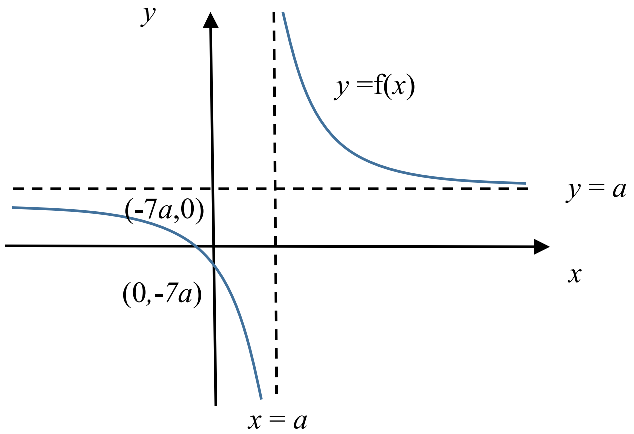
2021 H2 Math Paper 2 Preliminary Exam Solutions

<table>
<tr><th>Qn</th><th>Solutions</th></tr>
<tr><td>1(i)</td><td>

$$y=\frac{ax+7a^2}{x-a}=a+\frac{8a^2}{x-a}$$



Since **<u>every/all</u>** horizontal line $y=k, k\in\mathbb{R}\setminus\{a\}$, intersects the graph of $y=\mathrm{f}(x)$ **<u>exactly once</u>**, the function f is **<u>one-one</u>**. Hence f^{-1} exists.

Let $y=\frac{ax+7a^2}{x-a}$

$y(x-a)=ax+7a^2$

$x(y-a)=ay+7a^2$

$x=\frac{ay+7a^2}{y-a}$

Since $\mathrm{f}^{-1}(x)=\frac{ax+7a^2}{x-a},\ x\in\mathbb{R}, x\neq a,$

$D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}=(-\infty,a)\cup(a,\infty)=D_{\mathrm{f}}$

$\therefore \mathrm{f}^{-1}=\mathrm{f}$ and f is self-inverse (shown)

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<tr><td>(ii)</td><td>

Note that $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$..

$$\mathrm{f}^{241}\left(\frac{1}{a}\right)=\underbrace{\mathrm{fff...f}}_{241\text{ times}}\left(\frac{1}{a}\right)=\mathrm{f}\left(\underbrace{\mathrm{fff...f}}_{240\text{ times}}\left(\frac{1}{a}\right)\right)=\mathrm{f}\left(\underbrace{\mathrm{ff}^{-1}\mathrm{ff}^{-1}...\mathrm{ff}^{-1}}_{120\text{ times of ff}^{-1}}\left(\frac{1}{a}\right)\right)=\mathrm{f}\left(\frac{1}{a}\right)$$

$$\mathrm{f}\left(\frac{1}{a}\right)=\frac{a\left(\frac{1}{a}\right)+7a^2}{\frac{1}{a}-a}=\frac{a+7a^3}{1-a^2}$$

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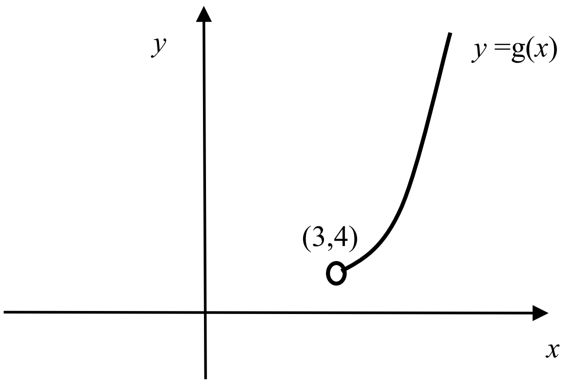
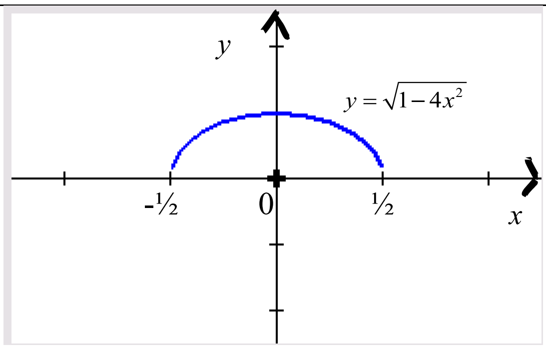
	Alternatively: $f(x) = f^{-1}(x) \Rightarrow f^2(x) = x$ $f^3(x) = ff^2(x) = f(x)$ $f^4(x) = ff^3(x) = ff(x) = x$ $\vdots$ $f^n(x) = f(x)$ when n is an odd integer $f^n(x) = x$ when n is an even integer $\therefore f^{241}\left(\frac{1}{a}\right) = f\left(\frac{1}{a}\right)$ $= \dfrac{a\left(\frac{1}{a}\right)+7a^2}{\frac{1}{a}-a} = \dfrac{a+7a^3}{1-a^2}$
(iii)	 Since $R_g = (4,\infty)$ and $D_f = (-\infty,\infty)\setminus\{a\}$, where $0 < a < 1$. $R_g \subseteq D_f$. $\therefore$ fg exists. $D_g = (3,\infty) \rightarrow (4,\infty) \rightarrow \left(a, \dfrac{4a+7a^2}{4-a}\right)$ $R_{fg} = \left(a, \dfrac{4a+7a^2}{4-a}\right)$

2(a)	 x-intercepts at $x=\frac{1}{2}$ and $x=-\frac{1}{2}$ $x=-\frac{1}{2}$, $\therefore \frac{1}{2}\sin u=-\frac{1}{2} \Rightarrow u=-\frac{\pi}{2}$ $x=\frac{1}{2}$, $\therefore \frac{1}{2}\sin u=\frac{1}{2} \Rightarrow u=\frac{\pi}{2}$ $x=\frac{1}{2}\sin u$ $\frac{\mathrm{d}x}{\mathrm{d}u}=-\frac{1}{2}\cos u$ Area $=\int_{-\frac{1}{2}}^{\frac{1}{2}}\sqrt{1-4x^2}\,\mathrm{d}x$ $=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{1-4\left(\frac{1}{2}\sin u\right)^2}\times\left(\frac{1}{2}\cos u\right)\ \mathrm{d}u$ $=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos u\times\cos u\ \mathrm{d}u$

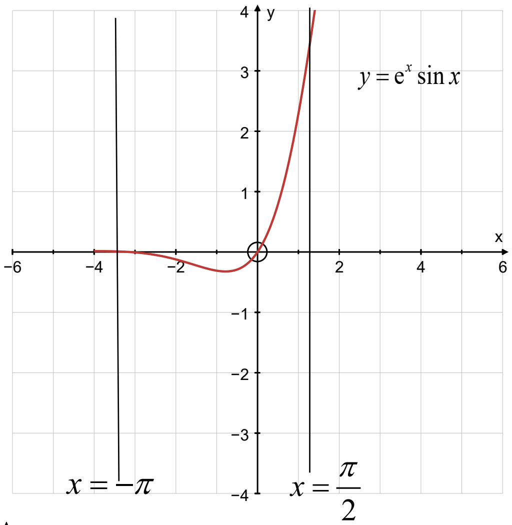
$$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos u)^2\ \mathrm{d}u$$

$$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+\cos 2u}{2}\ \mathrm{d}u$$

$$=\frac{1}{2}\left[\frac{u}{2}+\frac{\sin 2u}{4}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{1}{2}\left[\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right]=\frac{\pi}{4}\text{ units}^2$$

(bi)



Area

$$=\int_0^{\frac{\pi}{2}}e^x\sin x\ \mathrm{d}x+\int_{-\pi}^{0}\left(-e^x\sin x\right)\ \mathrm{d}x$$

Note:

$$\int \mathrm{e}^x\sin x\ \mathrm{d}x=\mathrm{e}^x\sin x-\int \mathrm{e}^x(\cos x)\ \mathrm{d}x$$

$$=\mathrm{e}^x\sin x-\left[\mathrm{e}^x\cos x-\int \mathrm{e}^x(-\sin x)\ \mathrm{d}x\right]$$

$$2\int \mathrm{e}^x\sin x\ \mathrm{d}x=\mathrm{e}^x\sin x-\mathrm{e}^x\cos x$$

$$\int \mathrm{e}^x\sin x\ \mathrm{d}x=\frac{1}{2}\left(\mathrm{e}^x\sin x-\mathrm{e}^x\cos x\right)+c$$

where c is an arbitrary constant.

Workings for integration by parts

Let $u_1=\sin x$ $\quad \frac{\mathrm{d}v_1}{\mathrm{d}x}=\mathrm{e}^x$

$\frac{\mathrm{d}u_1}{\mathrm{d}x}=\cos x$ $\quad v_1=\mathrm{e}^x$

	Let $u = \cos x \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x$ $\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x \qquad v = \mathrm{e}^x$ Thus area of region $= \left[\frac{1}{2}\left(\mathrm{e}^x \sin x - \mathrm{e}^x \cos x\right)\right]_0^{\frac{\pi}{2}} + \left[-\frac{1}{2}\left(\mathrm{e}^x \sin x - \mathrm{e}^x \cos x\right)\right]_{-\pi}^0$ $= \frac{1}{2}\left[\left(\mathrm{e}^{\frac{\pi}{2}} \sin\frac{\pi}{2}\right) - \left(-\mathrm{e}^0 \cos 0\right)\right]$ $+\left(-\frac{1}{2}\right)\left[\left(-e^0 \cos 0\right) - \left(-e^{-\pi} \cos(-\pi)\right)\right]$ $= \frac{1}{2}\left[e^{\frac{\pi}{2}} + 1\right] - \frac{1}{2}\left(-1 - e^{-\pi}\right)$ $= \left(\frac{1}{2}e^{\frac{\pi}{2}} + \frac{1}{2}e^{-\pi} + 1\right)$ units^2
(bii)	Volume $= \pi\int_{-\pi}^{\frac{\pi}{2}} \left(e^x \sin x\right)^2 \, \mathrm{d}x$ $= 8.6775\pi$ ≈ 27.3 units^3
3(i)	Let the acute angle between L and the x-axis be θ . $\cos\theta = \left(\left\lvert\frac{\begin{pmatrix}6\\3\\-2\end{pmatrix}.\begin{pmatrix}1\\0\\0\end{pmatrix}}{\sqrt{49}\sqrt{1}}\right\rvert\right)$ $\Rightarrow \theta = \cos^{-1}\left(\frac{6}{7}\right)$ $= 31.0^0$.
(ii)	Equation of the line L is $\mathbf{r} = \begin{pmatrix}-1\\2\\-4\end{pmatrix} + \lambda\begin{pmatrix}6\\3\\-2\end{pmatrix}$, $\lambda \in \mathbb{R}$. Let B be a point on the line L.

Then $\overrightarrow{OB} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix}$ for some $\lambda \in \mathbb{R}$

$$\overrightarrow{PB} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -7 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -7+6\lambda \\ -1+3\lambda \\ 1-2\lambda \end{pmatrix}$$

$$\left|\overrightarrow{PB}\right| = \sqrt{(-7+6\lambda)^2 + (-1+3\lambda)^2 + (1-2\lambda)^2}$$

$$= \sqrt{49\lambda^2 - 94\lambda + 51} \ldots\ldots.(1)$$

Given $\left|\overrightarrow{PB}\right| = \sqrt{59}$

$$\sqrt{49\lambda^2 - 94\lambda + 51} = \sqrt{59}$$

Square both sides and solve for λ,

$$49\lambda^2 - 94\lambda + 51 - 59 = 0$$

$$49\lambda^2 - 94\lambda - 8 = 0$$

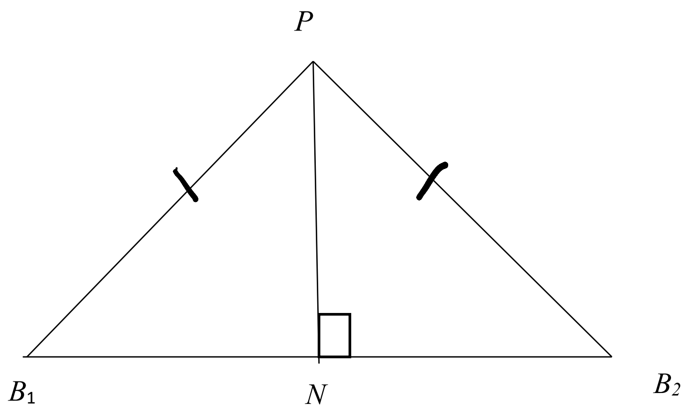
$$(49\lambda + 4)(\lambda - 2) = 0$$

$$\lambda = -\frac{4}{49} \text{ or } 2$$

$$\overrightarrow{OB_1} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} - \frac{4}{49}\begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} = \frac{1}{49}\begin{pmatrix} -73 \\ 86 \\ -188 \end{pmatrix} \quad \text{or} \quad \overrightarrow{OB_2} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} + 2\begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 11 \\ 8 \\ -8 \end{pmatrix}$$

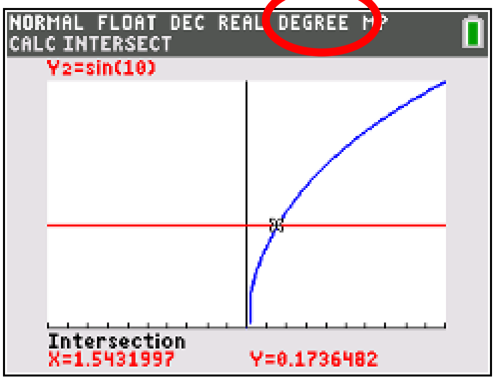
The points are $(11, 8, -8)$ or $\left(-\frac{73}{49}, \frac{86}{49}, -\frac{188}{49}\right)$.

Let the point on L closest to P be N.
Since $PB_1 = PB_2 \Rightarrow PB_1B_2$ is an isosceles triangle
Hence the perpendicular PN will bisect the base B_1B_2.
$\therefore$ N is the midpoint of B_1 and B_2.



	$\overrightarrow{ON}=\frac{1}{2}\left[\begin{pmatrix}11\\8\\-8\end{pmatrix}+\frac{1}{49}\begin{pmatrix}-73\\86\\-188\end{pmatrix}\right]=\frac{1}{98}\begin{pmatrix}466\\478\\-580\end{pmatrix}=\frac{1}{49}\begin{pmatrix}233\\239\\-290\end{pmatrix}$ Hence, the point closest to P is $\left(\frac{233}{49},\frac{239}{49},-\frac{290}{49}\right)$. **<u>Alternative Method 1(Hence)</u>** $\left\|\overrightarrow{PB}\right\|^2=49\lambda^2-94\lambda+51$ Differentiate with respect to λ, $\frac{d\left\|\overrightarrow{PB}\right\|^2}{d\lambda}=98\lambda-94$ Minimum distance means $\frac{d\left\|\overrightarrow{PB}\right\|^2}{d\lambda}=0$ $98\lambda-94=0$ $\lambda=\frac{47}{49}$ $\overrightarrow{ON}=\begin{pmatrix}-1\\2\\-4\end{pmatrix}+\frac{47}{49}\begin{pmatrix}6\\3\\-2\end{pmatrix}=\frac{1}{49}\begin{pmatrix}233\\239\\-290\end{pmatrix}$ Hence, the point closest to P is $\left(\frac{233}{49},\frac{239}{49},-\frac{290}{49}\right)$. **<u>Alternative Method 2 (Hence)</u>** $\left\|\overrightarrow{PB}\right\|^2=49\lambda^2-94\lambda+51=49(\lambda-\frac{47}{49})^2+\frac{290}{49}$ Hence minimum distance is obtained when $\lambda=\frac{47}{49}$. $\overrightarrow{ON}=\begin{pmatrix}-1\\2\\-4\end{pmatrix}+\frac{47}{49}\begin{pmatrix}6\\3\\-2\end{pmatrix}=\frac{1}{49}\begin{pmatrix}233\\239\\-290\end{pmatrix}$ Hence, the point closest to P is $\left(\frac{233}{49},\frac{239}{49},-\frac{290}{49}\right)$. **<u>Alternative Method 3 (Otherwise)</u>** Let N be the foot of perpendicular from the point P to the line L.

	Then $\overrightarrow{ON} = \begin{pmatrix}-1\\2\\-4\end{pmatrix} + \lambda\begin{pmatrix}6\\3\\-2\end{pmatrix}$ for some $\lambda \in \mathbb{R}$ $\overrightarrow{PN} = \begin{pmatrix}-1\\2\\-4\end{pmatrix} + \lambda\begin{pmatrix}6\\3\\-2\end{pmatrix} - \begin{pmatrix}6\\3\\-5\end{pmatrix} = \begin{pmatrix}-7\\-1\\1\end{pmatrix} + \lambda\begin{pmatrix}6\\3\\-2\end{pmatrix} = \begin{pmatrix}-7+6\lambda\\-1+3\lambda\\1-2\lambda\end{pmatrix}$ Since $\overrightarrow{PN}$ is perpendicular to line L, $\overrightarrow{PN} \bullet \begin{pmatrix}6\\3\\-2\end{pmatrix} = 0$ $\begin{pmatrix}-7+6\lambda\\-1+3\lambda\\1-2\lambda\end{pmatrix} \bullet \begin{pmatrix}6\\3\\-2\end{pmatrix} = 0$ $\lambda = \frac{47}{49}$ $\overrightarrow{ON} = \begin{pmatrix}-1\\2\\-4\end{pmatrix} + \frac{47}{49}\begin{pmatrix}6\\3\\-2\end{pmatrix} = \frac{1}{49}\begin{pmatrix}233\\239\\-290\end{pmatrix}$ Hence, the point closest to P is $\left(\frac{233}{49}, \frac{239}{49}, -\frac{290}{49}\right)$.
(iii)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix}6\\3\\-5\end{pmatrix} - \begin{pmatrix}-1\\2\\-4\end{pmatrix} = \begin{pmatrix}7\\1\\-1\end{pmatrix}$ A normal to the new plane is given by $\begin{pmatrix}7\\1\\-1\end{pmatrix} \times \begin{pmatrix}6\\3\\-2\end{pmatrix} = \begin{pmatrix}1\\8\\15\end{pmatrix}$ $\mathbf{r}.\begin{pmatrix}1\\8\\15\end{pmatrix} = \begin{pmatrix}-1\\2\\-4\end{pmatrix}\begin{pmatrix}1\\8\\15\end{pmatrix} = -45$ Cartesian equation: $x + 8y + 15z = -45$
4(i)	Since AB and AC are radii of the circle, $AB = AC$ $\therefore AB = x + y$

	By Pythagoras's Theorem, $BD=\sqrt{AB^2-AD^2}$ $BD=\sqrt{\left(x+y^2\right)-x^2}$ $=\sqrt{y^2+2xy}$ (Shown)
(ii)	$\sin\theta=\dfrac{BD}{AB}$ $=\dfrac{\sqrt{y^2+2xy}}{x+y}$ $=\dfrac{\sqrt{2xy}\sqrt{1+\frac{1}{2}\frac{y}{x}}}{x\left(1+\frac{y}{x}\right)}$ $=\sqrt{2\alpha}\left(1+\frac{1}{2}\alpha\right)^{\frac{1}{2}}(1+\alpha)^{-1}$ $=\sqrt{2\alpha}\left(1+\frac{1}{4}\alpha+...\right)(1-\alpha+...)$, since α is small $\approx\sqrt{2\alpha}\left(1-\frac{3}{4}\alpha\right)$ (Shown)
(iii)	$\sin 10^\circ=\sqrt{2\alpha}\left(1-\frac{3}{4}\alpha\right)$ (1) using GC, Draw $y=\sin 10^\circ$ and $y=\sqrt{\frac{2x}{100}}\left(1-\frac{3x}{400}\right)$  $\therefore y=1.54$ (2dp) Since $\alpha=\dfrac{y}{100}\Rightarrow\alpha=0.02$ (2dp) **Method 2**

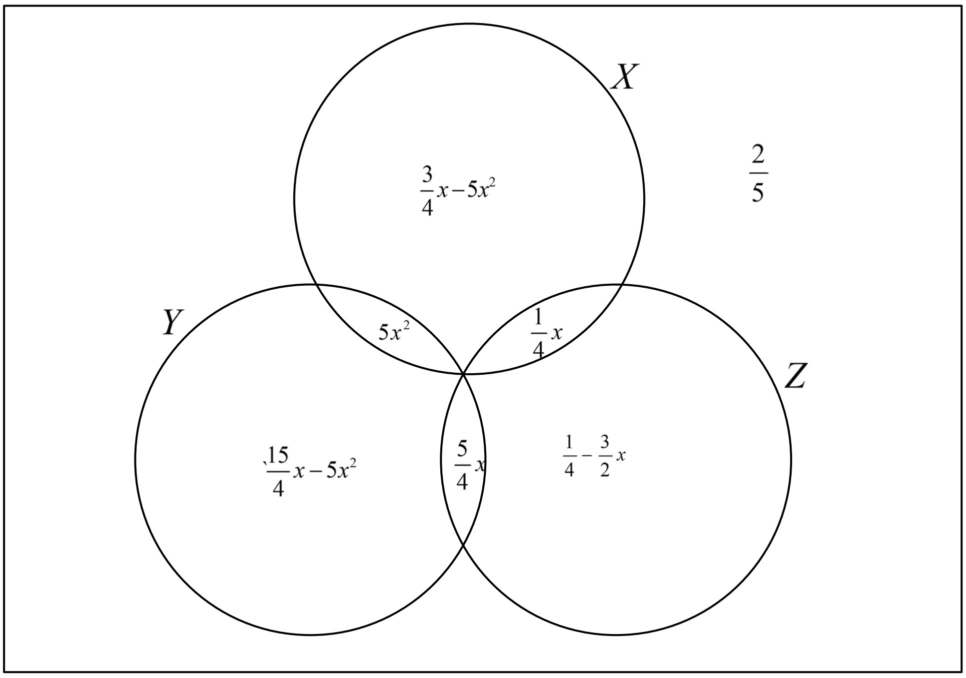
	Let $\sqrt{\alpha} = u$ $\Rightarrow \sin 10^\circ = \sqrt{2}u\left(1-\frac{3}{4}u^2\right)$ $\Rightarrow \frac{3\sqrt{2}}{4}u^3 - \sqrt{2}u + \sin 10^\circ = 0$ Using GC, $u = -1.2117907(rej), 1.0875651(rej), 0.12422559$ $\Rightarrow \alpha = 0.015432 = 0.02$ (2dp) $\Rightarrow y = 1.54$ (2dp) **Note to students: The solution closer to the y-axis is chosen because α is small.*
(iv)	The actual value of y using trigonometric methods: $AB = \frac{100}{\cos 10^\circ}$ $y = \frac{100}{\cos 10^\circ} - 100 = 1.54$ (2 d.p.)
(v)	The estimate of y is accurate up to 2 decimal places. The estimate using equation (1) is accurate because equation (1) holds only when α very close to zero (or when y is small compared to x. This is true as we found that $\alpha = 0.0154$ in part (iii)
5(i)	$\mathrm{P}(X \cap Z') = \mathrm{P}(X) - \mathrm{P}(X \cap Z)$ $= x - \frac{1}{4}x$ $= \frac{3}{4}x$ Alternatively, Since X and Z are independent, then X and Z' are independent. $\mathrm{P}(X \cap Z') = \mathrm{P}(X)\mathrm{P}(Z')$ $= x\left(1-\frac{1}{4}\right)$ $= \frac{3}{4}x$
(ii)	Since Y and X are independent, then Y' and X' are independent (*)

$$\mathrm{P}\left(Y'\mid X'\right)=\frac{\mathrm{P}\left(Y'\cap X'\right)}{\mathrm{P}\left(X'\right)}$$
$$=\frac{\mathrm{P}\left(Y'\right)P(X')}{\mathrm{P}\left(X'\right)}$$
$$=\frac{(1-y)(1-x)}{1-x}$$
$$=1-y$$

Alternatively, due to (*),

$\mathrm{P}\left(Y'\mid X'\right)=P(Y')=1-y$

(iii)



Sum of probabilities = 1

$$5x+x+\frac{1}{4}-5x^2-\frac{5}{4}x-\frac{1}{4}x+\frac{2}{5}=1$$
$$-5x^2+\frac{9}{2}x-\frac{7}{20}=0$$

$x=0.086$ or $x=0.814$

However, since P(Z) = 0.25 and $\mathrm{P}\left(\left(X\cup Y\cup Z\right)'\right)=\dfrac{2}{5}$,

Then $x \le 0.65$ (reason for rejecting x= 0.814)

$\therefore x=0.086.$

6(i) Possible outcomes:

Cases	Outcomes	Total Score	Probability
1	HHH	3	p^3
2	TTT	−3	$(1-p)^3$

3	HTT	-1	$3p(1-p)^2$
4	HHT	1	$3p^2(1-p)$

x	-3	-1	1	3
$\mathrm{P}(X=x)$	$(1-p)^3$	$3p(1-p)^2$	$3p^2(1-p)$	p^3

(ii)

$\mathrm{E}(X) = \sum_{all\ x} xP(X=x)$

$$
\begin{aligned}
&= (-3)(1-p)^3 + (-1)\left(3p(1-p)^2\right) + (1)\left(3p^2(1-p)\right) + (3)p^3 \\
&= -3(1-p)\left[(1-p)^2 + p(1-p) - p^2\right] + 3p^3 \\
&= -3(1-p)\left[1-2p+p^2+p-p^2-p^2\right] + 3p^3 \\
&= -3(1-p)\left[1-p-p^2\right] + 3p^3 \\
&= -3\left[1-p-p^2-p+p^2+p^3\right] + 3p^3 \\
&= -3(1-2p) \\
&= 6p-3
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{E}\left(\frac{X+3}{6}\right) &= \frac{1}{6}\mathrm{E}(X+3) = \frac{1}{6}\mathrm{E}(X) + \frac{1}{6}\mathrm{E}(3) = \frac{1}{6}\mathrm{E}(X) + \frac{1}{2} \\
&= \frac{1}{6}(6p-3) + \frac{1}{2} \\
&= p
\end{aligned}
$$

$\therefore \dfrac{X+3}{6}$ is an unbiased estimator of p. (Shown)

7(i)

Number of ways

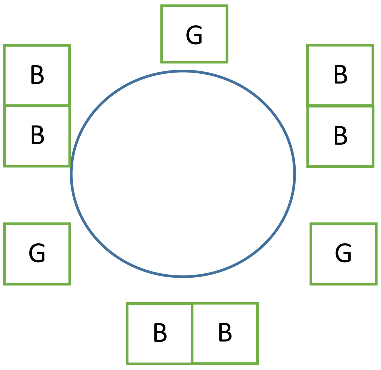
= Total number of ways without restriction - number of ways all 3 girls together

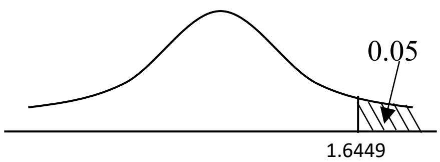
$$= \underbrace{(9-1)!}_{\text{9 people around a circle}} - \left[\underbrace{(7-1)!}_{\text{6 boys and 1 block of 3 girls}} \times \underbrace{3!}_{\text{3 girls}}\right]$$

$= 36000$

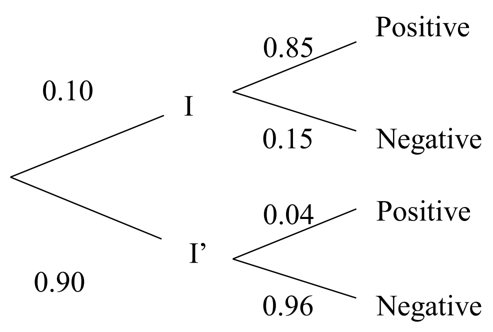
Alternative method

Number of ways

	$=$ number of ways 2 girls together, 1 separated $+$ number of ways all 3 girls separated $=\left(\underbrace{(5-1)!}_{\text{6 boys around a circle}} \times \underbrace{{}^{3}C_{2}}_{\text{3 girls choose 2 to be in a group}} \times \underbrace{{}^{6}P_{2}}_{\text{6 slots permute 2 groups of girls}} \times \underbrace{2!}_{\text{permute within the group of 2 girls}}\right)$ $+\left[\underbrace{(5-1)!}_{\text{6 boys around}} \times \underbrace{{}^{6}P_{3}}_{\text{6 slots permute 3 girls}}\right]$ $=36000$
(ii)	Number of ways $=\underbrace{(6-1)!}_{\text{6 remaining people around a circle}} \times \underbrace{{}^{6}C_{3}}_{\text{6 slots choose 3}} \times \underbrace{3!}_{\text{The 3 girls}}$ $=14400$
(iii)	Note: The arrangement must look like this  Number of ways without restrictions $=(9-1)!$ $=40320$ Number of ways for exactly 2 boys between any 2 girls $=\underbrace{(3-1)!}_{\text{Arranging 3 girls around a circle}} \times \underbrace{6!}_{\text{6 boys permutate within themselves}}$ $=1440$ Required prob. $=\dfrac{1440}{40320}$ $=\dfrac{1}{28}$ Alternative Method: Number of ways without restrictions

	$= (9-1)!$ $= 40320$ Number of ways for exactly 2 boys between any 2 girls $= \underbrace{(3-1)!}_{\text{Arranging 3 girls around a circle}} \times$ $\left[\underbrace{{}^{6}C_{2}}_{\text{6boys choose 2 to slot into first slot}} \times \underbrace{{}^{4}C_{2}}_{\text{4 boys choose 2 to slot into second slot}} \times {}^{2}C_{2} \times \underbrace{(2!)^{3}}_{\text{each group of boys permute within themselves}}\right]$ $= 1440$ Required prob. $= \dfrac{1440}{40320}$ $= \dfrac{1}{28}$
8(a)	Let X be the random variable denoting the amount of sodium in a randomly chosen packet of potato chips in mg and μ be the population mean amount of sodium in a packet of potato chips in mg. Test $H_0 : \mu = 798$ against $H_1 : \mu > 798$ at 5% level of significance Under H_0, since n is large, by Central Limit Theorem, $\overline{X} \sim N\left(798, \dfrac{6.5^2}{n}\right)$ approximately. Test statistic $Z = \dfrac{\overline{X} - 798}{\sqrt{\dfrac{6.5^2}{n}}} \sim N(0,1)$ approximately. Carry out 1-tailed z-test at the 5% level of significance. Since the company has understated the sodium content, we will reject H_0.  For H_0 to be rejected ,

	$z_{calc} \geq 1.6449$ $\dfrac{799.5-798}{\sqrt{\dfrac{6.5^2}{n}}} \geq 1.6449$ $\sqrt{\dfrac{6.5^2}{n}} \leq \dfrac{1.5}{1.6449}$ $n \geq \dfrac{6.5^2}{0.83158}$ $\Rightarrow n \geq 50.8$ Thus, set of n is $\left\{n \in \mathbb{Z}^+ : n \geq 51\right\}$.
(b)(i)	Let Y be the random variable denoting the amount of sodium in a randomly chosen packet of potato chips of the healthier recipe in mg and μ_1 be the population mean amount of sodium in a packet of potato chips of the healthier recipe in mg. Test $H_0 : \mu_1 = 798$ against $H_1 : \mu_1 < 798$ at 5% level of significance Unbiased estimate of the population variance s^2 $= \dfrac{n}{n-1} \times \text{sample variance}$ $= \dfrac{50}{49} \times 6.2^2$ $= 39.224$ Under H_0, since $n = 50$ is large, by Central Limit Theorem, $\overline{Y} \sim N\left(798, \dfrac{39.224}{50}\right)$ approximately. Test statistic $Z = \dfrac{\overline{Y} - 798}{\sqrt{\dfrac{39.224}{50}}} \sim N(0,1)$ approximately. Using a 1-tailed z-test, reject H_0 if p-value ≤ 0.05 Using GC, the test statistic value $\overline{x} = 796.3$ and $z_{\text{calc}} = -1.9194$ gives p-value = 0.0275 < 0.05 We reject H_0 and conclude that there is sufficient evidence at the 5% level of significance that the mean amount of sodium in each packet of potato chips is less than 798 mg.
(b)(ii)	The statement means that there is a probability of 0.05 of wrongly concluding that the mean amount of sodium in each packet of potato chips is less than 798 mg when in fact the mean amount of sodium in each packets is 798 mg.
9	COMMON LAST TOPIC
10(i)	1) The probability that a person is infected is constant at p for each person.

	2) The event of a person being infected is independent of any other people.
(ii)	Let X be the random variable " number of people who are infected, out of 25 people" $X \sim \mathrm{B}(25, p)$ $\mathrm{P}(X = r) = {}^{25}C_r p^r (1-p)^{25-r}, \ r = 0,1,2,...25.$ Since $\mathrm{P}(X \leq 2) = 0.0982$ $\binom{25}{0} p^0 (1-p)^{25} + \binom{25}{1} p^1 (1-p)^{24} + \binom{25}{2} p^2 (1-p)^{23} = 0.0982$ By GC, $p \approx 0.200.$
(iii)	Let D be the random variable denoting the no. of samples with at most 2 infected people out of 30 samples. $D \sim \mathrm{B}(30, 0.0982)$ $\mathrm{P}(D \geq 5)$ $= 1 - \mathrm{P}(D \leq 4)$ $= 0.16639$ $= 0.166$
(iv)	 Legend: I denote Infected $\mathrm{P(tested\ positive)} = 0.10 \times 0.85 + 0.90 \times 0.04$ $= 0.121$ $\mathrm{P(infected \mid tested\ positive)}$ $= \dfrac{\mathrm{P(tested\ positive\ and\ infected)}}{\mathrm{P(tested\ positive)}}$ $= \dfrac{0.10 \times 0.85}{0.121}$ $= 0.702$

(v)	P(not infected | tested positive) $= 1 - \text{P(infected} \mid \text{tested positive)}$ $= 1 - 0.70248$ $= 0.298$
(vi)	Amongst those who were tested positive, the proportion of population who are actually infected is 70.2%, hence it is worthwhile. [*Early diagnosis allow for timely treatment/Patients identified early can prevent the spread of the virus*] <u>Alternatively</u>, From part (v), the probability of a person not infected but test positive is close to 30%, which is not considered a low proportion. Hence the diagnostic test might not be worthwhile. [*These group of people would undergo unnecessary treatment which is a waste of resources/These group of people would unnecessarily be quarantined with the infected and might eventually be infected*]
11 (a)(i)	Let *X* be the random variable " volume of BBT dispensed in ml" $X \sim \text{N}(210, 5^2)$ $P(X > 220) = 0.02275 = 0.0228$ (3 s.f.) using GC
(ii)	Let *Y* be the random variable " no. of BBT cups out of 5 with overflow cups" $Y \sim \text{B}(5, 0.02275)$ $\text{P}(Y \le 1) = 0.995$ (3 s.f.) OR Required probability $= P(X < 220)^5 + \binom{5}{1}\left(P(X < 220)^4\right)\left(P(X > 220)\right)$ ≈ 0.995
(iii)	Let the mean volume of BBT dispensed by μ ml. $X \sim N(\mu, 5^2)$ $P(X < 212) \le 0.10$ $P(Z < \frac{212-\mu}{5}) \le 0.10$ $\Rightarrow \frac{212-\mu}{5} \le -1.2816$ $\Rightarrow \mu \ge 218.4$ Range of mean volume is $218 \le \mu \le 220$ (to 3 s.f.) (Accept $219 \le \mu \le 220$)

(iv)	$X_1 + X_2 + ... + X_n \sim N(210n, 25n)$ $P(X_1 + X_2 + ... + X_n > 211n)$ $= P(Z > \frac{211n - 210n}{5\sqrt{n}})$ $= P(Z > \frac{1}{5}\sqrt{n}) --- (*)$ Since n becomes very large , $P(Z > \frac{1}{5}\sqrt{n}) \to 0$.
(v)	The BBT seller would not obtain a random sample as she only interviewed customers who bought her BBT during lunch time. She would not be able to get feedback from customers who made the purchase during other hours, hence the **not all the customers have an equal chance of being selected.** The sample is not randomly selected as result.
(b)	Let T be the random variable denoting the volume of bubble tea served in ml. $\overline{T} \sim N(200, \frac{100^2}{60})$ approximately by Central Limit Theorem since the sample size $60 > 30$. Using GC, $P(\overline{T} < 198) = 0.0607$ (3s.f)