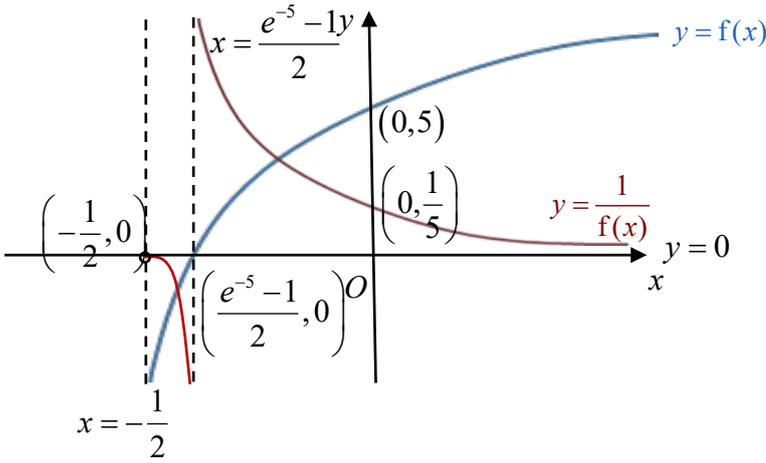


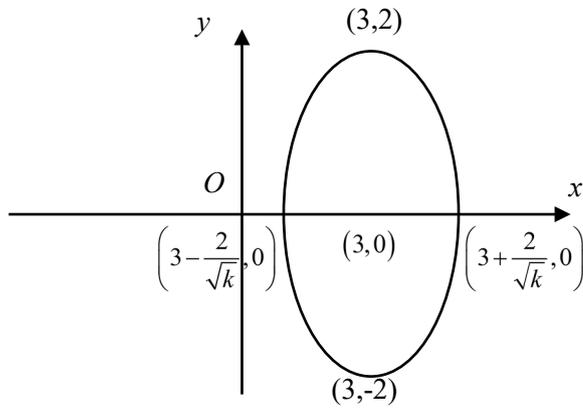
<p>1(i)</p>		
<p>1(ii)</p>	<p>In the equation $y = \ln(-2x)$, replace x with $(-x)$ gives us $y = \ln(2x)$. First, <u>reflect</u> the graph of $y = \ln(-2x)$ in the y-axis.</p> <p>In the equation $y = \ln(2x)$, replace x with $x + \frac{1}{2}$ gives us $y = \ln(2x + 1)$. Next, <u>translate</u> the graph of $y = \ln(2x)$, $\frac{1}{2}$ units in the negative x-direction.</p> <p>In the equation $y = \ln(2x + 1)$, replace y with $y - 5$ gives us $y = \ln(2x + 1) + 5$. Lastly, <u>translate</u> the graph of $y = \ln(2x + 1)$, 5 units in the positive y-direction.</p>	

2(a)

$$y^2 + k(x-3)^2 = 4$$

$$k > 0$$

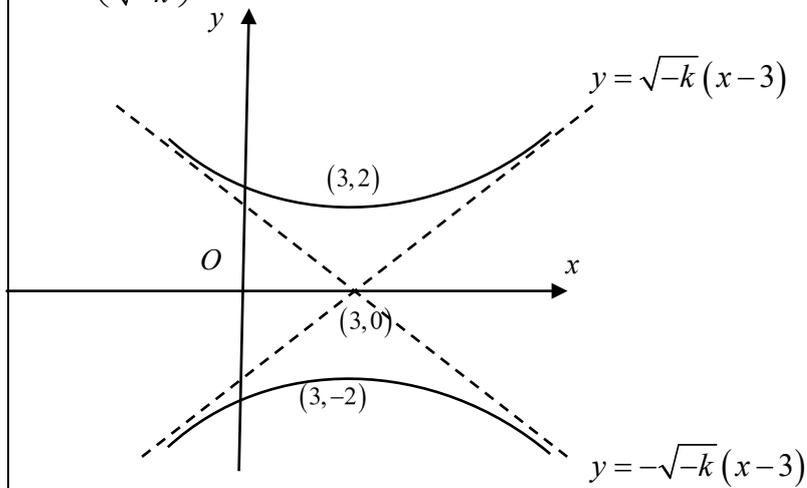
$$\frac{y^2}{2^2} + \frac{(x-3)^2}{\left(\frac{2}{\sqrt{k}}\right)^2} = 1$$



$$k < 0$$

$$y^2 - (-k)(x-3)^2 = 4$$

$$\frac{y^2}{2^2} - \frac{(x-3)^2}{\left(\frac{2}{\sqrt{-k}}\right)^2} = 1$$



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<p>(b) (i)</p>	<p>Volume</p> $= \pi \int_3^4 [4 - k(x-3)^2] dx$ $= \pi \left[4x - \frac{k}{3}(x-3)^3 \right]_3^4$ $= \pi \left(16 - \frac{k}{3} - 12 \right)$ $= \pi \left(4 - \frac{k}{3} \right)$	
<p>(ii)</p>	<p>Volume</p> $= \int_3^4 \left(\frac{1}{2} \right)^2 [4 - k(x-3)^2] dx$ $= \left(\frac{1}{2} \right)^2 \int_3^4 [4 - k(x-3)^2] dx$ $= \left(\frac{1}{2} \right)^2 \pi \left(4 - \frac{k}{3} \right)$ $= \frac{\pi}{4} \left(4 - \frac{k}{3} \right)$	

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<p>3 (a)</p>	$\frac{4+x^2}{\sqrt{4-x}} = (4+x^2)(4-x)^{-\frac{1}{2}}$ $= 4^{\frac{1}{2}}(4+x^2)\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ $= \frac{1}{2}(4+x^2)\left(1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\dots\right)$ $= \frac{1}{2}(4+x^2)\left(1+\frac{x}{8}+\frac{3}{128}x^2+\dots\right)$ $= \frac{1}{2}\left(4+\frac{1}{2}x+\frac{3}{32}x^2+x^2+\dots\right)$ $= 2+\frac{1}{4}x+\frac{35}{64}x^2+\dots$ <p>expansion is valid for</p> $\left -\frac{x}{4}\right < 1$ $\Rightarrow -4 < x < 4$ <p>$\therefore \{x \in \mathbb{R} : -4 < x < 4\}$</p>	
<p>3(b) (i)</p>	$\frac{dy}{dx} = \frac{\cos x}{1 + \sin x}$ <p>Differentiating $(1 + \sin x) \frac{dy}{dx} = \cos x$ w.r.t x,</p> $(1 + \sin x) \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} = -\sin x \quad (\text{shown})$	

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<p>3(ii)</p>	<p>Differentiating w.r.t. x,</p> $\cos x \frac{d^2 y}{dx^2} + (1 + \sin x) \frac{d^3 y}{dx^3} - \sin x \frac{dy}{dx} + \cos x \frac{d^2 y}{dx^2} = -\cos x$ <p>When $x = 0$,</p> $y = \ln(1 + 0) = 0$ $\frac{dy}{dx} = \frac{\cos 0}{1 + \sin 0} = 1$ $(1 + 0) \frac{d^2 y}{dx^2} + 1 = 0$ $\frac{d^2 y}{dx^2} = -1$ $(1)(-1) + (1 + 0) \frac{d^3 y}{dx^3} - (0)(1) + (1)(-1) = -1$ $\frac{d^3 y}{dx^3} = 1$ $y = 0 + (1)x + \frac{(-1)}{2!}x^2 + \frac{(1)}{3!}x^3 + \dots$ $= x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$	
<p>3(iii)</p>	$\ln\left(\frac{1 - \sin x}{2}\right) = \ln(1 + \sin(-x)) - \ln 2$ $= -\ln 2 + (-x) - \frac{1}{2}(-x)^2 + \frac{1}{6}(-x)^3 + \dots$ $= -\ln 2 - x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \dots$	
<p>4(i)</p>	<p>A normal to p_1 is $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix} = -3 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$.</p> <p>A normal to p_2 is $\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.</p> <p>Since $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} = -4 + 1 + 3 = 0$, p_1 is perpendicular to p_2.</p>	

<p>4(ii)</p>	<p>Show $(1, 0, -1)$ lies on l.</p> <p>Equation of p_1 is $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = -2$</p> <p>Since $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = -1 + 0 - 1 = -2$, $(1, 0, -1)$ lies on p_1.</p> <p>Since $4(1) + 0 + 3(-1) = 1$, $(1, 0, -1)$ lies on p_2.</p> <p>Since $(1, 0, -1)$ lies on both planes, it lies on l.</p>	
	<p>Find equation of line</p> <p>Line l is parallel to $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix}$.</p> <p>Equation of l: $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix}, t \in \mathbb{R}$</p>	
	<p><u>Alternative #1</u></p> <p>$p_1: \mathbf{r} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow p_1: -x + y + z = -2$ ----(1)</p> <p>$p_2: 4x + y + 3z = 1$ ----(2)</p> <p>Solving (1) and (2) simultaneously, by GC, equation of l:</p> <p>$\mathbf{r} = \begin{pmatrix} 3/5 \\ -7/5 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2/5 \\ -7/5 \\ 1 \end{pmatrix}, s \in \mathbb{R}$</p> <p>$\mathbf{r} = \begin{pmatrix} 3/5 \\ -7/5 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ -7 \\ 5 \end{pmatrix}, t \in \mathbb{R}$</p>	

(iii)

Let C be the point $(1, 0, -1)$ on l and F be the foot of perpendicular of B to l .

$$\overrightarrow{BC} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$$

$$\overrightarrow{BF} = \left(\overrightarrow{BC} \cdot \frac{\mathbf{n}_1}{|\mathbf{n}_1|} \right) \frac{\mathbf{n}_1}{|\mathbf{n}_1|}, \text{ where } \mathbf{n}_1 \text{ is a normal to } p_1$$

$$= \frac{(\overrightarrow{BC} \cdot \mathbf{n}_1) \mathbf{n}_1}{|\mathbf{n}_1|^2}$$

$$= \frac{1}{3} \left(\begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right) \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$$

$$\text{Area of } \triangle ABB' = 2 \times (\text{Area of } \triangle ABF)$$

$$= 2 \times \frac{1}{2} |\overrightarrow{BA} \times \overrightarrow{BF}|$$

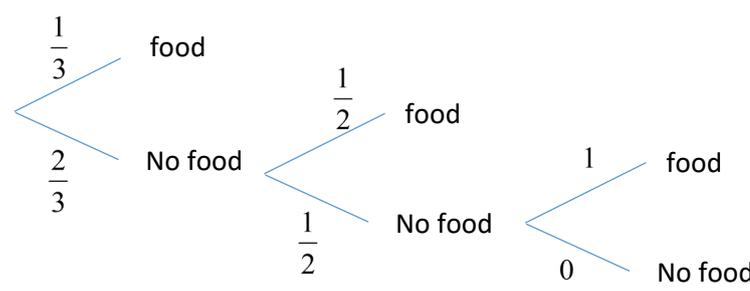
$$= \left| \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix} \times \frac{1}{3} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right| = \frac{1}{3} \left| \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} \right|$$

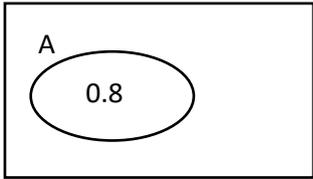
$$= \frac{\sqrt{26}}{3} \text{ units}^2$$

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	<p>Alternative (finding \overrightarrow{BF} using \overrightarrow{CF})</p> $\overrightarrow{CB} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{CF} = \frac{\left(\overrightarrow{CB} \cdot \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix} \right) \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix}}{4 + 49 + 25}$ $= \frac{1}{78} \left(\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix} \right) \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix}$ $\overrightarrow{BF} = \overrightarrow{CF} - \overrightarrow{CB} = \frac{1}{3} \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$	
(iv)	Q lies p_1 and p_2 , hence Q lies in the line l .	
(v)	$\left[\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix} \right] \cdot \begin{pmatrix} a \\ 3 \\ 1 \end{pmatrix} = -9$ $a - 1 + (2a + 16)t = -9 \quad \text{-----} (*)$ <p><u>Case 1:</u> If $a = -8$, (*) is true for all values of t.</p> <p><u>Case 2:</u> If $a \neq -8$, $t = -\frac{a+8}{2a+16} = -\frac{1}{2}$.</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \left(-\frac{1}{2}\right) \begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ -3.5 \\ 1.5 \end{pmatrix}$ <p>In both cases, we have $(0, -3.5, 1.5)$ lying on both l and p_3. Hence, coordinates of Q is $(0, -3.5, 1.5)$.</p>	
5i	<p>Number of ways = $\frac{8!}{3!}$</p> <p>= 6720</p>	
5ii	<p>Different cases: SxxxxSxxxxS, SxxxxSxxxxS, xSxxxxSxxxxS, and SxxxxSxxxxSx.</p> <p>Number of ways = $4 \times \frac{9!}{2!2!}$</p> <p>= 362880</p>	

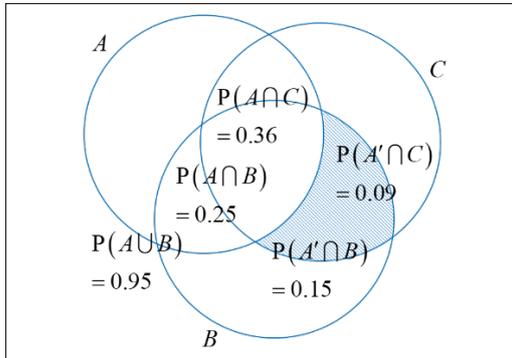
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	$P(\text{all 8 letters distinct}) = \frac{{}^2C_1 \times {}^3C_1 \times {}^2C_1 \times {}^5C_5}{{}^{12}C_8}$ $= \frac{4}{165}$ <p>Alternative</p> $P(\text{all 8 letters distinct})$ $= \left(\frac{2}{12}\right)\left(\frac{3}{11}\right)\left(\frac{2}{10}\right)\left(\frac{1}{9}\right)\left(\frac{1}{8}\right)\left(\frac{1}{7}\right)\left(\frac{1}{6}\right)\left(\frac{1}{5}\right) \times 8!$ $= \frac{4}{165}$	
6i	$P(X=1) = \frac{1}{3}$ $P(X=2) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ $P(X=3) = \frac{2}{3} \times \frac{1}{2} \times 1 = \frac{1}{3}$  <p>A probability tree diagram illustrating the outcomes for X. The tree starts with a root node on the left. The first branch splits into 'food' with probability $\frac{1}{3}$ and 'No food' with probability $\frac{2}{3}$. From the 'No food' node, the second branch splits into 'food' with probability $\frac{1}{2}$ and 'No food' with probability $\frac{1}{2}$. From the 'No food' node of the second branch, the third branch splits into 'food' with probability 1 and 'No food' with probability 0.</p>	

<p>6ii</p>	<p>(ii) $P(X = x) = \frac{1}{N}$, for $x = 1, 2, \dots, N$</p> <p>$E(X) = \frac{N+1}{2}$ (by symmetry)</p> <p><u>OR</u></p> <p>$E(X) = 1 \cdot \frac{1}{N} + 2 \cdot \frac{1}{N} + \dots + N \cdot \frac{1}{N}$</p> $= \frac{1}{N}(1 + 2 + \dots + N)$ $= \frac{1}{N} \left(\frac{N}{2}(1 + N) \right)$ $= \frac{N+1}{2}$ <p>$\text{Var}(X) = E(X^2) - [E(X)]^2$</p> $= \sum_{x=1}^N x^2 P(X = x) - \left(\frac{N+1}{2} \right)^2$ $= \frac{1}{N} \sum_{x=1}^N x^2 - \left(\frac{N+1}{2} \right)^2$ $= \frac{1}{N} \left[\frac{N(N+1)(2N+1)}{6} \right] - \left(\frac{N+1}{2} \right)^2$ $= \frac{N+1}{12} [2(2N+1) - 3(N+1)]$ $= \frac{(N+1)(N-1)}{12}$	
<p>7i</p>	<p>$P(A \cup B) = P(A) + P(B) - P(A \cap B)$</p> <p>$0.95 = 0.8 + 0.4 - P(A \cap B)$</p> <p>$P(A \cap B) = 0.25$</p> <p>$A$ and B are not independent because</p> <p>$P(A \cap B) = 0.25 \neq 0.32 = (0.8)(0.4) = P(A)P(B)$.</p>	
<p>7ii</p>	<p>$0.2 < c \leq 1$</p> <div style="display: flex; align-items: center; margin-top: 10px;"> <div style="border: 1px solid black; padding: 10px; margin-right: 20px;"> <p style="text-align: center;">A</p>  </div> <div style="border: 1px solid black; padding: 10px; flex-grow: 1;"> <p>Think: If $P(C) \leq 0.2$ then it is possible for $C \subseteq A'$. This means it is still possible for A and C to not intersect. But when $P(C) > 0.2$, this scenario won't happen!</p> </div> </div>	

7iii

$$\begin{aligned} P(A \cap C) &= P(C | A)P(A) \\ &= (0.45)(0.8) \\ &= 0.36 \end{aligned}$$



Note that $P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$

Note that $P((A \cup B)') = 1 - 0.95 = 0.05$ and

$$\begin{aligned} P(A' \cap C) &= P(C) - P(A \cap C) \\ &= 0.45 - 0.36 = 0.09 \end{aligned}$$

Hence,

$$\min(P(A' \cap B \cap C)) = 0.09 - 0.05 = 0.04 \text{ and}$$

$\max(P(A' \cap B \cap C)) = 0.09$ which occurs when

$$(A' \cap C) \subseteq (A' \cap B).$$

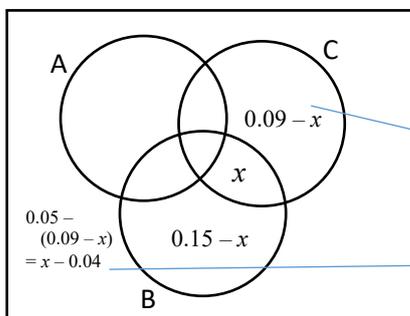
Alternative:

Let $P(A' \cap B \cap C) = x$. Note that

$P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$ and $P(A \cap C) = 0.36$. We also

know that $P((A \cup B)') = 1 - 0.95 = 0.05$. Using these to fill up the

venn diagram below:



Since all probabilities are at least 0 and at most 1, we must have

max value of $x = 0.09$

and

min value of $x = 0.04$

8(a)

$$B(n, p)$$

$$\text{Given: } E(X) = 2.75 \Rightarrow np = 2.75 \quad \text{--- (1)}$$

$$P(X < 2) = P(X \leq 1) = 0.13122$$

Method 1:

Using GC:

X	Y1	Y2
0	ERROR	0.1312
1	ERROR	0.1312
2	ERROR	0.1312
3	0.0197	0.1312
4	0.0935	0.1312
5	0.1312	0.1312
6	0.1535	0.1312
7	0.1682	0.1312
8	0.1786	0.1312
9	0.1863	0.1312
10	0.1923	0.1312

X=5

Hence $n = 5$

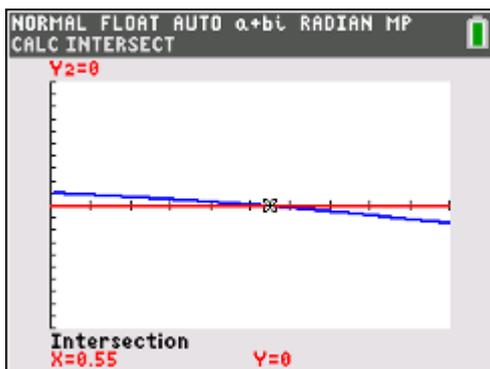
$$p = \frac{2.75}{5} = 0.55$$

Method 2:

$$P(X = 0) + P(X = 1) = 0.13122$$

$$(1-p)^n + np(1-p)^{n-1} = 0.13122$$

$$\text{From (1): } (1-p)^{\frac{2.75}{p}} + 2.75(1-p)^{\frac{2.75}{p}-1} = 0.13122$$



From the GC: $p = 0.55$

$$\therefore n = \frac{2.75}{0.55} = 5$$

bi

- (1) The probability that a resident uses Sharebike is $\frac{a}{100}$ for all the n residents.
 (2) The residents use Sharebike independently.

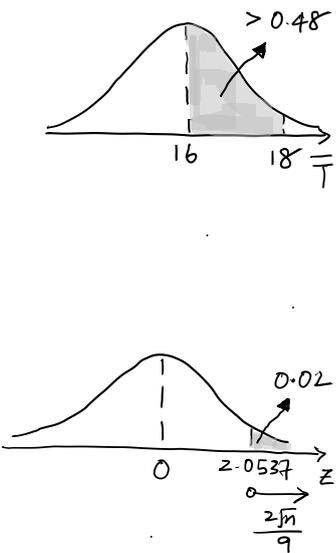
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bii	$X \sim B(45, 0.08)$ $P(9 \leq X \leq 13) = P(X \leq 13) - P(X \leq 8)$ $= 0.00843$	
biii	$X \sim B(12, r)$, where $r = \frac{a}{100}$ $P(X = 1) < P(X = 2)$ and $P(X = 2) > P(X = 3)$ $\binom{12}{1}r(1-r)^{11} < \binom{12}{2}r^2(1-r)^{10}$ and $\binom{12}{2}r^2(1-r)^{10} > \binom{12}{3}r^3(1-r)^9$ $r(1-r)^{10}[12(1-r) - 66r] < 0$ and $r^2(1-r)^9[66(1-r) - 220r] > 0$ Since $(1-r) > 0$ and $r > 0$, $12 - 78r < 0$ and $66 - 220r > 0$ $r > \frac{2}{13}$ and $r < \frac{3}{13}$ $\therefore \frac{2}{13} < r < \frac{3}{13}$ $\therefore \frac{200}{13} < a < \frac{300}{13}$	
9 (i)	Let $w = x - 425$ $\bar{x} = \bar{w} + 425 = 425 - \frac{136}{40} = 421.6$ (exact value) $s_x^2 = s_w^2 = \frac{1}{39} \left(4927.5 - \frac{(-136)^2}{40} \right) = \frac{4465.1}{39} = 114.49 \approx 114$	
(ii)	Let μ g be the population mean. $H_0 : \mu = 425$ $H_1 : \mu \neq 425$ Level of Significance: 5% Test Statistic: Since $n = 40$ is large, by Central Limit Theorem, \bar{X} is approximately normal. When H_0 is true, $Z = \frac{\bar{X} - 425}{S/\sqrt{n}} \sim N(0,1)$ approximately. $\text{Computation: } p\text{-value} = 0.044466 \text{ (or } z\text{-value} = -2.0097)$ Conclusion: Since $p\text{-value} = 0.0445 < 0.05$, (or $ -2.01 > 1.96$), H_0 is rejected at 5% level of significance. Hence there is sufficient evidence to conclude that the mean mass of a can of beans is not 425g.	
(iii)	No assumption is needed. Since the sample size is large, by Central Limit Theorem, the distribution of the sample mean mass of a can of beans, \bar{X} , is approximately normal.	
(iv)	There is a probability of 0.05 that the test concludes that the mean mass of a can of beans is not 425g which it is actually 425g.	

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(v)	<p>Let Y g be the mass of a packet of frozen corn and μ_Y g be the population mean.</p> <p>$H_0 : \mu_Y = 380$</p> <p>$H_1 : \mu_Y < 380$</p> <p>Level of Significance: $\alpha\%$</p> <p>Test Statistic:</p> <p>When H_0 is true, $Z = \frac{\bar{Y} - 380}{12/\sqrt{n}} \sim N(0,1)$</p> <p>Computation: $\bar{y} = 375$, p-value = 0.053292 (or z-value = -1.6137)</p> <p>For H_0 not to be rejected, p-value $> \frac{\alpha}{100}$</p> $\Rightarrow 0.053292 > \frac{\alpha}{100}$ <p>$\therefore \{\alpha \in \mathbb{R} : 0 < \alpha < 5.33\}$</p>	
10 (i)	<p>Let X and Y be the masses, in g, of a randomly chosen butter cookie and a randomly chosen chocolate cookie respectively.</p> <p>$X \sim N(15, 0.4^2)$ and $Y \sim N(20, 1.2^2)$</p> <p>$P(X > 15.5) = 0.10565 \approx 0.106$ (to 3 s.f.)</p>	
(ii)	<p>Let W be the number of butter cookies (out of 10) with mass more than 15.5g.</p> <p>$W \sim N(10, 0.10565)$</p> <p>$P(W \geq 4) = 1 - P(W \leq 3)$</p> $= 0.0155$	
(iii)	<p>Let $S = \frac{6}{100}(X_1 + \dots + X_{12}) + \frac{7.5}{100}(Y_1 + \dots + Y_{12})$</p> <p>$E(S) = \frac{6}{100}(12)(15) + \frac{7.5}{100}(12)(20)$</p> $= 28.8$ <p>$\text{Var}(S) = \left(\frac{6}{100}\right)^2 (12)(0.4)^2 + \left(\frac{7.5}{100}\right)^2 (12)(1.2)^2$</p> $= 0.104112 \text{ (exact value)}$ <p>$S \sim N(28.8, 0.104112)$</p> <p>$P(S < 29) = 0.732$</p>	

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<p>(iv)</p>	<p>We assume that the masses of cookies are independent of each other.</p> <p>Note: It is insufficient to say that “mass of a butter cookie is independent of the mass of a chocolate cookie”. This is because for the 12 butter cookies bought, we also need the condition that the mass of a butter cookie is independent of the mass of any other butter cookie. The same goes for the 12 chocolate cookies bought.</p>	
<p>(v)</p>	<p>Suppose T follows normal distribution i.e. $T \sim N(16, 9^2)$ $P(T < 0) = 0.037720$ which is not insignificant and time taken cannot be negative. Hence a normal distribution, with this mean and standard deviation, would not give a good approximation to the distribution of T.</p> <p>Alternatively,</p> <p>Due to the empirical rule of normal distribution, we would expect approximately 95% of the data to lie within 2 standard deviation from the mean. $16 \pm 2(9)$ have a range of values from -2 to 34.</p> <p>This would mean a non-negligible/significant 2.5% will have values below -2, which is impossible as time taken cannot be negative.</p> <p>Hence a normal distribution, with this mean and standard deviation, would not give a good approximation to the distribution of T.</p>	
<p>(vi)</p>	<p>Since $n \geq 30$, by Central Limit Theorem, $\bar{T} \sim N\left(16, \frac{9^2}{n}\right)$ approximately.</p> <p>$P(16 < \bar{T} < 18) > 0.48$ $P(\bar{T} > 18) < 0.5 - 0.48 = 0.02$</p> <p>Given $P\left(Z > \frac{18-16}{\frac{9}{\sqrt{n}}}\right) < 0.02$ $P\left(Z > \frac{2\sqrt{n}}{9}\right) < 0.02$</p> <p>From the GC, $P(Z > 2.0537) = 0.02$ $\frac{2\sqrt{n}}{9} > 2.0537$ $n > 85.4$ Least $n = 86$</p> 	

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