2021 SAJC H2 Maths Promo Paper Solution

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<td>1(a)</td>
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Let $S_n = an^2 + bn + c$

When $n = 1$, $1 = a + b + c$ --- (1)

When $n = 2$, $S_2 = u_1 + u_2$

$4a + 2b + c = 6$ ---- (2)

When $n = 12$, $276 = 144a + 12b + c$ --- (3)

Using GC to solve equations (1), (2) and (3),

$a = 2, b = -1, c = 0$.

$\therefore S_n = 2n^2 - n$

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<td>(b)</td>
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$$\sum_{n=2}^{N} \left(v_n - v_{n-1}\right) = \left(-\frac{3}{5}\right) \sum_{n=2}^{N} \left(\frac{2}{5}\right)^{n-2}$$

LHS:

$$\begin{aligned}
&\sum_{n=2}^{N} \left(v_n - v_{n-1}\right) \\
&= \cancel{v_2} - v_1 \\
&\quad + \cancel{v_3} - \cancel{v_2} \\
&\quad + \cancel{v_4} - \cancel{v_3} \\
&\quad + \ldots \\
&\quad + \cancel{v_{N-1}} - \cancel{v_{N-2}} \\
&\quad + v_N - \cancel{v_{N-1}} \\
&= v_N - 4
\end{aligned}$$

RHS:

$$\left(-\frac{3}{5}\right) \sum_{n=2}^{N} \left(\frac{2}{5}\right)^{n-2} = \left(-\frac{3}{5}\right) \frac{1\left[1 - \left(\frac{2}{5}\right)^{N-1}\right]}{1 - \frac{2}{5}} = \left(\frac{2}{5}\right)^{N-1} - 1$$

Therefore

$$v_N - 4 = \left(\frac{2}{5}\right)^{N-1} - 1$$

$$v_N = \left(\frac{2}{5}\right)^{N-1} + 3$$

As $N \to \infty, \left(\frac{2}{5}\right)^{N-1} \to 0$. Hence $v_n \to 3$, a **constant/unique finite value**.

Therefore the sequence is convergent.

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2(i)	$S_n = \frac{a^n}{2^{n-1}} - 2$; $S_{n-1} = \frac{a^{n-1}}{2^{n-2}} - 2$ For $n \geq 2$, $T_n = S_n - S_{n-1} = \frac{a^n}{2^{n-1}} - 2 - \left(\frac{a^{n-1}}{2^{n-2}} - 2\right)$ $= \left(\frac{a}{2}\right)^{n-1}\left(a - \frac{1}{2^{-1}}\right) = (a-2)\left(\frac{a}{2}\right)^{n-1}$ When $n = 1$, $T_1 = S_1 = \frac{a}{2^0} - 2 = a - 2 = (a-2)\left(\frac{a}{2}\right)^{1-1}$ which follows the form of $T_n = (a-2)\left(\frac{a}{2}\right)^{n-1}$ when $n = 1$. Thus, $T_n = (a-2)\left(\frac{a}{2}\right)^{n-1}$ for $n \geq 1$ (shown) $\frac{T_n}{T_{n-1}} = \frac{(a-2)\left(\frac{a}{2}\right)^{n-1}}{(a-2)\left(\frac{a}{2}\right)^{n-2}} = \frac{a}{2}$ (constant independent of n) Series is a geometric series. (shown)
(ii)	For the sum to infinity to exist, $\left\lvert\frac{a}{2}\right\rvert < 1$ $\therefore -2 < a < 2, a \neq 0$ [or $\therefore \lvert a\rvert < 2, a \neq 0$]
(iii)	Need to find the least value of n such that $\lvert S_n - S_\infty\rvert < 0.2$ $T_1 = -1,\ S_n = \frac{1}{2^{n-1}} - 2,$ Common ratio$=\frac{1}{2}$; $S_\infty = \frac{-1}{1-\frac{1}{2}} = -2$ For $\lvert S_n - S_\infty\rvert < 0.2$ $\left\lvert\frac{1}{2^{n-1}} - 2 - (-2)\right\rvert < 0.2$ $\left\lvert\frac{1}{2^{n-1}}\right\rvert < 0.2$ $\lvert 2^{n-1}\rvert > 5$ $2^{n-1} > 5$ $n - 1 > \frac{\ln 5}{\ln 2}$ $n > 3.32$ Least $n = 4$

	Alternatively, $T_1=-1,\ S_n=\dfrac{1}{2^{n-1}}-2,$ Common ratio$=\dfrac{1}{2}$ $S_\infty=\dfrac{-1}{1-\frac{1}{2}}=-2$ For $\left\|S_n-S_\infty\right\|<0.2$ $\left\|\dfrac{1}{2^{n-1}}-2-(-2)\right\|<0.2$ $\left\|\dfrac{1}{2^{n-1}}\right\|<0.2$

n	$\left\|\dfrac{1}{2^{n-1}}\right\|$
3	0.25>0.2
4	0.125<0.2
5	0.0625<0.2

Hence, the least $n=4$

3(i)	Let $y=\ln\left(\dfrac{e^x+1}{e^x-1}\right)$ $e^y=\dfrac{e^x+1}{e^x-1}$ $e^ye^x-e^y=e^x+1$ $e^ye^x-e^x=e^y+1$ $e^x\left(e^y-1\right)=e^y+1$ $e^x=\dfrac{e^y+1}{e^y-1}$ $x=\ln\left(\dfrac{e^y+1}{e^y-1}\right)$ Since $x=f^{-1}(y)=\ln\left(\dfrac{e^y+1}{e^y-1}\right)$, $f^{-1}(x)=\ln\left(\dfrac{e^x+1}{e^x-1}\right)=f\left(x\right)$
(ii)	Since $f\left(x\right)=f^{-1}\left(x\right)$, $ff(x)=ff^{-1}(x)=x$. Thus, $f^2(x)=x$. $f^{2021}\left(3\right)=f(3)=\ln\left(\dfrac{e^3+1}{e^3-1}\right)$

4(i)	Vectors **p** and **q** are parallel or either **p** or **q** (or both) is a zero vector.
(ii)	Since $\mathbf{r}\times\begin{pmatrix}-6\\3\\2\end{pmatrix}=\mathbf{0}$ $\mathbf{r}$ is parallel to the vector $\begin{pmatrix}-6\\3\\2\end{pmatrix}$. $\therefore \quad \mathbf{r}=\dfrac{1}{\sqrt{(-6)^2+3^2+2^2}}\begin{pmatrix}-6\\3\\2\end{pmatrix}$ $=\dfrac{1}{7}\begin{pmatrix}-6\\3\\2\end{pmatrix}$

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$\cos\theta = \dfrac{\left|\begin{pmatrix}-6\\3\\2\end{pmatrix}\bullet\begin{pmatrix}1\\0\\0\end{pmatrix}\right|}{\left|\sqrt{49}\right|\left|\sqrt{1}\right|}=\dfrac{\left|-6\right|}{\left|\sqrt{49}\right|\left|\sqrt{1}\right|}=\dfrac{6}{7}$

Using the trigonometric identity,
$\sin^2\theta+\cos^2\theta=1$

$\sin\theta=\sqrt{1-\cos^2\theta}$

$=\sqrt{1-\left(\dfrac{6}{7}\right)^2}$

$=\dfrac{1}{7}\sqrt{49-36}$

$=\dfrac{\sqrt{13}}{7}$

$\sin\theta>0$ since θ is an acute angle.

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Alternatively
Let θ be the required acute angle.

$\sin\theta = \dfrac{\left|\begin{pmatrix}-6\\3\\2\end{pmatrix}\times\begin{pmatrix}1\\0\\0\end{pmatrix}\right|}{\left|\sqrt{49}\right|\left|\sqrt{1}\right|}=\dfrac{\left|\begin{pmatrix}0\\2\\-3\end{pmatrix}\right|}{\left|\sqrt{49}\right|\left|\sqrt{1}\right|}$

$=\dfrac{\sqrt{13}}{7}$

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<tr><td>5(a)
(i)</td><td>$\frac{\mathrm{d}}{\mathrm{d}x}\left(3^{2x}\right) = 3^{2x}\left(2\right)\left(\ln 3\right)$
$= 9^x \ln 3^2$
$= 9^x \ln 9$ (Shown)</td><td>Alternatively,
Let $y = 3^{2x}$. Then, $\ln y = 2x\ln 3$
Differentiate both sides with respect to x,
$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\ln 3$
$\frac{\mathrm{d}y}{\mathrm{d}x} = 2y\ln 3 = 2\left(3^{2x}\right)\ln 3 = 3^{2x}\ln 9$</td></tr>
<tr><td>(a)
(ii)</td><td colspan="2">Let $y = \tan^{-1}\left(3^{2x}\right)$
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+\left(3^{2x}\right)^2}\left(9^x \ln 9\right) = \frac{9^x \ln 9}{1+3^{4x}}$</td></tr>
<tr><td>(b)</td><td colspan="2">$y^{2y} = \mathrm{e}^{3x+2\mathrm{e}}$
Taking ln on both sides, $2y\ln y = 3x + 2\mathrm{e}$
Differentiating both sides with respect to x,
$2\left[y\left(\frac{1}{y}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \left(\ln y\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right] = 3$
$2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(1+\ln y\right) = 3 - - - (1)$
Differentiating (1) with respect to x,
$2\left[\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{1}{y}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \left(1+\ln y\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\right] = 0$
$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\left(\frac{1}{y}\right) + \left(1+\ln y\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = 0$
$y\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = -\frac{1}{\left(1+\ln y\right)}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$
From (1):
$\frac{2}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{1}{\left(1+\ln y\right)}$
Hence,
$y\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = -\frac{1}{\left(1+\ln y\right)}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = -\frac{2}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$</td></tr>
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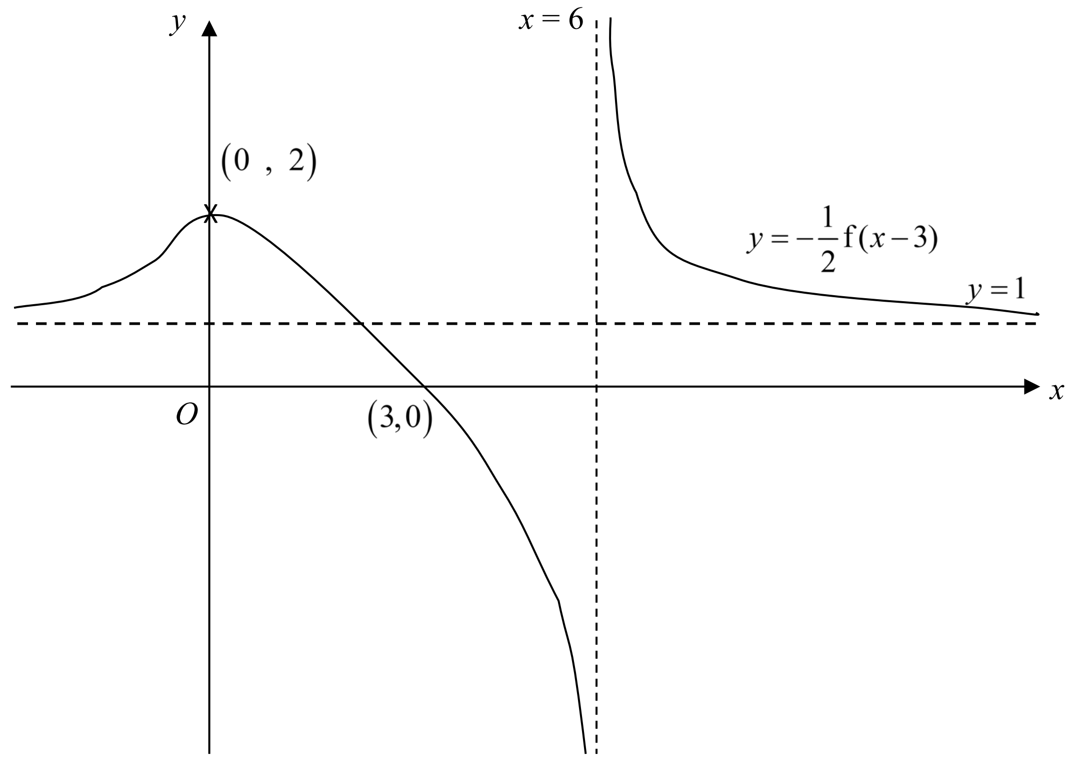
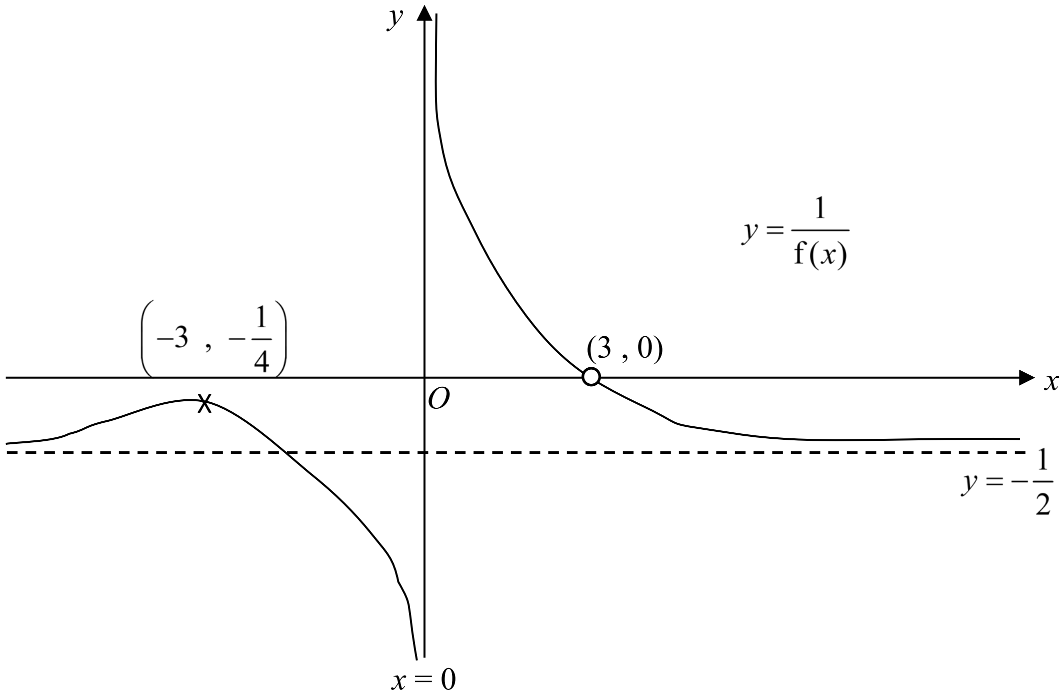
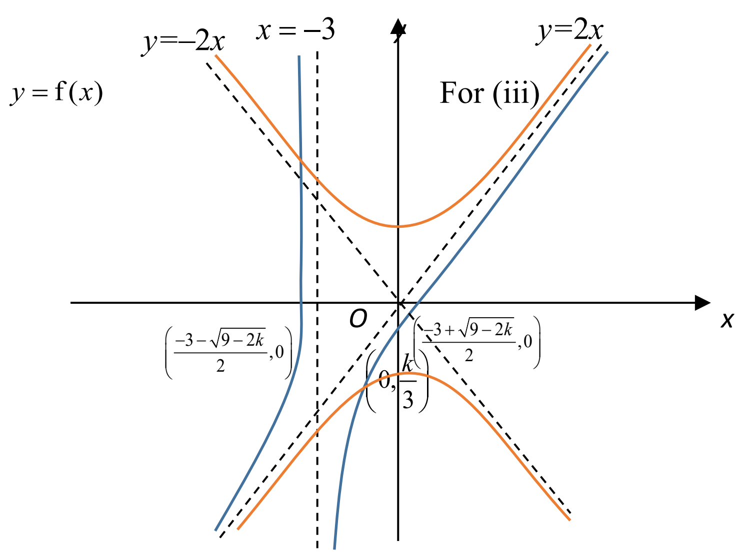
6(a) (i)	$\mathrm{h}'(x)=3\mathrm{e}^{3x}$ $\mathrm{g}(x)=\dfrac{1}{\mathrm{h}'(x)}=\dfrac{1}{3}\mathrm{e}^{-3x}$
(a) (ii)	$y=\mathrm{e}^{3x}$ $\downarrow$ Replace x with $-x$ $y=\mathrm{e}^{-3x}$ $\downarrow$ Replace y with $3y$ $y=\dfrac{1}{3}\mathrm{e}^{-3x}$ The graph of $y = \mathrm{h}(x)$ undergoes the transformations: 1. Reflection about the y-axis, followed by 2. A scaling parallel to the y axis with a scale factor of $\dfrac{1}{3}$ to obtain the graph of $y=\dfrac{1}{\mathrm{h}'(x)}$.
(b) (i) (a)	$y=\mathrm{f}(x)\xrightarrow{\text{replace } x \text{ by } x-3} y=\mathrm{f}(x-3)\xrightarrow{\text{replace } y \text{ by } -y} y=-\mathrm{f}(x-3)\xrightarrow{\text{replace } y \text{ by } 2y} y=-\dfrac{1}{2}\mathrm{f}(x-3)$ 

(b) (i) (b)	

(b) (ii)	$(-3\ ,\ 0)$ **[Note: (-3,0) on $y=f'(x)$ corresponds to the stationary point on $y=f(x)$, and there are no other *x*-intercept on $y=f'(x)$. However, there would be a *y*-intercept on $y=f'(x)$, but there is insufficient information from the question for us to determine the value of the *y*-intercept since we do not know the gradient of the graph $y=f(x)$ at *x* = 0.]**

7(i)	$y=\dfrac{2x^2+6x+k}{x+3}$ $\dfrac{dy}{dx}=\dfrac{(4x+6)(x+3)-(2x^2+6x+k)}{(x+3)^2}$ $=\dfrac{2x^2+12x+18-k}{(x+3)^2}>0$ $\Rightarrow 2x^2+12x+18-k>0$ for all real values of $x, x\neq -3$ $\Rightarrow$ Discriminant <0 $12^2-4(2)(18-k)<0$ $\Rightarrow k<0$	Alternatively, $y=\dfrac{2x^2+6x+k}{x+3}$ $y=2x+\dfrac{k}{x+3}$ $\dfrac{dy}{dx}=2-\dfrac{k}{(x+3)^2}$ For $\dfrac{dy}{dx}=2-\dfrac{k}{(x+3)^2}>0$, $k<0$
(ii)	$y=\dfrac{2x^2+6x+k}{x+3}$ $y=2x+\dfrac{k}{x+3}, k<0$ Equations of asymptotes: $y=2x,\ x=-3$ Intercepts: $x=0\Rightarrow y=\dfrac{k}{3}$ $y=0\Rightarrow 2x^2+6x+k=0$ $\Rightarrow x=\dfrac{-6\pm\sqrt{36-8k}}{4}=\dfrac{-3\pm\sqrt{9-2k}}{2}$  	

(iii)

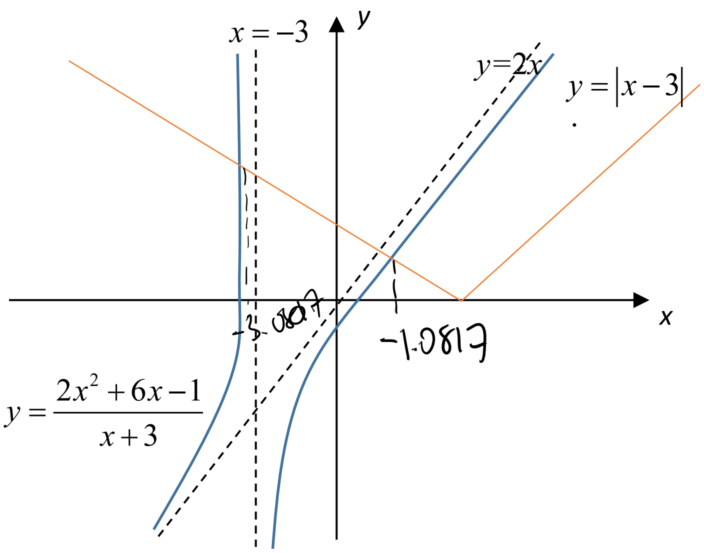
$$\frac{\left(\dfrac{2x^2+6x+k}{x+3}\right)^2}{4}-x^2=1 \text{ -----(*)}$$

Add the hyperbola $\dfrac{y^2}{2^2}-\dfrac{x^2}{1^2}=1$.

Asymptotes are $y=\pm 2x$ and y-intercepts are $(0,\pm 2)$

Since the graphs $y=\mathrm{f}(x)$ and $\dfrac{y^2}{2^2}-\dfrac{x^2}{1^2}=1$ have two intersection points, the equation (*) has two real roots.

(iv)



The x-coordinate of the intersection points are between the graphs $y=\dfrac{2x^2+6x-1}{x+3}$ and $y=|x-3|$ are –3.0817 and 1.0817.

For $\dfrac{2x^2+6x-1}{x+3}>|x-3|$, $\quad -3.08<x<-3$ or $x>1.08$.

8(i)

$$\frac{dx}{d\theta} = 2\cos\theta \quad , \quad \frac{dy}{d\theta} = -3\sin\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{1}{\frac{dx}{d\theta}} = (-3\sin\theta) \times \frac{1}{2\cos\theta} = -\frac{3}{2}\tan\theta$$

At P, $\sqrt{2}+2 = 2\sin\theta + 2 \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{2}}{2} \quad \Rightarrow \theta = \frac{\pi}{4}$

Check: $y = 3\cos\frac{\pi}{4} - 1 = \frac{3\sqrt{2}}{2} - 1$

Gradient of normal at P $= -\frac{1}{\left(-\frac{3}{2}\tan\frac{\pi}{4}\right)} = \frac{2}{3}$

Equation of normal at P: $y - \left(\frac{3}{2}\sqrt{2} - 1\right) = \frac{2}{3}\left[x - \left(\sqrt{2}+2\right)\right]$

$$y = \frac{2}{3}x - \frac{4}{3} + \frac{5}{3}\left(\frac{\sqrt{2}}{2}\right) - 1 = \frac{2}{3}x - \frac{7}{3} + \frac{5}{6}\sqrt{2}$$

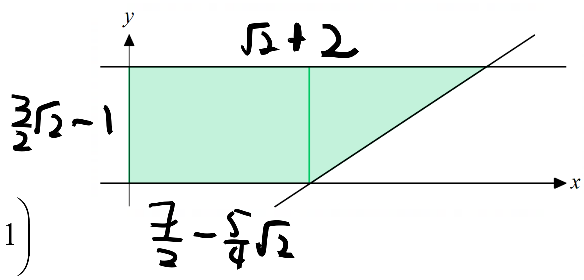
$$6y = 4x - 14 + 5\sqrt{2}$$

(ii)

When the normal at point P intersects the x-axis, $y = 0$

$0 = 4x - 14 + 5\sqrt{2}$

$x = \frac{14}{4} - \frac{5}{4}\sqrt{2} = \frac{7}{2} - \frac{5}{4}\sqrt{2}$



Area of quadrilateral

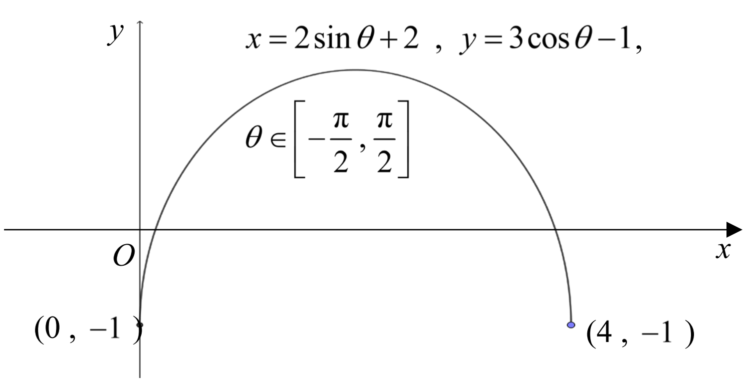
$= \frac{1}{2}\left[\left(\frac{7}{2} - \frac{5}{4}\sqrt{2}\right) + \left(\sqrt{2}+2\right)\right]\left(\frac{3}{2}\sqrt{2} - 1\right)$

$= 2.8854 = 2.885$ units^2 (3 d.p.)

(iii)

As $\theta \to \pm\frac{\pi}{2}$, $\frac{dy}{dx} = -\frac{3}{2}\tan\theta \to \pm\infty$. Hence, the tangents will become/approach vertical lines.

(iv)



9(i)	$\overrightarrow{OA}=\begin{pmatrix}0\\5\\15\end{pmatrix}$; $\overrightarrow{OB}=\begin{pmatrix}1\\-1\\3\end{pmatrix}$ $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}1\\-1\\3\end{pmatrix}-\begin{pmatrix}0\\5\\15\end{pmatrix}=\begin{pmatrix}1\\-6\\-12\end{pmatrix}$ Equation of line l_2 : $\mathbf{r}=\begin{pmatrix}0\\5\\15\end{pmatrix}+\mu\begin{pmatrix}1\\-6\\-12\end{pmatrix},\ \mu\in\mathbb{R}$
(ii)	Let the point of intersection between l_1 and l_2 be X. $\overrightarrow{OX}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and $\overrightarrow{OX}=\begin{pmatrix}0\\5\\15\end{pmatrix}+\mu\begin{pmatrix}1\\-6\\-12\end{pmatrix}$ for some $\lambda,\mu\in\mathbb{R}$ Since l_1 and l_2 intersect, $\begin{pmatrix}-2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}0\\5\\15\end{pmatrix}+\mu\begin{pmatrix}1\\-6\\-12\end{pmatrix}$ $-2-\lambda=\mu \quad\Rightarrow\quad \lambda+\mu=-2\ \ldots\ (1)$ $1+2\lambda=5-6\mu\Rightarrow\quad 2\lambda+6\mu=4\ \cdots\ (2)$ $3+3\lambda=15-12\mu\ \Rightarrow\ 3\lambda+12\mu=12\ldots\ (3)$ Solving (1) , (2) and (3) using GC, $\lambda=-4;\mu=2$ $\overrightarrow{OC}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+(-4)\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}2\\-7\\-9\end{pmatrix}$ Coordinates of C is $(2,-7,-9)$
(iii)	Since F is a point on l_1 : $\overrightarrow{OF}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\overrightarrow{AF}=\overrightarrow{OF}-\overrightarrow{OA}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}-\begin{pmatrix}0\\5\\15\end{pmatrix}=\begin{pmatrix}-2-\lambda\\-4+2\lambda\\-12+3\lambda\end{pmatrix}$ Since $\overrightarrow{AF}$ is perpendicular to l_1, $\overrightarrow{AF}\bullet\begin{pmatrix}-1\\2\\3\end{pmatrix}=0$

	$\begin{pmatrix} -2-\lambda \\ -4+2\lambda \\ -12+3\lambda \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = 0$ $2+\lambda-8+4\lambda-36+9\lambda=0$ $14\lambda=42$ $\lambda=3$ Therefore $\overrightarrow{OF} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \\ 12 \end{pmatrix}$	
(iv)	Let A' be the point of reflection of A along the line l_1 Using Ratio Theorem, $\overrightarrow{OF} = \dfrac{\overrightarrow{OA}+\overrightarrow{OA'}}{2}$ $\overrightarrow{OA'} = 2\overrightarrow{OF} - \overrightarrow{OA}$ $= 2\begin{pmatrix} -5 \\ 7 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 15 \end{pmatrix} = \begin{pmatrix} -10 \\ 9 \\ 9 \end{pmatrix}$ Let l_3 be the line of reflection of l_2 in the line l_1 $\overrightarrow{CA'} = \overrightarrow{OA'} - \overrightarrow{OC}$ $= \begin{pmatrix} -10 \\ 9 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ -7 \\ -9 \end{pmatrix}$ $= \begin{pmatrix} -12 \\ 16 \\ 18 \end{pmatrix} = 2\begin{pmatrix} -6 \\ 8 \\ 9 \end{pmatrix}$ Direction vector of $l_3 = \begin{pmatrix} -6 \\ 8 \\ 9 \end{pmatrix}$ equation of the line l_3: $\mathbf{r} = \begin{pmatrix} 2 \\ -7 \\ -9 \end{pmatrix} + \alpha\begin{pmatrix} -6 \\ 8 \\ 9 \end{pmatrix}, \ \alpha \in \mathbb{R}$	Alternatively, Let A' be the point of reflection of A along the line l_1 Using Ratio Theorem, $\overrightarrow{CF} = \dfrac{\overrightarrow{CA}+\overrightarrow{CA'}}{2}$ $\overrightarrow{CA'} = 2\overrightarrow{CF} - \overrightarrow{CA}$ $= 2\left[\overrightarrow{OF} - \overrightarrow{OC}\right] - \left[\overrightarrow{OA} - \overrightarrow{OC}\right]$ $= 2\overrightarrow{OF} - \overrightarrow{OC} - \overrightarrow{OA}$ $= 2\begin{pmatrix} -5 \\ 7 \\ 12 \end{pmatrix} - \begin{pmatrix} 2 \\ -7 \\ -9 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 15 \end{pmatrix}$ $= \begin{pmatrix} -12 \\ 16 \\ 18 \end{pmatrix} = 2\begin{pmatrix} -6 \\ 8 \\ 9 \end{pmatrix}$ Let l_3 be the line of reflection of l_2 in the line l_1 Direction vector of $l_3 = \begin{pmatrix} -6 \\ 8 \\ 9 \end{pmatrix}$ equation of the line l_3: $\mathbf{r} = \begin{pmatrix} 2 \\ -7 \\ -9 \end{pmatrix} + \alpha\begin{pmatrix} -6 \\ 8 \\ 9 \end{pmatrix}, \ \alpha \in \mathbb{R}$

(v)	The normal of plane = $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -6 \\ -12 \end{pmatrix} = \begin{pmatrix} -6 \\ -9 \\ 4 \end{pmatrix}$ Using $\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 15 \end{pmatrix}$ as a point on the plane $\mathbf{r} \bullet \begin{pmatrix} -6 \\ -9 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 15 \end{pmatrix} \bullet \begin{pmatrix} -6 \\ -9 \\ 4 \end{pmatrix}$ Cartesian of the required plane is $-6x - 9y + 4z = 15$

10(i)	By cosine rule, $z^2 = x^2 + y^2 - 2xy\cos\alpha$ ---------- (1) By considering the area of the triangle and the quadrilateral, $2\left(\frac{1}{2}xy\sin\alpha\right) = \frac{1}{2}(1.5)(1.8)\sin\alpha$ $xy = 1.35$ ---------- (2) $y = \frac{1.35}{x}$ --------- (3) Substitute (2) and (3) in (1): $z^2 = x^2 + \left(\frac{1.35}{x}\right)^2 - 2(1.35)\cos\alpha$ $z^2 = x^2 + \frac{1.8225}{x^2} - 2.7\cos\alpha$ (Shown)	
(ii)	Differentiating with respect to x, $2z\frac{dz}{dx} = 2x - \frac{3.645}{x^3}$ For the stationary values of z, $\frac{dz}{dx} = 0$ $2x - \frac{3.645}{x^3} = 0$ $2x^4 - 3.645 = 0$ $x^4 = 1.8225$ $x = \sqrt[4]{1.8225}$ $= 1.16189 = 1.162$ m (3 dp) or $x = -\sqrt[4]{1.8225}$ (N.A, $x > 0$)	**<u>Alternative solution</u>** $z = \left(x^2 + \frac{1.8225}{x^2} - 2.7\cos\alpha\right)^{\frac{1}{2}}$ $\frac{dz}{dx} = \frac{1}{2}\left(x^2 + \frac{1.8225}{x^2} - 2.7\cos\alpha\right)^{-\frac{1}{2}}\left(2x - \frac{2(1.8225)}{x^3}\right)$ $= \frac{1}{2}\frac{1}{\sqrt{x^2 + \frac{1.8225}{x^2} - 2.7\cos\alpha}}\left(2x - \frac{2(1.8225)}{x^3}\right)$ For stationary values of z, $\frac{dz}{dx} = 0$ $2x - \frac{3.645}{x^3} = 0$ $2x^4 - 3.645 = 0$ $x^4 = 1.8225$ $x = \sqrt[4]{1.8225} = 1.16189 = 1.162$ m (3 dp) or $x = -\sqrt[4]{1.8225}$ (N.A, $x > 0$)
	<u>Method</u> (Second Derivative Test) Differentiating with respect to x: $2\left(\frac{dz}{dx}\right)^2 + 2z\frac{d^2z}{dx^2} = 2 + \frac{10.935}{x^4}$ For the value of z to be minimum, $\frac{dz}{dx} = 0$ $2z\frac{d^2z}{dx^2} = 2 + \frac{10.935}{x^4}$	

	$\frac{\mathrm{d}^2z}{\mathrm{d}x^2}=\frac{1}{2z}\left(2+\frac{10.935}{x^4}\right)$ For $x=\sqrt[4]{1.8225}$, $\frac{\mathrm{d}^2z}{\mathrm{d}x^2}=\frac{1}{2z}\left(2+\frac{10.935}{1.8225}\right)$>0 since $z>0$ given that z is a length. Hence the length of MN is a minimum when $x=1.162$ m (3 decimal places)
(iii)