Qn	Solution	
1	**Integration Techniques**	
(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\frac{1}{x}}=-\frac{1}{x^2}\mathrm{e}^{\frac{1}{x}}$	
(ii)	$\int_1^2 \frac{1}{x^3}\mathrm{e}^{\frac{1}{x}}\,\mathrm{d}x=-\int_1^2\frac{1}{x}\left(-\frac{1}{x^2}\mathrm{e}^{\frac{1}{x}}\right)\mathrm{d}x$ $=-\left[\frac{1}{x}\mathrm{e}^{\frac{1}{x}}\right]_1^2+\int_1^2-\frac{1}{x^2}\mathrm{e}^{\frac{1}{x}}\,\mathrm{d}x$ $=-\left[\frac{1}{2}\mathrm{e}^{\frac{1}{2}}-\mathrm{e}\right]+\left[\mathrm{e}^{\frac{1}{x}}\right]_1^2$ $=-\frac{1}{2}\mathrm{e}^{\frac{1}{2}}+\mathrm{e}+\mathrm{e}^{\frac{1}{2}}-\mathrm{e}$ $=\frac{1}{2}\mathrm{e}^{\frac{1}{2}}$	

Qn	Solution	
2	**Sequences and Series**	
(i)	$\frac{1}{r!}-\frac{1}{(r+1)!}=\frac{(r+1)-1}{(r+1)!}=\frac{r}{(r+1)!}$	
(ii)	$\sum_{r=1}^{n}\frac{2r}{(r+1)!}=2\sum_{r=1}^{n}\left[\frac{1}{r!}-\frac{1}{(r+1)!}\right]$ $=2\left[\frac{1}{1!}-\frac{1}{2!}\right.$ $+\frac{1}{2!}-\frac{1}{3!}$ $+\frac{1}{3!}-\frac{1}{4!}$ • • • $+\frac{1}{(n-1)!}-\frac{1}{n!}$ $\left.+\frac{1}{n}-\frac{1}{(n+1)!}\right]$ $=2\left(1-\frac{1}{(n+1)!}\right)$	
(iii)	As $n\to\infty$, $\frac{1}{(n+1)!}\to 0.$ $\therefore \lim_{n\to\infty}\sum_{r=1}^{n}\frac{2r}{(r+1)!}=\lim_{n\to\infty}\left[2\left(1-\frac{1}{(n+1)!}\right)\right]=2$ Hence the series is convergent. Sum to infinity $=2$	
(iv)	$\frac{10}{6!}+\frac{12}{7!}+\frac{14}{8!}+...+\frac{50}{26!}=2\left(\frac{5}{6!}+\frac{6}{7!}+\frac{7}{8!}+...+\frac{25}{26!}\right)$ $=\sum_{r=5}^{25}\frac{2r}{(r+1)!}$ $=\sum_{r=1}^{25}\frac{2r}{(r+1)!}-\sum_{r=1}^{4}\frac{2r}{(r+1)!}$ $=2\left(1-\frac{1}{26!}\right)-2\left(1-\frac{1}{5!}\right)$ $=\frac{1}{60}-\frac{2}{26!}<\frac{1}{60}$	

Qn	Solution	
3	**Vectors**	
(i)	$2OC = 3CA \Rightarrow \dfrac{OC}{CA} = \dfrac{3}{2} \Rightarrow \overrightarrow{OC} = \dfrac{3}{5}\overrightarrow{OA} = \dfrac{3}{5}\mathbf{a}$ Equation of line BC: $\mathbf{r} = \overrightarrow{OB} + \lambda\overrightarrow{BC}$ $\mathbf{r} = \mathbf{b} + \lambda\left(\dfrac{3}{5}\mathbf{a} - \mathbf{b}\right)$ $\mathbf{r} = \dfrac{3}{5}\lambda\mathbf{a} + (1-\lambda)\mathbf{b} \quad, \lambda \in \mathbb{R}$ (shown)	
(ii)	$\overrightarrow{OD} = \dfrac{1}{2}\mathbf{b}$ Equation of line AD: $\mathbf{r} = \overrightarrow{OA} + \mu\overrightarrow{AD}$ $\mathbf{r} = \mathbf{a} + \mu\left(\dfrac{1}{2}\mathbf{b} - \mathbf{a}\right) \quad, \mu \in \mathbb{R}$ To find intersection point E, equate line BC and line AD. $\dfrac{3}{5}\lambda\mathbf{a} + (1-\lambda)\mathbf{b} = \mathbf{a} + \mu\left(\dfrac{1}{2}\mathbf{b} - \mathbf{a}\right)$ $\left(1 - \lambda - \dfrac{1}{2}\mu\right)\mathbf{b} = \left(1 - \mu - \dfrac{3}{5}\lambda\right)\mathbf{a}$ Since **a** and **b** are non-zero and non-parallel, $1 - \lambda - \dfrac{1}{2}\mu = 0 \Rightarrow \lambda + \dfrac{1}{2}\mu = 1$ --- (1) $1 - \mu - \dfrac{3}{5}\lambda = 0 \Rightarrow \dfrac{3}{5}\lambda + \mu = 1$ --- (2) Using GC to solve (1) and (2), $\lambda = \dfrac{5}{7}$ or $\mu = \dfrac{4}{7}$ Since $\lambda = \dfrac{5}{7}$, $\overrightarrow{OE} = \mathbf{b} + \dfrac{5}{7}\left(\dfrac{3}{5}\mathbf{a} - \mathbf{b}\right) = \dfrac{3}{7}\mathbf{a} + \dfrac{2}{7}\mathbf{b}$	
(iii)	Since $\overrightarrow{OF}$ is in the same direction as $\overrightarrow{AB}$, $\overrightarrow{OF} = p\overrightarrow{AB}$ $= p(\mathbf{b} - \mathbf{a})$ for some $p > 0$ Area of trapezium $OABF$ $=$ Area of triangle OAB + Area of triangle OBF $= \dfrac{13}{16}\lvert\mathbf{a} \times \mathbf{b}\rvert$	

	$\frac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OB}\right\|+\frac{1}{2}\left\|\overrightarrow{OF}\times\overrightarrow{OB}\right\|=\frac{13}{16}\|\mathbf{a}\times\mathbf{b}\|$ $\frac{1}{2}\|\mathbf{a}\times\mathbf{b}\|+\frac{1}{2}\|p(\mathbf{b}-\mathbf{a})\times\mathbf{b}\|=\frac{13}{16}\|\mathbf{a}\times\mathbf{b}\|$ $\frac{1}{2}\|\mathbf{a}\times\mathbf{b}\|+\frac{1}{2}\|p(\mathbf{b}\times\mathbf{b})-p(\mathbf{a}\times\mathbf{b})\|=\frac{13}{16}\|\mathbf{a}\times\mathbf{b}\|$ $\frac{1}{2}\|\mathbf{a}\times\mathbf{b}\|+\frac{1}{2}\|-p(\mathbf{a}\times\mathbf{b})\|=\frac{13}{16}\|\mathbf{a}\times\mathbf{b}\|$, $\mathbf{b}\times\mathbf{b}=\mathbf{0}$ $\frac{1}{2}\|\mathbf{a}\times\mathbf{b}\|+\frac{1}{2}p\|(\mathbf{a}\times\mathbf{b})\|=\frac{13}{16}\|\mathbf{a}\times\mathbf{b}\|$, $p>0$ $\frac{1}{2}+\frac{1}{2}p=\frac{13}{16}$ $p=\frac{5}{8}$ $\therefore \overrightarrow{OF}=\frac{5}{8}(\mathbf{b}-\mathbf{a})$ <u>Alternative method</u> Let G lie on OF extended such that $OABG$ is a parallelogram with area $\|\mathbf{a}\times\mathbf{b}\|$. Let $\overrightarrow{OF}=p\overrightarrow{AB}$ for some $p>0$. $\frac{1+p}{2}=\frac{13}{16}$ $p=\frac{5}{8}$ $\therefore \overrightarrow{OF}=\frac{5}{8}(\mathbf{b}-\mathbf{a})$	

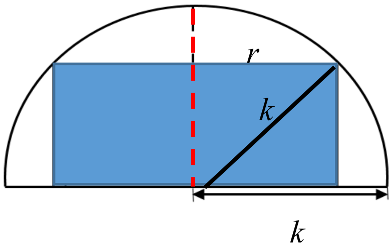
Qn	Solution	
4	**Maclaurin Series**	
(a)	$\left(b-\frac{x}{2}\right)^n=b^n\left(1-\frac{x}{2b}\right)^n$ $=b^n\left(1-\frac{xn}{2b}+\frac{n(n-1)}{2!}\left(\frac{x}{2b}\right)^2+..\right)$ $=b^n\left(1-\frac{n}{2b}x+\frac{n(n-1)}{8b^2}x^2+..\right)$ Since the coefficient of x is four times the coefficient of x^2, $-\frac{n}{2b}=\frac{4n(n-1)}{8b^2}$ $-1=\frac{n-1}{b}$ $-b=n-1$ $n=1-b$ Since the constant term in the expansion is $\frac{1}{2}$, $b^n=\frac{1}{2}$ Sub $n=1-b$ $b^{1-b}=\frac{1}{2}$ Using GC, $b=0.346$ $\left(\text{rejected because } b \text{ is an integer}\right)$ or $b=2$ $\therefore\ b=2$ and $n=-1$	
(bi)	Let $\mathrm{f}(x)=\ln\left(2x^2\right)$. As $\mathrm{f}(0)$ is undefined, it is not possible to obtain a Maclaurin series for $\ln\left(2x^2\right)$.	
(bii)	$\mathrm{f}(x)=\ln\left(2x^2\right)$ $\mathrm{f}'(x)=\frac{4x}{2x^2}=\frac{2}{x}$ $\mathrm{f}''(x)=-\frac{2}{x^2}$ When $x=2$, $\mathrm{f}(2)=\ln 8$, $\mathrm{f}'(2)=1$, $\mathrm{f}''(2)=-\frac{1}{2}$ $\therefore \ln\left(2x^2\right)=\ln 8+1(x-2)+\frac{\left(\frac{-1}{2}\right)}{2!}(x-2)^2+...$ $=\ln 8+(x-2)-\frac{1}{4}(x-2)^2+...$	

<table>
<tr><th>Qn</th><th>Solution</th><th></th></tr>
<tr><td>5</td><td>Differentiation Max/Min Problems</td><td></td></tr>
<tr><td>(i)</td><td>

Height of cylinder, $h=\sqrt{k^2-r^2}$

$V=\pi r^2\left(\text{height}\right)$

$\quad =\pi r^2\sqrt{k^2-r^2}$



</td><td></td></tr>
<tr><td>(ii)</td><td>

$$\frac{\mathrm{d}V}{\mathrm{d}r}=\pi\left[r^2.\frac{1}{2}\left(k^2-r^2\right)^{-\frac{1}{2}}\left(-2r\right)+\sqrt{k^2-r^2}\left(2r\right)\right]$$

$$=\frac{\pi r\left[-r^2+2\left(k^2-r^2\right)\right]}{\sqrt{x^2-r^2}}$$

$$=\frac{\pi r\left[2k^2-3r^2\right]}{\sqrt{k^2-r^2}}$$

Let $\frac{\mathrm{d}V}{\mathrm{d}r}=0$

$2k^2-3r^2=0 \quad \left(\because r\neq 0\right)$

$3r^2=2k^2$

$r=\sqrt{\frac{2}{3}}k \quad \left(\because r>0\right)$

To show maximum V:

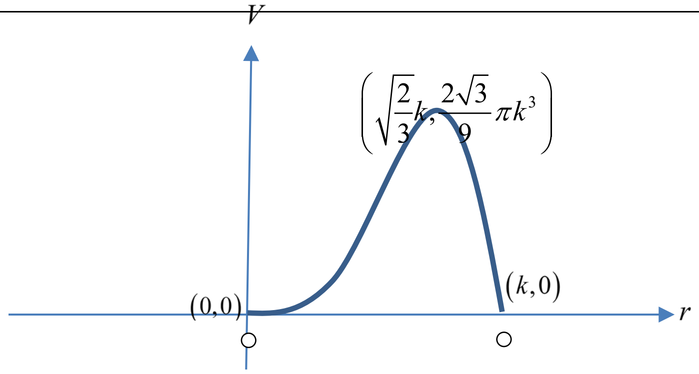
1st derivative Test

<table>
<tr><td>r</td><td>$\left(\sqrt{\frac{2}{3}}k\right)^-$</td><td>$\sqrt{\frac{2}{3}}k$</td><td>$\left(\sqrt{\frac{2}{3}}k\right)^+$</td></tr>
<tr><td>Sign of $\frac{\mathrm{d}V}{\mathrm{d}r}$</td><td>+ve</td><td>0</td><td>-ve</td></tr>
</table>

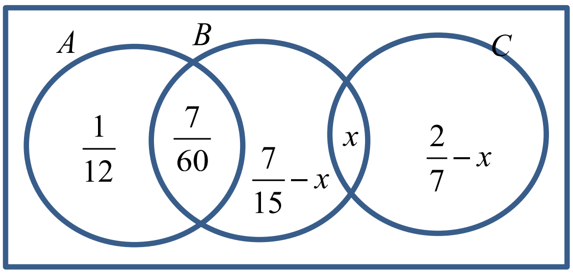
Hence V is a maximum at $r=\sqrt{\frac{2}{3}}k$.

Maximum volume $=\pi\frac{2\sqrt{3}}{9}k^3\ \text{cm}^3$

</td><td></td></tr>
</table>

(iii)		
(iv)	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ $= \frac{\sqrt{k^2 - (0.5k)^2}}{\pi(0.5k)\left[2k^2 - 3(0.5k)^2\right]}\left(\sqrt{3}\pi k^2\right)$ $= 2.4$ cm/min	

Qn	Solution	
6	**Permutation and Combination**	
(i)	Number of ways = ${}^{16}C_3$ $= 560$	
(ii)	Number of ways if Alex is in = ${}^{9}C_4 + {}^{7}C_1\,{}^{9}C_3$ $= 714$ Number of ways if Betty is in = ${}^{9}C_4$ $= 126$ Total number of ways $= 840$	

Qn	Solution	
7	**Probability**	
(i)	Since A and B are independent, $\mathrm{P}(A\mid B)=\mathrm{P}(A)=\frac{1}{5}\quad -(1)$ $\mathrm{P}(A\cap B)=\frac{1}{5}\mathrm{P}(B)\quad -(2)$ $\mathrm{P}(A\cup B)=\frac{2}{3}$ $\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)=\frac{2}{3}\quad -(3)$ Sub (1) and (2) into (3), $\frac{1}{5}+\mathrm{P}(B)-\frac{1}{5}\mathrm{P}(B)=\frac{2}{3}$ $\frac{4}{5}\mathrm{P}(B)=\frac{7}{15}$ $\mathrm{P}(B)=\frac{7}{12}$ $\mathrm{P}(A\cap B')=\mathrm{P}(A\cup B)-\mathrm{P}(B)$ $=\frac{2}{3}-\frac{7}{12}$ $=\frac{1}{12}$	
(ii)	 Let $x=\mathrm{P}(B\cap C)$. Since total probability is 1, $\mathrm{P}(B'\cap C')+\frac{7}{60}+\frac{7}{15}-x+x+\frac{2}{7}-x=1$ $\mathrm{P}(B'\cap C')=\frac{11}{84}+x$ Since $0\le x\le\frac{2}{7}$, $\frac{11}{84}\le\frac{11}{84}+x\le\frac{5}{12}$ $\frac{11}{84}\le\mathrm{P}(B'\cap C')\le\frac{5}{12}$	

Qn	Solution	
8	**Discrete Random Variable**	
(i)	X can take values 0, 1, 2, 3 $\mathrm{P}(X=0)=\dfrac{r(r-1)}{(r+3)(r+2)}$ $\mathrm{P}(X=1)=\dfrac{3r(r-1)\times 2}{(r+3)(r+2)(r+1)}=\dfrac{6r(r-1)}{(r+3)(r+2)(r+1)}$ $\mathrm{P}(X=2)=\dfrac{3\times 2\times r(r-1)\times 3}{(r+3)(r+2)(r+1)r}=\dfrac{18(r-1)}{(r+3)(r+2)(r+1)}$ $\mathrm{P}(X=3)=\dfrac{3\times 2\times 1\times r(r-1)\times 4}{(r+3)(r+2)(r+1)r(r-1)}=\dfrac{24}{(r+3)(r+2)(r+1)}$	

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{r(r-1)}{(r+3)(r+2)}$	$\frac{6r(r-1)}{(r+3)(r+2)(r+1)}$	$\frac{18(r-1)}{(r+3)(r+2)(r+1)}$	$\frac{24}{(r+3)(r+2)(r+1)}$

(ii)

$$\mathrm{E}(X)=\frac{6r(r-1)}{(r+3)(r+2)(r+1)}+\frac{36(r-1)}{(r+3)(r+2)(r+1)}+\frac{72}{(r+3)(r+2)(r+1)}$$

$$=\frac{6(r^2+5r+6)}{(r+3)(r+2)(r+1)}$$

$$=\frac{6(r+2)(r+3)}{(r+3)(r+2)(r+1)}$$

$$=\frac{6}{r+1}$$

$$\mathrm{E}(X^2)=\frac{6r(r-1)}{(r+3)(r+2)(r+1)}+\frac{72(r-1)}{(r+3)(r+2)(r+1)}+\frac{216}{(r+3)(r+2)(r+1)}$$

$$=\frac{6(r^2+11r+24)}{(r+3)(r+2)(r+1)}$$

$$=\frac{6(r+3)(r+8)}{(r+3)(r+2)(r+1)}$$

$$=\frac{6(r+8)}{(r+2)(r+1)}$$

$$\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$$

$$\mathrm{Var}(X)=\frac{6(r+8)}{(r+2)(r+1)}-\left(\frac{6}{r+1}\right)^2$$

$$=\frac{6(r+8)(r+1)-36(r+2)}{(r+2)(r+1)^2}$$

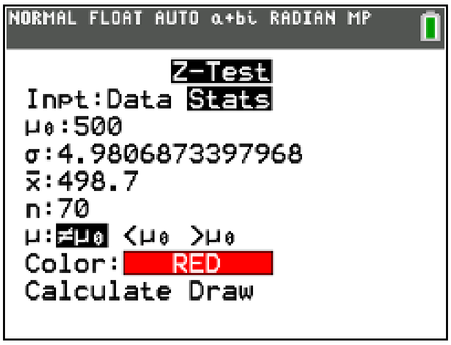
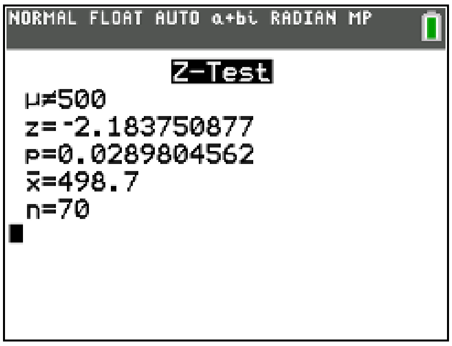
$$=\frac{6(r^2+3r-4)}{(r+2)(r+1)^2}\quad\text{(shown)}$$

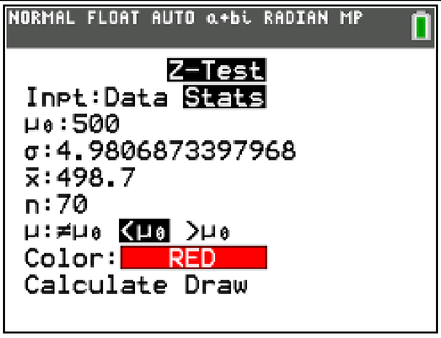
$$\therefore \mathrm{g}(r)=r^2+3r-4$$

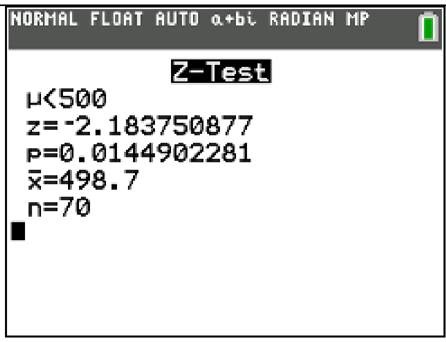
(iii)	Given that $\text{Var}(X)=\dfrac{11}{16}$ $\dfrac{6(r^2+3r-4)}{(r+2)(r+1)^2}=\dfrac{11}{16}$ Solve using GC, $r=7$	

Qn	Solution	
9	**Normal Distribution**	
(i)	Let X be the mass of a randomly chosen carrot in g $\bar{X} = \frac{X_1 + X_2 + X_3 + X_4}{4}$ $\mathrm{E}\left(\bar{X}\right) = 140$, $\mathrm{Var}\left(\bar{X}\right) = \frac{20^2}{4}$ $\bar{X} \sim \mathrm{N}\left(140, \frac{20^2}{4}\right)$ $\mathrm{P}(\bar{X} > 150) = 0.159$ (3 s.f)	
(ii)	Let Y be the mass of a randomly chosen tomato in g Let $S = Y_1 + Y_2 + Y_3 + Y_4 + Y_5$ $S \sim \mathrm{N}\left(5\mu, 5\left(7^2\right)\right)$ <u>Method 1</u> $\mathrm{P}(S > 520) = 0.17$ $\mathrm{P}(Z > \frac{520 - 5\mu}{\sqrt{5(7^2)}}) = 0.17$ $\frac{520 - 5\mu}{\sqrt{5(49)}} = 0.95417$ $520 - 5\mu = 0.95417\sqrt{5(49)}$ $\mu = \frac{520 - 0.95417\sqrt{5(49)}}{5}$ $= 101$ (3 s.f.) <u>Method 2</u> $\mathrm{P}(S > 520) = 0.17$ Using GC, $\mu = 101$ $\left(3 \text{ s.f.}\right)$	

(iii)	Let C be the total cost of four randomly chosen carrots and five randomly chosen tomatoes $C=\frac{0.15}{100}\left(X_1+X_2+X_3+X_4\right)+\frac{0.33}{100}\left(Y_1+Y_2+Y_3+Y_4+Y_5\right)$ $\mathrm{E}\left(C\right)=\frac{0.15}{100}(4)(140)+\frac{0.33}{100}\left(5\right)(100)$ $=2.49$ $\mathrm{Var}\left(C\right)=\left(\frac{0.15}{100}\right)^2(4)(20^2)+\left(\frac{0.33}{100}\right)^2\left(5\right)(7^2)$ $=0.00626805$ $C\sim\mathrm{N}\left(2.49,0.00626805\right)$ $\mathrm{P}(C\le 2.50)=0.550$ (to 3 s.f)	

Qn	Solution	
10	**Hypothesis Testing**	
(i)	Unbiased estimate of population mean is $\bar{x} = \frac{-91}{70} + 500$ $= 498.7$ (exact) Unbiased estimate of population variance is $s^2 = \frac{1}{69}\left(1830 - \frac{(-91)^2}{70}\right)$ $= \frac{17117}{690}$ $= 24.807$ $= 24.8$ (3 s.f.)	
(ii)	Let X be the mass of a randomly chosen pack of popcorn (in grams). Let μ denote the population mean mass of the packs of popcorn (in grams). $H_0 : \mu = 500$ $H_1 : \mu \neq 500$ Under H_0, since $n = 70$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(500, \frac{24.807}{70}\right)$ approximately. Test statistics: $Z = \frac{\bar{X} - 500}{\sqrt{\frac{24.807}{70}}}$ Level of significance: 2% Reject H_0 if $p-$value < 0.02 Using GC, $p-$value $= 0.0290$ (3 s.f)   Since $p-$value $= 0.0290 > 0.02$ we do not reject H_0 and conclude that there is insufficient evidence at 2% significance level, that population mean mass of packs of popcorn is not 500 grams. Thus the manager's claim is valid at 2% level of significance.	
(iii)	$H_0 : \mu = 500$ $H_1 : \mu < 500$	





From two-tailed to one-tailed test, the new

$p-\text{value}=\frac{1}{2}(0.0290)=0.0145.$

Since $p-\text{value}=0.0145<0.02$ we reject H_0 and conclude that there is sufficient evidence at 2% significance level, that population mean mass of packs of popcorn is less than 500 grams. Thus the manager has overstated his claim at 2% level of significance.

(iv)

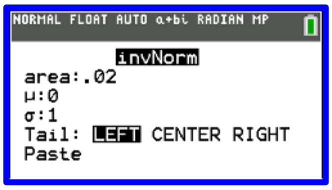
$s^2=\frac{n}{n-1}(\text{sample variance})=\frac{70}{69}(6.1^2)=37.749\quad(5\text{ s.f.})$

$H_0:\mu=500$

$H_1:\mu<500$

Under H_0, since $X\sim N\left(500,\frac{70}{69}(6.1^2)\right)$,

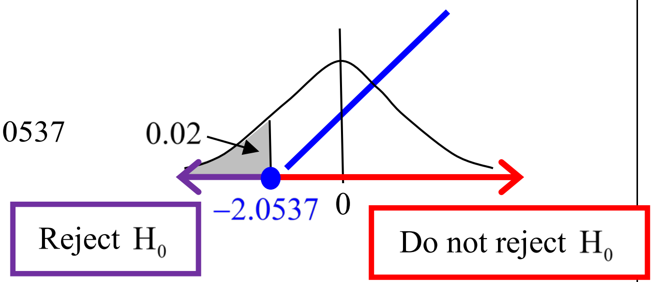
$\overline{X}\sim N\left(500,\frac{1}{70}\left(\frac{70}{69}(6.1^2)\right)\right).$



Test statistics: $Z=\dfrac{\overline{X}-500}{\frac{6.1}{\sqrt{69}}}$

Level of significance: 2%

Reject H_0 if $z-\text{value}<-2.0537$



$z-\text{value}=\dfrac{m-500}{\frac{6.1}{\sqrt{69}}}$

Since mean mass of packs of popcorn is not overstated at 2% level of significance, there is insufficient evidence that the mean mass of packs of popcorn is less than 500 grams. Thus, H_0 is not rejected

$z-\text{value}>-2.0537$

$\dfrac{m-500}{\frac{6.1}{\sqrt{69}}}>-2.0537$

$m>498.48936$

Therefore, $m>498.49$ (2 d.p.)

Qn	Solution	
11	**Binomial Distribution**	
(i)	Let X be the number of bulbs, out of 50, that are defective. $X \sim \mathrm{B}(50, 0.012)$ $\mathrm{P}(X=3)=0.019203$ $=0.0192 \ (3 \text{ s.f.})$	
(ii)	Required Prob $=\mathrm{P}(X=2)^2 \times \mathrm{P}(X=3) \times \mathrm{P}(X \le 1) \times \dfrac{4!}{2!}$ $=0.0019777$ $=0.00198 \ (3 \text{ s.f.})$	
(iii)	Let Y be the number of bulbs, out of n, that are defective. $Y \sim \mathrm{B}(n, 0.012)$ Given $\mathrm{P}(Y>2)>0.25 \quad \Rightarrow \quad 1-\mathrm{P}(Y \le 2)>0.25$, Using GC, when $n=144, 1-\mathrm{P}(Y \le 2)=0.2497<0.25$ when $n=145, 1-\mathrm{P}(Y \le 2)=0.2529>0.25$ Least number of bulbs = 145	
(iv)	Let W be the number of bulbs, out of 25, that are defective. $W \sim \mathrm{B}(25, 0.012)$ $\mathrm{P}(\text{box is accepted as satisfactory})$ $=\mathrm{P}(X \le 2)+\mathrm{P}(X=3) \times \mathrm{P}(W_1=0)+\mathrm{P}(X=3) \times \mathrm{P}(W_1=1) \times \mathrm{P}(W_2=0)$ $=0.99511$ $=0.995 \ (3 \text{ s.f.})$	
(v)(a)	Since the probability of accepting a box as satisfactory is 0.99511 is high, the inspection scheme is not stringent.	
(v)(b)	Since the expected number of defective bulbs in the first sample is $50 \times 0.012=0.6$ is significantly lower than the rejection criterion of 4 or more defective bulbs (and the criterion for the testing of the 2nd sample), the probability of accepting a box as satisfactory would be high regardless of the outcomes of the 2nd or 3rd samples. Thus, the inspection scheme is considered not stringent.	
(vi)	Let N be the number of bulbs tested. $\mathrm{P}(N=50)=1-\mathrm{P}(X=3)=0.9807967$ $\mathrm{P}(N=75)=\mathrm{P}(X=3) \times (1-\mathrm{P}(W=1))=0.0148914$ $\mathrm{P}(N=100)=\mathrm{P}(X=3) \times \mathrm{P}(W=1)=0.0043119$ $\mathrm{E}(N)=50 \times \mathrm{P}(N=50)+75 \times \mathrm{P}(N=75)+100 \times \mathrm{P}(N=100)$ $=50.588$ $=50.6 \ (3 \text{ s.f.})$	