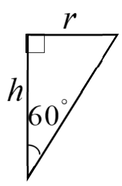
RAFFLES INSTITUTION
H2 Mathematics (9758)
2021 Year 6

2021 Year 6 H2 Math Preliminary Paper 1: Solutions

Qn 1: Solution	
[4]	$\mathrm{f}(x)=x^4+ax^2+bx+c$ $\mathrm{f}(2)=0 \quad \Rightarrow 16+4a+2b+c=0 \quad \Rightarrow 4a+2b+c=-16 \ldots(1)$ $\mathrm{f}(3)=12 \quad \Rightarrow 81+9a+3b+c=12 \quad \Rightarrow 9a+3b+c=-69 \ldots(2)$ $\mathrm{f}(4)=26 \quad \Rightarrow 256+16a+4b+c=26 \Rightarrow 16a+4b+c=-230 \ldots(3)$ Solving (1), (2) and (3), $a=-54,\ b=217,\ c=-234$

Qn 2: Solution	
[4]	Volume of water in the conical container at time t seconds, $V=\frac{1}{3}\pi r^2 h$ $\tan 60^\circ=\frac{r}{h} \Rightarrow r=h\sqrt{3}$  Therefore $\quad V=\pi h^3$ $\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t}=3\pi h^2\frac{\mathrm{d}h}{\mathrm{d}t}$ When t =15, $V=94\pi-(2\pi)(15)=64\pi$, $\pi h^3=64\pi \Rightarrow h=4$ $\therefore \frac{\mathrm{d}h}{\mathrm{d}t}=-2\pi \div 3\pi(4)^2=-\frac{1}{24}$ $\therefore$Rate at which h is decreasing at the instant when t =15 is $\frac{1}{24}$ cms^{-1}.

Qn 3: Solutions	
(a) [3]	$$\begin{aligned}\int x\tan^{-1}x\,\mathrm{d}x &= \left(\frac{x^2}{2}\tan^{-1}x\right)-\int \frac{x^2}{2}\left(\frac{1}{1+x^2}\right)\mathrm{d}x\\ &= \frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int 1-\frac{1}{1+x^2}\,\mathrm{d}x\\ &= \frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left[x-\tan^{-1}x\right]+c\\ &= \frac{x^2+1}{2}\tan^{-1}x-\frac{x}{2}+c\end{aligned}$$
(b) (i) [2]	Given the substitution $u=\frac{1}{x}$, we have $\frac{\mathrm{d}u}{\mathrm{d}x}=-\frac{1}{x^2}$ $$\begin{aligned}\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x &= -\int \sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\mathrm{d}x\\ &= -\int \sin u\,\mathrm{d}u\\ &= \cos u+c\\ &= \cos\left(\frac{1}{x}\right)+c\end{aligned}$$ OR $$\begin{aligned}\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x &= -\int \left(-\frac{1}{x^2}\right)\sin\left(\frac{1}{x}\right)\mathrm{d}x\\ &= \cos\left(\frac{1}{x}\right)+c\end{aligned}$$
(b) (ii) [3]	$$\begin{aligned}\pi\int_{\frac{1}{(n+1)\pi}}^{\frac{1}{n\pi}} \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x &= \pi\left[\cos\left(\frac{1}{x}\right)\right]_{\frac{1}{(n+1)\pi}}^{\frac{1}{n\pi}}\\ &= \pi\left[\cos(n\pi)-\cos\left((n+1)\pi\right)\right]\end{aligned}$$

	If n is even, $\pi\int_{\frac{1}{(n+1)\pi}}^{\frac{1}{n\pi}} \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x = \pi\left[1-(-1)\right] = 2\pi \quad a=2$ If n is odd, $\pi\int_{\frac{1}{(n+1)\pi}}^{\frac{1}{n\pi}} \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x = \pi\left[-1-1\right] = -2\pi \quad a=-2$ *OR* $a = 2(-1)^n$

Qn 4: Solutions

(a)
[5]

$$(2r+1)^3-(2r-1)^3$$
$$=\left[8r^3+12r^2+6r+1\right]-\left[8r^3-12r^2+6r-1\right]$$
$$=24r^2+2$$
$$\therefore k=2$$

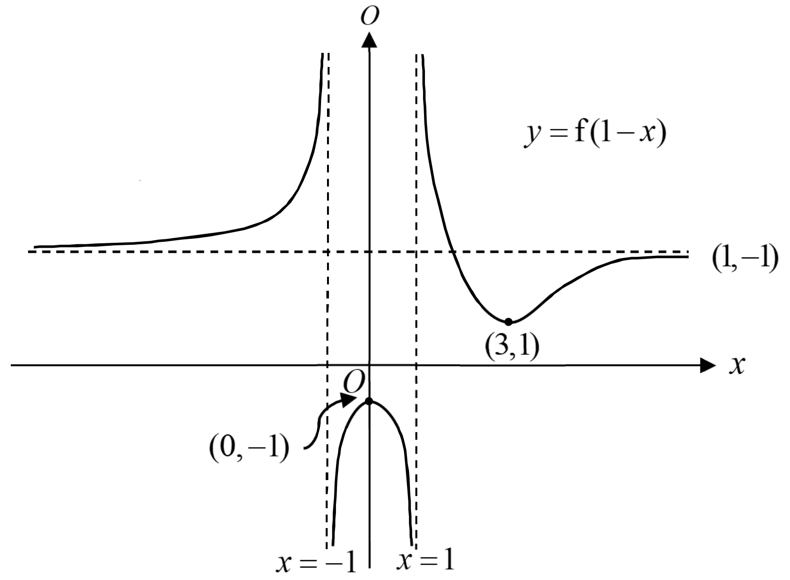
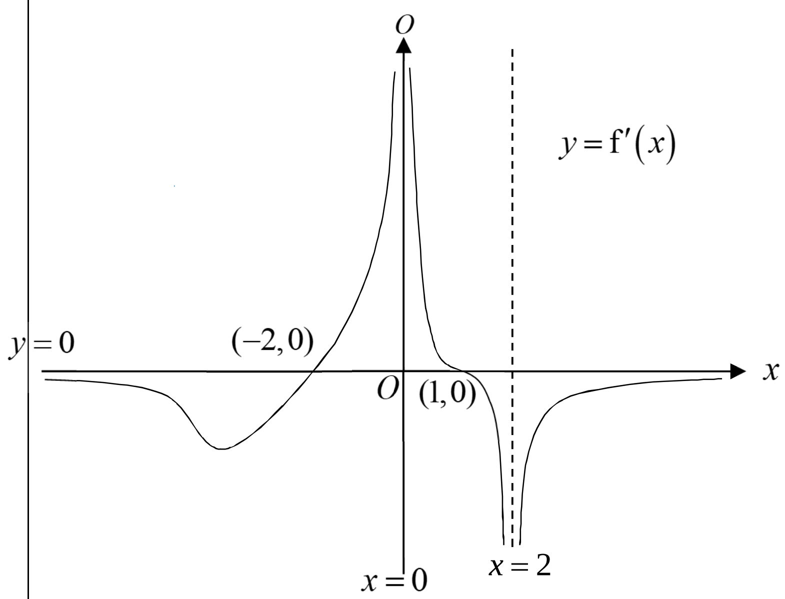
OR

$$(2r+1)^3-(2r-1)^3$$
$$=\left[(2r+1)-(2r-1)\right]\left[(2r+1)^2+(2r+1)(2r-1)+(2r-1)^2\right]$$
$$=2\left[4r^2+4r+1+4r^2-1+4r^2-4r+1\right]$$
$$=2\left(12r^2+1\right)$$
$$=24r^2+2$$
$$\therefore k=2$$

$$\sum_{r=1}^{n}\left(24r^2+2\right)=\sum_{r=1}^{n}\left((2r+1)^3-(2r-1)^3\right)$$

$$\begin{aligned}24\sum_{r=1}^{n}r^2+2\sum_{r=1}^{n}1 &= 3^3 - 1^3\\ &+5^3 - 3^3\\ &+7^3 - 5^3\\ &+ \quad \ldots\\ &+(2n-1)^3-(2n-3)^3\\ &+(2n+1)^3-(2n-1)^3\\ &=(2n+1)^3 - 1\end{aligned}$$

	$24\sum_{r=1}^{n} r^2 + 2n = (2n+1)^3 - 1$ $\sum_{r=1}^{n} r^2 = \frac{1}{24}\left[(2n+1)^3 - (2n+1)\right]$ $= \frac{(2n+1)}{24}\left[(2n+1)^2 - 1\right]$ $= \frac{(2n+1)}{24}(2n+2)(2n)$ $= \frac{(2n+1)}{24}(2n+2)(2n)$ $= \frac{1}{6}n(n+1)(2n+1)$ $\therefore p = \frac{1}{6}, \quad q = 1$
(b) **[3]**	Let $a_n = \frac{1}{n^3}$. $n\left(\frac{a_n}{a_{n+1}} - 1\right) = n\left[\frac{\left(\frac{1}{n^3}\right)}{\left(\frac{1}{(n+1)^3}\right)} - 1\right]$ $= n\left[\frac{(n+1)^3}{n^3} - 1\right]$ $= n\left(\frac{n^3 + 3n^2 + 3n + 1 - n^3}{n^3}\right)$ $= \frac{3n^2 + 3n + 1}{n^2}$ $= 3 + \frac{3}{n} + \frac{1}{n^2}$ $\lim_{n\to\infty}\left[n\left(\frac{a_n}{a_{n+1}} - 1\right)\right] = \lim_{n\to\infty}\left(3 + \frac{3}{n} + \frac{1}{n^2}\right) = 3 > 1$. Therefore $\sum_{r=1}^{\infty}\frac{1}{r^3}$ converges.

Qn 5: Solutions	
(a) (i) [3]	
(a) (ii) [3]	
(b) [3]	$y=\ln\left(1-\frac{x}{2}\right) \to y=\ln(1-x) \to y=-\ln(1-x)=\ln\left(\frac{1}{1-x}\right)$ $\to y=\ln\left(\frac{1}{1-x}\right)+\ln 2=\ln\left(\frac{2}{1-x}\right)$ Scale the graph of $y=\ln\left(1-\frac{x}{2}\right)$ parallel to the *x*-axis by factor $\frac{1}{2}$, followed by a reflection about the ***x*-axis**, followed by a translation of ln 2 units in the positive *y*-direction.

	OR $y=\ln\left(1-\frac{x}{2}\right)=\ln\left(\frac{2-x}{2}\right)\to y=-\ln\left(\frac{2-x}{2}\right)=\ln\left(\frac{2}{2-x}\right)$ $\to y=\ln\left(\frac{2}{1-x}\right)$ Reflect the graph of $y=\ln\left(1-\frac{x}{2}\right)$ about the ***x*-axis**, followed by a translation of 1 unit in the **negative** *x*-direction.

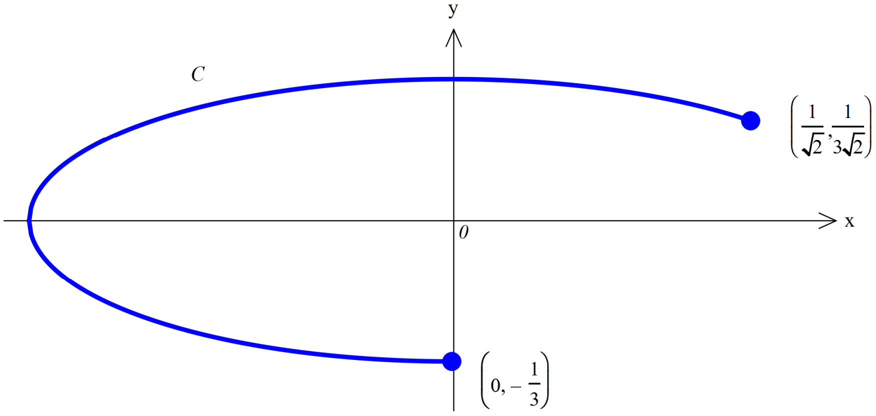
Qn 6: Solutions	
(a) **[2]**	Let $\mathrm{f}(z) = z^3 + 2z + 4\mathrm{i}$ $\mathrm{f}(2\mathrm{i}) = (2\mathrm{i})^3 + 2(2\mathrm{i}) + 4\mathrm{i}$ $= 8\mathrm{i}^3 + 4\mathrm{i} + 4\mathrm{i}$ $= -8\mathrm{i} + 8\mathrm{i}$ $= 0$ $z = 2\mathrm{i}$ is a root of the equation $z^3 + 2z + 4\mathrm{i} = 0$.
[3]	$z^3 + 2z + 4\mathrm{i} = 0$ $(z - 2\mathrm{i})(z^2 + 2\mathrm{i}z - 2) = 0$ $z = 2\mathrm{i}$ or $z = \dfrac{-2\mathrm{i} \pm \sqrt{-4+8}}{2}$ $z = 2\mathrm{i}$ or $z = -\mathrm{i} \pm 1$ The other roots are $1 - \mathrm{i}$, $-1 - \mathrm{i}$.
(b) **[4]**	We have $\arg(w_1) = \dfrac{5\pi}{6}$ and $\arg(w_2) = \dfrac{\pi}{4}$. Hence $\arg\left(\dfrac{w_2}{w_1}\right) = \arg(w_2) - \arg(w_1)$ $= \dfrac{\pi}{4} - \dfrac{5\pi}{6}$ $= -\dfrac{7\pi}{12}$ $\arg\left(\dfrac{w_2}{w_1}\right)^n = n\arg\left(\dfrac{w_2}{w_1}\right)$ $= -\dfrac{7n\pi}{12}$ Hence we need to find the least positive integer n such that $\dfrac{-7n\pi}{12} = -\dfrac{\pi}{2} + m(2\pi) = \dfrac{(4m-1)\pi}{2}$, $m \in \mathbb{Z}$. Rearranging, $n = \dfrac{6 - 24m}{7} = \dfrac{6(1-4m)}{7}$.

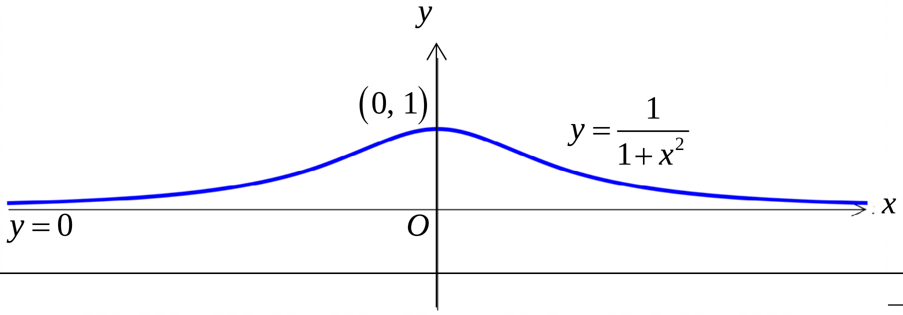
	Method 1 Therefore we need to have an integer m such that $1-4m$ is positive (and thus negative m) and a multiple of 7. Checking through the negative integer values of m, we have $1-4m=5,9,13,17,\underline{21},\ldots$. The corresponding least value of n is therefore 18. **Method 2**

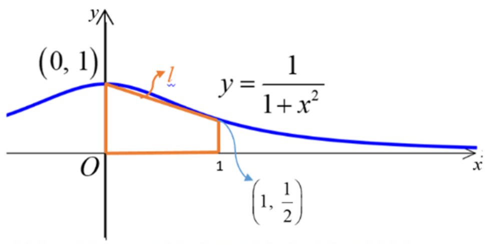
m	$n=\dfrac{6(1-4m)}{7}$
-1	$\dfrac{30}{7}$
-2	$\dfrac{54}{7}$
-3	$\dfrac{78}{7}$
-4	$\dfrac{102}{7}$
-5	18

Hence smallest $n=18$, corresponding to when $m=-5$.

Qn 7: Solutions	
(i) **[3]**	$\dfrac{dx}{dt}=\cos t, \quad \dfrac{dy}{dt}=-\dfrac{1}{3}\sin t$ $\dfrac{dy}{dx}=-\dfrac{1}{3}\tan t$ At P, $\dfrac{dy}{dx}=-\dfrac{1}{3}\tan p$ Gradient of normal $=3\cot p$ Equation of normal at P: $y-\dfrac{1}{3}\cos p=(3\cot p)(x-\sin p)$ $y=(3\cot p)x-\dfrac{8}{3}\cos p$
(ii) **[4]**	When $t=-\dfrac{\pi}{4}$, equation of normal: $y=(-3\cot\dfrac{\pi}{4})x-\dfrac{8}{3}\cos\dfrac{\pi}{4}$ $y=-3x-\dfrac{8}{3\sqrt{2}}$ ------ (*) For normal to cut C again, substitute $x=\sin t$, $y=\dfrac{1}{3}\cos t$ into (*)

	$\frac{1}{3}\cos t = -3\sin t - \frac{8}{3\sqrt{2}}$ $\frac{1}{3}\cos t + 3\sin t = -\frac{8}{3\sqrt{2}}$ From GC, $t = -2.5775$ (4 d.p.) $x = \sin(-2.5775) = -0.53$ (2 d.p.) $y = \frac{1}{3}\cos(-2.5775) = -0.28$ (2 d.p.) The coordinates of point A is $(-0.53, -0.28)$.
(iii) **[2]**	
(iv) **[2]**	Area of required region $= \int_0^{\frac{1}{\sqrt{2}}} y \, dx$ $= \int_0^{\frac{\pi}{4}} \frac{1}{3}\cos t \frac{dx}{dt} \, dt$ $= \frac{1}{3}\int_0^{\frac{\pi}{4}} \cos^2 t \, dt$ $= 0.214$ units^2 (3sf)

Q8: Solutions	
(i) **[2]**	

(ii) **[4]**	Now, $y=\dfrac{1}{1+x^2} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-2x}{\left(1+x^2\right)^2}$. At (c, d), we have $d=\dfrac{1}{1+c^2}$ and $\dfrac{d-1}{c-0}=\dfrac{-2c}{\left(1+c^2\right)^2} \Rightarrow d=\dfrac{-2c^2}{\left(1+c^2\right)^2}+1$. Hence we have $\dfrac{-2c^2}{\left(1+c^2\right)^2}+1=\dfrac{1}{1+c^2}$ $\Rightarrow -2c^2+\left(1+c^2\right)^2=\left(1+c^2\right)$ $\Rightarrow c^4-c^2=0$ $\Rightarrow c^2\left(c^2-1\right)=0$ $\Rightarrow c=0$ or $c=\pm 1$. Since $c>0$, $c=1$ and $d=\dfrac{1}{2}$. Equation of l is $y-\dfrac{1}{2}=\dfrac{-2(1)}{\left(1+1^2\right)^2}(x-1) \Rightarrow y=-\dfrac{x}{2}+1$.
(iii) **[2]**	Area of $R=\int_0^1 \dfrac{1}{1+x^2}\,\mathrm{d}x=\left[\tan^{-1}x\right]_0^1=\dfrac{\pi}{4}$. Area of $R>$ Area of trapezium $\Rightarrow \dfrac{\pi}{4}>\dfrac{1}{2}\left(1+\dfrac{1}{2}\right)(1)=\dfrac{3}{4}$ $\Rightarrow \pi>3$ (Shown).  Note: Besides using the formula for area of trapezium, we can also use the following: $\int_0^1 -\dfrac{x}{2}+1\,\mathrm{d}x=\dfrac{3}{4}$.

(iv) **[3]**	$\pi\int_{\frac{1}{2}}^{1} x^2\ \mathrm{d}y = \pi\int_{\frac{1}{2}}^{1} \frac{1}{y} - 1\ \mathrm{d}y$ $= \pi\left[\ln y - y\right]_{\frac{1}{2}}^{1}$ $= \pi\left[\left(\ln 1 - 1\right) - \left(\ln\frac{1}{2} - \frac{1}{2}\right)\right]$ $= \pi\left(-\ln\frac{1}{2} - \frac{1}{2}\right) = \pi\left(\ln 2 - \frac{1}{2}\right).$ Now, Volume obtained > Volume of cone with radius 1 and height $\frac{1}{2}$ $\Rightarrow \pi\int_{\frac{1}{2}}^{1} x^2\ \mathrm{d}y > \frac{1}{3}\pi\left(1^2\right)\left(\frac{1}{2}\right)$ $\Rightarrow \pi\left(\ln 2 - \frac{1}{2}\right) > \frac{\pi}{6}$ $\Rightarrow \ln 2 > \frac{1}{6} + \frac{1}{2} = \frac{4}{6} = \frac{2}{3}$ (shown). Note: Besides using the formula for volume of cone, can also consider $\pi\int_{\frac{1}{2}}^{1} \left(2 - 2y\right)^2\ \mathrm{d}y = \frac{\pi}{6}$.

Qn 9: Solutions	
(i) **[4]**	Where l_{AN} : $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ intersects $\pi : \mathbf{r}.\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 10$, $\left[\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right]\cdot\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 10$ $\left(2 - 1 - 3\right) + \lambda\left(1 + 1 + 1\right) = 10 \Rightarrow 3\lambda = 12 \quad \Rightarrow \quad \lambda = 4$ $\overrightarrow{ON} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + 4\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix}$ The coordinates of N is $(6, -3, 1)$.

(ii) **[2]**	$\overrightarrow{OB}=\overrightarrow{OA}+2\overrightarrow{AN}$ $=\overrightarrow{OA}+2\overrightarrow{ON}-2\overrightarrow{OA}$ $=2\overrightarrow{ON}-\overrightarrow{OA}$ $=2\begin{pmatrix}6\\-3\\1\end{pmatrix}-\begin{pmatrix}2\\1\\-3\end{pmatrix}=\begin{pmatrix}10\\-7\\5\end{pmatrix}$
(iii) **[4]**	Equation of line $l:\mathbf{r}=\begin{pmatrix}-1\\1\\-4\end{pmatrix}+s\begin{pmatrix}3\\0\\1\end{pmatrix},\quad s\in\mathbb{R}$ When $s=1$, $\mathbf{r}=\begin{pmatrix}-2\\1\\-3\end{pmatrix}=\overrightarrow{OA}$, so A lies on l Let point of intersection of l and p_1 be X. When l intersects p_1, $\left[\begin{pmatrix}-1\\1\\-4\end{pmatrix}+s\begin{pmatrix}3\\0\\1\end{pmatrix}\right].\begin{pmatrix}1\\-1\\1\end{pmatrix}=10\ \Rightarrow s=4$ $\overrightarrow{OX}=\begin{pmatrix}11\\1\\0\end{pmatrix}$ Equation of reflected line $l':\mathbf{r}=\begin{pmatrix}10\\-7\\5\end{pmatrix}+\mu\left(\begin{pmatrix}10\\-7\\5\end{pmatrix}-\begin{pmatrix}11\\1\\0\end{pmatrix}\right)$ $l':\mathbf{r}=\begin{pmatrix}10\\-7\\5\end{pmatrix}+\mu\begin{pmatrix}-1\\-8\\5\end{pmatrix},\quad \mu\in\mathbb{R}$
(iv) **[2]**	Distance between p_1 and $p_2=\frac{1}{2}AB=\frac{1}{2}\left\|\begin{pmatrix}8\\-8\\8\end{pmatrix}\right\|=4\sqrt{3}$ OR Distance between p_1 and $p_2=BN=AN=\left\|4\begin{pmatrix}1\\-1\\1\end{pmatrix}\right\|=4\sqrt{3}$ OR $p_2:\mathbf{r}.\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}10\\-7\\5\end{pmatrix}.\begin{pmatrix}1\\-1\\1\end{pmatrix}=22$

	Distance between p_1 and $p_2 = \dfrac{22-10}{\sqrt{1^2+1^2+1^2}} = \dfrac{12}{\sqrt{3}} = 4\sqrt{3}$ OR Distance between p_1 and $p_2 = \dfrac{\left\lvert \overrightarrow{XB}.\begin{pmatrix}1\\-1\\1\end{pmatrix}\right\rvert}{\sqrt{1^2+1^2+1^2}} = \dfrac{\left\lvert \begin{pmatrix}-1\\-8\\5\end{pmatrix}\begin{pmatrix}1\\-1\\1\end{pmatrix}\right\rvert}{\sqrt{1^2+1^2+1^2}} = \dfrac{12}{\sqrt{3}} = 4\sqrt{3}$

Qn 10: Solutions	
[8]	Since the sum of money owed to the bank increases at a rate proportional to the sum owed and Bob repays the bank at a constant rate r, $\frac{dx}{dt} = kx - r$, where $k > 0$. When $x = a$, interest and repayment balance. Then $\frac{dx}{dt} = 0 = ka - r \Rightarrow k = \frac{r}{a}$ Therefore $\frac{dx}{dt} = \frac{r}{a}(x) - r = \frac{r}{a}(x-a)$ $\frac{dx}{dt} = \frac{r}{a}(x-a)$ $\int \frac{1}{x-a}\,dx = \int \frac{r}{a}\,dt$ $\ln\lvert x-a\rvert = \frac{rt}{a} + C \quad , C \in \mathbb{R}$ $\lvert x-a\rvert = e^{\frac{rt}{a}} e^{C}$ $x - a = Be^{\frac{rt}{a}}$ where $B = \pm e^{C}$ $x = Be^{\frac{rt}{a}} + a$ When $t = 0$, $x = A$, $A = B + a \Rightarrow B = A - a$ $x = (A-a)e^{\frac{rt}{a}} + a$
[4]	For the loan to be repaid in a finite time T, $x = (A-a)e^{\frac{rt}{a}} + a$ must be a decreasing function as t increases. So $A - a < 0 \Rightarrow A < a$ When the loan is repaid, $x = 0$. $0 = (A-a)e^{\frac{rT}{a}} + a$ $\Rightarrow \frac{a}{a-A} = e^{\frac{rT}{a}}$ $\Rightarrow T = \frac{a}{r}\ln\left(\frac{a}{a-A}\right)$ (Shown)

Qn 11: Solutions	
(i) **[2]**	$V=\pi r^2 h \Rightarrow h=\dfrac{V}{\pi r^2}$. Now, $S=2\pi r^2+2\pi rh$ $=2\pi r^2+2\pi r\left(\dfrac{V}{\pi r^2}\right)$ $=2\pi r^2+\dfrac{2V}{r}$ (Shown).
(ii) **[7]**	$S=2\pi r^2+\dfrac{2V}{r}\Rightarrow\dfrac{\mathrm{d}S}{\mathrm{d}r}=4\pi r-\dfrac{2V}{r^2}$. $\dfrac{\mathrm{d}S}{\mathrm{d}r}=0\Rightarrow 4\pi r-\dfrac{2V}{r^2}=0$ $\Rightarrow 4\pi r^3-2V=0$ $\Rightarrow r=\left(\dfrac{V}{2\pi}\right)^{\frac{1}{3}}$. $\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2}=4\pi+\dfrac{4V}{r^3}\Rightarrow\left.\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2}\right\|_{r=\left(\frac{V}{2\pi}\right)^{\frac{1}{3}}}=4\pi+\dfrac{4V}{\left(\frac{V}{2\pi}\right)}=12\pi>0.$ So S is minimum when $r=\left(\dfrac{V}{2\pi}\right)^{\frac{1}{3}}$. $\therefore S=2\pi r^2+\dfrac{2V}{r}$ $=2\pi\left(\dfrac{V}{2\pi}\right)^{\frac{2}{3}}+\dfrac{2V}{\left(\frac{V}{2\pi}\right)^{\frac{1}{3}}}$ $=(2\pi)^{\frac{1}{3}}V^{\frac{2}{3}}+2^{\frac{4}{3}}\pi^{\frac{1}{3}}V^{\frac{2}{3}}$ $=\left(2\pi V^2\right)^{\frac{1}{3}}(1+2)$ $=3\left(2\pi V^2\right)^{\frac{1}{3}}$, where $k=3$ and $m=2$. When $r=\left(\dfrac{V}{2\pi}\right)^{\frac{1}{3}}$, $\dfrac{r}{h}=\dfrac{r}{\left(\frac{V}{\pi r^2}\right)}=\dfrac{\pi r^3}{V}=\dfrac{\pi\left(\frac{V}{2\pi}\right)}{V}=\dfrac{1}{2}$.

	Therefore $r : h = 1 : 2$.
(iii) **[2]**	Largest spherical shaped ice that can be carved out has radius, $R = r = \left(\dfrac{V}{2\pi}\right)^{\frac{1}{3}}$, since when S is minimum, $h = 2r$. Hence the volume of the largest spherical ice is $\dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi\left(\dfrac{V}{2\pi}\right) = \dfrac{2V}{3}$ (Shown).
(iv) **[1]**	No, the manufacturer should not proceed as the spherical shaped ice has volume at least $\dfrac{2}{3}V$ and so $\dfrac{1}{3}$ of the volume of the cylindrical shaped ice will go to waste which is quite a lot. OR Yes, the manufacturer should proceed even though the spherical shaped ice has volume at least $\dfrac{2}{3}V$ as the $\dfrac{1}{3}V$ of crushed ice that is leftover during carving can be used for other drinks which require crushed ice.