further differentiation of the result in **(i)**, find the Maclaurin series for y, up to and including the term in x^3. [4]

(iii) By using appropriate standard series expansions from the List of Formulae (MF26), verify the correctness of the Maclaurin series for $y=\sin\left(\ln(1+\mathrm{e}x)\right)$ found in part **(ii)**. [2]

9 A curve C has equation $6x^2-4y^2=3xy^2$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12x-3y^2}{8y+6xy}$. [2]

The points A and B on C each has x-coordinate 2. The tangents to C at A and B meet at the point M.

(ii) Find the exact coordinates of M. [5]

10 A curve C has parametric equations

$$x = a\cos^2\theta,$$
$$y = a\cos^2\theta\sin\theta,$$

for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $a > 0$.

(i) Sketch C and state the Cartesian equation of the line of symmetry. [2]

(ii) Find the values of θ at the points where C meets the x-axis. [1]

(iii) Show that the area enclosed by the x-axis, and the part of C above the x-axis, is given by $\int_{\theta_1}^{\theta_2} 2a^2\cos^3\theta\sin^2\theta\,\mathrm{d}\theta$ where θ_1 and θ_2 should be stated. [3]

(iv) Hence, by expressing $\cos^3\theta$ as $\cos\theta\left(1-\sin^2\theta\right)$, find in terms of a, the exact total area enclosed by C. [3]

(v) It is given that the point $P\left(a\cos^2 p, a\cos^2 p\sin p\right)$ is on C. The point F is the midpoint of OP, where O is the origin. Find a Cartesian equation of the curve traced by F as p varies. [3]

11 A team of scientists is researching on the growth rate of a certain seaweed in an ocean. The length x metres, of the seaweed, at time t days (during a period of its growth) is proportional to the amount of water it contains. The seaweed absorbs water at a rate proportional to the length of the seaweed and loses water at a rate proportional to the square of the length of the seaweed. It is observed that the growth rates of the length of the seaweed are 0.49 metres per day and 0.96 metres per day when the lengths of the seaweed are 1 metre and 2 metres respectively.

(i) Show that x and t are related by the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{100}x(50-x)$. [3]

(ii) Given that the initial length of the seaweed is 0.5 metres, find an expression for x in terms of t. Hence find the time taken for the seaweed to reach a length of 45 metres. [7]

(iii) Sketch the graph of x against t. [2]

12 In order to train for the Open Water Swimming event in the University Sports Meet, Carol swims away from a shore towards the ocean. In her first minute of swimming, she covered a distance of 80 metres. Due to fatigue, the distance covered for each subsequent minute is 1% less than that in the previous minute.

However, there were ocean waves that pushed Carol back towards the shore. In her first minute of swimming, the waves pushed her back by a distance of 4 metres. As she swam further away from the shore, the waves got weaker and for each subsequent minute, she was pushed back by 0.05 metres less than that in the previous minute.

(i) Find the distance between Carol and the shore after the first 5 minutes. [3]

It is now given that the ocean waves became weaker away from the shore until they pushed Carol back towards the shore at 2 metres per minute from nth minute onwards.

(ii) Show that $n = 41$. [1]

(iii) Hence find the time taken for the distance between Carol and the shore to be at least 5 kilometres, giving your answer correct to the nearest minute. [5]

(iv) Find the greatest distance between Carol and the shore, and the time it took for her to reach this distance. Describe what happened to the distance Carol swam per minute after she had reached this greatest distance. [3]