1 Use the substitution $u=\sqrt{x}$ to find the exact value of $\displaystyle\int_1^9 \frac{1}{x\left(2\sqrt{x}-1\right)}\,\mathrm{d}x$. [4]

2 **(i)** Given that a is a positive constant, sketch the curve with equation $y=\dfrac{x+a}{x-a}$. State the equations of any asymptotes and the coordinates of the points where the curve crosses the axes. [3]

(ii) **(a)** Solve the inequality $\left|\dfrac{x+a}{x-a}\right|>1$. [2]

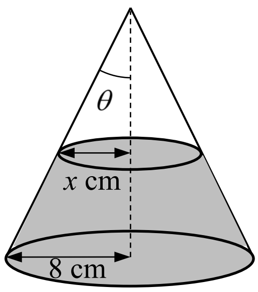
(b) Solve the inequality $\dfrac{|x|+a}{|x|-a}>1$. [1]

3 **(i)** By considering $\dfrac{1}{(r-2)!}-\dfrac{1}{r!}$ for $r\geq 2$, find an expression for $\displaystyle\sum_{r=1}^{n}\frac{r^2-r-1}{r!}$ in terms of *n*. [3]

(ii) Give a reason why the series $\displaystyle\sum_{r=1}^{\infty}\frac{r^2-r-1}{r!}$ converges and write down its value. [2]

(iii) Show that $\displaystyle\sum_{r=1}^{n}\frac{r-2}{(r-1)!}<1$ for all values of $n\geq 1$. [2]

4



Water is poured at a rate of 8 cm^3 per second into a conical container with base radius 8 cm. The semi-vertical angle of the cone is θ, where $\tan\theta=0.4$. At time *t* seconds after the start, the radius of the water surface is *x* cm (see diagram). Find the rate of increase of the depth of water when the depth is 10 cm.

[The volume of a cone of base radius *r* and height *h* is given by $V=\dfrac{1}{3}\pi r^2 h$.] [7]

5 With reference to the origin *O*, three points *A*, *B* and *C* have non-zero position vectors **a**, **b** and **c** respectively, such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$.

(i) Show that $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}$. [2]

(ii) If $\angle AOB=180^\circ$, describe the geometrical relationship between the points *A*, *B* and *C*. [1]

It is given instead that $0^\circ<\angle AOB<90^\circ$ and $\angle BOC=2\angle AOB$.

(iii) By considering the magnitudes of the vectors on both sides of the equation in part **(i)**, show that $(\mathbf{a}+2\mathbf{c}).(\mathbf{a}-2\mathbf{c})<0$. [4]

The points A, B and C lie in the plane p which has a normal parallel to the unit vector $\mathbf{n}$.

(iv) Given that p has equation $\mathbf{r}.\mathbf{n} = k$, state the value of k. Justify your answer. [1]

6 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_{n+1} = Au_n + Bn + C$, where A, B and C are constants, $A \neq 0$ and $n \geqslant 1$. It is given that $u_1 = 4$.

(a) **(i)** If $A = 1$ and $B = 0$, find the value of $\sum_{r=11}^{30} u_r$ in terms of C. [3]

(ii) If instead the sequence is a geometric progression, state the values of B and C, and find the inequality satisfied by A such that $u_{20} > 2000$. [3]

(b) Given instead that $u_2 = 16$, $u_3 = 70$ and $u_4 = 334$, find u_5. [3]

7 The function f is defined by $\mathrm{f}: x \mapsto \dfrac{1}{1+x^2}, x \in \mathbb{R}, 0 < x \le k$ where k is a real constant.

(i) Show that f has an inverse. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

It is further given that $k < \dfrac{1}{1+k^2}$.

(iii) Sketch the graph of $y = \mathrm{f}^{-1}(x)$. Your sketch should indicate the position of the graph in relation to the line $y = x$. [3]

(iv) Deduce the number of solutions of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [1]

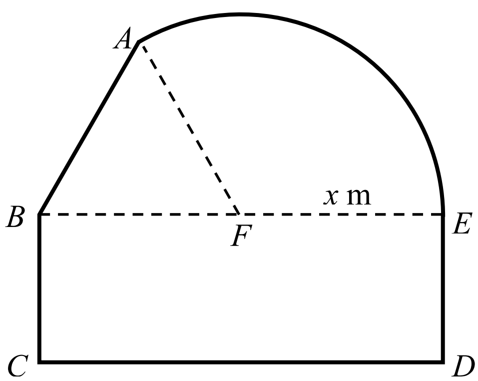
8 **(a)** The complex number w is given by $w = r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \le \theta \le \dfrac{\pi}{2}$.

(i) Given that $z = \left(1 - \mathrm{i}\sqrt{3}\right)w$, find $|z|$ in terms of r and $\arg(z)$ in terms of θ. [2]

(ii) Given that $\left(\dfrac{z^2}{z^*}\right)^2 = 36\mathrm{i}$, find r and all possible values of θ. [5]

(b) The equation $z^3 + pz^2 + 17z - 13 = 0$, where p is real, has a root $z = 2 + q\mathrm{i}$ where q is a real positive constant. Find the values of p and q, showing your working. [5]

9



The diagram above shows a plan view of a livestock enclosure, $ABCDEA$, consisting of a rectangle $BCDE$ joined to an equilateral triangle BFA and a sector of a circle with radius x metres and centre F. The points B, F and E lie on a straight line with $FE = x$ metres.

(i) Find the exact area of sector FEA, giving your answer in terms of x and in a simplified form. [2]

To ensure sufficient living space is given to the livestock, it is desired to have the area of the enclosure as 1000 m^2. At the same time, to save on the cost required for fencing the enclosure, the perimeter of the enclosure, P metres, should be made as small as possible.

(ii) Show that

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right).$$ [4]

(iii) Use calculus to find the minimum value of P, giving your answer to one decimal place. Justify, by further differentiation, that the value of P you have found is a minimum. [6]

10 **(i)** Find $\int \frac{1}{2 + x - x^2}\, dx$, given that $0 \le x \le \frac{1}{2}$. [3]

(ii) The mass, x grams, of a substance, X, present in a chemical reaction at time t minutes satisfies the differential equation

$$\frac{dx}{dt} = k(2 + x - x^2)$$

where $0 \le x \le \frac{1}{2}$ and k is a constant. It is given that $x = \frac{1}{2}$ and $\frac{dx}{dt} = -\frac{1}{9}$ when $t = 0$

(a) Find t in terms of x. [3]

(b) Find the exact time for there to be none of the substance X present in the chemical reaction. [1]

The mass, y grams, of another substance, Y, present in the chemical reaction at time t minutes satisfies the differential equation $\frac{d^2y}{dt^2} = \frac{1}{t+1}$. It is given that $y = 0$ and $\frac{dy}{dt} = 0$ when $t = 0$

(c) Find y in terms of t. [5]

11 A curve C has parametric equations

$$x = 3\cos t - \cos 3t, \quad y = 3\sin t - \sin 3t,$$

where $0 \le t \le \frac{\pi}{2}$. The line l is the normal to C at the point where $t = \frac{\pi}{3}$.

(i) Find the equation of l. [3]

(ii) On the same diagram, sketch C and l. [2]

(iii) Find the exact area of the region bounded by C, l and the y-axis. [9]