Section A: Pure Mathematics [44 marks]

1 The function f is defined by

$$\mathrm{f} : x \mapsto \ln(2x+1)+5, \quad x \in \mathbb{R}, x > -\frac{1}{2}.$$

(i) Sketch on the same diagram the graphs of $y=\mathrm{f}(x)$ and $y=\dfrac{1}{\mathrm{f}(x)}$, giving the equations of any asymptotes and the coordinates of any points where the graphs cross the axes. [6]

(ii) Describe a sequence of three transformations which would transform the curve $y=\ln(-2x)$ onto the curve $y=\mathrm{f}(x)$. [3]

2 A curve C has equation $y^2+k(x-3)^2=4$, where k is a real constant.

(a) Sketch, on separate clearly labelled diagrams, the graphs of C for both $k>0$ and $k<0$. State, in terms of k, the coordinates of any points where the curves cross the x-axis, the equations of any asymptotes and the coordinates of any points of intersection of the asymptotes and stationary points. [5]

(b) Assume now that $0<k<1$.

(i) The region S is bounded by C and the lines $x=3$ and $x=4$. Find the volume of the solid of revolution formed when S is rotated about the x-axis through 180°, giving your answer in terms of k. [3]

The curve C is stretched with scale factor $\dfrac{1}{2}$ parallel to the y-axis to form the curve D.

(ii) Without integration, state the volume of the solid of revolution formed when the region bounded by D and the lines $x=3$ and $x=4$ is rotated about the x-axis through $180°$, giving your answer in terms of k. [1]

3 **(a)** Expand $\dfrac{4+x^2}{\sqrt{4-x}}$ in ascending powers of x, up to and including the term in x^2. [3]

State the set of values of x for which the expansion is valid. [1]

(b) It is given that $y=\ln(1+\sin x)$.

(i) Show that $(1+\sin x)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+(\cos x)\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\sin x$. [2]

(ii) By further differentiation of the result in part **(i)**, find the Maclaurin series for y, up to and including the term in x^3. [4]

(iii) Hence deduce the Maclaurin series for $\ln\left(\dfrac{1-\sin x}{2}\right)$, up to and including the term in x^3. [2]

4 The planes p_1 and p_2 have equations $\mathbf{r}=\begin{pmatrix}0\\3\\-5\end{pmatrix}+\lambda\begin{pmatrix}1\\3\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$ and $4x+y+3z=1$ respectively, where λ and μ are parameters. These two planes meet in the line *l*.

(i) Show that p_1 is perpendicular to p_2. [2]

(ii) Explain why *l* passes through the point $(1,0,-1)$ and find a vector equation for *l*. [3]

(iii) The points $A(0,3,-5)$ and $B(2,2,-3)$ lie in p_1 and p_2 respectively and the point B' is the reflection of point *B* in the line *l*. Find the exact area of triangle ABB'. [5]

A third plane p_3 has equation $\mathbf{r}.\begin{pmatrix}a\\3\\1\end{pmatrix}=-9$, where *a* is a constant. The point *Q* lies in all three planes p_1, p_2 and p_3.

(iv) Explain why *Q* lies on the line *l*. [1]

(v) Hence or otherwise, find the coordinates of *Q*, showing your working. [3]

Section B: Statistics [56 marks]

5 Find the number of ways in which all twelve letters of the word MISSPELLINGS can be arranged if

(i) both the I's are placed at the beginning and both the L's are placed at the end, [1]

(ii) between any two S's, there must be at least 4 other letters. [3]

A group of 8 letters is randomly selected from the letters of the word MISSPELLINGS. Find the probability that all 8 letters are distinct. [2]

6 A scientist is interested to study the behaviour of raccoons.

N boxes are placed in a room with exactly 1 of them containing food. Alex, the raccoon, will randomly open one of the boxes to see if it contains food. It will stop once it finds food, else, it will randomly open another box until it finds food. You may assume that Alex is trained such that it will not attempt to open the same box more than once. Let *X* be the number of boxes that Alex opens.

(i) If $N=3$, determine the probability distribution of *X*. [2]

(ii) State $\mathrm{E}(X)$ and show that $\mathrm{Var}(X)=\dfrac{(N-1)(N+1)}{12}$. [4]

[You may use the result $\sum_{r=1}^{n} r^2=\dfrac{n(n+1)(2n+1)}{6}$.]

7 For events A, B and C it is given that $\mathrm{P}(A)=0.8$, $\mathrm{P}(B)=0.4$, $\mathrm{P}(C)=c$ and $\mathrm{P}(A\cup B)=0.95$.

(i) Find $\mathrm{P}(A\cap B)$ and determine with reason, if events A and B are independent. [2]

(ii) State the range of values of c which necessarily implies that events A and C are **not** mutually exclusive. [1]

It is now given that $\mathrm{P}(C)=\mathrm{P}(C\mid A)=0.45$.

(iii) Find $\mathrm{P}(A\cap C)$ and state the greatest and least possible values of $\mathrm{P}(A'\cap B\cap C)$. [3]

8 **(a)** The random variable Y has the distribution $\mathrm{B}(n,p)$ with mean 2.75. Given that $\mathrm{P}(Y<2)=0.13122$, find the value of n and the value of p. [3]

(b) On average $a\%$ of the residents in a city use the bicycle-sharing platform, ShareBike. A sample of n residents is taken and the random variable X denotes the number of residents in the sample who use ShareBike.

(i) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X has a binomial distribution.

(ii) Given that $n=45$ and $a=8$, find the probability that at least 9 but not more than 13 residents use ShareBike. [2]

(iii) It is given instead that $n=12$ and the modal number of residents who use ShareBike in the sample is 2. Use this information to find exactly the range of values that a can take. [4]

9 A factory supplies beans in small cans. The mass of one can of beans is denoted by X grams. A random sample of 40 cans of beans was taken and the masses are summarised as follows.

$$\sum(x-425)=-136 \quad \text{and} \quad \sum(x-425)^2=4927.5$$

(i) Calculate unbiased estimates of the population mean and variance of the mass of cans of beans. [2]

(ii) Test, at the 5% significance level, the claim that the mean mass of a can of beans is 425 grams. You should state your hypotheses and define any symbols you use. [5]

(iii) State, giving a reason, whether any assumptions about the population are needed in order for the test to be valid. [1]

(iv) Explain, in the context of the question, the meaning of ‘at the 5% significance level’. [1]

The factory also supplies frozen corn in packets. The mass of a randomly chosen packet of frozen corn has a normal distribution with standard deviation 12 grams. The factory claims that the mean mass of the packets of frozen corn is 380 grams. However, a random sample of 15 packets of frozen corn is taken and the mean mass of the sample is found to be 375 grams.

(v) Given that a one-tail test at the $\alpha\,\%$ significance level concludes that there is insufficient evidence to reject the factory’s claim, find the set of possible values of α. [3]

10 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.

A certain bakery bakes two types of cookies; butter cookies and chocolate cookies. The masses of butter cookies have the distribution $\mathrm{N}\left(15, 0.4^2\right)$ and the masses of chocolate cookies have the distribution $\mathrm{N}\left(20, 1.2^2\right)$. The units for mass are grams.

(i) Find the probability that the mass of a randomly chosen butter cookie is more than 15.5 grams. [1]

(ii) 10 butter cookies are randomly chosen. Find the probability that at least 4 of them each has mass more than 15.5 grams. [3]

The cookies are sold by weight. Butter cookies cost \$6 per 100 grams and chocolate cookies cost \$7.50 per 100 grams.

(iii) Miss Lee bought 12 butter cookies and 12 chocolate cookies for her family. Find the probability that she paid less than \$29. [4]

(iv) State an assumption needed for your calculations in part **(iii)**. [1]

The waiting time, T minutes, before a customer is served at the bakery has a mean of 16 minutes and a standard deviation of 9 minutes.

(v) Give a reason why a normal distribution, with this mean and standard deviation, would not give a good approximation to the distribution of T. [1]

(vi) The waiting times of n randomly chosen customers at the bakery are taken, where $n > 30$. Given that the probability that the average waiting time of these n customers is between 16 minutes and 18 minutes is more than 0.48, find the least value of n. [5]