Calculator Model:
(if applicable)

Paya Lebar Methodist Girls' School (Secondary)
Preliminary Examination 2021
Secondary 4 Express

Candidate Name: ____________________________ (   )   Class: ________

Centre Number	S				

Index Number				

ADDITIONAL MATHEMATICS 4049/02

Paper 2

30 August 2021

Candidates answer on the Question Paper.

Additional Materials: Writing Paper (1 sheet)

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 **(i)** Differentiate $3xe^{2x+1}$ with respect to x. [2]

(ii) Hence find $\int 9xe^{2x+1}\mathrm{d}x$. [3]

2 **(i)** Prove that $\dfrac{1-\tan^2\theta}{\sec^2\theta+2\tan\theta}=\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$. [4]

(ii) Hence solve the equation $1-\tan^2\theta = 2\sec^2\theta + 4\tan\theta$ for $-180° \le \theta \le 180°$. [4]

3 **(i)** Show that $x+a$ is a factor of $2x^3-3ax^2-3a^2x+2a^3$ where a is a constant. [1]

(ii) Express $\dfrac{3}{2x^3-3ax^2-3a^2x+2a^3}$ as the sum of three partial fractions. [7]

4 **(a)** Express $2\log_3 x = 2 + \log_3(x-3)$ as a quadratic equation in *x* and explain why there are no real solutions. [4]

(b) **Without using a calculator**, solve the equation $(\ln x)(\ln x - \pi - 2) = -2\pi$ using the substitution $y = \ln x$. [3]

5 **(a)** Find the values of x and y which satisfy the equations

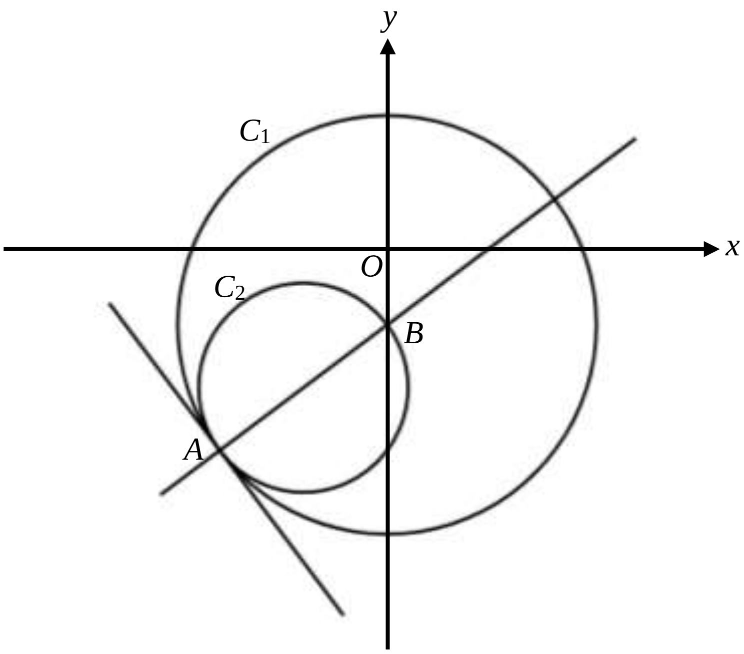
$$2^{x-y} = \sqrt[4]{\frac{1}{16}},$$

$$\frac{5^x}{25^{-y}} = \left(\frac{1}{5}\right)^{-\frac{1}{3}}.$$

[4]

(b) A trapezium of area $\left(2+15\sqrt{3}\right)$ cm^2 has a perpendicular height of $\left(2+4\sqrt{3}\right)$ cm and the length of one of the parallel sides of $\left(2+\sqrt{3}\right)$ cm. **Without using a calculator,** obtain an expression for the length of the other parallel side in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers. [5]

6



The diagram shows two circles C_1 and C_2.
The equation of circle C_1, with centre B, is $x^2 + y^2 + 8y - 84 = 0$.

The tangent to circle C_1 at the point A has a gradient of $-\frac{4}{3}$.

Circle C_2 has diameter AB.

(i) Find the radius of circle C_1 and the coordinates of its center. [2]

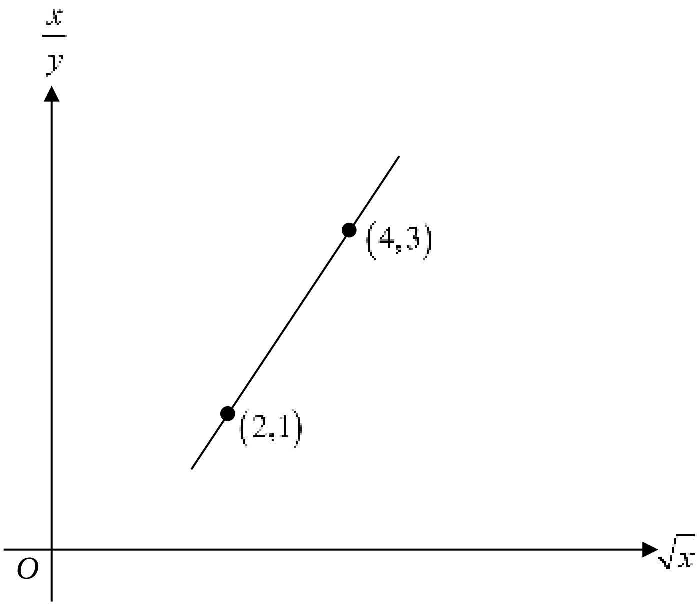
(ii) Find the coordinates of A. [4]

(iii) Find the equation of another circle C_3 which is a reflection of circle C_2 about the y-axis. [3]

7 $\mathrm{f}(x)$ is such that $\mathrm{f}''(x) = -2x + 2 + 2\cos 2x$. Given that $\mathrm{f}'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} - 1$ and $\mathrm{f}(0) = \frac{3}{2}$,

show that $\mathrm{f}\left(\frac{\pi}{2}\right) + \frac{\pi^3}{96} = \frac{\pi^2 + 10}{4}$. [9]

8(a)



The diagram shows part of a straight line graph which passes through $(2,1)$ and $(4,3)$.

Find the equation of the straight line in the form $y=\dfrac{x}{a+b\sqrt{x}}$, where a and b are constants. [3]

8(b) The mass, m grams, of a certain bacteria, t hours after observations began, are recorded in the table below.

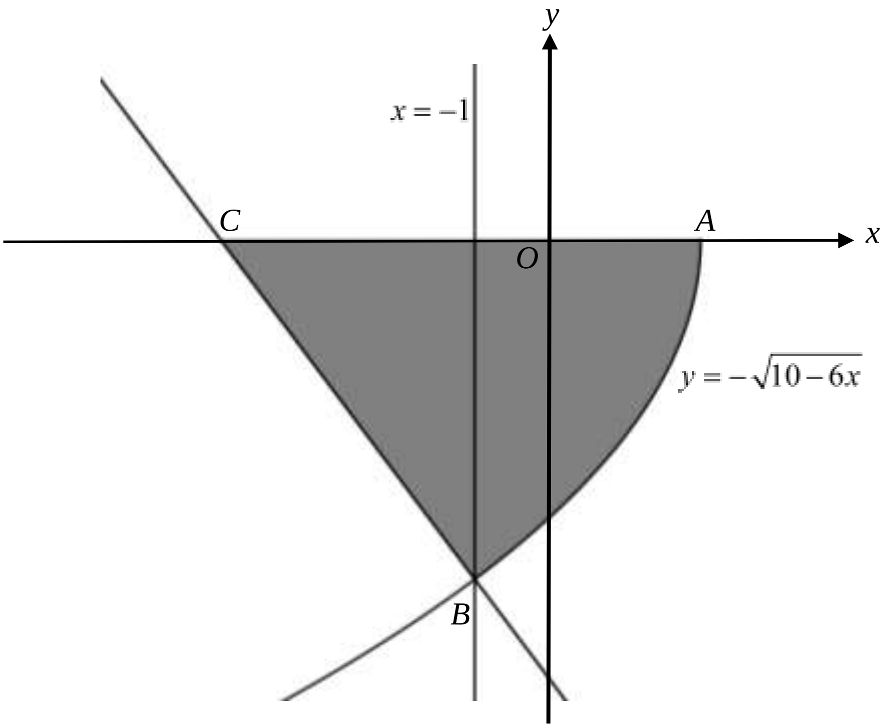
t (hours)	2	4	6	8	10
m (grams)	7.39	20.09	54.60	148.41	403.43

(i) On the grid below plot $\ln m$ against t and draw a straight line graph. [2]

(ii) Find the gradient of your straight line and hence express m in the form Ae^{kt}, where A and k are constants [4]

(iii) Estimate the time taken for the bacteria to gain ten times its original mass. [3]

9

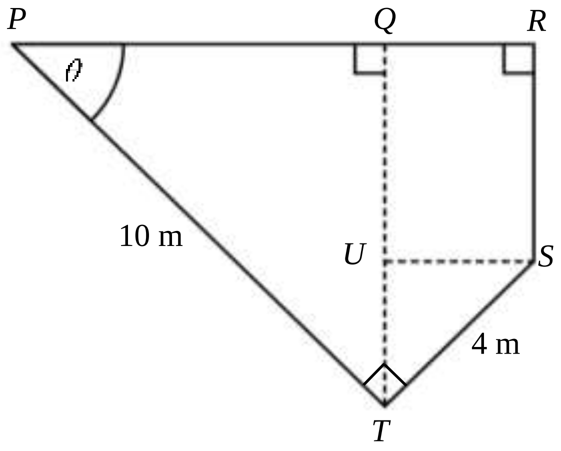


The diagram shows part of the curve $y = -\sqrt{10 - 6x}$ meeting the x-axis at the point A. The line $x = -1$ intersects the curve at the point B. The normal to the curve at B meets the x-axis at the point C.

Find the area of the shaded region. [11]

Continuation of working space for Question 9.

10



The diagram shows a vaccination facility in the shape of a quadrilateral in which angles *PQT*, *PTS* and *PRS* are right angles. *SU* is parallel to *PR*. The lengths of *PT* and *ST* are 10 m and 4 m respectively. The acute angle *QPT* is θ radians.

(i) Show that the perimeter, *W* m, is given by $W = 14 + 14\sin\theta + 6\cos\theta$. [2]

(ii) Find the value of *R* when $14\sin\theta + 6\cos\theta$ is expressed as $R\sin(\theta+\alpha)$, where *R* and α are constants and hence state the maximum perimeter of the vaccination facility. [3]

(iii) Section *QRST* will be converted into a quarantine facility. Show that the area of the quarantine facility, A m^2, is given by $A = 40\sin^2\theta - 4\sin 2\theta$. [2]

(iv) Given that θ can vary, find the value of θ which gives a stationary value of A and determine the nature of this stationary value. [5]

End of Paper

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Qn No	Working
1(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(3xe^{2x+1}\right)=3e^{2x+1}+6xe^{2x+1}$
1(ii)	$\int 9xe^{2x+1}\mathrm{d}x=\frac{9}{2}xe^{2x+1}-\frac{9}{4}e^{2x+1}+c$
2(ii)	$1-\tan^2\theta=2\sec^2\theta+4\tan\theta$ $\tan\theta=-\frac{1}{3}$ $\alpha=18.43°$ $\theta=-18.4°,161.6°$
3(i)	Let $\mathrm{f}(x)=2x^3-3ax^2-3a^2x+2a^3$ Remainder $=\mathrm{f}(-a)=0$ Since the remainder is 0, by Factor theorem, $x+a$ is a factor of $\mathrm{f}(x)$.
(ii)	$\frac{3}{2x^3-3ax^2-3a^2x+2a^3}=\frac{1}{3a^2(x+a)}-\frac{4}{3a^2(2x-a)}+\frac{1}{3a^2(x-2a)}$
4(a)	$2\log_3 x=2+\log_3(x-3)$ $x^2-9x+27=0$ $b^2-4ac=(-9)^2-4(1)(27)$ $=81-108$ $=-27<0$ Since $b^2-4ac<0,$ there are no real solutions.
4(b)	$x=e^{\pi}$ or $x=e^2$
5(a)	$x=-\frac{5}{9},y=\frac{4}{9}$
5(b)	Let the other base be b, $b=6-2\sqrt{3}$
6(i)	Centre is $\left(0,\frac{8}{-2}\right)=(0,-4)$ Radius $=\sqrt{0^2+(-4)^2-(-84)}=10$ units
6(ii)	coordinates of A is $(-8,-10)$
(iii)	Radius of $C_3=\frac{10}{2}=5$ units Centre of circle $C_2=\left(\left(\frac{0-8}{2}\right),\frac{-4-10}{2}\right)$ $=(-4,-7)$ Centre of circle C_3 $(4,-7)$

	$(x-4)^2+(y+7)^2=5^2$ or $x^2-8x+y^2+14y+40=0$
7	$f'(x)=-x^2+2x+\sin 2x+\dfrac{\pi^2}{16}$ $f(x)=-\dfrac{x^3}{3}+x^2-\dfrac{\cos 2x}{2}+\dfrac{\pi^2}{16}x+2$ $f\left(\dfrac{\pi}{2}\right)+\dfrac{\pi^3}{96}=\dfrac{\pi^2+10}{4}$
8a	$y=\dfrac{x}{-1+\sqrt{x}}$
8b(i)	Refer to attached graph
8b(ii)	$A=e, k=\dfrac{1}{2}$
8b(iii)	the time taken is about 4.6 ± 0.1 hours
9	$A\left(\dfrac{5}{3},0\right)$ $B(-1,-4)$ $\dfrac{dy}{dx}=\dfrac{3}{\sqrt{10-6x}}$ at $B, \dfrac{dy}{dx}=\dfrac{3}{4}$ Equation of normal at B: $y=-\dfrac{4}{3}x-\dfrac{16}{3}$ $C(-4,0)$ Area of shaded region $=-\int_{-1}^{\frac{5}{3}}-\sqrt{10-6x}\ dx+\dfrac{1}{2}(-1-(-4))(4)$ $=\dfrac{118}{9}$ units^2
10(i)	$\angle UTS=\theta$ $RS=QT-UT$ $=10\sin\theta-4\cos\theta$ $W=PT+TS+RS+QR+PQ$ $=10+4+10\sin\theta-4\cos\theta+4\sin\theta+10\cos\theta$ $=14+14\sin\theta+6\cos\theta$
10(ii)	$R=2\sqrt{58}$ Max $W=29.2$ m

10(iii)	$A = 40\sin^2\theta - 4\sin 2\theta$
10(iv)	$A = 40\sin^2\theta - 4\sin 2\theta$ $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 40\sin 2\theta - 8\cos 2\theta$ $\theta = 0.0987$ $\frac{\mathrm{d}^2 A}{\mathrm{d}\theta^2} = 80\cos 2\theta + 16\sin 2\theta$ $\frac{\mathrm{d}^2 A}{\mathrm{d}\theta^2} = 81.5843 > 0$ $\therefore \theta = 0.0987$ gives a minimum Area.