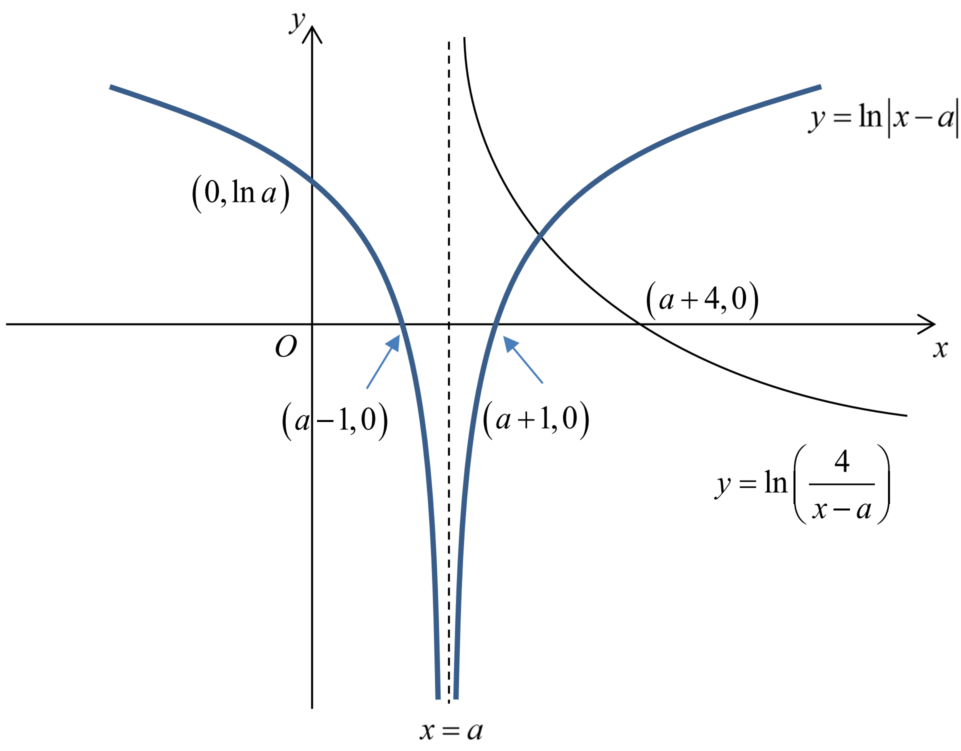
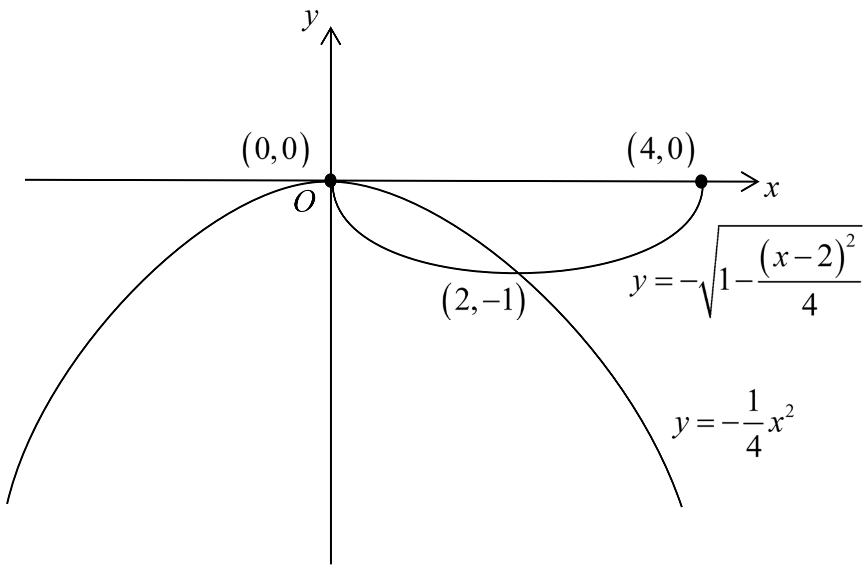
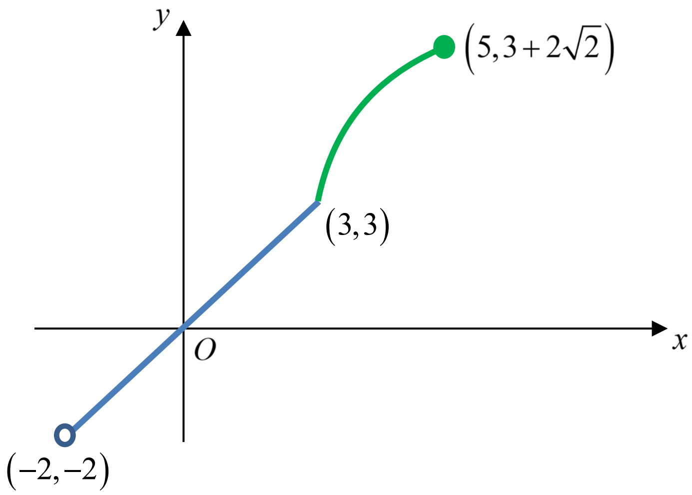
2021 H2 MATH (9758/01) JC 2 PRELIMINARY EXAMINATION – SOLUTION

Qn	Solution	
1	**Equations and Inequalities**	
	$y=a(2x+1)^2+bx+c\mathrm{e}^{2x}$ Differentiate wrt x, $\frac{\mathrm{d}y}{\mathrm{d}x}=2a(2x+1)2+b+2c\mathrm{e}^{2x}\ldots$ (*) Sub $(1, 17-\mathrm{e}^2)$ into C $9a+b+c\mathrm{e}^2=17-\mathrm{e}^2$ ..... (1) Sub $(0,1)$ into C $a+0b+c=1$ ..... (2) Sub $(0,1)$ and gradient 5 into (*) $4a+b+2c=5$ ..... (3) Using GC, $a=2, b=-1, c=-1$ Equation of C is $y=2(2x+1)^2-x-\mathrm{e}^{2x}$	

Qn	Solution	
2	**Graphing Techniques**	
		
	To find intersection between the 2 graphs, $\ln\left(\frac{4}{x-a}\right)=\ln(x-a)$ $\quad\quad$ (since $x>a$) $(x-a)^2=4$ $x=a+2$ or $x=a-2$ (rej. $\because x>a$) $\ln\left(\frac{4}{x-a}\right)\geq\ln\lvert x-a\rvert$ $\therefore a<x\leq a+2$	

Qn	Solution	
3	**Definite Integral**	
(i)		
(ii)	Volume generated = $\pi\int_0^2\left(-\sqrt{1-\frac{(x-2)^2}{4}}\right)^2-\left(-\frac{1}{4}x^2\right)^2\mathrm{d}x$ $=\frac{14}{15}\pi$ units^3 or 2.93 units^3 (3 sf)	

Qn	Solution	
4	**Complex Numbers**	
(i)	Since the polynomial $\mathrm{P}(z)$ has real coefficients and $r\mathrm{e}^{\mathrm{i}\theta}$ is a root of $\mathrm{P}(z)=0 \quad \Rightarrow \quad r\mathrm{e}^{-\mathrm{i}\theta}$ is also a root. Thus, a second root for $\mathrm{P}(z)=0$ is $r\mathrm{e}^{-\mathrm{i}\theta}$. A quadratic factor for $\mathrm{P}(z)$ is $\left(z-r\mathrm{e}^{\mathrm{i}\theta}\right)\left(z-r\mathrm{e}^{-\mathrm{i}\theta}\right)$ $=z^2-r\mathrm{e}^{\mathrm{i}\theta}z-r\mathrm{e}^{-\mathrm{i}\theta}z+r^2\mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{-\mathrm{i}\theta}$ $=z^2-rz\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)+r^2$ $=z^2-rz\left(\cos\theta+\mathrm{i}\sin\theta+\cos\theta-\mathrm{i}\sin\theta\right)+r^2$ $=z^2-rz\left(2\cos\theta\right)+r^2$ $=z^2-\left(2r\cos\theta\right)z+r^2 \quad$ (shown)	
(ii)	Since the polynomial $\mathrm{P}(z)$ has real coefficients, $\sqrt{3}\mathrm{e}^{-\mathrm{i}\frac{\pi}{6}}$ and $\sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}$ are the other 2 roots. From **(i)**, the quadratic factor of $\mathrm{P}(z)$ is $z^2-\left(2r\cos\theta\right)z+r^2$ The solutions $\sqrt{3}\mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$ and $\sqrt{3}\mathrm{e}^{-\mathrm{i}\frac{\pi}{6}}$ give the quadratic factor $z^2-2\left(\sqrt{3}\right)\left(\cos\frac{\pi}{6}\right)z+3=z^2-3z+3$ The solutions $\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ and $\sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}$ give the quadratic factor $z^2-2\left(\sqrt{2}\right)\left(\cos\frac{\pi}{4}\right)z+2=z^2-2z+2$ Thus, $\mathrm{P}(z)=\left(z^2-3z+3\right)\left(z^2-2z+2\right)$ $\quad=z^4-2z^3+2z^2-3z^3+6z^2-6z+3z^2-6z+6$ $\quad=z^4-5z^3+11z^2-12z+6$ Therefore, $a=-5, b=11, c=-12, d=6$	

Qn	Solution	
5	**Functions**	
(i)	For f to be continuous, $x=\sqrt{(x^2+kx+3)}+3$ at $x=3$ $3=\sqrt{(3^2+k(3)+3)}+3$ $12+3k=0$ $k=-4$ (shown)	
(ii)	 The line $y=a, a\in\mathbb{R}$ cuts the graph of f at most once. Hence f is a one-one function and thus the inverse of f exists.	
(iii)	Let $y=\sqrt{(x^2+kx+3)}+3$ $(y-3)^2=x^2-4x+3$ $(y-3)^2=(x-2)^2-4+3$ $(x-2)^2=(y-3)^2+1$ $x=2\pm\sqrt{(y-3)^2+1}$ Since $3<x\le 5$, $x=2+\sqrt{(y-3)^2+1}$ OR $x=2+\sqrt{y^2-6y+10}$ $\mathrm{f}^{-1}(x)=\begin{cases} x, & \text{for } -2<x\le 3, \\ 2+\sqrt{x^2-6x+10}, & \text{for } 3<x\le 3+2\sqrt{2}. \end{cases}$	
(iv)	$\{x\in\mathbb{R}: -2<x\le 3\}$	

Qn	Solution	
6	**Differentiation Tangents and Normals**	
(i)	$x^3 - 9xy + y^3 = 0$ -------- (1) Differentiate wrt x, $3x^2 - 9\left(y + x\frac{\mathrm{d}y}{\mathrm{d}x}\right) + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $3x^2 - 9y - 9x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $3y^2\frac{\mathrm{d}y}{\mathrm{d}x} - 9x\frac{\mathrm{d}y}{\mathrm{d}x} = 9y - 3x^2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9y - 3x^2}{3y^2 - 9x} = \frac{3y - x^2}{y^2 - 3x}$ At $(4,2)$, gradient of tangent is $\frac{3(2)-(4)^2}{(2)^2 - 3(4)} = \frac{5}{4}$ Equation of tangent is $y - 2 = \frac{5}{4}(x-4)$ $y = \frac{5}{4}x - 3$	
(ii)	Tangent parallel to the y-axis, $y^2 - 3x = 0$ $x = \frac{y^2}{3}$ Substitute into equation (1), $\left(\frac{y^2}{3}\right)^3 - 9\left(\frac{y^2}{3}\right)y + y^3 = 0$ $\frac{y^6}{27} - 3y^3 + y^3 = 0$ $y^3\left(\frac{y^3}{27} - 2\right) = 0$ $y = 0$ or $y = \sqrt[3]{54} = 3\sqrt[3]{2}$ (Reject) $x = \frac{1}{3}\left(3\sqrt[3]{2}\right)^2 = 3(2^{\frac{2}{3}})$ Point is $\left(3(2^{\frac{2}{3}}), 3(2^{\frac{1}{3}})\right)$	

Qn	Solution	
7	**Arithmetic and Geometric Series**	
(i)	Let b be the first term of the arithmetic progression. $a=b+d \quad -(1)$ $ar^2=b+2d \quad -(2)$ $ar^7=b+4d \quad -(3)$ From equations (1), (2) and (3), $ar^2-a=\frac{ar^7-ar^2}{2}(=d)$ $2r^2-2=r^7-r^2$ $r^7-3r^2+2=0$	
(ii)	Using GC, $r=1$ or $r=0.93725$ or $r=-0.77986$ (5 d.p.) (rejected since d is non-zero) Since $\lvert r\rvert<1$, sum to infinity of geometric progression exists.	
(iii)	The first even-numbered term of the A.P. is $a=12$. $d=ar^2-a=12\left(0.93725^2-1\right)=12(-0.12156)=-1.4587$ $\lvert E-S_\infty\rvert<1000$ $\left\lvert\frac{n}{2}\left[2a+(n-1)(2d)\right]-\frac{a}{1-r}\right\rvert<1000$ $\left\lvert\frac{n}{2}\left[2(12)+(n-1)(-2.9174)\right]-\frac{12}{1-0.93725}\right\rvert<1000$ $\left\lvert\frac{n}{2}\left[24+(n-1)(-2.9174)\right]-191.23\right\rvert<1000$ Using GC, When $n=28$, $\text{LHS}=958<1000$ When $n=29$, $\text{LHS}=1028>1000$ Hence largest value of $n=28$.	

Qn	Solution	
8	**Complex Numbers**	
(i)	$z^3-8=(z-2)(z^2+2z+4)=0$ $z=2$ or $z=\dfrac{-2\pm\sqrt{-12}}{2}=-1\pm\mathrm{i}\sqrt{3}$ For $z=-1+\mathrm{i}\sqrt{3}$, $\lvert z\rvert=\sqrt{(-1)^2+(\sqrt{3})^2}=2$ $\arg(z)=\pi-\tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right)=\dfrac{2}{3}\pi$ For $z=-1-\mathrm{i}\sqrt{3}$, $\arg(z)=-\pi+\tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right)=-\dfrac{2}{3}\pi$ $\therefore z=2\mathrm{e}^{\mathrm{i}0}$ or $2\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$ or $2\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}\right)}$	
(ii)	$z^6-64=(z^3-8)(z^3+8)=0 \Rightarrow z^3=8$ or $z^3=-8$ For $z^3=-8$, we replace z by $-z$ in $z^3=8$ and we have $z=-2\mathrm{e}^{\mathrm{i}0}$ or $-2\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$ or $-2\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}\right)}$ $=2\mathrm{e}^{\mathrm{i}\pi}$ or $2\mathrm{e}^{\mathrm{i}\frac{5\pi}{3}}$ or $2\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$ $\equiv 2\mathrm{e}^{\mathrm{i}\pi}$ or $2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}$ or $2\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$ (Since $-r\mathrm{e}^{\mathrm{i}\theta}=-1\times r\mathrm{e}^{\mathrm{i}\theta}=\mathrm{e}^{\mathrm{i}\pi}\times r\mathrm{e}^{\mathrm{i}\theta}=r\mathrm{e}^{\mathrm{i}(\theta+\pi)}$) Therefore the six roots are $z=2\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}\right)}$ or $2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}$ or $2\mathrm{e}^{\mathrm{i}0}$ or $2\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$ or $2\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$ or $2\mathrm{e}^{\mathrm{i}\pi}$	
(iii)	$\mathrm{f}(w)=2^{n+1}+2^n w+2^{n-1}w^2+\ldots+2^2 w^{n-1}+2w^n+w^{n+1}$ $=\dfrac{2^{n+1}\left(1-\left(\frac{w}{2}\right)^{n+2}\right)}{1-\frac{w}{2}}$ $=\dfrac{2^{n+1}\left(1-\left(\frac{w}{2}\right)^{2}\right)}{1-\frac{w}{2}}$ $\because w^n=2^n \Rightarrow \left(\dfrac{w}{2}\right)^n=1$ $=\dfrac{2^{n+1}\left(1-\frac{w}{2}\right)\left(1+\frac{w}{2}\right)}{1-\frac{w}{2}}$ $=2^{n+1}\left(1+\dfrac{w}{2}\right)$	

(iv)	The roots of $z^n = 2^n$ lie on a circle of radius 2 centred about the origin since $\lvert z\rvert = 2$. Therefore, for any root w, $\dfrac{\mathrm{f}(w)}{2^{n+1}} = 1 + \dfrac{w}{2}$ will lie on a circle of radius 1, centred about $(1,0)$. Hence the cartesian equation of C is $(x-1)^2 + y^2 = 1$.	

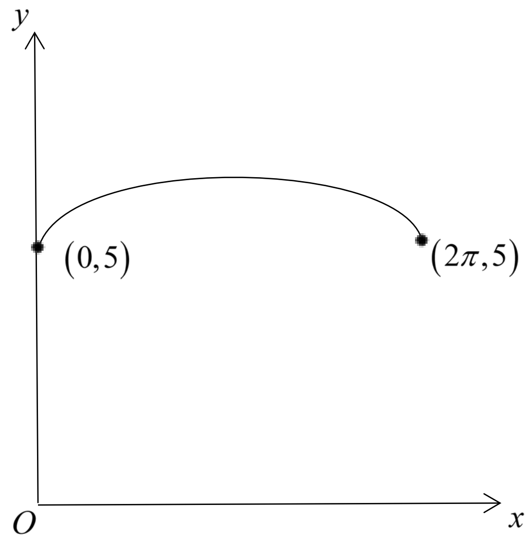
Qn	Solution	
9	**Vectors**	
(i)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}2\\-1\\3\end{pmatrix}-\begin{pmatrix}1\\-2\\3\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}$ Normal vector of plane π_1 : $\begin{pmatrix}1\\1\\0\end{pmatrix}\times\begin{pmatrix}4\\0\\-1\end{pmatrix}=\begin{pmatrix}-1\\1\\-4\end{pmatrix}=-\begin{pmatrix}1\\-1\\4\end{pmatrix}$ Equation of plane π_1: $\mathbf{r}\bullet\begin{pmatrix}1\\-1\\4\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\4\end{pmatrix}=15$ $\therefore$ cartesian equation of plane π_1 is $x-y+4z=15$ (shown).	
(ii)	Equation of line l, $\mathbf{r}=\begin{pmatrix}1\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}4\\0\\-1\end{pmatrix}$, $\lambda\in\mathbb{R}$ Since F is on l, $\overrightarrow{OF}=\begin{pmatrix}1+4\lambda\\-2\\3-\lambda\end{pmatrix}$ for some $\lambda\in\mathbb{R}$. $\overrightarrow{BF}=\overrightarrow{OF}-\overrightarrow{OB}=\begin{pmatrix}1+4\lambda\\-2\\3-\lambda\end{pmatrix}-\begin{pmatrix}2\\-1\\3\end{pmatrix}=\begin{pmatrix}-1+4\lambda\\-1\\-\lambda\end{pmatrix}$ Since $BF\perp l$, $\overrightarrow{BF}\bullet\mathbf{d}=0$ $\therefore\begin{pmatrix}-1+4\lambda\\-1\\-\lambda\end{pmatrix}\bullet\begin{pmatrix}4\\0\\-1\end{pmatrix}=0$ $-4+16\lambda+\lambda=0$ $\lambda=\frac{4}{17}$ $\therefore\overrightarrow{OF}=\begin{pmatrix}1+4\left(\frac{4}{17}\right)\\-2\\3-\left(\frac{4}{17}\right)\end{pmatrix}$ $\therefore$ coordinates of $F=\left(\frac{33}{17},-2,\frac{47}{17}\right)$	

(iii)	$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}8\\1\\-3\end{pmatrix}-\begin{pmatrix}1\\-2\\3\end{pmatrix}=\begin{pmatrix}7\\3\\-6\end{pmatrix}$ Shortest distance from C to π_1 $=\dfrac{\left\lvert\begin{pmatrix}7\\3\\-6\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\4\end{pmatrix}\right\rvert}{\sqrt{1^2+(-1)^2+4^2}}$ $=\dfrac{20}{\sqrt{18}}$ units or $\dfrac{10\sqrt{2}}{3}$ units	
(iv)	$\overrightarrow{AF}=\overrightarrow{OF}-\overrightarrow{OA}=\begin{pmatrix}\frac{33}{17}\\-2\\\frac{47}{17}\end{pmatrix}-\begin{pmatrix}1\\-2\\3\end{pmatrix}=\begin{pmatrix}\frac{16}{17}\\0\\-\frac{4}{17}\end{pmatrix}$ Area of triangle ABF $=\dfrac{1}{2}\left\lvert\overrightarrow{AB}\times\overrightarrow{AF}\right\rvert$ $=\dfrac{1}{2}\left\lvert\begin{pmatrix}1\\1\\0\end{pmatrix}\times\begin{pmatrix}\frac{16}{17}\\0\\-\frac{4}{17}\end{pmatrix}\right\rvert$ $=\dfrac{2}{17}\left\lvert\begin{pmatrix}1\\1\\0\end{pmatrix}\times\begin{pmatrix}4\\0\\-1\end{pmatrix}\right\rvert$ $=\dfrac{2}{17}\left\lvert\begin{pmatrix}-1\\1\\-4\end{pmatrix}\right\rvert$ $=\dfrac{2}{17}\sqrt{(-1)^2+1^2+4^2}$ $=\dfrac{2\sqrt{18}}{17}$ units^2 Volume of $CABF$ $=\dfrac{1}{3}\times\dfrac{2\sqrt{18}}{17}\times\dfrac{10\sqrt{2}}{3}$ $=\dfrac{40}{51}$ units^3 <u>Alternative method</u> Note that triangle ABF is a right angle triangle. $\overrightarrow{AB}=\begin{pmatrix}1\\1\\0\end{pmatrix}$	

	$\left\|\overrightarrow{AF}\right\| = \dfrac{\left\|\begin{pmatrix}1\\1\\0\end{pmatrix}\cdot\begin{pmatrix}4\\0\\-1\end{pmatrix}\right\|}{\sqrt{17}} = \dfrac{4}{\sqrt{17}}$ $\left\|\overrightarrow{BF}\right\| = \sqrt{\left(\sqrt{2}\right)^2 - \left(\dfrac{4}{\sqrt{17}}\right)^2} = \sqrt{2-\dfrac{16}{17}} = \sqrt{\dfrac{18}{17}}$ Area $= \dfrac{1}{2}\left\|\overrightarrow{BF}\right\|\left\|\overrightarrow{AF}\right\|$ $= \dfrac{1}{2}\times\sqrt{\dfrac{18}{17}}\times\dfrac{4}{\sqrt{17}}$ $= \dfrac{2\sqrt{18}}{17}$ Volume of $CABF = \dfrac{1}{3}\times\dfrac{2\sqrt{18}}{17}\times\dfrac{10\sqrt{2}}{3}$ $= \dfrac{40}{51}$ units^3	
(v)	Equation of plane π_1 : $\mathbf{r}\bullet\begin{pmatrix}1\\-1\\4\end{pmatrix} = 15 \Rightarrow \mathbf{r}\bullet\dfrac{1}{\sqrt{18}}\begin{pmatrix}1\\-1\\4\end{pmatrix} = \dfrac{15}{\sqrt{18}}$ $\therefore$ Equation of plane π_2 : $\mathbf{r}\bullet\dfrac{1}{\sqrt{18}}\begin{pmatrix}1\\-1\\4\end{pmatrix} = \dfrac{15}{\sqrt{18}} + \dfrac{20}{\sqrt{18}}$ $\left(\text{from (iii)}, -\dfrac{20}{\sqrt{18}} \text{ would contain } C\right)$ $= \dfrac{35}{\sqrt{18}}$ π_2 : $\mathbf{r}\bullet\begin{pmatrix}1\\-1\\4\end{pmatrix} = 35$	

Qn	Solution	
10	**Differential Equations**	
(i)	$\frac{dx}{dt}=\frac{m}{x}-hx, \quad \text{where } m,h>0$ When $x=2, \frac{dx}{dt}=0$. $\frac{m}{2}-2h=0$ $m=4h$ $\frac{dx}{dt}=\frac{m}{x}-hx$ $=\frac{4h}{x}-hx$ $=h\left(\frac{4-x^2}{x}\right)$ $=k\left(\frac{4-x^2}{x}\right), \quad \text{where } k=h$	
(ii)	$\frac{dx}{dt}=k\left(\frac{4-x^2}{x}\right)$ $\int \frac{x}{4-x^2}\ dx=\int k\ dt$ $-\frac{1}{2}\int \frac{-2x}{4-x^2}\ dx=\int k\ dt$ $-\frac{1}{2}\ln\left\|4-x^2\right\|=kt+c, \quad \text{where } c\in\mathbb{R}$ $\ln\left\|4-x^2\right\|=-2kt-2c$ $4-x^2=Ae^{-2kt}, \quad A=\pm e^{-2c}$ $x=\sqrt{4-Ae^{-2kt}}, \quad \text{since } x\geq 0$ When $t=0, x=1$. $1=\sqrt{4-Ae^{-2k(0)}}\Rightarrow A=3$ $\therefore x=\sqrt{4-3e^{-2kt}}$ The number of Kawaii otters **increases** and **tend towards** 2000.	

(iii)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{10}{4+t}\ln\left(1+\frac{1}{4}t\right)$ $\int 1\ \mathrm{d}x = \int \frac{10}{4+t}\ln\left(1+\frac{1}{4}t\right)\ \mathrm{d}t$ $\int 1\ \mathrm{d}x = 10\int \frac{1}{4+t}\ln\left(1+\frac{1}{4}t\right)\ \mathrm{d}t$ $\int 1\ \mathrm{d}x = 10\int \frac{\frac{1}{4}}{1+\frac{1}{4}t}\ln\left(1+\frac{1}{4}t\right)\ \mathrm{d}t$ $x = 5\left(\ln\left(1+\frac{1}{4}t\right)\right)^2 + D, \quad \text{where } D \in \mathbb{R}$	
(iv)	Ben's model might not be appropriate as his model suggests that the population of Kawaii otters tends towards infinity in the long run, which is not likely to happen due to space constraints and limited resources in Otterland.	

Qn	Solution	
11	**Definite Integral**	
(i)		
(ii)	$\int \sin^2 t\left(1-\cos 2t\right)\,\mathrm{d}t$ $=\frac{1}{2}\int\left(1-\cos 2t\right)^2\,\mathrm{d}t$ $=\frac{1}{2}\int 1-2\cos 2t+\cos^2 2t\,\mathrm{d}t$ $=\frac{1}{2}\int 1-2\cos 2t+\frac{1}{2}\left(1+\cos 4t\right)\mathrm{d}t$ $=\frac{1}{2}\left(\frac{3}{2}t-\sin 2t+\frac{1}{8}\sin 4t\right)+C,\ C\in\mathbb{R}$	
(iii)	$x=2t-\sin 2t$ $\frac{\mathrm{d}x}{\mathrm{d}t}=2-2\cos 2t$ $\text{Area}=\int_0^{2\pi} y\,\mathrm{d}x$ $=\int_0^{\pi}\left(5+2\sin^2 t\right)\left(2-2\cos 2t\right)\mathrm{d}t$ $=2\int_0^{\pi} 5-5\cos 2t+2\sin^2 t\left(1-\cos 2t\right)\mathrm{d}t$ $=2\left[5t-\frac{5}{2}\sin 2t+\frac{3}{2}t-\sin 2t+\frac{1}{8}\sin 4t\right]_0^{\pi}$ $\left(\text{from part (ii)}\right)$ $=13\pi\ \mathrm{m}^2$ <u>Alternative method</u> $\text{Area}=5\times 2\pi+\int_0^{2\pi} y-5\,\mathrm{d}x$ $=10\pi+\int_0^{\pi}\left(2\sin^2 t\right)\left(2-2\cos 2t\right)\mathrm{d}t$ $=10\pi+2\left[\frac{3}{2}t-\sin 2t+\frac{1}{8}\sin 4t\right]_0^{\pi}$ $\left(\text{from part (ii)}\right)$ $=13\pi\ \mathrm{m}^2$	

(iv)	$y=5+2\sin^2 t$ $\dfrac{\mathrm{d}y}{\mathrm{d}t}=4\sin t\cos t=2\sin 2t$ Surface area $=\dfrac{\pi}{4}\displaystyle\int_0^{\pi} y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$ $=\dfrac{\pi}{4}\displaystyle\int_0^{\pi}\left(5+2\sin^2 t\right)\sqrt{\left(2-2\cos 2t\right)^2+\left(2\sin 2t\right)^2}\ \mathrm{d}t$ Note that $\sqrt{\left(2-2\cos 2t\right)^2+\left(2\sin 2t\right)^2}=2\sqrt{1-2\cos 2t+\cos^2 2t+\sin^2 2t}$ $=2\sqrt{2-2\cos 2t}$ $=2\sqrt{2-2\left(1-2\sin^2 t\right)}$ $=4\sqrt{\sin^2 t}$ $=4\sin t \quad \left(\text{since } \sin t\geq 0 \text{ for } 0\leq t\leq \pi\right)$ Surface area $=\pi\displaystyle\int_0^{\pi}\left(5+2\sin^2 t\right)\sin t\ \mathrm{d}t$ $=\pi\displaystyle\int_0^{\pi}\left(7-2\cos^2 t\right)\sin t\ \mathrm{d}t$ $=\pi\displaystyle\int_0^{\pi}7\sin t-2\sin t\cos^2 t\ \mathrm{d}t$ $=\pi\left[-7\cos t+\dfrac{2}{3}\cos^3 t\right]_0^{\pi}$ $=\pi\left(7-\dfrac{2}{3}-\left(-7+\dfrac{2}{3}\right)\right)$ $=\dfrac{38}{3}\pi\ \ \mathrm{m}^2$	