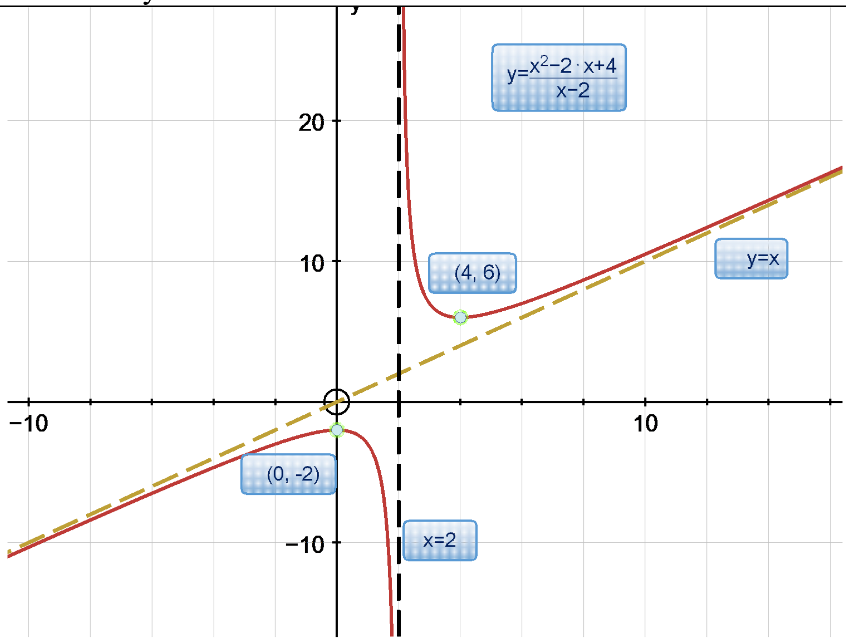
St Andrew's Junior College

2021 H2 Math Prelim Exam Paper 1 Solutions

<table>
<tr><th>Qn</th><th>Suggested Solutions</th></tr>
<tr><td>1(i)</td><td>$y=\dfrac{ax^2-2ax+4}{x-b}$
Equation of vertical asymptote: $x-b=0 \Rightarrow x=b$
Thus $b=2$

$y=\dfrac{ax^2-2ax+4}{x-2}=ax+\dfrac{4}{x-2}$
Equation of horizontal asymptote: $y=ax$
Thus $a=1$</td></tr>
<tr><td>1(ii)</td><td>$y=\dfrac{x^2-2x+4}{x-2}$
$y(x-2)=x^2-2x+4$
$x^2+(-2-y)x+(4+2y)=0---(*)$
For y not able to take any values, the equation (*) will have no real roots, therefore discriminant < 0

$b^2-4ac<0$
$(-2-y)^2-4(1)(4+2y)<0$
$4+4y+y^2-16-8y<0$
$y^2-4y-12<0$
$(y-6)(y+2)<0$
$-2<y<6$
Therefore y cannot take values between -2 and 6.</td></tr>
<tr><td>1(iii)</td><td>
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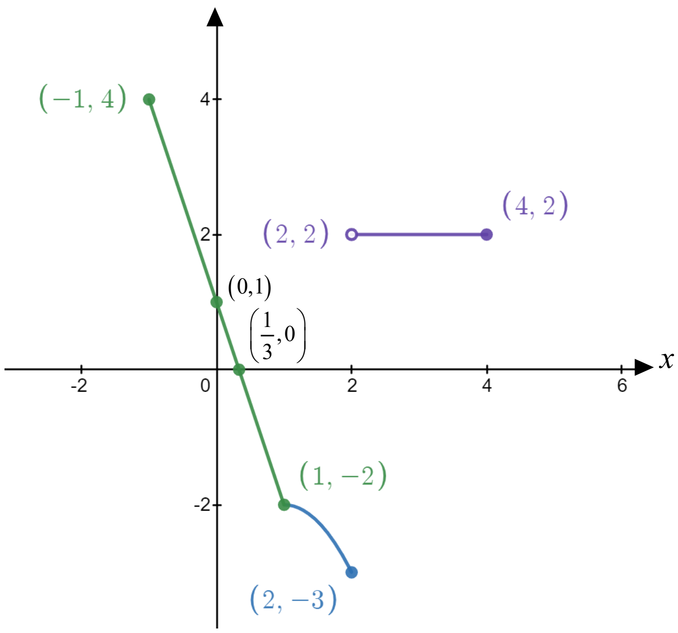
Qn	Suggested Solutions
2(a)	 Range of f = [–3, 4]
(b)	C': Reflect about the y-axis B': Scale parallel to x-axis by scale factor of $\frac{1}{5}$ A': Translate 1 unit in the negative x-direction $y=e^{x-2}-x$ $\downarrow C'$: Replace x by $-x$ $y=e^{-x-2}+x$ $y=x+e^{-x-2}$ $\downarrow B'$: Replace x by $5x$ $y=5x+e^{-5x-2}$ $\downarrow A'$: Replace x by $x+1$ $y=5(x+1)+e^{-5(x+1)-2}$ $y=5x+5+e^{-5x-7}$
3(a)(i)	$u_n=S_n-S_{n-1}$ $=A(2^n)+Bn^2+C-\left[A(2^{n-1})+B(n-1)^2+C\right]$ $=A\left(2^n-2^{n-1}\right)+B\left(n^2-(n-1)^2\right)$ $=A\left(2^{n-1}\right)(2-1)+B\left(2n-1\right)$ $=A\left(2^{n-1}\right)+B\left(2n-1\right)$

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<tr><td>(a) (ii)</td><td>

<u>Method 1 (Form 3 equations to solve)</u>
$u_1 = A\left(2^{1-1}\right) + B\left(2(1)-1\right) = 7$
$u_2 = A\left(2^{2-1}\right) + B\left(2(2)-1\right) = 18$
$S_2 = A(2^2) + B(2)^2 + C = 25$ (or $S_1 = A\left(2^1\right) + B\left(1\right)^2 + C = 7$)

By GC, $A = 3$, $B = 4$, $C = -3$

Method 2 (Use 2 equations to find A and B first)
$u_1 = A\left(2^{1-1}\right) + B\left(2(1)-1\right) = 7$
$u_2 = A\left(2^{2-1}\right) + B\left(2(2)-1\right) = 18$
By GC, $A = 3, B = 4$

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$S_n = \sum_{r=1}^{n}\left[A\left(2^{r-1}\right) + B\left(2r-1\right)\right]$
$= A\sum_{r=1}^{n} 2^{r-1} + 2B\sum_{r=1}^{n} r - B\sum_{r=1}^{n} 1$
$= A\frac{1(2^n-1)}{2-1} + 2B\frac{n(n+1)}{2} - Bn$
$= A\left(2^n\right) - A + Bn^2$
$\therefore C = -A$
Therefore, $C = -3$

</td><td>

$S_2 = 3(2^2) + 4(2)^2 + C = 25$
(or $S_1 = A\left(2^1\right) + B\left(1\right)^2 + C = 7$)

Therefore, $C = -3$

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<tr><td>(b)</td><td>

$r(r+1)(r+2) - (r-2)(r-1)r$
$= r[(r+1)(r+2) - (r-2)(r-1)]$
$= r[r^2 + 3r + 2 - (r^2 - 3r + 2)]$
$= r[6r]$
$= 6r^2$
$\therefore k = 6$

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$$\sum_{r=1}^{n} 6r^2 = \sum_{r=1}^{n}\left[r(r+1)(r+2)-(r-2)(r-1)r\right]$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}\sum_{r=1}^{n}\left[r(r+1)(r+2)-(r-2)(r-1)r\right]$$

$$\begin{aligned}
=\frac{1}{6}[&(1)(2)(3) &&-(-1)(0)(1)\\
+&[(2)(3)(4) &&-(0)(1)(2)\\
+&[(3)(4)(5) &&-(1)(2)(3)\\
+&[(4)(5)(6) &&-(2)(3)(4)\\
+&\\
&\vdots\\
+&(n-3)(n-2)(n-1) &&-(n-5)(n-4)(n-3)\\
+&(n-2)(n-1)(n) &&-(n-4)(n-3)(n-2)\\
+&(n-1)(n)(n+1) &&-(n-3)(n-2)(n-1)\\
+&(n)(n+1)(n+2) &&-(n-2)(n-1)(n)]
\end{aligned}$$

$$=\frac{1}{6}\left[-(-1)(0)(1)-(0)(1)(2)+(n-1)(n)(n+1)+(n)(n+1)(n+2)\right]$$

$$=\frac{(n-1)(n)(n+1)+(n)(n+1)(n+2)}{6}$$

$$=\frac{n(n+1)\left[(n-1)+(n+2)\right]}{6}$$

$$=\frac{n(n+1)(2n+1)}{6} \text{ (Shown)}$$

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<tr><td>4 (i)</td><td>

$y=1+\dfrac{a-2}{x-a}, \quad x\neq a$

Asymptotes: $x=a$, $y=1$

When $x=0$, $y=\dfrac{2}{a}$.

When $y=0$, $x=2$

$y=-\dfrac{1}{a}x+\dfrac{2}{a}$

When $x=0$, $y=\dfrac{2}{a}$

When $y=0$, $x=2$

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	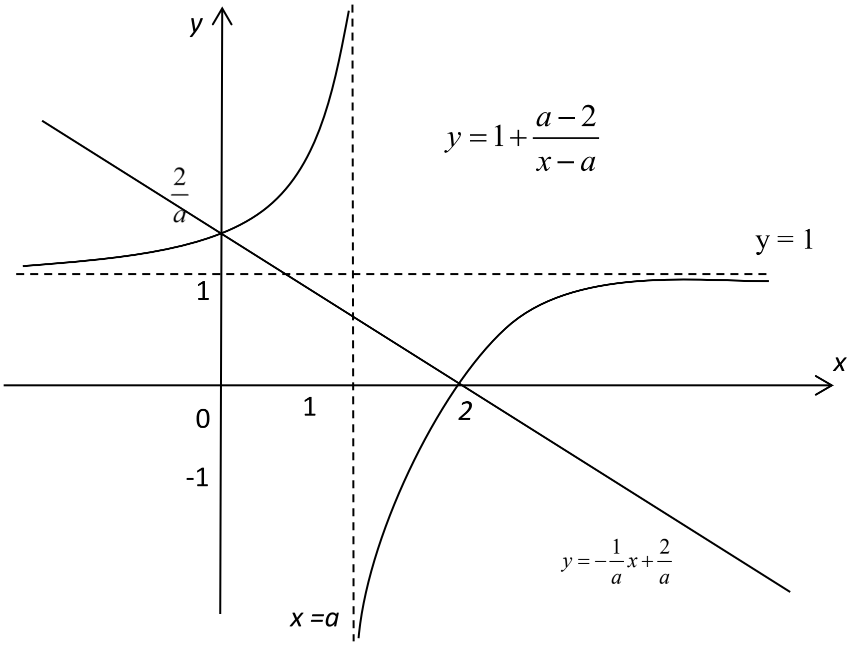
(i)	From the graph, $1 + \frac{a-2}{x-a} > -\frac{1}{a}x + \frac{2}{a}$. $\text{Ans}: \ 0 < x < a \text{ or } x > 2$
(ii)	$1 + \frac{ax-2x}{1-ax} > -\frac{1}{ax} + \frac{2}{a}$. $1 + \frac{x(a-2)}{x(\frac{1}{x} - a)} > -\frac{1}{a}\left(\frac{1}{x}\right) + \frac{2}{a}$ $1 + \frac{a-2}{\frac{1}{x} - a} > -\frac{1}{a}\left(\frac{1}{x}\right) + \frac{2}{a}$ Let $y = \frac{1}{x}$, $1 + \frac{a-2}{y-a} > -\frac{1}{a}y + \frac{2}{a}$. From (i), $0 < y < a \text{ or } y > 2$ $0 < \frac{1}{x} < a \text{ or } \frac{1}{x} > 2$ $\therefore x > \frac{1}{a} \text{ or } 0 < x < \frac{1}{2}$

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Qn	Suggested Solutions
5 (a)	$u_n = S_n - S_{n-1}$ $= (e^n - 1) - (e^{n-1} - 1)$ $= e^n - e^{n-1}$ $\dfrac{u_n}{u_{n-1}} = \dfrac{e^n - e^{n-1}}{e^{n-1} - e^{n-2}}$ $= \dfrac{e^{n-1}(e-1)}{e^{n-2}(e-1)}$ $= e$ that is a constant independent of n. Hence the sequence is a geometric progression.
(b)(i)	Let T_n be the decrease in area that has happened for the nth year. $T_n = 64 + (n-1)(-3)$ $= 67 - 3n$ The decrease in area of island S forms an AP with common difference – 3. Since the decrease in area stops when the decrease is less than 5 km^2, $67 - 3n < 5$ $n > 20\dfrac{2}{3}$ Least $n = 21$. Hence the total decrease in area from the first year to the 21st year is $S_{21} = \dfrac{21}{2}[2(64) + (21-1)(-3)]$ $= 714$ Hence the area of Island S when the decrease is less than 5 km^2 is $2880 - 714 = 2166\text{km}^2$
(b)	According to Company B, the decrease in area now follows a Geometric Progression with first term 64 and common ratio $\dfrac{5}{6}$. Hence the long run/theoretical decrease in area of Island B is $S_\infty = \dfrac{64}{1 - \frac{5}{6}}$ $= 384$ Hence the theoretical area in Island B in the long run is $\text{Area} = 2880 - 384$ $= 2496\text{km}^2$

Qn	Suggested Solutions
6 (i)	$\dfrac{4-x^2y^2}{x^2+y^2}=\dfrac{1}{2}$ $8-2x^2y^2=x^2+y^2$ Differentiating with respect to x, $-2\left(2xy^2+x^2\left(2y\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\right)=2x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $-2xy^2-2x^2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=x+y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $-\left(y+2x^2y\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=x+2xy^2$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{x+2xy^2}{y+2x^2y}\quad(\text{shown})$
(ii)	When $x=2$, $\dfrac{4-(2)^2y^2}{(2)^2+y^2}=\dfrac{1}{2}$ $8-8y^2=4+y^2$ $y^2=\dfrac{4}{9}$ $y=\dfrac{2}{3}\qquad(y\geq 0)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{2+2(2)\left(\frac{2}{3}\right)^2}{\left(\frac{2}{3}\right)+2(2)^2\left(\frac{2}{3}\right)}=-\dfrac{17}{27}$ **Equation of PQ:** $y-\dfrac{2}{3}=-\dfrac{17}{27}(x-2)$ when $y=0, x=\dfrac{52}{17}$ $Q\left(\dfrac{52}{17},0\right)$

Qn	Suggested Solutions
(iii)	when $y=0,\ \theta=\frac{\pi}{2}, x=k^2$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=k^2\cos\theta$ Bounded area = Area of triangle OPQ $\int_0^{k^2} y\ \mathrm{d}x = \frac{1}{2}\left(\frac{52}{17}\right)\left(\frac{2}{3}\right)$ $\int_0^{\frac{\pi}{2}} k\cos\theta\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta = \frac{1}{2}\left(\frac{52}{17}\right)\left(\frac{2}{3}\right)$ $\int_0^{\frac{\pi}{2}} k^3\cos^2\theta\ \mathrm{d}\theta = \frac{52}{51}$ $k^3 = \frac{52}{51\int_0^{\frac{\pi}{2}}\cos^2\theta\ \mathrm{d}\theta} = 1.298205$ $k=1.09$
7(a)	Let $z=1+\sqrt{3}i$ is a root of the equation $3z^3+az^2+bz-8=0$. **Since *a* and *b* are real numbers, the coefficients of all the terms of the equation are real, hence complex roots exist in conjugate pairs. This implies that since $z=1+\sqrt{3}i$ is a root then its conjugate $z^*=1-\sqrt{3}i$ is also a root of the equation.** We can form a quadratic factor $(z-(1+\sqrt{3}i)(z-(1-\sqrt{3}i)= z^2-2z+4$. $\therefore 3z^3+az^2+bz-8=(3z-2)(z^2-2z+4)$ Comparing the coefficient of $z^2: a=-6-2$ $\Rightarrow a=-8$ Comparing the coefficient of $z: b=12+4$ $\Rightarrow b=16$ The remaining root is $z=\frac{2}{3}$.
7(b) (i)	Since $\mathrm{e}^{\mathrm{i}\theta}=\cos\theta+\mathrm{i}\sin\theta$, L.H.S. $=(\cos\theta+\mathrm{i}\sin\theta)^4$ $=\left(\mathrm{e}^{\mathrm{i}\theta}\right)^4$ $=\mathrm{e}^{\mathrm{i}(4\theta)}$ $=\cos 4\theta+\mathrm{i}\sin 4\theta$ (Shown)

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(ii)	$\tan 4\theta = \dfrac{\sin 4\theta}{\cos 4\theta}$ $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$ $\cos^4\theta + 4\cos^3\theta(i\sin\theta) + 6\cos^2\theta(i\sin\theta)^2 +$ $4\cos\theta(i\sin\theta)^3 + (i\sin\theta)^4 = \cos 4\theta + i(\sin 4\theta)$ Comparing real parts, $\cos 4\theta = \cos^4\theta - 6\cos^2\theta(\sin^2\theta) + (\sin^4\theta)$ Comparing imaginary parts, $\sin 4\theta = 4\cos^3\theta(\sin\theta) - 4\cos\theta(\sin^3\theta)$ $\therefore \tan 4\theta = \dfrac{4\cos^3\theta(\sin\theta) - 4\cos\theta(\sin^3\theta)}{\cos^4\theta - 6\cos^2\theta(\sin^2\theta) + (\sin^4\theta)}$ $= \dfrac{\cos^4\theta(4\tan\theta - 4\tan^3\theta)}{\cos^4\theta(1 - 6\tan^2\theta + \tan^4\theta)}$ $= \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ (Shown)
(iii)	$\tan 4\theta = 4$ $\Rightarrow \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} = 4$ $\Rightarrow \tan\theta - \tan^3\theta = 1 - 6\tan^2\theta + \tan^4\theta$ $\Rightarrow \tan^4\theta + \tan^3\theta - 6\tan^2\theta - \tan\theta + 1 = 0 - - - (*)$ Using GC The possible values of $\tan\theta$ are $-2.91,\ 2.05,\ -0.488$ and 0.344
8 (i)	Since the point P lines on AB between A and B such that AP: PB = $(1-\lambda):\lambda$, by ratio theorem, $\overrightarrow{OP} = \lambda\overrightarrow{OA} + (1-\lambda)\overrightarrow{OB}$, $0 < \lambda < 1, \lambda \in \mathbb{R}$

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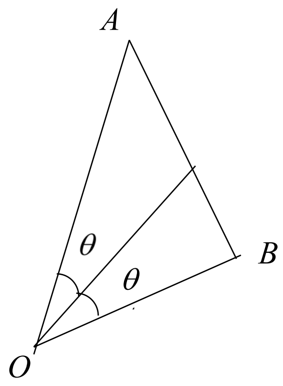

Let the $\angle AOP = \angle BOP = \theta$.
$\cos\theta = \dfrac{\underset{\sim}{p}\bullet\underset{\sim}{a}}{ap} = \dfrac{\underset{\sim}{p}\bullet\underset{\sim}{b}}{bp} --- (*)$
$\Rightarrow \dfrac{\left[\lambda\underset{\sim}{a}+(1-\lambda)\underset{\sim}{b}\right]\bullet\underset{\sim}{a}}{a} = \dfrac{\left[\lambda\underset{\sim}{a}+(1-\lambda)\underset{\sim}{b}\right]\bullet\underset{\sim}{b}}{b}$
$\Rightarrow \dfrac{\lambda a^2+(1-\lambda)\left(\underset{\sim}{a}\bullet\underset{\sim}{b}\right)}{a} = \dfrac{\lambda\underset{\sim}{a}\bullet\underset{\sim}{b}+(1-\lambda)b^2}{b}$
$\Rightarrow \left(\underset{\sim}{a}\bullet\underset{\sim}{b}\right)\left[\dfrac{(1-\lambda)}{a}-\dfrac{\lambda}{b}\right] = (1-\lambda)b-\lambda a --- \ (\#)$
Method 1a
$\Rightarrow \left(\underset{\sim}{a}\bullet\underset{\sim}{b}\right)[b(1-\lambda)-\lambda a] = ab[(1-\lambda)b-\lambda a]$
$\Rightarrow \left(\underset{\sim}{a}\bullet\underset{\sim}{b}\right)[\lambda(a+b)-b] = ab[\lambda(a+b)-b]$
$\Rightarrow [\lambda(a+b)-b]\left(\underset{\sim}{a}\bullet\underset{\sim}{b}-ab\right) = 0$
Since O, A and B are not collinear,
$\underset{\sim}{a}\bullet\underset{\sim}{b} \neq ab$ --- (@)
$\therefore \lambda(a+b)-b=0$
$\Rightarrow \lambda = \dfrac{b}{a+b}$ (Shown)
Method 1b
$\Rightarrow \left(ab\cos 2\theta\right)\left[\dfrac{(1-\lambda)}{a}-\dfrac{\lambda}{b}\right] = (1-\lambda)b-\lambda a$
Since (#) is true for all θ, we compare terms independent of θ
$\Rightarrow \left(1-\lambda\right)b-\lambda a = 0$
$\Rightarrow \lambda = \dfrac{b}{a+b}$ (Shown)
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<tr><td>(iii)</td><td>Since AP = BQ</td></tr>
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	$\Rightarrow AQ:QB=\lambda:(1-\lambda)$ $\Rightarrow \overrightarrow{OQ}=(1-\lambda)\underset{\sim}{a}+\lambda\underset{\sim}{b}$ $OQ^2=\left((1-\lambda)\underset{\sim}{a}+\lambda\underset{\sim}{b}\right)\bullet\left((1-\lambda)\underset{\sim}{a}+\lambda\underset{\sim}{b}\right)$ $=(1-\lambda)^2a^2+\lambda^2b^2+2\lambda(1-\lambda)\left(\underset{\sim}{a}\bullet\underset{\sim}{b}\right)$ $OP^2=\left(\lambda\underset{\sim}{a}+(1-\lambda)\underset{\sim}{b}\right)\bullet\left(\lambda\underset{\sim}{a}+(1-\lambda)\underset{\sim}{b}\right)$ $=\lambda^2a^2+(1-\lambda)^2b^2+2\lambda(1-\lambda)\left(\underset{\sim}{a}\bullet\underset{\sim}{b}\right)$ $\therefore OQ^2-OP^2=\left(2\lambda-1\right)(b^2-a^2)$ $=\left(2\left(\dfrac{b}{a+b}\right)-1\right)(b^2-a^2)$ $=\left(\dfrac{b-a}{a+b}\right)(b-a)(b+a)$ $=(b-a)^2$
9(i)	$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}=9.8-0.2\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2$ $v=\dfrac{\mathrm{d}x}{\mathrm{d}t}\Rightarrow\dfrac{\mathrm{d}v}{\mathrm{d}t}=\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$ Hence, $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}=9.8-0.2\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2\Rightarrow\dfrac{\mathrm{d}v}{\mathrm{d}t}=9.8-0.2v^2$ (shown) To solve $\dfrac{\mathrm{d}v}{\mathrm{d}t}=9.8-0.2v^2$: $\dfrac{\mathrm{d}v}{\mathrm{d}t}=9.8-0.2v^2$ $\displaystyle\int\frac{1}{9.8-0.2v^2}\,\mathrm{d}v=\int 1\,\mathrm{d}t$ $\dfrac{1}{0.2}\displaystyle\int\frac{1}{49-v^2}\,\mathrm{d}v=t+C$, where C is an arbitrary constant $\dfrac{1}{0.2}\left(\dfrac{1}{2(7)}\right)\ln\left\|\dfrac{7+v}{7-v}\right\|=t+C$ $\dfrac{5}{14}\ln\left\|\dfrac{7+v}{7-v}\right\|=t+C$ $\ln\left\|\dfrac{7+v}{7-v}\right\|=\dfrac{14}{5}t+D,$ where $D=\dfrac{14}{5}C$

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	$\dfrac{7+v}{7-v}=A\mathrm{e}^{\frac{14}{5}t}$, where $A=\pm\mathrm{e}^{D}$ When $t=0, v=0$ $\dfrac{7+0}{7-0}=A\mathrm{e}^{0}\Rightarrow A=1$ $\therefore \dfrac{7+v}{7-v}=\mathrm{e}^{\frac{14}{5}t}\Rightarrow \dfrac{7-v}{7+v}=\mathrm{e}^{-\frac{14}{5}t}$ $7-v=7\mathrm{e}^{-\frac{14}{5}t}+\mathrm{e}^{-\frac{14}{5}t}v$ $7\left(1-\mathrm{e}^{-\frac{14}{5}t}\right)=v\left(1+\mathrm{e}^{-\frac{14}{5}t}\right)$ $\therefore v=\dfrac{7\left(1-\mathrm{e}^{-\frac{14}{5}t}\right)}{1+\mathrm{e}^{-\frac{14}{5}t}}$ (shown)
(ii)	$\dfrac{\mathrm{d}v}{\mathrm{d}t}=9.8-kv,\quad k>0$ $\int \dfrac{1}{9.8-kv}\,\mathrm{d}v=\int 1\,\mathrm{d}t$ $-\dfrac{1}{k}\int \dfrac{-k}{9.8-kv}\,\mathrm{d}v=t+C$, where C is an arbitrary constant $-\dfrac{1}{k}\ln\lvert 9.8-kv\rvert=t+C$ $\ln\lvert 9.8-kv\rvert=-kt+D$, where $D=-Ck$ $9.8-kv=Ae^{-kt}$, where $A=\pm\mathrm{e}^{D}$ When $t=0, v=0$ $9.8-k(0)=Ae^{0}\Rightarrow A=9.8$ $\therefore 9.8-kv=9.8e^{-kt}$ $v=\dfrac{9.8(1-e^{-kt})}{k}$

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(iii)	$v=\dfrac{7\left(1-\mathrm{e}^{-\frac{14}{5}t}\right)}{1+\mathrm{e}^{-\frac{14}{5}t}}$ As $t\to\infty$, $\mathrm{e}^{-\frac{14}{5}t}\to 0$ $\therefore \dfrac{1-\mathrm{e}^{-\frac{14}{5}t}}{1+\mathrm{e}^{-\frac{14}{5}t}}\to 1$ $\lim\limits_{t\to\infty}\dfrac{7\left(1-\mathrm{e}^{-\frac{14}{5}t}\right)}{1+\mathrm{e}^{-\frac{14}{5}t}}=7$ $\therefore$ Terminal velocity for object in Model 1 is 7 $\lim\limits_{t\to\infty}\dfrac{9.8(1-e^{-kt})}{k}=7$ As $t\to\infty$, $e^{-kt}\to 0$ $\therefore \dfrac{9.8}{k}=7\Rightarrow k=1.4$
(iv)	For model 1: When $v=7\times 0.8=5.6$ $5.6=\dfrac{7\left(1-\mathrm{e}^{-\frac{14}{5}t}\right)}{1+\mathrm{e}^{-\frac{14}{5}t}}$ Using GC, t = 0.785 (3 s.f.) For model 2: When $v=7\times 0.8=5.6$ $5.6=\dfrac{9.8(1-e^{-(1.4)t})}{1.4}$ Using GC, $t=1.15$ s Object in Model 1 reaches 80% of its terminal velocity earlier.

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11(i)	When the projectile to hits x_T, $y=0$ $(v\sin\theta)t-\frac{1}{2}gt^2=0$ $t\left(v\sin\theta-\frac{1}{2}gt\right)=0$ $t=0$ (rejected as $t>0$) or $v\sin\theta-\frac{1}{2}gt=0$ $t=\frac{2v\sin\theta}{g}$ (shown)
(ii)	$A=\int_0^{x_T} y\,\mathrm{d}x$ $=\int_0^{\frac{2v\sin\theta}{g}} y\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\mathrm{d}t$ $=\int_0^{\frac{2v\sin\theta}{g}}\left((v\sin\theta)t-\frac{1}{2}gt^2\right)(v\cos\theta)\,\mathrm{d}t$ $=(v\cos\theta)\int_0^{\frac{2v\sin\theta}{g}}\left((v\sin\theta)t-\frac{1}{2}gt^2\right)\mathrm{d}t$ $=(v\cos\theta)\left[(v\sin\theta)\frac{t^2}{2}-\frac{1}{2}g\left(\frac{t^3}{3}\right)\right]_0^{\frac{2v\sin\theta}{g}}$ $=(v\cos\theta)\left(\frac{(v\sin\theta)}{2}\left(\frac{2v\sin\theta}{g}\right)^2-\frac{1}{6}g\left(\frac{2v\sin\theta}{g}\right)^3\right)$ $=(v\cos\theta)\left(\frac{2(v\sin\theta)^3}{g^2}-\frac{1}{6g^2}(8)(v\sin\theta)^3\right)$ $=(v\cos\theta)\left(\frac{2}{3}\frac{(v\sin\theta)^3}{g^2}\right)$ $=\frac{2v^4\sin^3\theta\cos\theta}{3g^2}$
(iii)	$A=\frac{2v^4\sin^3\theta\cos\theta}{3g^2}$ $\frac{\mathrm{d}A}{\mathrm{d}\theta}=\frac{2v^4}{3g^2}\left(\sin^3\theta(-\sin\theta)+\cos\theta(3\sin^2\theta\cos\theta)\right)$ $=\frac{2v^4}{3g^2}\left(3\sin^2\theta\cos^2\theta-\sin^4\theta\right)$ For stationary values,

Qn	Suggested Solutions
	$\frac{\mathrm{d}A}{\mathrm{d}\theta}=0$ $\frac{2v^4}{3g^2}\left(3\sin^2\theta\cos^2\theta-\sin^4\theta\right)=0$ $\sin^2\theta\left(3\cos^2\theta-\sin^2\theta\right)=0$ $\sin^2\theta=0$ or $3\cos^2\theta-\sin^2\theta=0$ $\sin\theta=0$ or $\tan^2\theta=3$ $\theta=0$ or $\tan\theta=\pm\sqrt{3}$ (rejected θ>0) $\tan\theta=\sqrt{3}$ or $\tan\theta=-\sqrt{3}$ $\theta=\frac{\pi}{3}$ (rejected as θ is acute, so $\tan\theta$>0) Hence when $\theta=\frac{\pi}{3}$, $A=\frac{2v^4\sin^3\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)}{3g^2}=\frac{2v^4\left(\left(\frac{\sqrt{3}}{2}\right)^3\right)\left(\frac{1}{2}\right)}{3g^2}=\frac{\sqrt{3}}{8}\frac{v^4}{g^2}$ $\frac{\mathrm{d}A}{\mathrm{d}\theta}=\frac{2v^4}{3g^2}\left(3\sin^2\theta\cos^2\theta-\sin^4\theta\right)$ $=\frac{2v^4}{3g^2}\left(\frac{3}{4}(4\sin^2\theta\cos^2\theta)-\sin^4\theta\right)$ $=\frac{2v^4}{3g^2}\left(\frac{3}{4}\left(\sin 2\theta\right)^2-\sin^4\theta\right)$ $\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2}=\frac{2v^4}{3g^2}\left(\frac{3}{4}\left(4\sin 2\theta\cos 2\theta\right)-4\sin^3\theta\cos\theta\right)$ $=\frac{2v^4}{3g^2}\left(\frac{3}{2}\left(\sin 4\theta\right)-4\sin^3\theta\cos\theta\right)$ When $\theta=\frac{\pi}{3}$

Qn	Suggested Solutions
	$\frac{\mathrm{d}^2 A}{\mathrm{d}\theta^2}=\frac{2v^4}{3g^2}\left(\frac{3}{2}(\sin 4\theta)-4\sin^3\theta\cos\theta\right)$ $=\frac{2v^4}{3g^2}\left(\frac{3}{2}\left(-\frac{\sqrt{3}}{2}\right)-4\left(\frac{\sqrt{3}}{2}\right)^3\frac{1}{2}\right)$ $=\frac{2v^4}{3g^2}\left(-\frac{3\sqrt{3}}{4}-\frac{3\sqrt{3}}{4}\right)$ $=-\frac{2v^4}{3g^2}\left(\frac{3\sqrt{3}}{2}\right)$ $=-\frac{\sqrt{3}v^4}{g^2}$ Hence maximum $A=\frac{\sqrt{3}v^4}{8g^2}$