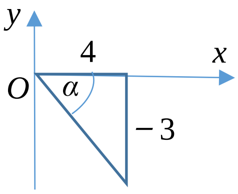
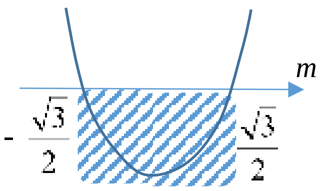
Preliminary Examination 2021
Secondary 4 Express
Additional Mathematics

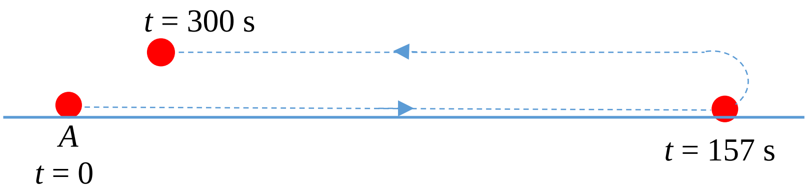
Qn No	Working	Total
1(i)	Given $x=\tan^{-1}\left(-\frac{3}{4}\right)$ $\therefore\ \tan x=-\frac{3}{4}$ Since principal values are $-90^\circ<\tan^{-1}x<90^\circ$, hence angle in 4th quad. $\cos x=\frac{4}{5}$  <u>ALTERNATIVE</u> $\sec^2 x=1+\tan^2 x$ $\sec x=\sqrt{1+\left(-\frac{3}{4}\right)^2}$ $=\sqrt{\frac{25}{16}}$ $=\frac{5}{4}$ $\therefore \cos x=\frac{4}{5}$	1
1(ii)	$\because \cos x=1-2\sin^2\frac{x}{2}$ $\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$ Sub $\cos x=\frac{4}{5},\therefore$ $\sin\frac{x}{2}=\pm\sqrt{\frac{1-\frac{4}{5}}{2}}$ $\sin\frac{x}{2}=\pm\sqrt{\frac{1}{10}}$ $\sin\frac{x}{2}=\frac{1}{\sqrt{10}}$ (reject) or $\sin\frac{x}{2}=-\frac{1}{\sqrt{10}}$ $=-\frac{\sqrt{10}}{10}$	2

2	$3^{2x^2} - 28\left(3^{x^2+1}\right) + 243 = 0$ $\left(3^{x^2}\right)^2 - 28\left(3^{x^2}\right)(3) + 243 = 0$ $\left(3^{x^2}\right)^2 - 84\left(3^{x^2}\right) + 243 = 0$ Let $y = 3^{x^2}$, $y^2 - 84y + 243 = 0$ $(y-81)(y-3) = 0$ $(y-81) = 0$ or $(y-3) = 0$ $y = 81$ $y = 3$ Hence, $3^{x^2} = 81$ or $3^{x^2} = 3$ $3^{x^2} = 3^4$ By comparison, $x^2 = 4$ or $x^2 = 1$ $x = \pm 2$ or $x = \pm 1$	4
3	$y = mx + 1$ .... (1) $y = \dfrac{x^2 - 2x + 1}{x - 2}$ .... (2) (1) = (2), $mx + 1 = \dfrac{x^2 - 2x + 1}{x - 2}$ $(mx+1)(x-2) = x^2 - 2x + 1$ $mx^2 - 2mx + x - 2 = x^2 - 2x + 1$ $(m-1)x^2 + (3-2m)x - 3 = 0$ $b^2 - 4ac = (3-2m)^2 - 4(m-1)(-3)$ $= 9 - 12m + 4m^2 + 12m - 12$ $= 4m^2 - 3$ Since curve doesn't meet line, there's no real roots. $\therefore b^2 - 4ac < 0$ Hence, $4m^2 - 3 < 0$ $(2m)^2 - \left(\sqrt{3}\right)^2 < 0$ $\left(2m - \sqrt{3}\right)\left(2m + \sqrt{3}\right) < 0$  $\therefore -\dfrac{\sqrt{3}}{2} < m < \dfrac{\sqrt{3}}{2}$	5

4	$\angle ABF = \angle FEB = \angle y$ (Alt seg theoreom) $\angle CBD = \angle BED = \angle x$ (Alt seg theoreom) $\angle CBD = \angle FDB = \angle x$ (Alt angles, $FD // AC$) $\angle FEB = \angle BED$ ($\angle$s in the same seg) $= \angle CBD$ i.e. $\angle y = \angle x$ $\because \angle FEB = \angle CBD$, hence, BE bisects $\angle DEF$.	4
5(i)	Given $\frac{dA}{dt} = -24$ cm^2/s, $A = 4\pi r^2$ $\frac{dA}{dr} = 8\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $-24 = 8\pi r \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{-24}{8\pi r}$ $\frac{dr}{dt} = \frac{-3}{\pi r}$ cm/s	3
5(ii)	$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi r^2 \times \left(\frac{-3}{\pi r}\right)$ $\frac{dV}{dt} = -12r$ When $r = 10$ cm, $\frac{dV}{dt} = -120$ cm^3/s $\therefore$ Rate of decrease in volume is 120 cm^3/s.	3
6(i)	When $t = 0$,	2

	$N=\dfrac{95\ 000}{2+498e^{-0.6(0)}}$ $N=\dfrac{95\ 000}{2+498(1)}$ $N=190$ Initially, there are 190 people who were infected by the virus.	
6(ii)	When $N = 0.08(95\ 000) = 7600$, $\dfrac{95\ 000}{2+498e^{-0.6t}}\geq 7600$ $95\ 000\geq 7600\left(2+498e^{-0.6t}\right)$ $\dfrac{95\ 000}{7600}\geq 2+498e^{-0.6t}$ $10.5\geq 498e^{-0.6t}$ $e^{-0.6t}\leq \dfrac{7}{332}$ $\ln e^{-0.6t}\leq \ln\dfrac{7}{332}$ $-0.6t\leq \ln\dfrac{7}{332}$ $t\geq \dfrac{\ln\frac{7}{332}}{-0.6}$ $t\geq 6.432041366$ The authority will issue home quarantine orders after 7 days.	3
6(ii)	When t is very large, $e^{-0.6t}=0$, $N=\dfrac{95\ 000}{2+0}=47500$ Hence, all 95 000 people will never be infected after a long period of time.	2
7(a)(i)	$(2-3x)^5=32+5(16)(-3x)+10(8)(9x^2)+10(4)(-27x^3)+\cdots$	2

	$= 32 - 240x + 720x^2 - 1080x^3 + \ldots$	
7(a)(ii)	$\left(1+2x^2\right)\left(2-3x\right)^5$ $=\left(1+2x^2\right)\left(32 - 240x + 720x^2 - 1080x^3 + \ldots\right)$ $= 2x^2\left(-240x\right)+1\left(-1080x^3\right)+ \ldots$ $=-480x^3 - 1080x^3 + \ldots$ $=-1560x^3 + \ldots$ coefficient of $x^3 = -1560$	2
7(b)	$\left(x-\frac{1}{2x^2}\right)^n$ General Term $=\binom{n}{r}\left(x\right)^{n-r}\left(-\frac{1}{2x^2}\right)^r$ $=\binom{n}{r}\left(x\right)^{n-r}\left(-\frac{1}{2}\right)^r\left(x^{-2}\right)^r$ $=\binom{n}{r}\left(-\frac{1}{2}\right)^r\left(x\right)^{n-r-2r}$ $=\binom{n}{r}\left(-\frac{1}{2}\right)^r\left(x\right)^{n-3r}$ Since the expansion is dependent of x, $\therefore n-3r \neq 0$. Hence, $n \neq 3r$ Therefore, n is any positive integer which is not a multiple of 3.	4
8(i)	Maximum speed is 9.2 m/s	1

8(ii)	$v = 9.2\sin 0.02t$ $\sin 0.02t = 0$ $0.02t = 0, \pi, 2\pi, \ldots$ $t = 0, 50\pi, 100\pi$ Since $0 \le t \le 300$, Megan made a turn at $50\pi = 157$ seconds. (3.s.f)	2
8(iii)	Megan only made 1 turn in her run at 50π s and completed her run at 300s.  $s = \int v$ $s = \int (9.2\sin 0.02t)\ \mathrm{d}t$ $s = -460\cos 0.02t + c$ when $t = 0$, $s = 0$. $0 = -460\cos 0 + c$ $\therefore\ c = 460$ Hence, $s = -460\cos 0.02t + 460$ when $t = (50\pi)$ s, $s = -460\cos 0.02(50\pi) + 460$ $s = -460(-1) + 460$ $s = 920$ m when $t = (300 - 50\pi)$ s, $s = -460\cos 0.02(300 - 50\pi) + 460$ $s = 441.67833 + 460$ $s = 901.6783319$ m Total $s = 901.6783319 + 920 = 1821.678$ m Total dist = 1820 m **<u>Alternative</u>** When t = 300, $s = -460\cos[0.02(300)] + 460 = 18.321668$ m Dist for 2nd phase = 920 – 18.321668 = 901.6783319 m **<u>Alternative</u>**	3

	$s = \int_0^{50\pi} 9.2\sin 0.02t\,dt + \left\|\int_{50\pi}^{300} 9.2\sin 0.02t\,dt\right\|$ $= \left[-\frac{9.2\cos 0.02t}{0.02}\right]_0^{50\pi} + \left[-\frac{9.2\cos 0.02t}{0.02}\right]_{50\pi}^{300}$ $= [-460\cos 0.02t]_0^{50\pi} + [-460\cos 0.02t]_{50\pi}^{300}$ $= [-460\cos 0.02(50\pi) + 460\cos 0]$ $+ \ \|-460\cos 0.02(300) + 460\cos 0.02(50\pi)\|$ $= (460 + 460) + \|-441.67833 - 460\|$ $= 920 + 901.67833$ $= 1821.67833$ $= 1820\ \text{m}$ Total dist = 1820 m	
8(iv)	$a = \frac{dv}{dt}$ $a = \frac{d}{dt}9.2\sin 0.02t$ $a = 9.2(0.02)\cos 0.02t$ $a = 0.184\cos 0.02t$ or $a = \frac{23}{125}\cos\frac{t}{50}$ When t = 200 s, $a = -0.1202704262$ $a = -0.120\ \text{m/s}^2$ (3 s.f)	2
9(i)	$y = x + \frac{1}{4x}$	4

$$\frac{dy}{dx} = 1 + \frac{1}{4}(-1)\left(x^{-2}\right)$$

$$\frac{dy}{dx} = 1 - \frac{1}{4x^2}$$

For y to an increasing function, $\frac{dy}{dx} > 0$.

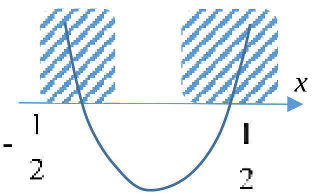
Hence, $1 - \frac{1}{4x^2} > 0$

$$1 > \frac{1}{4x^2}$$

$$4x^2 > 1$$

$$4x^2 - 1 > 0$$

$$(2x-1)(2x-1) > 0$$



$\therefore x < -\frac{1}{2}$ or $x > \frac{1}{2}$

9(ii)

For any stationary points, $\frac{dy}{dx} = 0$

$$1 - \frac{1}{4x^2} = 0$$

$$1 = \frac{1}{4x^2}$$

$$4x^2 = 1$$

$$x^2 = \frac{1}{4}$$

$$x = \pm\frac{1}{2}$$

2

9(iii)

x	0.5^-	0.5	0.5^+
Sign of $\frac{dy}{dx}$	–	0	+
Slope	\	–	/

$\frac{dy}{dx}$ changes from – to + as x increases through 0.5, hence $x = \frac{1}{2}$ is a minimum point.

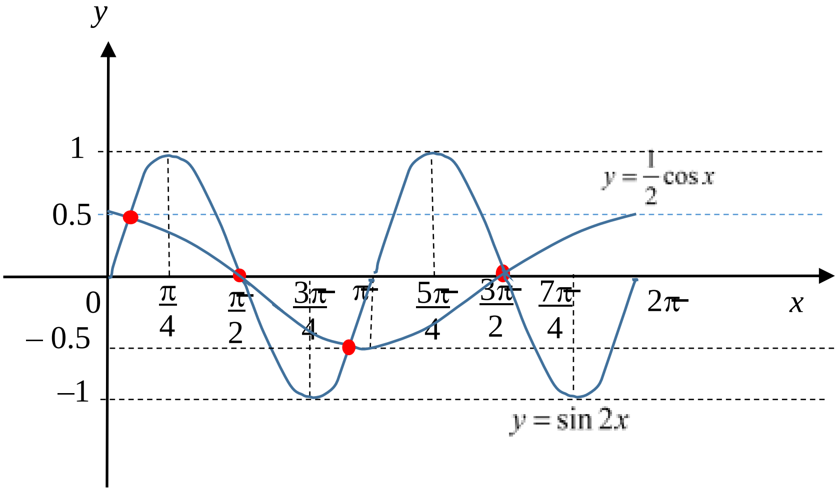
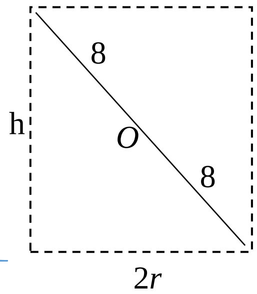
x	-0.5^-	-0.5	-0.5^+
Sign of $\frac{dy}{dx}$	+	0	–
Slope	/	–	\

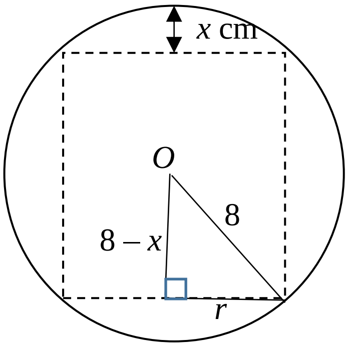
$\frac{dy}{dx}$ changes from + to – as x increases through – 0.5, hence $x = -\frac{1}{2}$ is a maximum point.

3

10(a) $0 \le \cos^{-1} x \le \pi$ 1

10(b)(i) period of $g(x) = \pi$ or 180^o 1

10(b)(ii)	period of $\mathrm{f}(x) = 2\pi$ or 360°	1
10(b)(iii)		4
10(b)(iv)	$4\sin 2x = 2\cos x$ $\therefore \sin 2x = \frac{1}{2}\cos x$ From the sketch, No. of solutions = 4	1
10(b)(v)	When g(x) is shifted down by 0.5 units, it intersects f(x) 5 times. Hence, m = 0.5	1
11(i)	Let the ht bet the of cylinder be h cm. $16^2 = h^2 + (2r)^2$ $h = \sqrt{256 - 4r^2}$ $h = \sqrt{4(64 - r^2)}$ $h = 2\sqrt{64 - r^2}$ cylinder's ht = $2\sqrt{64 - r^2}$ cm (shown)  <u>Alternative</u> By Pythagoras' Theorem,	2

	$\frac{1}{2}\text{ht} = \sqrt{8^2 - r^2}$ cylinder's ht = $2\sqrt{64 - r^2}$ cm (shown) <u>Alternative</u> Let the ht bet the top of cylinder and sphere be x cm. cylinder's ht = $2(8) - 2x$ $= (16 - 2x)$ cm  $(8 - x)^2 + r^2 = 8^2$ $(8 - x)^2 = 8^2 - r^2$ $8 - x = \sqrt{8^2 - r^2}$ $\therefore x = 8 - \sqrt{8^2 - r^2}$ cylinder's ht $= (16 - 2x)$ cm $= 16 - 2\left(8 - \sqrt{8^2 - r^2}\right)$ $= 16 - 16 + 2\sqrt{64 - r^2}$ $= 2\sqrt{64 - r^2}$ cm (shown)	
11(ii)	Curved surface area of cylinder, $A = 2\pi rh$ $A = 2\pi r\left(2\sqrt{64 - r^2}\right)$ $A = 4\pi r\sqrt{64 - r^2}$ $\frac{dA}{dr} = (4\pi)\left(\sqrt{64 - r^2}\right) + (4\pi r)\left[\frac{1}{2}\left(64 - r^2\right)^{-\frac{1}{2}}(-2r)\right]$ $\frac{dA}{dr} = 4\pi\left(64 - r^2\right)^{\frac{1}{2}} - 4\pi r^2\left(64 - r^2\right)^{-\frac{1}{2}}$ $\frac{dA}{dr} = 4\pi\left(64 - r^2\right)^{-\frac{1}{2}}\left[\left(64 - r^2\right) - r^2\right]$ $\frac{dA}{dr} = \frac{4\pi\left(64 - 2r^2\right)}{\left(64 - r^2\right)^{\frac{1}{2}}}$ $\frac{dA}{dr} = \frac{4\pi(2)\left(32 - r^2\right)}{\left(64 - r^2\right)^{\frac{1}{2}}}$ $\frac{dA}{dr} = \frac{8\pi\left(32 - r^2\right)}{\sqrt{64 - r^2}}$ (shown)	4

<table>
<tr>
<td>11(iii)</td>
<td>
when $2\sqrt{64-r^2}=2r$
$\sqrt{64-r^2}=r$
$64-r^2=r^2$
$2r^2=64$
$r^2=32$
$r=\sqrt{32}=4\sqrt{2}$ cm

<table>
<tr><td>r</td><td>$4\sqrt{2}^-$</td><td>$4\sqrt{2}$</td><td>$4\sqrt{2}^+$</td></tr>
<tr><td>Sign of $\dfrac{dA}{dr}$</td><td>+</td><td>0</td><td>–</td></tr>
<tr><td>Slope</td><td>/</td><td>–</td><td>\</td></tr>
<tr><td colspan="4">$\dfrac{dA}{dr}$ changes from – to + as r increases through $4\sqrt{2}$, hence $r=4\sqrt{2}$ is a maximum point.</td></tr>
</table>

Therefore, maximum curved surface area of the cylinder,
A cm^2 is obtained when the height of the cylinder is equal to its diameter. (shown)
</td>
<td>3</td>
</tr>
<tr>
<td>12(iii)</td>
<td><u>Alternative</u></td>
<td>3</td>
</tr>
</table>

	let $\frac{dA}{dr} = 0$, $8\pi\left(32 - r^2\right)\left(64 - r^2\right)^{-\frac{1}{2}} = 0$ $32 - r^2 = 0$ $r = \sqrt{32} = 4\sqrt{2}$ cm (r > 0) when $r = \sqrt{32}$, $h = 2\sqrt{64 - \left(\sqrt{32}\right)^2}$ $h = 2\sqrt{64 - 32}$ $h = 2\sqrt{32}$ $h = 2r$ $\frac{d^2A}{dr^2} = 8\pi\left[\left(32 - r^2\right)\left(-\frac{1}{2}\right)\left(64 - r^2\right)^{-\frac{3}{2}} + \left(-2r\right)\left(64 - r^2\right)^{-\frac{1}{2}}\right]$ $\frac{d^2A}{dr^2} = -\frac{8}{2}\pi\left(64 - r^2\right)^{-\frac{3}{2}}\left[\left(32 - r^2\right) + 4r\left(64 - r^2\right)\right]$ $\frac{d^2A}{dr^2} = -4\pi\left(64 - r^2\right)^{-\frac{3}{2}}\left[32 - r^2 + 256r - 4r^3\right]$ $\frac{d^2A}{dr^2} = \frac{-4\pi\left(-4r^3 - r^2 + 256r + 32\right)}{\sqrt{\left(64 - r^2\right)^3}}$ $\frac{d^2A}{dr^2} = -50.26548246$ $< 0 \quad (\therefore \text{ max } A)$ Therefore, maximum curved surface area of the cylinder, A cm^2 is obtained when the height of the cylinder is equal to its diameter. (shown)	
12(iii)	<u>Alternative</u>	3

	$A = 4\pi r\sqrt{64-r^2}$ $\frac{dA}{dr} = 8\pi\left(32-r^2\right)\left(64-r^2\right)^{-\frac{1}{2}}$ $\frac{d^2A}{dr^2} = 8\pi\left[\left(32-r^2\right)\left(-\frac{1}{2}\right)\left(64-r^2\right)^{-\frac{3}{2}} + \left(-2r\right)\left(64-r^2\right)^{-\frac{1}{2}}\right]$ $\frac{d^2A}{dr^2} = -\frac{8}{2}\pi\left(64-r^2\right)^{-\frac{3}{2}}\left[\left(32-r^2\right)+4r\left(64-r^2\right)\right]$ $\frac{d^2A}{dr^2} = -4\pi\left(64-r^2\right)^{-\frac{3}{2}}\left[32-r^2+256r-4r^3\right]$ $\frac{d^2A}{dr^2} = \frac{-4\pi\left(-4r^3-r^2+256r+32\right)}{\sqrt{\left(64-r^2\right)^3}}$ when $2\sqrt{64-r^2} = 2r$ $\sqrt{64-r^2} = r$ $64-r^2 = r^2$ $2r^2 = 64$ $r^2 = 32$ $r = \sqrt{32} = 4\sqrt{2}$ cm $\frac{d^2A}{dr^2} = -98.96016959$ < 0 $(\therefore \max A)$ Therefore, maximum curved surface area of the cylinder, A cm^2 is obtained when the height of the cylinder is equal to its diameter. (shown)	

12	Given $f'(x) = e^{-2x} + \sin 4x$, $\therefore f''(x) = -2e^{-2x} + 4\cos 4x$ & $f(x) = \int \left(e^{-2x} + \sin 4x\right) dx$ $f(x) = \dfrac{e^{-2x}}{-2} - \dfrac{\cos 4x}{4} + c$ $\therefore \quad f(x) = -\dfrac{1}{2e^{2x}} - \dfrac{1}{4}\cos 4x + c$ Given $f(0) = \dfrac{1}{4}$, $\dfrac{1}{4} = -\dfrac{1}{2} - \dfrac{1}{4} + c$ $\therefore c = 1$ Hence, $f(x) = -\dfrac{1}{2e^{2x}} - \dfrac{1}{4}\cos 4x + 1$ *LHS* $= f''(x) - 4f(x) - 1$ $= -2e^{-2x} + 4\cos 4x - 4\left(-\dfrac{1}{2e^{2x}} - \dfrac{1}{4}\cos 4x + 1\right) - 1$ $= -2e^{-2x} + 4\cos 4x + 2e^{-2x} + \cos 4x - 4 - 1$ $= 5\cos 4x - 5$ $= 5(\cos 4x - 1)$ $= 5\left[\left(1 - 2\sin^2 2x\right) - 1\right]$ $= 5\left(-2\sin^2 2x\right)$ $= -10\sin^2 2x$ (shown)	8

13(i)	$m_{AB}=\frac{1-0}{5-2}=\frac{1}{3}$ $m_{AB}=m_{DC}=\frac{1}{3}$ $y=\frac{1}{3}x+c$ Using $C(7,5)$, $5=\frac{1}{3}(7)+c$ $c=\frac{8}{3}$ $\therefore$ equation of line DC is $y=\frac{1}{3}x+\frac{8}{3}$. ....(1) $m_{AD}\left(\frac{1}{3}\right)=-1$ $\therefore m_{AD}=-3$ $y=-3x+c$ Using $A(2,0)$, $0=-3(2)+c$ $c=6$ $\therefore$ equation of line AD is $y=-3x+6$. ....(2) $(1)=(2)$, $\frac{1}{3}x+\frac{8}{3}=-3x+6$ $\frac{10}{3}x=\frac{10}{3}$ $x=1$ When $x=1$, $y=-3(1)+6=3$ $\therefore D(1,3)$	5

13(ii)	Let $E(x,y)$ $\left(\frac{1+5}{2},\frac{3+1}{2}\right)=\left(\frac{x+2}{2},\frac{y+0}{2}\right)$ $(3,2)=\left(\frac{x+2}{2},\frac{y}{2}\right)$ By comparison, $3=\frac{x+2}{2}$ or $2=\frac{y}{2}$ $x=4$ or $y=4$ $\therefore E(4,4)$	2
13(iii)	$A_{ABCD}=\frac{1}{2}\begin{vmatrix}2 & 5 & 7 & 1 & 2\\ 0 & 1 & 5 & 3 & 0\end{vmatrix}$ $=\frac{1}{2}\left[(2+25+21+0)-(0+7+5+6)\right]$ $=\frac{1}{2}[48-18]$ $=15 \text{ units}^2$ Alternative, $CD=\sqrt{(5-3)^2+(7-1)^2}=\sqrt{40} \text{ units}$ $AB=\sqrt{(5-2)^2+(1-0)^2}=\sqrt{10} \text{ units}$ $AD=\sqrt{(2-1)^2+(0-3)^2}=\sqrt{10} \text{ units}$ $\therefore A_{ABCD}=\frac{1}{2}\left(\sqrt{10}\right)\left(\sqrt{40}+\sqrt{10}\right)$ $=15 \text{ units}^2$	2

13(iv)	$m_{AB}\ \dfrac{1}{3}$ Since AB is a reflection of BX, $\therefore m_{BX} = -\dfrac{1}{3}$ $y = -\dfrac{1}{3}x + c$ Using $B(5,1)$, $1 = -\dfrac{1}{3}(5) + c$ $c = \dfrac{8}{3}$ $\therefore$ equation of line BX is $y = -\dfrac{1}{3}x + \dfrac{8}{3}$.	1