Promo Practice Paper 5 [TMJC 2022] 100m

1 **RI JC1 Promo 9758/2019/Q10**

The complex numbers z and w are such that

$$z = -1 + \mathrm{i}a \quad \text{and} \quad w = b + \mathrm{i},$$

where a and b are real numbers.

(i) Given that $zz^* + w^2 = 4 - 2\mathrm{i}$, where z^* is the complex conjugate of z, find the exact values of a and b. [4]

2 A local tour agent brought 4 groups of 5 tourists each to a durian shop for a durian tasting tour. He paid for the durians ordered by each group A - D as shown in the following table:

	A	*B*	*C*	*D*
Type of durian	**Weight of durians**			
Mao Shan Wang (in kg)	4.5	3	6	7.5
Red Prawn (in kg)	4	4.5	3	3
D24 (in kg)	4	2	3	6
Total amount paid (in $)	218.50	162.50	219	*k*

(i) The tour agent could not remember the price per kilogram for each type of durian and he lost the receipt for group D. Find k, the amount paid for group D. [4]

The tour agent found a new durian shop which offers a durian buffet. Durian lovers can feast on as many durians (Mao Shan Wang, Red Prawn, and D24 durians) as they want within 90 minutes. The buffet is priced at $50 per person.

(ii) Give a reason why the tour agent should not bring future tourist groups to the new durian shop for durian tasting. [1]

3 Find, by differentiation, the coordinates of the stationary point of the curve

$$y = 3x^2 - k^2 \ln\left(\frac{x}{4}\right),$$

where $x > 0$ and k is a positive constant. Hence determine the nature of the stationary point. [6]

4 **(i)** Without using a calculator, find the exact solution set for the inequality

$$\frac{7}{x^2-2x-6} \geq -1.$$ [4]

(ii) Hence solve the inequality $\dfrac{7}{x^2-2|x|-6} \geq -1$. [3]

5 **(i)** Expand $\dfrac{\sqrt{1-3x}}{2+4x}$ in ascending powers of x, up to and including the term in x^2 [4]

(ii) Find the range of values of x for which the expansion is valid. [2]

(iii) By putting $x=-\dfrac{1}{4}$ into your result in part **(i)**, show that $\sqrt{7} \approx \dfrac{p}{q}$, where p and q are integers to be determined. [2]

6 Given that $a > 0$, functions f and g are defined by

$$\mathrm{f} : x \mapsto x+2+\frac{4}{x-1} \qquad \text{for } x \in \mathbb{R}, x \neq 1,$$
$$\mathrm{g} : x \mapsto \ln(x+a) \qquad \text{for } x \in \mathbb{R}, x > 0.$$

(i) Explain clearly why gf does not exist. [2]

(ii) Find the range of values of a, in exact form, such that fg exists. [2]

For the rest of the question, assume that the function fg exists.

(iii) Define fg in a similar form, in terms of a. [2]

(iv) Given that $a > \mathrm{e}^3$, find the range of fg, in terms of a. [2]

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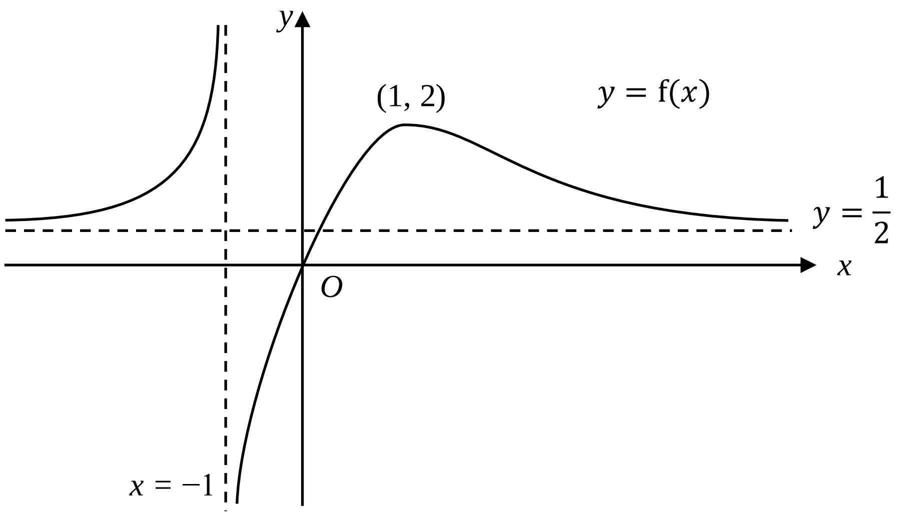

The diagram shows the graph of $y = \mathrm{f}\left(x\right)$. The curve has an axial intercept at $\left(0,0\right)$, a turning point at $\left(1,2\right)$ and asymptotes $x = -1$ and $y = \frac{1}{2}$.

Sketch, on separate clearly labelled diagrams, the graphs of

(i) $y = \mathrm{f}\left(|x| - 1\right)$, [3]

(ii) $y = \mathrm{f}\,'(x)$, [3]

(iii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

giving the equations of any asymptotes and coordinates of any x-intercepts and turning points, where applicable.

8 Given that $y = \mathrm{f}(x) = \ln\left(1 + \sin x\right)$, show that $\mathrm{e}^{y}\left[\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right] = -\sin\left(x\right)$. [2]

(i) By further differentiation of this result, find the Maclaurin series of y, up to and including the term in x^3. [4]

(ii) Let the answer in part **(i)** be g(x). Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ on the same diagram, where $0 < x < 3$. Hence or otherwise, explain why the approximation of $\mathrm{f}(x)$ using $\mathrm{g}(x)$ is not accurate when $x = 2$. [4]

[Turn Over

9 A curve C has parametric equations

$$x = \sqrt{2}\sin\theta - 1, \quad y = 2 + 3\cos\theta, \quad \text{for } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}.$$

(i) Using differentiation, find the coordinates of the point(s) on the curve at which the tangent is parallel to the y-axis. [7]

(ii) Sketch C, showing clearly the features of the curve at the points where $\theta = -\frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$ and coordinates of any turning point(s). [3]

(iii) Find the cartesian equation of C in the form $y = \mathrm{f}(x)$. [3]

10 Referred to origin O, the points P, Q, R and S have position vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ and $\mathbf{s}$ respectively.

(i) Given that $\alpha\mathbf{p} + \beta\mathbf{q} + \mathbf{r} = \mathbf{0}$ and $\alpha + \beta + 1 = 0$, where α and β are non-zero constants. Show that P, Q and R are collinear. [3]

It is given that the point R lies on PQ produced. The point T lies on line RS produced such that $RT : ST = 3 : 2$.

(ii) Find the position vector of the point T in terms of $\mathbf{r}$ and $\mathbf{s}$. [2]

(iii) Give a geometrical interpretation of $\dfrac{|(\mathbf{p}-\mathbf{r})\cdot(\mathbf{s}-\mathbf{r})|}{|\mathbf{s}-\mathbf{r}|}$. [1]

(iv) Find the area of triangle PRT in the form $\gamma|\mathbf{p}\times\mathbf{s} - \mathbf{p}\times\mathbf{r} - \mathbf{r}\times\mathbf{s}|$, where γ is a constant to be determined. [3]

11 ACJC Promo 9758/2021/Q12

Mrs Tan plans to start a business which requires a start-up capital of \$700,000. She decided to first save \$200,000 by depositing money every month into a savings plan. For the remaining \$500,000, she intends to take a loan from a finance company.

She deposited \$3000 into the savings plan in the first month and on the first day of each subsequent month, she deposited \$100 more than the previous month. Mrs Tan will continue depositing money into the savings plan until the total amount in her savings plan reaches \$200,000. It is given that this savings plan pays no interest.

(i) Find the month in which Mrs Tan's monthly deposit will exceed \$6,550. [2]

(ii) Find the number of months that it will take for Mrs Tan to save \$200,000 and hence find the amount that she would have deposited in the last month. [4]

After Mrs Tan has saved \$200,000, she took a loan of \$500,000 from a finance company. To repay the loan from the finance company, Mrs Tan would have to pay a monthly payment of $\$x$ at the beginning of each month, starting from the first month. An interest of 0.3% per month will be charged on the outstanding loan amount at the end of the month.

(iii) Show that the outstanding amount at the end of n^{th} month, after the interest has been charged, is $A\left(1.003^n\right)-Bx\left(1.003^n-1\right)$, where A and B are exact constants to be determined. [3]

(iv) Find the amount of $\$x$, to 2 decimal places, if Mrs Tan wants to fully repay her loan in 8 years. [2]

Using the value of x found in part **(iv)**, calculate the total interest that the finance company will earn from Mrs Tan at the end of 8 years. [2]

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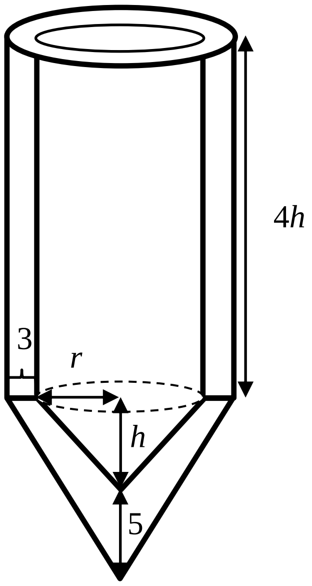

The diagram shows a storage well with fixed capacity k m^3. The storage well is made up of two parts, a right circular cone and a cylinder. The right circular cone has radius r m and height h m. The cylinder has radius r m and height $4h$ m. The walls of the storage well are made of a special material. The walls of the cylindrical part of the storage well are 3 m thick. The height from the vertex of the inner wall to the outer wall of the right circular cone is 5 m. The volume of the special material used to make the storage well is V m^3.

(i) Show that the volume of the special material used to make the storage well is given by $V = (r+3)^2\left(\frac{k}{r^2}\right) + \frac{5}{3}\pi(r+3)^2 - k$. [2]

[It is given that the volume of a right circular cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.]

(ii) Use differentiation to find the exact value of r where V is minimum. (You need not show that the volume is a minimum.) [4]

(iii) It is given that the capacity of the storage well is 700 m^3 and $2 < r \leq 10$. Sketch the graph showing V as r varies. [2]

(iv) It is given instead that $h = 10$ and $r = 5$. Liquid is poured into the storage well at a constant rate of 0.1 m^3 per minute. The depth of the liquid in the storage well at time t minutes, is x m. Find the rate of change of the depth of the liquid in the storage well when the volume of liquid in the storage well is 200 m^3. [5]

End of Paper

Answers

1	$a=\pm\sqrt{3}\,,\ b=-1$
2	(i) 298.50
3	$\left(\dfrac{\sqrt{6}k}{6},k^2\left(\dfrac{1}{2}-\ln\sqrt{6}k+\ln 24\right)\right)$
4	(i) $\left\{x\in\mathbb{R}:\ x<1-\sqrt{7}\quad \text{or} \quad x=1 \quad \text{or} \quad x>1+\sqrt{7}\right\}$ (ii) $x>1+\sqrt{7}$ or $x<-1-\sqrt{7}$
5	(i) $\dfrac{1}{2}-\dfrac{7}{4}x+\dfrac{47}{16}x^2+\cdots$ (ii) $-\dfrac{1}{3}<x<\dfrac{1}{3}$ (iii) $p=287,\quad q=128$
6	(ii) $a\geq \mathrm{e}$ (iii) $\mathrm{fg}:x\mapsto \ln(x+a)+2+\dfrac{4}{\ln(x+a)-1}$ for $x\in\mathbb{R}, x>0$ (iv) $\mathrm{R}_{\mathrm{fg}}=\left(\ln a+2+\dfrac{4}{\ln a-1},\infty\right)$
7	
8	(i) $y\approx x-\dfrac{1}{2}x^2+\dfrac{1}{6}x^3$
9	(i) $(-\sqrt{2}-1,2)$, $(\sqrt{2}-1,2)$ (iii) $y=2+\sqrt{9-\dfrac{9(x+1)^2}{2}}$
10	(ii) $\overrightarrow{OT}=3\mathbf{s}-2\mathbf{r}$, (iv) $\gamma=\dfrac{3}{2}$
11	(i) 37th month (ii) 41 months, \$2000 (iv) \$5984.09 (v) \$74,472.64
12	(iv) 0.00152

[Turn Over