TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ______________________________________

CIVICS GROUP: ______________________________________

H2 MATHEMATICS 9758

Paper 2

20 SEPTEMBER 2021

3 hours

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and Civics Group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

For Examiners' Use	
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Total	

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 29 printed pages and 1 blank pages.

Section A: Pure Mathematics [40 marks]

1 **(i)** Differentiate $e^{\frac{1}{x}}$ with respect to x. [1]

(ii) Hence find the exact value of

$$\int_1^2 \frac{1}{x^3} e^{\frac{1}{x}} \, dx \ .$$ [3]

2 **(i)** Show that $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$. [1]

(ii) Hence find $\sum_{r=1}^{n} \frac{2r}{(r+1)!}$ in terms of n. [3]

(iii) Give a reason why the series $\sum_{r=1}^{\infty} \frac{2r}{(r+1)!}$ converges, and write down the value of the sum to infinity. [2]

(iv) Using the result in part **(ii)**, show that

$$\frac{10}{6!} + \frac{12}{7!} + \frac{14}{8!} + \ldots + \frac{50}{26!} < \frac{1}{60}.$$ [2]

3

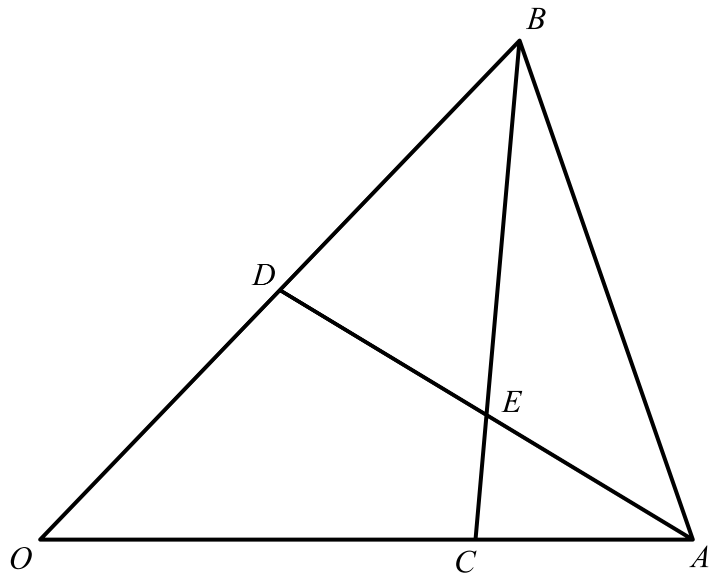

Figure 1

With reference to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Point D is the mid-point of OB and point C lies on OA such that $2OC = 3CA$. The lines AD and BC intersect at point E (see Figure 1).

(i) Show that the vector equation of the line BC can be written as $\mathbf{r} = \frac{3}{5}\lambda\mathbf{a} + (1-\lambda)\mathbf{b}$, where λ is a real parameter. [1]

(ii) Find the position vector of E in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

(iii) Point F is such that $\overrightarrow{OF}$ is in the same direction as $\overrightarrow{AB}$. Given that the area of trapezium $OABF$ is $\frac{13}{16}|\mathbf{a}\times\mathbf{b}|$ units^2, find the position vector of F in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

4 **(a)** Expand $\left(b-\frac{x}{2}\right)^n$ in ascending powers of x, where b is a positive constant and n is a negative constant, up to and including the term in x^2. [2]

It is given that the coefficient of x is four times the coefficient of x^2 and the constant term in the expansion is $\frac{1}{2}$. Find the **integer** values of b and n. [3]

(b) **(i)** Explain why it is not possible to obtain a Maclaurin series for $\ln\left(2x^2\right)$. [1]

(ii) A Taylor series is an expansion of a real function $\mathrm{f}(x)$ about a point $x=a$ and it is defined by

$$\mathrm{f}(x)=\mathrm{f}(a)+\mathrm{f}'(a)(x-a)+\frac{\mathrm{f}''(a)}{2!}(x-a)^2+\ldots+\frac{\mathrm{f}^{(n)}(a)}{n!}(x-a)^n+\ldots$$

where $\mathrm{f}^{(n)}(a)$ is the value of the nth derivative of $\mathrm{f}(x)$ when $x=a$.

Find the first three exact non-zero terms of the Taylor series for $\ln\left(2x^2\right)$ about the point $x=2$. [You need not simplify your answer.] [3]

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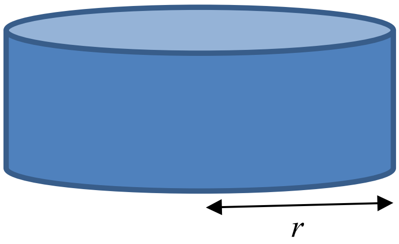

Figure 1

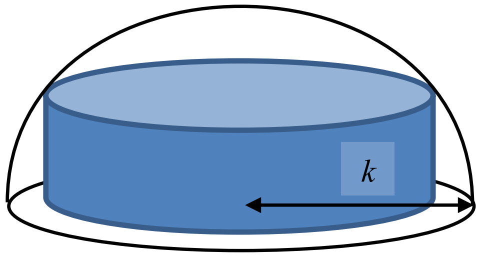

Figure 2

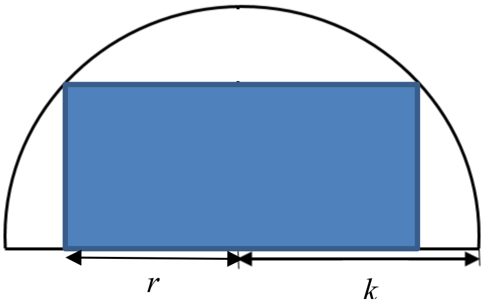

Figure 3

Figure 1 shows a cylinder with radius r cm. As shown in Figure 2, the cylinder can be inscribed in a hemisphere with fixed radius k cm such that the circumference of the top surface of the cylinder is in contact with the curved surface of the hemisphere. A cross-sectional view of the cyclinder and hemisphere is shown in Figure 3.

(i) Show that the volume of the cylinder, V cm^3, is given by $V = \pi r^2\sqrt{k^2 - r^2}$. [1]

(ii) Use differentiation to find, in terms of k, the value of r which gives a maximum value of V, justifying that this value indeed gives a maximum V. Hence write down the exact maximum volume of the cylinder in terms of k. [5]

(iii) Sketch the graph showing the volume of the cylinder as the radius of the cylinder varies. [3]

(iv) It is given that the volume of the cylinder increases at a constant rate of $\sqrt{3}\pi k^2$ cm^3 per minute. Find the rate at which the radius of the cylinder is increasing when $r = 0.5k$. [2]

Section B: Probability and Statistics [60 marks]

6 A committee of five people is chosen from a group consisting of ten men and eight women. One of the men is Alex and one of the women is Betty.

(i) Find the number of ways such that Alex and Betty are both in the committee. [1]

(ii) It is decided that either Alex or Betty will be in the committee, but not both. Find the number of ways such that there is at most one woman in the committee. [3]

7 A and B are independent events such that $\mathrm{P}(A \mid B) = \frac{1}{5}$ and $\mathrm{P}(A \cup B) = \frac{2}{3}$.

(i) Find the exact value of $\mathrm{P}(A \cap B')$. [4]

For a third event C, it is given that $\mathrm{P}(C) = \frac{2}{7}$ and that A and C are mutually exclusive.

(ii) Find the range of values of $\mathrm{P}(B' \cap C')$. [3]

8 A bag contains 3 white balls and r red balls, where $r \geq 2$. Balls are drawn one at a time, at random from the bag and without replacement. The random variable X is the number of white balls drawn until 2 red balls are drawn.

(i) Show that $\mathrm{P}(X = 1) = \dfrac{6r(r-1)}{(r+3)(r+2)(r+1)}$, and find the probability distribution of X. [4]

(ii) Find the expectation of X and show that the variance of X can be expressed in the form $\dfrac{6\mathrm{g}(r)}{(r+2)(r+1)^2}$, where $\mathrm{g}(r)$ is a quadratic polynomial to be determined. [5]

(iii) Given that the variance of X is $\dfrac{11}{16}$, find the value of r. [1]

9 **In this question, you should state clearly the values of the parameters of any normal distribution you use.**

The masses, in grams, of carrots and tomatoes sold at a supermarket have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean (grams)	Standard deviation (grams)
Carrots	140	20
Tomatoes	μ	7

(i) Find the probability that the mean mass of four randomly chosen carrots is more than 150 grams. [2]

(ii) If the probability that the total mass of five randomly chosen tomatoes exceeds 520 grams is 0.17, find the value of μ. [3]

For the rest of the question, let $\mu = 100$.

At the supermarket, carrots are sold at \$0.15 per 100 grams and tomatoes are sold at \$0.33 per 100 grams.

(iii) Find the probability that the total cost of four randomly chosen carrots and five randomly chosen tomatoes is at most \$2.50. [5]

10 The manager of a factory producing popcorn claims that the mean mass of packs of popcorn is 500 grams. To test this claim, a random sample of 70 packs of popcorn is weighed and the masses, x grams, of each pack of popcorn is recorded. The results are summarised by

$$\sum(x-500) = -91 \qquad \sum(x-500)^2 = 1830.$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Test, at the 2% significance level, whether the manager's claim is valid. [4]

(iii) Hence, determine the conclusion at the 2% significance level if the manager has overstated the mean mass of packs of popcorn. [2]

(iv) The factory purchases new machinery to pack the popcorn and the mass of packs of popcorn is now known to have a normal distribution. A new random sample of 70 packs of popcorn is taken and found to have mean mass of m grams with standard deviation of 6.1 grams. Find the range of values of m to conclude that the mean mass of packs of popcorn is not overstated at the 2% significance level. Leave your answer to 2 decimal places. [5]

11 A factory manufactures a large number of bulbs every day and they are packed into boxes by thousands for distribution to bulbs suppliers. Due to a manufacturing fault, 1.2% of these bulbs are defective. A random sample of 50 bulbs is taken from each box and tested.

(i) Find the probability that exactly 3 bulbs are defective in a random sample of 50 bulbs. [1]

(ii) Find the probability that in four random samples of 50 bulbs each, there are exactly two samples with exactly 2 defective bulbs each, one sample with exactly 3 defective bulbs, and one sample with fewer than 2 defective bulbs. [3]

(iii) Determine the least number of bulbs tested in a sample such that the probability of having more than 2 defective bulbs is more than 0.25. [2]

A three-step inspection scheme is devised as follows to determine if a box of bulbs can be accepted as satisfactory or rejected:

- Step 1: A random sample of 50 bulbs is tested and if it contains fewer than 3 defective bulbs, the box is accepted as satisfactory. If there is more than 3 defective bulbs in the sample of 50, then the box is rejected. If there is exactly 3 defective bulbs, a second random sample of 25 bulbs is tested.
- Step 2: If the second sample contains no defective bulb, the box is accepted as satisfactory. If there is more than 1 defective bulb in the second sample, then the box is rejected. If there is exactly 1 defective bulb, a third random sample of 25 bulbs is tested.
- Step 3: If the third sample contains no defective bulb, the box is accepted as satisfactory. Otherwise, the box is rejected.

(iv) Find the probability that a randomly chosen box is accepted as satisfactory. [3]

(v) An inspection scheme is considered to be stringent if the likelihood of accepting a box as satisfactory is not high.

(a) Using your answer in part **(iv)**, explain, in context, whether the inspection scheme is stringent. [1]

(b) By considering the expected number of defective bulbs for the first sample, suggest clearly, in context, whether the inspection scheme is stringent. [2]

(vi) Calculate the expected number of bulbs to be tested under this inspection scheme, showing your working clearly. [3]

End of Paper