Preliminary Examination 2021 Paper 2
4049/02 Secondary 4Express
Additional Mathematics

Qn No	Working
1(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(3xe^{2x+1}\right)=3e^{2x+1}+3x(2)e^{2x+1}$ $=3e^{2x+1}+6xe^{2x+1}$
1(ii)	$\int\left(3e^{2x+1}+6xe^{2x+1}\right)\mathrm{d}x=3xe^{2x+1}+c_1$ $\int 6xe^{2x+1}\mathrm{d}x=3xe^{2x+1}-\int 3e^{2x+1}\mathrm{d}x+c_1$ $\int\left(\frac{2}{3}\right)9xe^{2x+1}\mathrm{d}x=3xe^{2x+1}-\frac{3}{2}e^{2x+1}+c_2$ $\frac{2}{3}\int 9xe^{2x+1}\mathrm{d}x=3xe^{2x+1}-\frac{3}{2}e^{2x+1}+c_2$ $\int 9xe^{2x+1}\mathrm{d}x=\left(3xe^{2x+1}-\frac{3}{2}e^{2x+1}+c_2\right)\div\left(\frac{2}{3}\right)$ $=\left(3xe^{2x+1}-\frac{3}{2}e^{2x+1}+c_2\right)\times\left(\frac{3}{2}\right)$ $=\frac{9}{2}xe^{2x+1}-\frac{9}{4}e^{2x+1}+c$
2(i)	$\text{LHS}=\frac{1-\tan^2\theta}{\sec^2\theta+2\tan\theta}$ $=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}+\frac{2\sin\theta}{\cos\theta}}$ $=\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{1+2\sin\theta\cos\theta}{\cos^2\theta}}$ $=\frac{\cos^2\theta-\sin^2\theta}{1+2\sin\theta\cos\theta}$ $=\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}$ $=\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)^2}$ $=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$ $=\text{RHS}$

	OR $\text{LHS}=\dfrac{1-\tan^2\theta}{\sec^2\theta+2\tan\theta}$ $=\dfrac{(1-\tan\theta)(1+\tan\theta)}{1+\tan^2\theta+2\tan\theta}$ $=\dfrac{(1-\tan\theta)(1+\tan\theta)}{(1+\tan\theta)^2}$ $=\dfrac{1-\tan\theta}{1+\tan\theta}$ $=\dfrac{\cos\theta-\sin\theta}{\cos\theta}\times\dfrac{\cos\theta}{\cos\theta+\sin\theta}$ $=\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$
2(ii)	$1-\tan^2\theta=2\sec^2\theta+4\tan\theta$ $1-\tan^2\theta=2\left(\sec^2\theta+2\tan\theta\right)$ $\dfrac{1-\tan^2\theta}{\sec^2\theta+2\tan\theta}=2$ $\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=2$ $\cos\theta-\sin\theta=2\cos\theta+2\sin\theta$ $3\sin\theta=-\cos\theta$ $\tan\theta=-\dfrac{1}{3}$ $\alpha=18.43°$ $\theta=-18.4°,161.6°$
3(i)	Let $f(x)=2x^3-3ax^2-3a^2x+2a^3$ When $x+a$ is the divisor, by Remainder Theorem, Remainder $=f(-a)$ $=2(-a)^3-3a(-a)^2-3a^2(-a)+2a^3$ $=-2a^3-3a^3+3a^3+2a^3$ $=0$ Since the remainder is 0, by Factor theorem, $x+a$ is a factor of $f(x)$.

(ii)

$2x^3-3ax^2-3a^2x+2a^3=(x+a)(2x^2+bx+2a^2)$

Comparing coefficient of x,

$ab+2a^2=-3a^2$

$b=-5a$

Hence $2x^3-3ax^2-3a^2x+2a^3=(x+a)(2x^2-5ax+2a^2)$

$=(x+a)(2x-a)(x-2a)$

$$\frac{3}{2x^3-3ax^2-3a^2x+2a^3}=\frac{A}{x+a}+\frac{B}{2x-a}+\frac{C}{x-2a}$$

$3=A(2x-a)(x-2a)+B(x+a)(x-2a)+C(x+a)(2x-a)$

When $x=-a$,

$A(-3a)(-3a)=3$

$A=\dfrac{1}{3a^2}$

When $x=\dfrac{a}{2}$,

$B\left(\dfrac{3a}{2}\right)\left(-\dfrac{3a}{2}\right)=3$

$B=-\dfrac{4}{3a^2}$

When $x=2a$,

$C(3a)(3a)=3$

$C=\dfrac{1}{3a^2}$

$$\frac{3}{2x^3-3ax^2-3a^2x+2a^3}=\frac{1}{3a^2(x+a)}-\frac{4}{3a^2(2x-a)}+\frac{1}{3a^2(x-2a)}$$

4(a)

$2\log_3 x=2+\log_3(x-3)$

$\log_3 x^2-\log_3(x-3)=2$

$\log_3\left(\dfrac{x^2}{x-3}\right)=2$

$\dfrac{x^2}{x-3}=3^2$

$x^2-9x+27=0$

$a=1, b=-9, c=27$

$b^2-4ac=(-9)^2-4(1)(27)$

$=81-108$

$=-27<0$

Since $b^2-4ac<0,$ there are no real solutions.

	OR $2\log_3 x = 2 + \log_3(x-3)$ $2\log_3 x = 2\log_3 3 + \log_3(x-3)$ $\log_3 x^2 = \log_3\left[9(x-3)\right]$ $x^2 = 9x - 27$ $x^2 - 9x + 27 = 0$ $a = 1,\ b = -9, c = 27$ $b^2 - 4ac = (-9)^2 - 4(1)(27)$ $= 81 - 108$ $= -27 < 0$ Since $b^2 - 4ac < 0,$ there are no real solutions.
4(b)	$(\ln x)(\ln x - \pi - 2) = -2\pi$ $y(y - \pi - 2) = -2\pi$ $y^2 + y(-\pi - 2) + 2\pi = 0$ $(y - \pi)(y - 2) = 0$ $y = \pi$ or $y = 2$ $\ln x - \pi = 0$ or $\ln x - 2 = 0$ $x = e^{\pi}$ or $x = e^2$ **OR** $\ln x(\ln x - \pi - 2) = -2\pi$ $y(y - \pi - 2) = -2\pi$ $y^2 + y(-\pi - 2) + 2\pi = 0$ $y = \dfrac{-(-\pi - 2) \pm \sqrt{(-\pi - 2)^2 - 4(1)(2\pi)}}{2(1)}$ $= \dfrac{\pi + 2 \pm \sqrt{(\pi - 2)^2}}{2}$ $y = \dfrac{\pi + 2 + \pi - 2}{2}$ or $\dfrac{\pi + 2 - \pi + 2}{2}$ $= \pi$ or 2 $\ln x = \pi$ or $\ln x = 2$ $x = e^{\pi}$ or $x = e^2$

5(a)

$$2^{x-y}=\sqrt[4]{\frac{1}{16}}$$

$$2^{x-y}=\left(2^{-4}\right)^{\frac{1}{4}}$$

$$x-y=-1---------(1)$$

$$\frac{5^x}{25^{-y}}=\left(\frac{1}{5}\right)^{-\frac{1}{3}}$$

$$\frac{5^x}{5^{-2y}}=\left(5^{-1}\right)^{-\frac{1}{3}}$$

$$x+2y=\frac{1}{3}---------(2)$$

$$(2)-(1),\ 3y=\frac{4}{3}$$

$$y=\frac{4}{9}$$

$$\text{when } y=\frac{4}{9},\ x-\frac{4}{9}=-1$$

$$x=-\frac{5}{9}$$

$$\therefore x=-\frac{5}{9},\ y=\frac{4}{9}$$

5(b)

Let the other base be b,

$$\frac{1}{2}\left(2+4\sqrt{3}\right)\left(2+\sqrt{3}+b\right)=2+15\sqrt{3}$$

$$\left(1+2\sqrt{3}\right)\left(2+\sqrt{3}+b\right)=2+15\sqrt{3}$$

$$2+\sqrt{3}+b=\frac{2+15\sqrt{3}}{1+2\sqrt{3}}\times\frac{1-2\sqrt{3}}{1-2\sqrt{3}}$$

$$=\frac{\left(2+15\sqrt{3}\right)\left(1-2\sqrt{3}\right)}{(1)^2-\left(2\sqrt{3}\right)^2}$$

$$=\frac{2-4\sqrt{3}+15\sqrt{3}-30(3)}{1-12}$$

$$=\frac{-88+11\sqrt{3}}{-11}$$

$$=8-\sqrt{3}$$

$$b=8-2-\sqrt{3}-\sqrt{3}$$

$$=6-2\sqrt{3}$$

	OR $\frac{1}{2}\left(2+4\sqrt{3}\right)\left(2+\sqrt{3}+a+b\sqrt{3}\right)=2+15\sqrt{3}$ $\left(1+2\sqrt{3}\right)\left[\left(2+a\right)+\left(b+1\right)\sqrt{3}\right]=2+15\sqrt{3}$ $2+a+\left(b+1\right)\sqrt{3}+\left(4+2a\right)\sqrt{3}+6\left(b+1\right)=2+15\sqrt{3}$ $\left(2+a+6b+6\right)+\left(4+2a+b+1\right)\sqrt{3}=2+15\sqrt{3}$ $\left(8+a+6b\right)+\left(5+2a+b\right)\sqrt{3}=2+15\sqrt{3}$ By comparison, $8+a+6b=2$ $a+6b=-6$ $2a+12b=-12----(1)$ $5+2a+b=15$ $2a+b=10------(2)$ $(1)-(2),$ $11b=-22$ $b=-2$ when $b=-2,\ a=-6-6\left(-2\right)$ $=6$ Length of the other parallel base is $\left(6-2\sqrt{3}\right)$cm.
6(i)	$x^2+y^2+8y-84=0$ $\left(x-0\right)^2+\left(y+4\right)^2-16-84=0$ $\left(x-0\right)^2+\left(y+4\right)^2=10^2$ Centre is $\left(0,-4\right)$ and radius is 10 units. **OR** Centre is $\left(0,\frac{8}{-2}\right)=\left(0,-4\right)$ Radius $=\sqrt{0^2+\left(-4\right)^2-\left(-84\right)}=10$ units
6(ii)	Equation of line *AB:*

	$\frac{y+4}{x}=\frac{3}{4}$ $3x=4y+16$ $y=\frac{3}{4}x-4$ $x^2+\left(\frac{3}{4}x-4\right)^2+8\left(\frac{3}{4}x-4\right)-84=0$ $x^2+\frac{9x^2}{16}-6x+16+6x-32-84=0$ $\frac{25}{16}x^2=100$ $x^2=\frac{1600}{25}$ $=64$ $x=\pm 8$ Since $x<0, x=-8$ $y=\frac{3}{4}(-8)-4$ $=-10$ coordinates of A is $(-8,-10)$
(iii)	Radius of $C_3=\frac{10}{2}=5$ units Centre of circle C_2 $=\left(\left(\frac{0-8}{2}\right),\frac{-4-10}{2}\right)$ $=(-4,-7)$ Centre of circle C_3 $(4,-7)$ $(x-4)^2+(y+7)^2=5^2$ or $x^2-8x+y^2+14y+40=0$

7

$$f''(x) = -2x + 2 + 2\cos 2x$$

$$f'(x) = \int (-2x + 2 + 2\cos 2x)\,dx$$

$$= -x^2 + 2x + \sin 2x + c_1$$

Given $f'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} - 1$

$$-\frac{\pi^2}{16} - \frac{\pi}{2} - 1 + c_1 = -\frac{\pi}{2} - 1$$

$$c_1 = \frac{\pi^2}{16}$$

$$\therefore f'(x) = -x^2 + 2x + \sin 2x + \frac{\pi^2}{16}$$

$$f(x) = \int -x^2 + 2x + \sin 2x + \frac{\pi^2}{16}\,dx$$

$$= -\frac{x^3}{3} + x^2 - \frac{\cos 2x}{2} + \frac{\pi^2}{16}x + c_2$$

Given $f(0) = \frac{3}{2}$

$$0 + 0 - \frac{1}{2} + c_2 = \frac{3}{2}$$

$$c_2 = 2$$

$$f(x) = -\frac{x^3}{3} + x^2 - \frac{\cos 2x}{2} + \frac{\pi^2}{16}x + 2$$

$$f\left(\frac{\pi}{2}\right) + \frac{\pi^3}{96} = -\frac{\left(\frac{\pi}{2}\right)^3}{3} + \left(\frac{\pi}{2}\right)^2 - \frac{\cos 2\left(\frac{\pi}{2}\right)}{2} + \frac{\pi^2}{16}\left(\frac{\pi}{2}\right) + 2 + \frac{\pi^3}{96}$$

$$= -\frac{\pi^3}{24} + \frac{\pi^2}{4} + \frac{1}{2} + \frac{\pi^3}{32} + 2 + \frac{\pi^3}{96}$$

$$= -\frac{\pi^3}{96} + \frac{\pi^2}{4} + \frac{5}{2} + \frac{\pi^3}{96}$$

$$= \frac{\pi^2 + 10}{4}$$

8a

$$\text{gradient} = \frac{3-1}{4-2}$$

$$= 1$$

$$\frac{x}{y} = \sqrt{x} + c$$

sub point $(2,1)$,

$$1 = 2 + c$$

$$c = -1$$

$$\frac{x}{y} = \sqrt{x} - 1$$

$$y = \frac{x}{-1 + \sqrt{x}}$$

<u>OR</u>

	$\dfrac{\frac{x}{y}-1}{\sqrt{x}-2}=\dfrac{3-1}{4-2}$ $\dfrac{x}{y}-1=\sqrt{x}-2$ $\dfrac{x}{y}=\sqrt{x}-1$ $y=\dfrac{x}{-1+\sqrt{x}}$
8b(i)	Refer to attached graph
8b(ii)	$m=\dfrac{6-2}{10-2}=\dfrac{1}{2}$ $\ln m=\dfrac{1}{2}t+1$ $m=e^{\frac{1}{2}t+1}$ $=\left(e^{1}\right)\left(e^{\frac{1}{2}t}\right)$ $\therefore A=e, k=\dfrac{1}{2}$
8b(iii)	When $t=0$, $\ln m_0=1$ $m_0=e^1$ When new mass $=$ $m=e+10e$, $\ln 11e=\ln 29.9011$ $=3.397895$ From graph, when $\ln m=3.397895, t=4.8$ $\therefore$ the time taken is about 4.8 ± 0.1 hours

9	when $y=0$, $-\sqrt{10-6x}=0$ $x=\frac{5}{3}$ $\therefore A\left(\frac{5}{3},0\right)$ when $x=-1$, $y=-\sqrt{10-6(-1)}$ $=-4$ $\therefore B(-1,-4)$ $\frac{dy}{dx}=-\left(\frac{1}{2}\right)(10-6x)^{-\frac{1}{2}}(-6)$ $=\frac{3}{\sqrt{10-6x}}$ at B, $\frac{dy}{dx}=\frac{3}{\sqrt{10-6(-1)}}$ $=\frac{3}{4}$ Equation of normal at B: $\frac{y+4}{x+1}=-\frac{4}{3}$ $y=-\frac{4}{3}x-\frac{16}{3}$ at C, $y=0$, $-\frac{4}{3}x-\frac{16}{3}=0$ $x=-4$ $\therefore C(-4,0)$ Area of shaded region $=-\int_{-1}^{\frac{5}{3}}-\sqrt{10-6x}\ dx+\frac{1}{2}(-1-(-4))(4)$ $=\left[\frac{(10-6x)^{\frac{3}{2}}}{\frac{3}{2}(-6)}\right]_{-1}^{\frac{5}{3}}+6$ $=-\frac{1}{9}\left\{[0]-[16]^{\frac{3}{2}}\right\}+6$ $=\frac{64}{9}+6$ $=\frac{118}{9}$ units^2

10(i)	$\angle UTS = \theta$ $RS = QT - UT$ $= 10\sin\theta - 4\cos\theta$ $W = PT + TS + RS + QR + PQ$ $= 10 + 4 + 10\sin\theta - 4\cos\theta + 4\sin\theta + 10\cos\theta$ $= 14 + 14\sin\theta + 6\cos\theta$
10(ii)	$R = \sqrt{14^2 + 6^2} = \sqrt{232} = 2\sqrt{58}$ Max $W = 14 + 2\sqrt{58}$ $= 29.23$ $= 29.2$ m
10(iii)	$A = \frac{1}{2}[10\sin\theta + 10\sin\theta - 4\cos\theta]4\sin\theta$ $= 2\sin\theta[20\sin\theta - 4\cos\theta]$ $= 40\sin^2\theta - 8\sin\theta\cos\theta$ $= 40\sin^2\theta - 4\sin 2\theta$ **<u>OR</u>** $A = 4\sin\theta(10\sin\theta - 4\cos\theta) + \frac{1}{2}(4\sin\theta)(4\cos\theta)$ $= 40\sin^2\theta - 16\sin\theta\cos\theta + 8\sin\theta\cos\theta$ $= 40\sin^2\theta - 8\sin\theta\cos\theta$ $= 40\sin^2\theta - 4\sin 2\theta$
10(iv)	$A = 40\sin^2\theta - 4\sin 2\theta$ $\frac{dA}{d\theta} = 80\sin\theta\cos\theta - 8\cos 2\theta$ $= 40\sin 2\theta - 8\cos 2\theta$ For maximum area, $\frac{dA}{d\theta} = 0$ $40\sin 2\theta - 8\cos 2\theta = 0$ $\tan 2\theta = \frac{8}{40}$ $\alpha = 0.1973955598$ $2\theta = 0.1974$ $\theta = 0.0987$ $\frac{dA}{d\theta} = 40\sin 2\theta - 8\cos 2\theta$ $\frac{d^2A}{d\theta^2} = 80\cos 2\theta + 16\sin 2\theta$ $\frac{d^2A}{d\theta^2} = 81.5843 > 0$ $\therefore \theta = 0.0987$ gives a minimum Area.