Calculator Model: *(if applicable)*

Paya Lebar Methodist Girls' School (Secondary)
Preliminary Examination 2021
Secondary 4 Express

Name: ____________________________ (   )   Class: ________

Centre Number	S				

Index Number				

ADDITIONAL MATHEMATICS — 4049/01

Paper 1 — 27 August 2021

Candidates answer on the Question Paper. — 2 hours 15 mintues

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number, name and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 x is the principal value of $\tan^{-1}\left(-\frac{3}{4}\right)$. **Without using a calculator**, find the value of

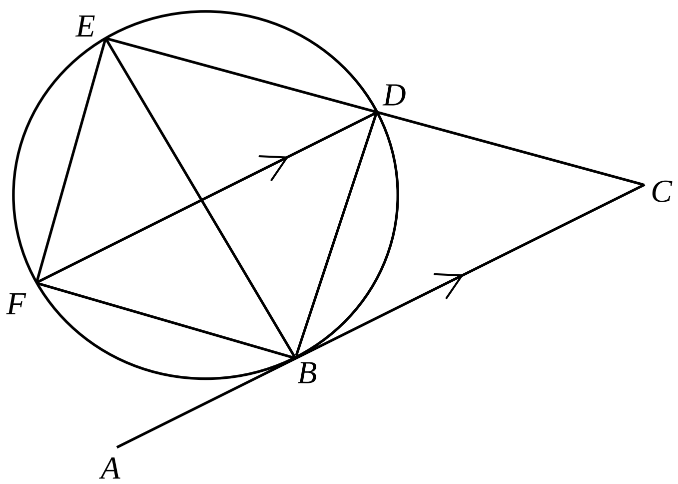
(i) $\cos x$, [1]

(ii) $\sin\frac{x}{2}$. [2]

2 Solve the equation $3^{2x^2} - 28\left(3^{x^2+1}\right) + 243 = 0$. [4]

3 Find the set of values of the constant m for which the line $y = mx + 1$ does not meet the curve $y = \dfrac{x^2 - 2x + 1}{x - 2}$. Express your answer in exact form. [5]

4



The diagram shows a circle passing through the points B, D, E and F. The straight line ABC is a tangent to the circle. CDE is a straight line and FD is parallel to AC. Prove that EB bisects angle DEF. [4]

5 The surface area of a spherical balloon is decreasing at the rate of 24 cm^2/s.

(i) If the radius of the balloon at time *t* seconds is *r* cm, find an expression, in terms of *r*, for the rate of change of the radius. [3]

(ii) Find the rate of decrease in the volume of the balloon when the radius is 10 cm. [3]

6 A town has a population of 95 000. During a pandemic, the total number of people, N, infected t days after the virus has begun, is modelled by $N = \dfrac{95\ 000}{2 + 498e^{-0.6t}}$.

(i) Find the number of people who were initially infected by the virus. [2]

(ii) The authority decided to issue home quarantine orders when 8% of the population are infected. After how many days will the authority issue the order? [3]

Continuation of working space for question **6(ii)**.

(iii) Explain based on the model whether all 95 000 people will become infected after a long period of time. [2]

7 **(a)** **(i)** Write down and simplify the first four terms in the expansion, in ascending powers of x, of $(2-3x)^5$. [2]

(ii) Hence find the coefficient of x^3 in the expansion of $(1+2x^2)(2-3x)^5$. [2]

(b) Every term in the expansion of $\left(x - \frac{1}{2x^2}\right)^n$ is **dependent** on x.

By considering the general term in the binomial expansion of $\left(x - \frac{1}{2x^2}\right)^n$, give two comments on the possible values of n. [4]

8 Megan runs back and forth along a straight line as she trains for her physical fitness test. Her velocity, v m/s, t seconds after passing a fixed point A, is given by $v = 9.2\sin 0.02t$, where $0 \le t \le 300$.

(i) State Megan's maximum speed. [1]

(ii) Find the time when Megan makes a turn after passing A. [2]

(iii) Find the total distance Megan covered for her entire run. [3]

Continuation of working space for question **8(iii)**.

(iv) Find Megan's acceleration when $t = 200$. [2]

9 The equation of a curve is $y = x + \dfrac{1}{4x}$.

(i) Find the values of x for which y is increasing. [4]

(ii) Find the x-coordinates of each of the stationary points of the curve. [2]

(iii) **Using the first derivative test**, find the nature of each stationary point. [3]

10 **(a)** State the values between which the principal value of $\cos^{-1} x$ lie. [1]

(b) It is given that $f(x) = \sin 2x$ and $g(x) = \frac{1}{2}\cos x$.

(i) State the period of $f(x)$. [1]

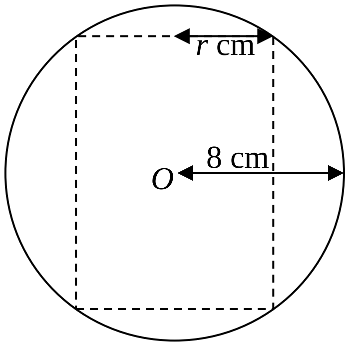
(ii) State the period of $g(x)$. [1]

(iii) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$. [4]

(iv) Hence, state the number of solutions of the equation $4\sin 2x = 2\cos x$ for $0 \le x \le 2\pi$. [1]

(v) The number of solutions of the equation $\sin 2x = \frac{1}{2}\cos x - m$, where m is a constant, for $0 \le x \le 2\pi$ is exactly 5. Find the value of m. [1]

11 The diagram shows a container in the form of a sphere, radius 8 cm. A right cylinder with base radius r cm, is inscribed in the sphere. The circumferences of the circular ends of the cylinder are in contact with the inner surface of the sphere. The **curved surface area** of the cylinder is A cm^2.



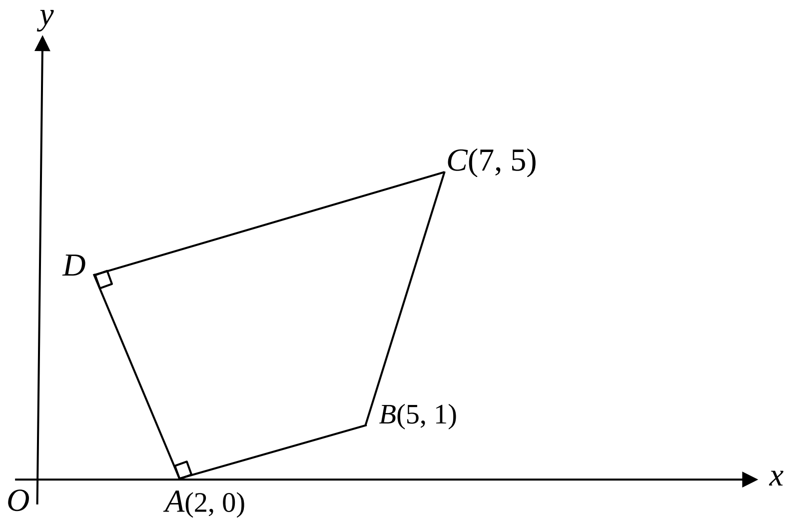
(i) Show the height of the cylinder is $2\sqrt{64-r^2}$ cm. [2]

(ii) Show that $\dfrac{dA}{dr} = \dfrac{8\pi\left(32 - r^2\right)}{\sqrt{64 - r^2}}$. [4]

(iii) Given that r can vary, show that the curved surface area of the cylinder is maximum when the height of the cylinder is equal to its diameter. [3]

12 Given that $f'(x) = e^{-2x} + \sin 4x$ and $f(0) = \frac{1}{4}$, show that $f''(x) - 4f(x) - 1 = -10\sin^2 2x$. [8]

13



The diagram shows a quadrilateral *ABCD* with vertices $A(2, 0)$, $B(5, 1)$ and $C(7, 5)$.

Angle BAD = angle $ADC = 90°$.

(i) Find the coordinates of *D*. [5]

(ii) A point E lies on CD such that $ABED$ is a square. Find the coordinates of E. [2]

(iii) Find the area of the quadrilateral $ABCD$. [2]

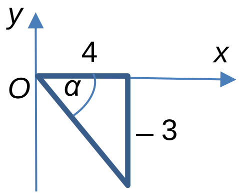
(iv) A point X on the x-axis is such that angle BAX = angle BXA. Find the equation of BX. [1]

End of Paper

BLANK PAGE

<table>
<tr><th>Qn No</th><th>Working</th></tr>
<tr><td>1(i)</td><td>

$\cos x = \frac{4}{5}$



<u>ALTERNATIVE</u>

$\sec^2 x = 1 + \tan^2 x$

$\sec x = \sqrt{1 + \left(-\frac{3}{4}\right)^2}$

$= \sqrt{\frac{25}{16}}$

$= \frac{5}{4}$

$\therefore \cos x = \frac{4}{5}$

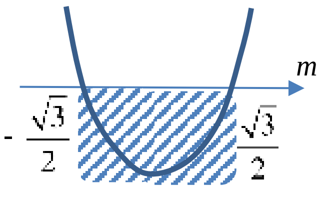
</td></tr>
<tr><td>1(ii)</td><td>

$\sin \frac{x}{2} = -\frac{1}{\sqrt{10}}$

</td></tr>
<tr><td>2</td><td>

$x = \pm 2$ or $x = \pm 1$

</td></tr>
<tr><td>3</td><td>



$\therefore -\frac{\sqrt{3}}{2} < b < \frac{\sqrt{3}}{2}$

</td></tr>
<tr><td>5(i)</td><td>

$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{-3}{\pi r}$ cm/s

</td></tr>
<tr><td>5(ii)</td><td>

$\frac{\mathrm{d}V}{\mathrm{d}t} = -120 \text{ cm}^3/\text{s}$

$\therefore$ Rate of decrease in volume is 120 cm^3/s.

</td></tr>
<tr><td>6(i)</td><td>

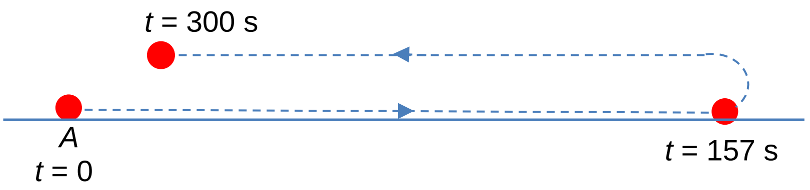
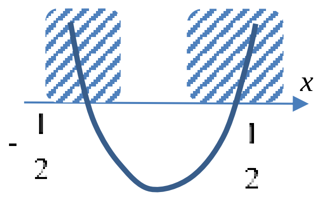
When $t = 0$,
$N = 190$
Initially, there are 190 people who were infected by the virus.

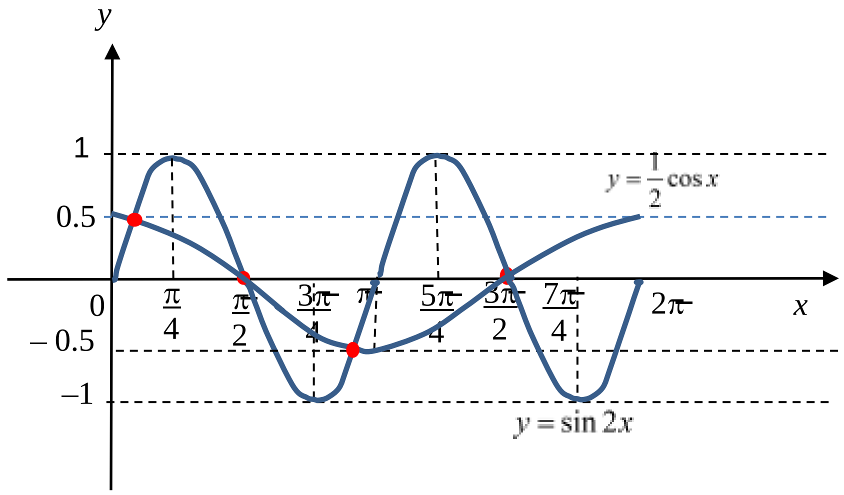
</td></tr>
<tr><td>6(ii)</td><td>

When $N = 0.08(95\,000) = 7600$,

$t \geq 6.432041366$

</td></tr>
</table>

	The authority will issue home quarantine orders after 7 days.
6(ii)	When t is very large, $e^{-0.6t}$ approaches 0, Hence, all 95 000 people will never be infected after a long period of time.
7(a)(i)	$(2-3x)^5 = 32 + 5(16)(-3x) + 10(8)(9x^2) + 10(4)(-27x^3) + \cdots$ $= 32 - 240x + 720x^2 - 1080x^3 + \ldots$
7(a)(ii)	coefficient of $x^3 = -1560$
7(b)	n is any positive integer which is not a multiple of 3.
8(i)	Maximum speed is 9.2 m/s
8(ii)	Megan made a turn at $50\pi = 157$ seconds. (3.s.f)
8(iii)	Megan only made 1 turn in her run at 50π s and completed her run at 300s.  Total dist = 1820 m
8(iv)	When $t = 200$ s, $a = -0.120$ m/s^2 (3 s.f)
9(i)	$y = x + \frac{1}{4x}$ $\frac{dy}{dx} = 1 - \frac{1}{4x^2}$ For y to an increasing function, $\frac{dy}{dx} > 0$.  $\therefore x < -\frac{1}{2}$ or $x > \frac{1}{2}$
9(ii)	For any stationary points, $\frac{dy}{dx} = 0$ $x = \pm\frac{1}{2}$
9(iii)	

	x	0.5^-	0.5	0.5^+	x	-0.5^-	-0.5	-0.5^+
	Sign of $\frac{dy}{dx}$	–	0	+	Sign of $\frac{dy}{dx}$	+	0	–
	Slope	\	–	/	Slope	/	–	\
	$\frac{dy}{dx}$ changes from – to + as x increases through 0.5, hence $x=\frac{1}{2}$ is a minimum point.				$\frac{dy}{dx}$ changes from + to – as x increases through – 0.5, hence $x=-\frac{1}{2}$ is a maximum point.			
10(a)	$0 \le \cos^{-1} x \le \pi$							
10(b)(i)	period of $g(x) = \pi$ or 180°							
10(b)(ii)	period of $f(x) = 2\pi$ or 360°							
10(b)(iii)								
10(b)(iv)	From the sketch, No. of solutions = 4							
10(b)(v)	$m = 0.5$							
11(ii)	Curved surface area of cylinder, $A = 2\pi rh$ $A = 2\pi r\left(2\sqrt{64-r^2}\right)$ $A = 4\pi r\sqrt{64-r^2}$ $\frac{dA}{dr} = (4\pi)\left(\sqrt{64-r^2}\right)+(4\pi r)\left[\frac{1}{2}\left(64-r^2\right)^{-\frac{1}{2}}(-2r)\right]$							
11(iii)	when $2\sqrt{64-r^2} = 2r$ $r = \sqrt{32} = 4\sqrt{2}$ cm							

r	$4\sqrt{2}^-$	$4\sqrt{2}$	$4\sqrt{2}^+$
Sign of $\frac{dA}{dr}$	+	0	–

<table>
<tr><td></td><td>
<table>
<tr><td>Slope</td><td>/</td><td>–</td><td>\</td></tr>
<tr><td colspan="4">$\frac{dA}{dr}$ changes from – to + as r increases through $4\sqrt{2}$, hence
$r = 4\sqrt{2}$ is a maximum point.</td></tr>
</table>

Therefore, maximum curved surface area of the cylinder,
A cm^2 is obtained when the height of the cylinder is equal to its diameter. (shown)</td></tr>
<tr><td>12</td><td>$f''(x) = -2e^{-2x} + 4\cos 4x$
$f(x) = -\frac{1}{2e^{2x}} - \frac{1}{4}\cos 4x + 1$</td></tr>
<tr><td>13(i)</td><td>equation of line DC is $y = \frac{1}{3}x + \frac{8}{3}$. ....(1)
equation of line AD is $y = -3x + 6$. ....(2)
$\therefore D(1,3)$</td></tr>
<tr><td>13(ii)</td><td>Let $E(x, y)$
$(3,2) = \left(\frac{x+2}{2}, \frac{y}{2}\right)$
$\therefore E(4,4)$</td></tr>
<tr><td>13(iii)</td><td>$A_{ABCD} = \frac{1}{2}\begin{vmatrix} 2 & 5 & 7 & 1 & 2 \\ 0 & 1 & 5 & 3 & 0 \end{vmatrix}$
$= 15$ units^2</td></tr>
<tr><td>13(iv)</td><td>equation of line BX is $y = -\frac{1}{3}x + \frac{8}{3}$.</td></tr>
</table>