TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2021
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS

Paper 2

4048/02

Monday 23 Aug 2021
2 hours and 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Curved surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all questions.

1   **(a)**   Write as a single fraction in its simplest form.

     (i) $\dfrac{3t^2}{w} \div \dfrac{9t^2}{w^3}$

Answer ……………..…………….. [1]

     (ii) $\dfrac{3}{y-1} - \dfrac{5}{y+6}$

Answer ……………..…………….. [2]

   (b)   Simplify $\dfrac{2v^2 - 5v - 12}{16 - v^2}$

Answer ……………..…………….. [3]

2   In stall A, one Chicken pie costs $1.50, one Mushroom pie costs $1.30 and one Tuna pie costs $1.80.
In stall B, one Chicken pie costs $0.20 more, one Mushroom pie costs $0.30 less and one Tuna pie costs $0.10 less.

The information can be represented by the matrix $\mathbf{P} = \begin{pmatrix} \overset{C}{1.5} & \overset{M}{1.3} & \overset{T}{1.8} \\ 0.2 & -0.3 & -0.1 \end{pmatrix} \begin{matrix} \text{Stall A} \\ \text{Stall B} \end{matrix}$

(a)   Simon buys 50 Chicken pies and 20 Tuna pies.
Ivy buys 40 Chicken pies, 20 Mushroom pies and 30 Tuna pies.
Represent their purchases in a 3 × 2 Matrix **Q**.

Answer **Q** =   [1]

(b)   Evaluate the matrix **R = PQ**.

Answer **R** =   [2]

(c)   Use your answer in **(b)** to explain whether it is better for Simon to buy from stall A or stall B.

Answer

Stall ………… because …………………………………………………………………..………

……………………………………………………………………………………………………….....   [1]
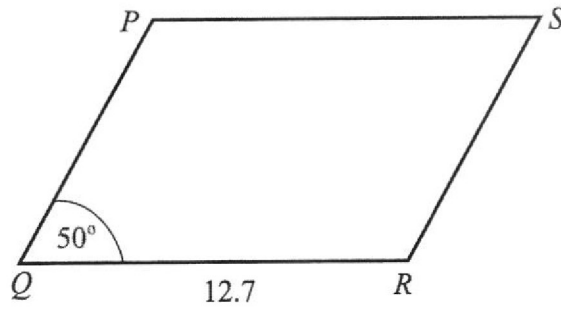
(d)   Stall B has a promotion of 30% off on all pies while prices of pies in stall A has increased by 10%.
Using your answer in **(b)** or otherwise, calculate the **lowest** total amount both Simon and Ivy will pay for the pies.

Answer $ …………….…………..…….   [3]

3 **(a)** In the parallelogram $PQRS$, $QR = 12.7$ cm and angle $PQR = 50°$.

The area of the parallelogram is 52.6 cm^2.
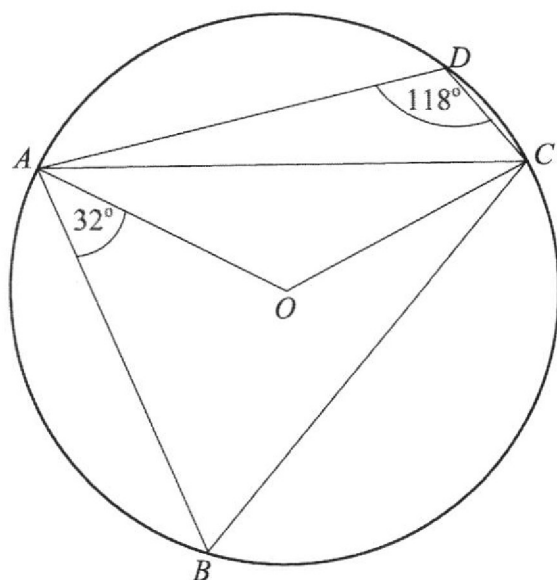
(i) Show that the length of $RS = 5.407$ cm.

Answer

[2]

(ii) Hence or otherwise, calculate the length of the longer diagonal of the parallelogram $PQRS$.

Answer .... cm [2]

(b) In the diagram, the points A, B, C and D lie on a circle, centre O.
$\angle ADC = 118°$ and $\angle BAO = 32°$.

Find, giving reasons for each answer,
(i) angle ABC,

Answer .... [1]

(ii) angle BCO.

Answer .... [2]

4
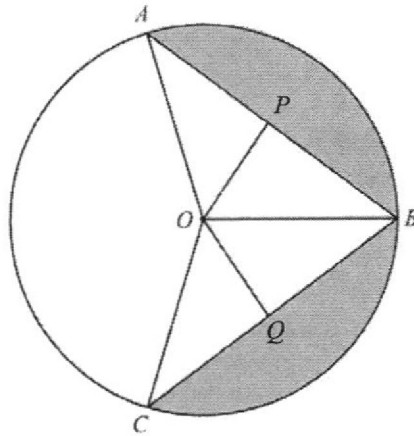
A, B and C are points on the circle centre O and $AB = BC$.
P is the midpoint of chord AB and Q is the midpoint of chord BC.

(a) Prove that triangle OAP is congruent to triangle OCQ.
Give a reason for each statement you make.

Answer

....

....

....

.... [3]

(b) Given that the radius of the circle is 6 cm and the obtuse angle $AOC = \dfrac{7\pi}{9}$,

calculate the shaded area.

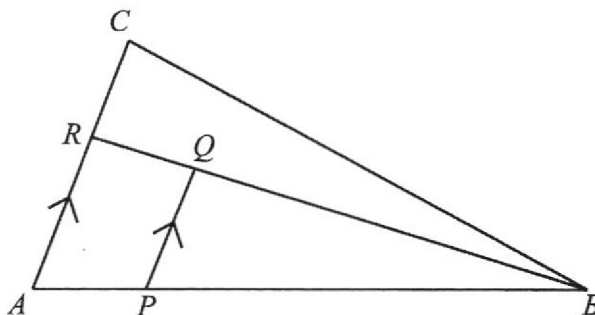*Answer* .... cm² [4]

[Turn over

5   **(a)**   A cuboid has a volume of 250 cm^3, correct to the nearest cubic centimetre.
The height of the cuboid is 8.4 cm, correct to 1 decimal place.

Calculate the greatest possible base area of the cuboid.

Answer   ……..…….……… cm² [2]

(b)   In the figure, *AC* and *PQ* are parallel lines. *P* lies on *AB* such that $AP : PB = 1 : 5$ and
R lies on *AC* such that $AR : RC = 3 : 2$.

(i)   Explain why triangles *ABR* and *PBQ* are similar.

Answer

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

…………………………………………………………………………………….. [2]

(ii) Show that the ratio of area of triangle PBQ to the area of trapezium $APQR$ is $25 : 11$.

Answer

[1]

(iii) If the area of the trapezium $APQR$ is 22 cm², calculate the area of triangle ABC.

Answer .... cm² [3]

6 (a) The diagram shows a cylindrical container used to dispense coffee in a hotel.
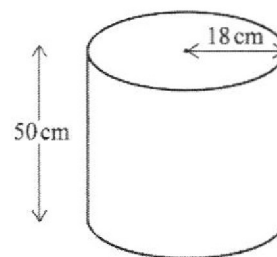
The container has a height of 50 cm and a radius of 18 cm.
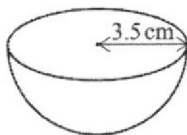
(i) Calculate the volume of the cylinder.

Answer ……...…….…..……… cm³ [1]

(ii) 25 litres of coffee are poured into the empty container.
Work out the height, h, of the empty space in the container.
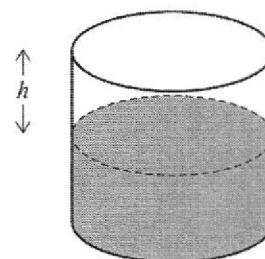
Answer $h =$ ……….…..……… cm [2]

**(iii)** Cups in the shape of a hemisphere of radius 3.5 cm are filled with coffee from the container.

Work out the maximum number of these cups that can be completely filled from the 25 litres of coffee in the container.

Answer .... [2]

(b) A solid shape consists of a cube with a pyramid on top has a total height of $10x$ cm.
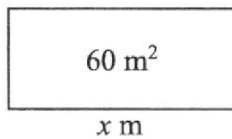The pyramid sits perfectly on one surface of the cube.

Each side of the cube is $6x$ cm.
Find an expression, in terms of x, for the surface area of the solid.
Give your answer in its simplest form.

$10x$

$6x$
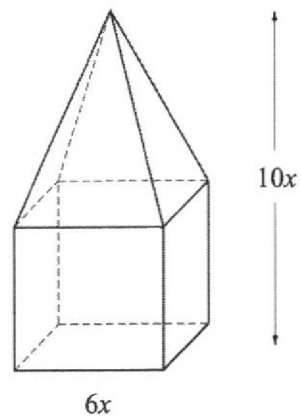
Answer .... cm^2 [4]

7   Mabel wants to fence off some land as an enclosure for her chickens.
The enclosure will be a rectangle with an area of 60 m².

$$60 \text{ m}^2$$

x m

(a)   The enclosure is x m long.
Show that the perimeter of fencing, P m, required for the enclosure is given by
$P = 2x + \dfrac{120}{x}$.   [1]

The table below shows some values of x and the corresponding values of P for the fencing.

x	2	4	6	8	10	12	14
P	k	38	32	31	32	34	36.6

(b)   Find the value of k.

Answer $k =$ ....[1]

(c)   On the grid, plot the points given and draw the graph of $P = 2x + \dfrac{120}{x}$ for $2 \le x \le 14$.

(d)   Mabel only has 35 m of fencing.
Use your graph to find the **range** of values of x that she can choose.

Answer   ....[2]

(e)   Mabel would like to use the graph to estimate the length and width of the enclosure when it is a square. Suggest an equation of the straight line that Mabel should draw.

Answer   ....[1]
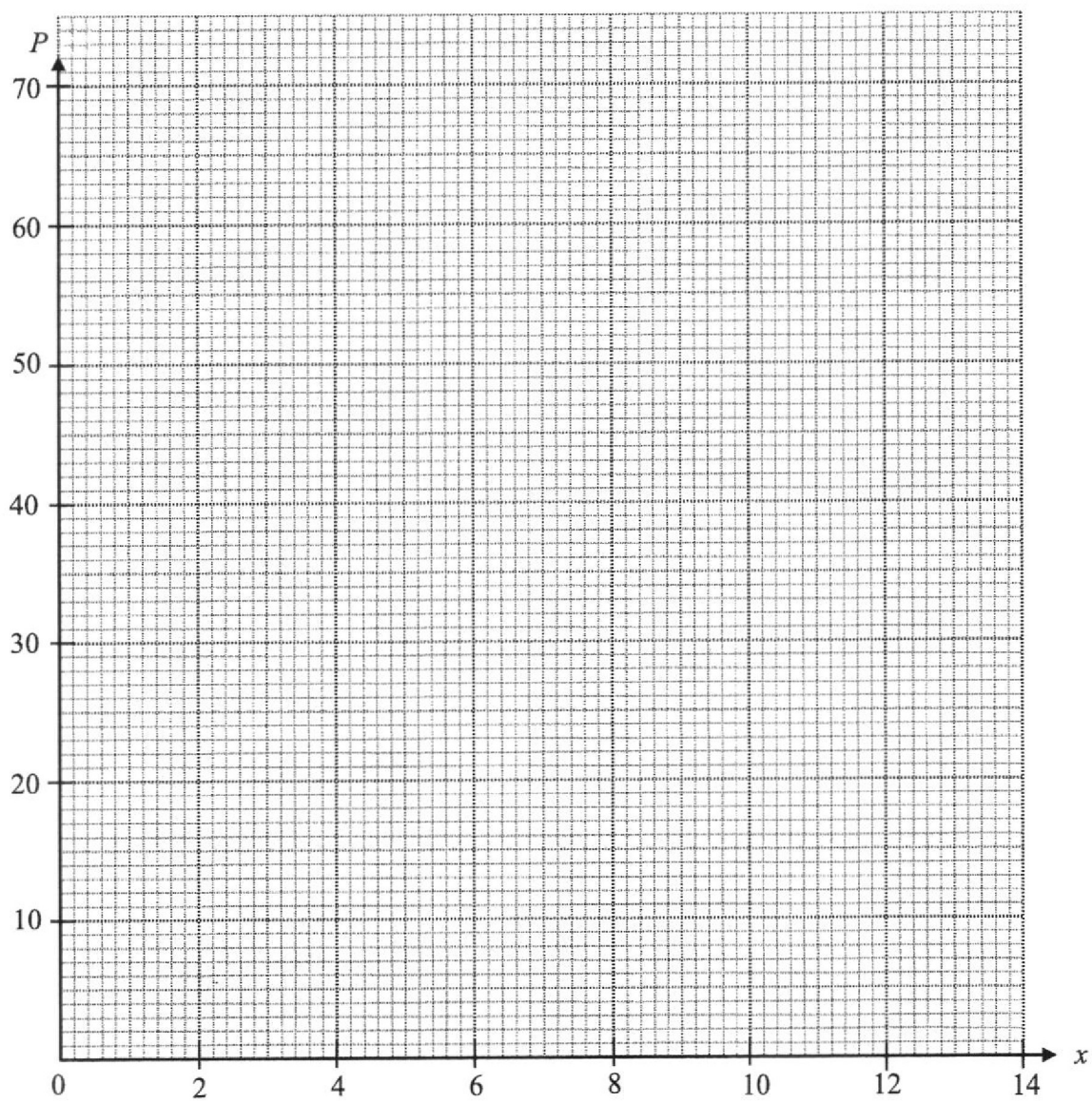
[3]

8   The table below shows part of Ahmad's personal income tax bill.

	S'PORE ($)	OTHER COUNTRIES ($)	TOTAL ($)
EMPLOYMENT	123, 419.00		123, 419.00
TOTAL INCOME	**123, 419.00**		**123, 419.00**
LESS: Approved Donations			1, 543.00
ASSESSABLE INCOME			*p*
LESS: PERSONAL RELIEFS			
Earned Income		1, 000.00	
NS-man/wife/parent		1, 500.00	
Life Insurance		19, 318.00	
TOTAL PERSONAL RELIEFS			*q*
CHARGEABLE INCOME			**100, 058.00**

(a)   Calculate the values of *p* and *q*.

Answer p = ...

q = ...…............ [2]

(b)   The tax rate for the year is given in the table below.

	Chargeable Income ($)	Rate (%)	Gross Tax Payable ($)
On the first	20,000	0	0
On the next	10,000	2.0	200
On the first	30,000		200
On the next	10,000	3.5	350
On the first	40,000		550
On the next	40,000	7.0	2,800
On the first	80,000		3,350
On the next	40,000	11.5	4,600
On the first	120,000		7,950
On the next	40,000	15	6,000

https://www.iras.gov.sg/irashome/Individuals/Locals/Working-Out-Your-Taxes/Income-Tax-Rates/

(i) Show that Ahmad's income tax payable is $5656.67.

Answer

[2]

(ii) In the same year of tax assessment, Angie's income tax payable is 0.55 of Ahmad's. Angie claims that her chargeable income is also 0.55 of Ahmad's chargeable income. Do you agree? Support your stand with calculations.

....

.... [5]

9 Small triangles are formed by placing rods between dots as shown in the diagrams.
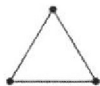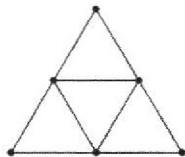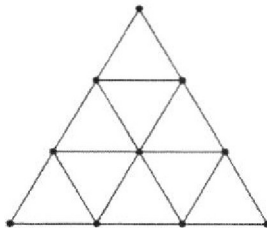
Diagram 1   Diagram 2   Diagram 3   Diagram 4
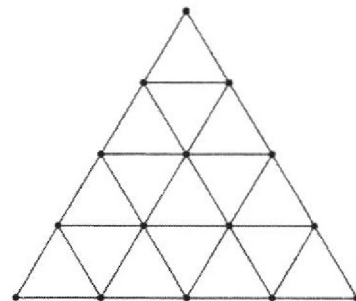
(a) Complete the table below.

Diagram n	1	2	3	4	5
Number of small triangles (T)	1	4	9	16	
Number of dots (D)	3	6	10	15	
Number of rods (R)	3	9	18	30	45

[2]

(b) Explain why it is not possible to have 1025 small triangles.

....

.... [1]

(c) Given that $R = D + T - 1$, find the value of n when $D = 561$ and $R = 1584$.

Answer $n =$ .... [2]

(d) A sequence is 1, 3, 6, 10, 15 ...

    (i) The nth term of the above sequence is $\frac{1}{2}n(n+1)$.

        Write an expression for R in terms of n.

Answer .... [1]

    (ii) How many rods are there in Diagram 16?

Answer .... [1]

(e) Find an expression for D in terms of n.

Answer .... [1]

10 The masses of 40 oranges were measured.
    The cumulative frequency curve below shows the distribution of the masses.
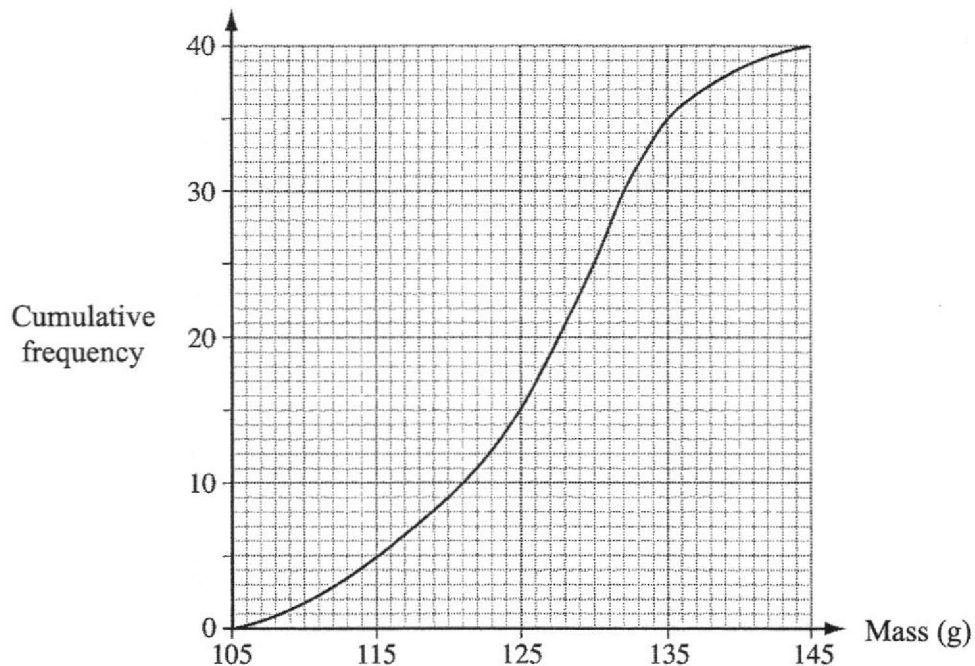
The box-and-whisker below shows the distribution of the masses of 40 apples.

Mass of apples (g)
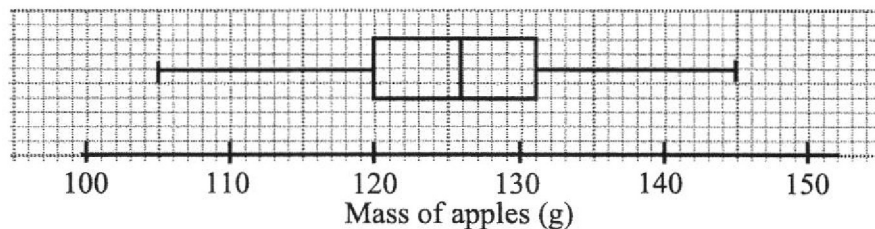
(a) Use the two diagrams to complete this table for the two types of fruits.

Type	Lower quartile	Median	Upper quartile	Inter-quartile range
Orange	g	g	g	11 g
Apple	g	126 g	g	11 g

[3]

(b) Describe how the cumulative frequency curve for the apples may differ from the curve for the oranges.

....

.... [1]

(c) Below are two statements comparing the distributions of the masses of oranges and apples.

For each statement, write **True** or **False**. Give a reason for each answer, stating clearly which statistics you use to make your decision.

(i) The apples are heavier than the oranges.

.... because ....

.... [1]

(ii) A greater percentage of oranges weigh more than 131g than apples.

.... because ....

.... [1]

(d) The grouped frequency table for the masses of the oranges is given below.

Mass (m g)	$105 \leq m < 115$	$115 \leq m < 125$	$125 \leq m < 135$	$135 \leq m < 145$
Frequency	5	10	20	5

(i) Calculate an estimate of the mean mass.

Answer .... g [1]

(ii) Calculate an estimate of the standard deviation.

Answer .... g [1]

(iii) 2 oranges are chosen at random without replacement.
Calculate the probability that at least one of the oranges weigh at least 125 g.

Answer .... [2]

11 (a) On Monday, Dev goes on a 3.6 km run.

(i) His average speed for the first 1.2 km is x km/h.

Simplify and write down an expression, in terms of x, for the time taken for the first 1.2 km.

Answer ……………..…………….. minutes [1]

(ii) His average speed for the last 2.4 km of the run is 2 km/h slower than the first 1.2 km.

Simplify and write down an expression, in terms of x, for the time taken for the final 2.4 km.

Answer ……………..…………….. minutes [1]

(iii) Dev takes 25 minutes to complete the full 3.6 km run.
Form an equation in x and show that it simplifies to $25x^2 - 266x + 144 = 0$.

[2]

(iv) Solve the equation $25x^2 - 266x + 144 = 0$, leaving your answers correct to 3 decimal places.

Answer $x = $ ....……................... [3]

(b) On Friday, Dev completed a 4 km run on the same average speed that he ran for the last 2.4 km of the 3.6 km run on Monday.

Calculate the time Dev took to run 4 km on Friday.
Give your answer in minutes and seconds, correct to the nearest seconds.

Answer ....……....... [2]

12 **(a)** On the map, 3 MRT stations, Braddell (NS18), Caldecott (CC17) and Toa Payoh (NS19) are joined to form a triangle.
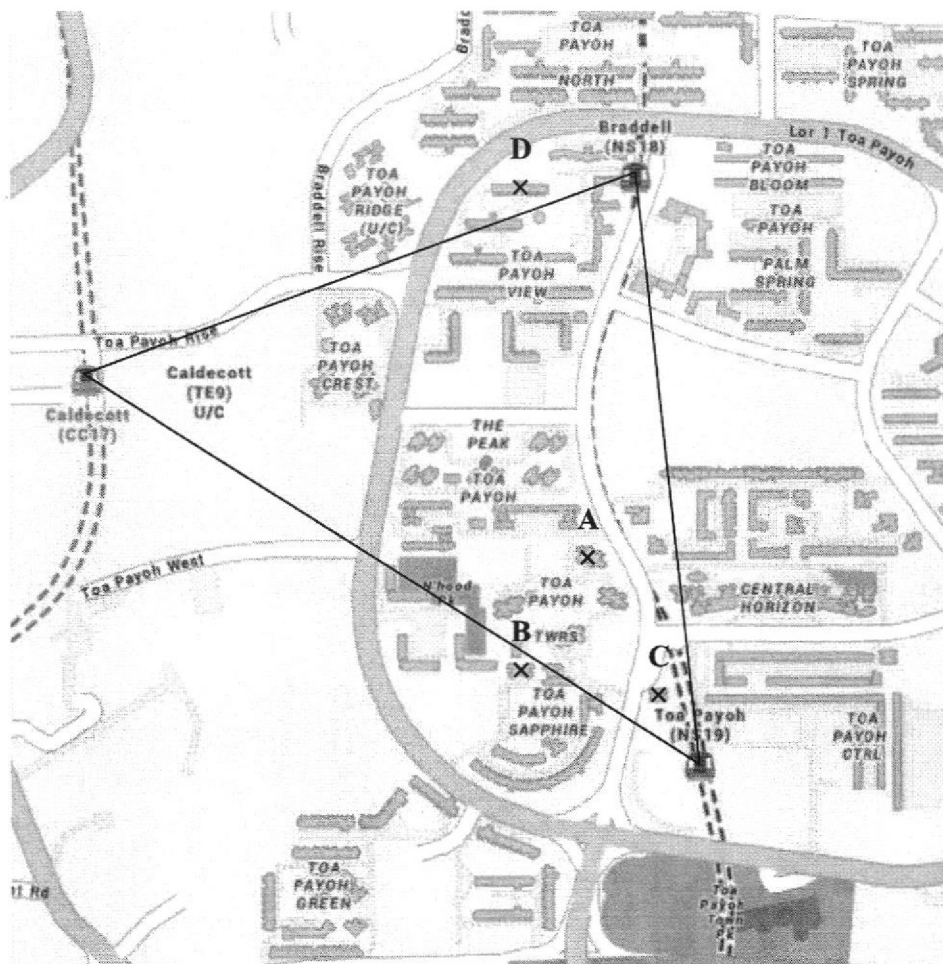
On the map,

(i) construct the perpendicular bisector of the line connecting NS18 and NS19. [1]

(ii) construct the angle bisector at NS18. [1]

(iii) shade the region inside the triangle that is closer to the line joining NS18 and CC17 and is closer to NS18 than NS19. [1]

John and Chieh, both Singaporeans, are looking at buying a re-sale unit in Toa Payoh.
They found the following units below and marked them **A**, **B**, **C** and **D** on the map on page 22.

A	B
Blk 153 Toa Payoh Sapphire 4 Room · Model A 101 sqm / $770 psf(Built) **$838,000** 🛏3 🛁2	Blk 163 Lorong 1 Toa Payoh 3 Room · Improved 67 sqm / $483 psf(Built) **$348,338** 🛏2 🛁1
C	D
Blk 175 Lorong 2 Toa Payoh 4 Room · Improved 84 sqm / $514 psf(Built) **$465,000** 🛏3 🛁2	Blk 116 Lorong 2 Toa Payoh 3 Room · Improved 63 sqm / $457 psf(Built) **$310,000** 🛏2 🛁2

Adapted from: https://www.srx.com.sg/singapore-property-listings/hdb-for-sale

Note: sqm = square metre (m²)
      psf = per square foot (psf)

(b) Which unit is the most value for money? Explain.

....

.... [1]

(c) Both John and Chieh are first-time HDB applicants as a married couple, they want to purchase a unit that is closest to the MRT station. Their combined monthly income is $7 500 and they wish to complete financing their home in 15 years' time using the HDB loan.

John's friend, Janet, also Singaporean, is looking at purchasing unit **B**, which is 2 km away from her mum's place, under the Single's scheme. Janet is 38 this year and her monthly salary is $6 500.

Assuming that they receive **all** the relevant grants and take up the maximum loan amount, suggest the number of years Janet should take to service the bank loan such that her interest paid is lower than John's and Chieh's.

Justify any decisions you make and show your calculations clearly.

....

....

.... [8]

Table 1: Comparison between HDB and Bank Loan

	HDB Loan	Bank Loan
Maximum loan	90% of purchase price	75% of purchase price
Interest Rate (p.a)	2.6%	1.8%

Table 2: Grants that can be used to offset the purchase price of the flat

Enhanced CPF Housing Grant (EHG) [Only for Singaporeans]

Who Is Eligible?	Income Ceiling	Grant Amount
Singles	$4,500	$40,000
Couples/families	$9,000	$80,000

Proximity Housing Grant (PHG) [stay within 4km from parents*]

Who Is Eligible?	Income Ceiling	Grant Amount
Singles	No income ceiling!	$10,000 — $20,000*
Couples/families		$15,000 — $30,000*

Singles Grant

Who Is Eligible?	Income Ceiling	Grant Amount
Singles (35 years old and above)	$7,000 (purchase under Single Singapore Citizen Scheme)	2 to 4-room $50,000
	$14,000 (purchase under Non-Citizen Family Scheme)	Min 5-room $40,000

Family Grant

Who Is Eligible?	Income Ceiling	Grant Amount
Couples/families	$14,000	2 to 4-room $50,000
	$21,000 (if buy with extended family)	Min 5-room $40,000

End of Paper