1 **(i)** Show that $\dfrac{1}{r(r+1)}-\dfrac{1}{(r+1)(r+2)}=\dfrac{A}{r(r+1)(r+2)}$, where A is a constant to be found. [1]

(ii) Hence find $\displaystyle\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)}$. (There is no need to express your answer as a single algebraic fraction.) [3]

(iii) Given a reason why the series $\displaystyle\sum_{r=1}^{\infty}\frac{1}{r(r+1)(r+2)}$ converges, and write down its value. [2]

2 **(i)** Given that $y=\sin^{-1}\left(x^2\right)$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. [1]

(ii) Evaluate $\displaystyle\int_2^3 \ln\left[\tan^{-1}x\right]\,\mathrm{d}x$, giving your answer correct to 3 decimal places. [1]

(iii) Given that $k>0$ and $\displaystyle\int_0^k x^2\sin^{-1}\left(x^2\right)\,\mathrm{d}x=\int_2^3 \ln\left[\tan^{-1}x\right]\,\mathrm{d}x$, show that

$$\int_0^k \frac{x^4}{\sqrt{1-x^4}}\,\mathrm{d}x=ak^3\sin^{-1}k^2+b,$$

where a and b are constants to be determined. [4]

3 **(a)** **Do not use a calculator in answering this question.**

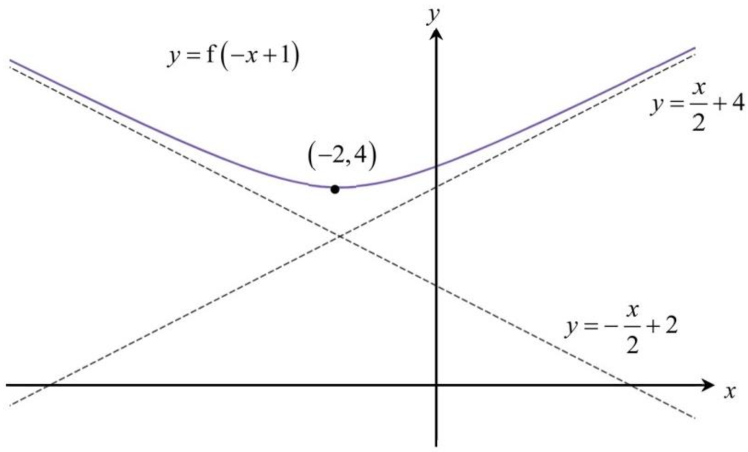
The equation $z^2+(-5+2\mathrm{i})z+(21-\mathrm{i})=0$ has a root $z=3+\mathrm{i}\,c$, where c is a real constant. Find the value of c and hence find the second root of the equation in Cartesian form, $a+\mathrm{i}b$, showing your working. [5]

(b) The complex number z is such that $8z^3+125=0$.

(i) Given that one possible value of z is $-\frac{5}{2}$, use a **non-calculator method** to find the other possible values of z. Give your answers in the form $a+ib$, where a and b are exact values. [3]

(ii) Write these values of z in modulus argument form and represent them on an Argand diagram. [4]

4 The diagram below shows the graph of $y=\mathrm{f}(-x+1)$ which has a minimum point at $(-2,4)$ and has lines $y=\frac{x}{2}+4$ and $y=-\frac{x}{2}+2$ as asymptotes.



(i) The diagram above shows a part of the curve with equation $\frac{(y+a)^2}{m}-\frac{(x+b)^2}{k}=1$. Find the values of a, b and k. [4]

(ii) Sketch the graph of $y=\mathrm{f}(x)$.

5 Given that $\mathrm{f}(\theta)=\tan\left(\frac{1}{2}\theta\right)$, show that $\mathrm{f}'(\theta)=\frac{1}{2}\left\{1+\left[\mathrm{f}(\theta)\right]^2\right\}$. [2]

The equation $\cot\left(\frac{\pi}{2}-\frac{1}{2}\theta\right)-\frac{5\sqrt{35}}{59}\theta=0$ has a positive root close to zero. Use the expansion above to obtain the approximate value of the root, correct to 4 decimal places. [3]

6 **(a)** Sketch the graph of $y=\frac{x+3}{(2x-1)(x+2)}$. Give the equations of the asymptotes, the coordinates of the point(s) where the curve crosses either axis and the coordinates of the two stationary points. [4]

(b) The curve C has equation $\frac{(3y-3)^2}{a^2}+\frac{(4-2x)^2}{b^2}=1$, where a and b are positive constants. Given that C is a circle and the x-axis is a tangent to C, state the values of a and b. [2]

Describe a sequence of transformations that transforms the graph of C to $\frac{(y-3)^2}{a^2}+\frac{(5-2x)^2}{b^2}=1$. [2]

7 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}-\frac{1}{2\sqrt{x+8}}$ and P is a point that moves along the curve. The y-coordinate of P is decreasing at 0.3 units per second at $(1,0)$.

(i) Find the rate of decrease of the x-coordinate of P at $(1,0)$. [2]

(ii) Find the equation of the curve, leaving your answer in the form $y=\mathrm{f}(x)$.

Mr Tan owns a company that manufactures and sells chipsets for smartphones. He models the average profit from one chipset, \$$y$, $y \geqslant 0$, when the selling price of each chipset is \$$x$, with the equation $y=\mathrm{f}(x)$.

(iii) Find algebraically, the selling price of each chipset which gives the maximum average profit. [3]

(iv) Sketch the graph showing the average profit from one chipset as the selling price of each chip set varies. State a suitable range of values that Mr Tan should set for the selling price of a chipset. [2]

8 **(i)** Show that $\frac{2x^2+5}{\left(x^2+1\right)^2}=\frac{A}{x^2+1}+\frac{B}{\left(x^2+1\right)^2}$, where A and B are constants to be determined. [1]

(ii) Using the substitution $x=\tan u$, show that $\int \frac{1}{\left(x^2+1\right)^2}\,\mathrm{d}x=\frac{ax}{x^2+1}+b\tan^{-1}x+C$, where a and b are constants to be found. [5]

(iii) Hence, find $\int \frac{2x^2+7x+5}{\left(x^2+1\right)^2}\,\mathrm{d}x$. [4]

9 A curve C has parametric equations

$$x=\frac{1}{t}+t \quad , \qquad y=\frac{1}{t}-t, \qquad \text{where } t\neq 0.$$

(i) Show that the gradient of the normal to the curve at $P\left(\frac{1}{p}+p, \frac{1}{p}-p\right)$ is given by $\frac{p^2-1}{p^2+1}$. [2]

(ii) Given that $p=\frac{1}{\sqrt{3}}$, determine the acute angle between the normal and the line $y=x+3$. [3]

(iii) Point Q on C has parameter q. Show that the gradient of PQ is $\frac{1+pq}{1-pq}$. [2]

(iv) The normal at P cuts the curve again at point R with parameter r. Show that $r=-\frac{1}{p^3}$. [3]

10 Crypto currency Miners are rewarded certain amounts of the currency at varying time intervals when they carry out validation tasks of transactions using their personal computers.

A crypto currency, Shockcoin (SHC), has a reward payment schedule such that the first reward payout is 20 days after a Miner commences mining and the second reward payout is 28 days after the first. The duration of each subsequent payout is 8 days more than the duration between the two preceding payouts. For example, if the duration between the last and current payout is 100 days, then it would take 108 days to the next reward payout. Albert plans to commence mining on 1 January 2022.

(i) After receiving his first reward payout on 21 January 2022, on what day and month will he receive his second reward payout? [1]

(ii) Show that from the start of mining, the number of days Albert takes to receive his n^{th} reward payout is given by $4n^2+16n$. [2]

The rewards for Miners are paid in Shockcoin (SHC).

The first reward for a Miner is SHC 120 000. For all subsequent reward payouts, the reward amount is 85% of the previous payout.

[Give non-exact numerical answers correct to 2 decimal places.]

(iv) Find the amount, in SHC Albert is expected to receive in his last payout in 2023. [2]

It is estimated that it would cost Albert 25 SHC per day for the electricity to keep the computer on to mine SHC.

(vi) Show that the total net gain for Albert immediately after receiving n reward payouts is given by $a\left(8000\left(1-0.85^n\right)-bn^2-cn\right)$ where a, b and c are constants to be found. [2]

(v) In order to maximise his total net gain, Albert should stop mining after the kth payout. Find the value of k and the maximum total net gain. [2]

11 **(i)** The complex number w can be expressed as $\mathrm{e}^{\mathrm{i}\theta}$.

(a) Find $w^n+\dfrac{1}{w^n}$ and $w^n-\dfrac{1}{w^n}$ in simplified trigonometric form. [3]

(b) By considering the binomial expansion of $\left[\left(w+\dfrac{1}{w}\right)\left(w-\dfrac{1}{w}\right)\right]^3$ and the results from **part (a)**, show that $\sin^3\theta\cos^3\theta=\dfrac{3}{32}\sin 2\theta-\dfrac{1}{32}\sin 6\theta$. [4]

(ii) Hence find the first two non-zero terms of the Maclaurin series for $\sin^3\theta\cos^3\theta$. [2]

End of Paper