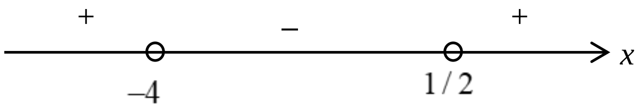
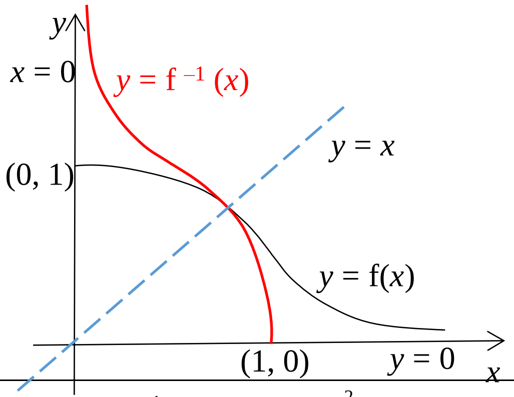
Promo Practice Paper 2 [NYJC 2021] Solutions

Q1	Suggested Answers
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}x+x\sqrt{1-x^2}\right)=\frac{1}{\sqrt{1-x^2}}+\sqrt{1-x^2}+x\left(\frac{-2x}{2\sqrt{1-x^2}}\right)$ $=\frac{1-x^2}{\sqrt{1-x^2}}+\sqrt{1-x^2}$ $=\sqrt{1-x^2}+\sqrt{1-x^2}$ $=2\sqrt{1-x^2}$ $\int\sqrt{1-x^2}\ \mathrm{d}x=\frac{1}{2}\left(\sin^{-1}x+x\sqrt{1-x^2}\right)+C$
(b)	$\int\frac{x^2}{\sqrt{4x^3+1}}\ \mathrm{d}x=\frac{1}{12}\int 12x^2\left(4x^3+1\right)^{-\frac{1}{2}}\ \mathrm{d}x$ $=\frac{1}{12}\left(\frac{\left(4x^3+1\right)^{\frac{1}{2}}}{1/2}\right)+C$ $=\frac{1}{6}\sqrt{4x^3+1}+C$

Q2	Suggested Answers
	$\frac{x+3}{x+4}\le\frac{5}{1-2x},\ x\ne -4,\frac{1}{2}$ $\frac{x+3}{x+4}-\frac{5}{1-2x}\le 0$ $\frac{(x+3)(1-2x)-5(x+4)}{(x+4)(1-2x)}\le 0$ $\frac{-2x^2-10x-17}{(x+4)(1-2x)}\le 0$ $\frac{-2\left(x^2+5x+\frac{17}{2}\right)}{-(x+4)(2x-1)}\le 0$ $\frac{2\left(\left(x+\frac{5}{2}\right)^2+\frac{9}{4}\right)}{(x+4)(2x-1)}\le 0$ Since $\left(x+\frac{5}{2}\right)^2+\frac{9}{4}>0$ for all $x\in\mathbb{R}$, $\frac{2}{(x+4)(2x-1)}\le 0$ 

	$-4 < x < \frac{1}{2}$
	$\frac{x^2+3}{x^2+4} \le \frac{5}{1-2x^2}$ Replace x with x^2 in inequality in (i), $-4 < x^2 < \frac{1}{2}$ Since $x^2 \ge 0$, $0 \le x^2 < \frac{1}{2} \Rightarrow x^2 - \frac{1}{2} < 0$ $\left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right) < 0$ $\therefore -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Q3	Suggested Answers
	Since $OP = 2PA$, $\overrightarrow{OP} = \frac{2}{3}\mathbf{a}$ Using Ratio Theorem, $\overrightarrow{OQ} = \frac{5\mathbf{a}+4\mathbf{b}}{9} = \frac{5}{9}\mathbf{a} + \frac{4}{9}\mathbf{b}$ $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = -\frac{1}{9}\mathbf{a} + \frac{4}{9}\mathbf{b} = \frac{1}{9}(4\mathbf{b}-\mathbf{a})$ Since the line passes through P and is // to $4\mathbf{b}-\mathbf{a}$ therefore an equation of l is $\mathbf{r} = \frac{2}{3}\mathbf{a} + \lambda(4\mathbf{b}-\mathbf{a})$ where / Î $\mathbb{R}$
	Since X lies on l, we have $\overrightarrow{OX} = \frac{2}{3}\mathbf{a} + t(4\mathbf{b}-\mathbf{a})$ for a particular value of t Since AX is perpendicular to l, $\overrightarrow{AX} \bullet (4\mathbf{b}-\mathbf{a}) = 0$ $\left(\frac{2}{3}\mathbf{a} + t(4\mathbf{b}-\mathbf{a}) - \mathbf{a}\right) \bullet (4\mathbf{b}-\mathbf{a}) = 0$ $\left(\left(-\frac{1}{3} - t\right)\mathbf{a} + 4t\mathbf{b}\right) \bullet (4\mathbf{b}-\mathbf{a}) = 0$ Since $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, $\therefore \mathbf{a} \bullet \mathbf{b} = 0$ $\therefore \left(\frac{1}{3} + t\right)\lvert\mathbf{a}\rvert^2 + 16t\lvert\mathbf{b}\rvert^2 = 0$, $\lvert\mathbf{a}\rvert = \sqrt{3}$, $\lvert\mathbf{b}\rvert = \frac{1}{2}$ $\therefore \left(\frac{1}{3} + t\right)3 + 4t = 0$, $t = -\frac{1}{7}$ $\therefore \overrightarrow{OX} = \frac{2}{3}\mathbf{a} - \frac{1}{7}(4\mathbf{b}-\mathbf{a}) = \frac{1}{21}(17\mathbf{a} - 12\mathbf{b})$

<table>
<tr><th>Q4</th><th>Suggested Answers</th></tr>
<tr><td>(i)</td><td>$k = 0$</td></tr>
<tr><td>(ii)</td><td>
</td></tr>
<tr><td>(iii)</td><td>

$$\text{fg}(x) = \frac{1}{\left(\frac{x^2+1}{x}\right)^2 + 1} = \frac{x^2}{x^4+3x^2+1}$$

$$\text{g}(x) = \text{f}^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \text{fg}(x) = \frac{1}{5}$$

$$\frac{x^2}{x^4+3x^2+1} = \frac{1}{5}$$

$$x^4 - 2x^2 + 1 = 0$$

$$(x^2-1)^2 = 0$$

$$x^2 = 1$$

Since $D_g = (0, \infty)$, $x = -1$ is rejected. $\therefore\ x = 1$</td></tr>
<tr><td>(iii)</td><td>Alternatively,

$$\text{fg}(x) = \frac{1}{\left(\frac{x^2+1}{x}\right)^2 + 1}$$

$$\text{g}(x) = \text{f}^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \text{fg}(x) = \frac{1}{5}$$

$$\frac{1}{\left(\frac{x^2+1}{x}\right)^2 + 1} = \frac{1}{5}$$

$$\left(\frac{x^2+1}{x}\right)^2 = 4$$</td></tr>
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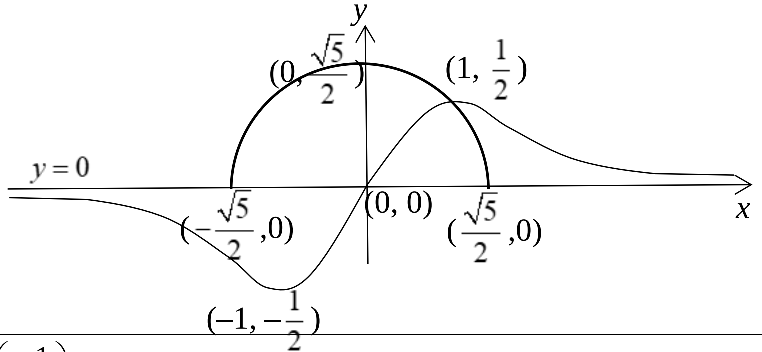
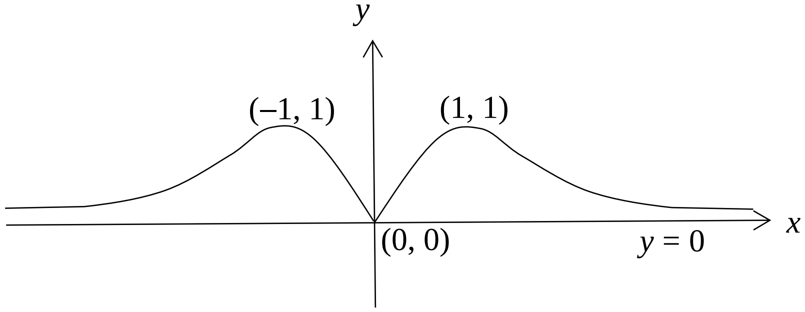
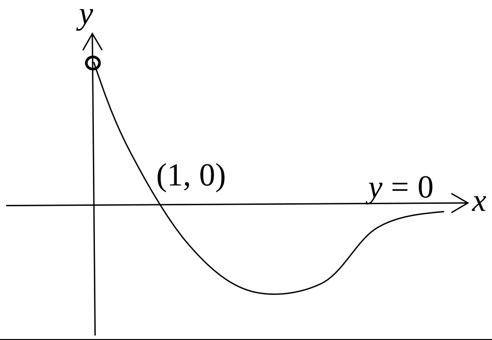
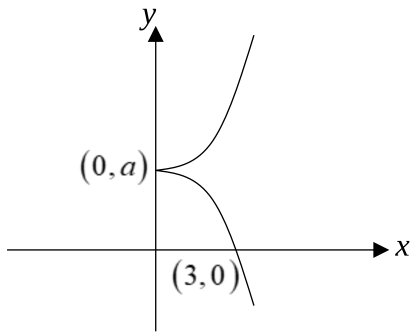
	$\left(\dfrac{x^2+1}{x}\right)=2$ or $\left(\dfrac{x^2+1}{x}\right)=-2$ $x^2-2x+1=0$ or $x^2+2x+1=0$ $(x-1)^2=0$ or $(x+1)^2=0$ $x=1$ or $x=-1$ Since $D_g=(0,\infty)$, $x=-1$ is rejected. $\therefore$ $x=1$
(iii)	Otherwise method: Let $y=\dfrac{1}{x^2+1}$ $x=\pm\sqrt{\dfrac{1}{y}-1}$ Since $x\geq 0$ $x=\sqrt{\dfrac{1}{y}-1}$ $f^{-1}(x)=\sqrt{\dfrac{1}{x}-1}$ $f^{-1}\left(\dfrac{1}{5}\right)=\sqrt{5-1}=2$ $\dfrac{x^2+1}{x}=2$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$

Q5	Suggested Answers
(a)(i)	
(ii)	$\left(1,\dfrac{1}{2}\right)$

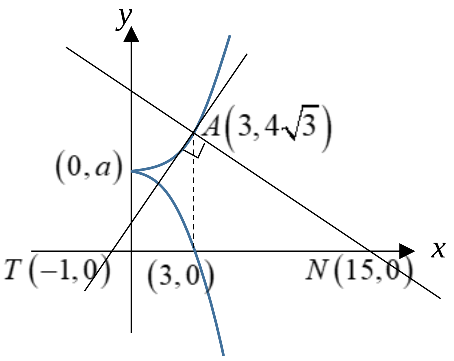
(iii)	$1 \le x \le \frac{\sqrt{5}}{2}$
(b)(i)	
(ii)	

Q6	Suggested Answers
(i)	$y = \sqrt{1 + \ln(1 + \sin 2x)}$ $y^2 = 1 + \ln(1 + \sin 2x)$ $2y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos 2x}{1 + \sin 2x}$ $y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos 2x}{1 + \sin 2x}$
(ii)	$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{-2\sin 2x(1 + \sin 2x) - 2\cos 2x(\cos 2x)}{(1 + \sin 2x)^2}$ $= \frac{-2\sin 2x - 2\sin^2 2x - 2\cos^2 2x}{(1 + \sin 2x)^2}$ $= \frac{-2(\sin 2x + 1)}{(1 + \sin 2x)^2} = \frac{-2}{(1 + \sin 2x)}$
(iii)	$y\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{4\cos 2x}{(1 + \sin 2x)^2}$ Let $\mathrm{f}(x) = \sqrt{1 + \ln(1 + \sin 2x)}$ $\mathrm{f}(0) = 1$ $\mathrm{f}'(0) = 1$ $\mathrm{f}''(0) = -3$ $\mathrm{f}'''(0) = 13$ The Maclaurin expansion of

	$y=\sqrt{1+\ln(1+\sin 2x)}=1+x-\frac{3x^2}{2!}+\frac{13x^3}{3!}+\ldots$ $=1+x-\frac{3}{2}x^2+\frac{13}{6}x^3+\ldots$
(iv)	$\left(1+x-\frac{3}{2}x^2+\frac{13}{6}x^3+\ldots\right)^2$ $=\left(1+x-\frac{3}{2}x^2+\frac{13}{6}x^3+\ldots\right)\left(1+x-\frac{3}{2}x^2+\frac{13}{6}x^3+\ldots\right)$ $=1+x-\frac{3}{2}x^2+\frac{13}{6}x^3$ $+x+x^2-\frac{3}{2}x^3$ $-\frac{3}{2}x^2-\frac{3}{2}x^3$ $+\frac{13}{6}x^3+\ldots$ $=1+2x-2x^2+\frac{4}{3}x^3+\ldots$ Using standard series expansion from MF 26, $y^2=1+\ln(1+\sin 2x)=1+\ln\left[1+\left(2x-\frac{(2x)^3}{3!}+\ldots\right)\right]$ $=1+\left(2x-\frac{8x^3}{6}\right)-\frac{1}{2}\left(2x-\frac{8x^3}{6}\right)^2+\frac{1}{3}\left(2x-\frac{8x^3}{6}\right)^3+\ldots$ $=1+2x-\frac{4}{3}x^3-\frac{1}{2}(2x)^2+\frac{1}{3}(2x)^3+..$ $=1+2x-2x^2+\frac{4}{3}x^3+\ldots$ Alternatively, $1+\ln(1+\sin 2x)=1+\sin 2x-\frac{1}{2}(\sin 2x)^2+\frac{1}{3}(\sin 2x)^3+\ldots$ $=1+\left(2x-\frac{(2x)^3}{3!}\right)-\frac{1}{2}\left(2x-\frac{(2x)^3}{3!}\right)^2+\frac{1}{3}\left(2x-\frac{(2x)^3}{3!}\right)^3+\ldots$ $=1+2x-\frac{8}{6}x^3-\frac{1}{2}(2x)^2+\frac{1}{3}(2x)^3+..$ $=1+2x-2x^2+\frac{4}{3}x^3+\ldots$ Since the expansion of y^2 from the standard series is same as the earlier result, therefore the first 4 terms in the series for y is correct.

Q7	Suggested Answers
(i)	When $x=0$, $3t^2=0 \Rightarrow t=0$ $y=a$ When $y=0$, $a(t^3+1) \Rightarrow t=-1$ $x=3$ 
(ii)	$x=3t^2 \qquad y=a(t^3+1)$ $\frac{dx}{dt}=6t \qquad \frac{dy}{dt}=3at^2$ $\frac{dy}{dx}=\frac{dy}{dt}\Big/\frac{dx}{dt}=\frac{1}{2}at$ At $A(3,2a)$, $2a=a(t^3+1)$ $2=t^3+1$ $t^3=1$ $t=1$ Hence $\frac{dy}{dx}=\frac{1}{2}a$ $\frac{1}{2}a=\tan\frac{\pi}{3}=\sqrt{3}$ Hence $a=2\sqrt{3}$ Equation of the tangent at A, $y-2a=\frac{1}{2}a(x-3)$ $y-4\sqrt{3}=\sqrt{3}(x-3)$ $y=\sqrt{3}x+\sqrt{3}$
(ii)	<u>Alternative method using cartesian equation:</u> $y=a(t^3+1) \quad \Rightarrow \quad t=\left(\frac{y}{a}-1\right)^{\frac{1}{3}}$ -------(1) Substitute (1) into $x=3t^2$:

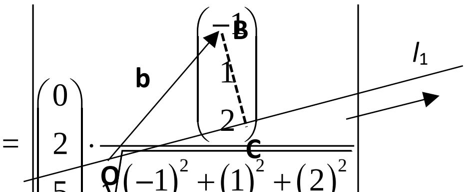
	$x=3\left(\frac{y}{a}-1\right)^{\frac{2}{3}}$ ------(2) Differentiate (2) with respect to *x*: $1=2\left(\frac{y}{a}-1\right)^{-\frac{1}{3}}\left(\frac{1}{a}\right)\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a}{2}\left(\frac{y}{a}-1\right)^{\frac{1}{3}}$ At $A(3,2a)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a}{2}\left(\frac{2a}{a}-1\right)^{\frac{1}{3}}$ $=\frac{a}{2}$ $\frac{1}{2}a=\tan\frac{\pi}{3}=\sqrt{3}$ Hence $a=2\sqrt{3}$ Equation of the tangent at *A*, $y-2a=\frac{1}{2}a(x-3)$ $y-4\sqrt{3}=\sqrt{3}(x-3)$ $y=\sqrt{3}x+\sqrt{3}$

(iii)	Equation of the normal at A, $$y-4\sqrt{3}=-\frac{1}{\sqrt{3}}(x-3)$$ $$y=-\frac{1}{\sqrt{3}}x+5\sqrt{3}$$ When $y=0$, $0=\sqrt{3}x+\sqrt{3} \Rightarrow x=-1$ $\quad T(-1,0)$ $0=-\frac{1}{\sqrt{3}}x+5\sqrt{3} \Rightarrow x=15$ $\quad N(15,0)$ Hence area of triangle ATN $=\frac{1}{2}(15-(-1))(4\sqrt{3})$ $=32\sqrt{3}$ sq. units 

Q8	Suggested Answers
(a)	Given $z=3-7\text{i}$ is a root, substitute it into $zz^*+2\text{i}z=a+6\text{i}$, $\Rightarrow (3-7\text{i})(3+7\text{i})+2\text{i}(3-7\text{i})=a+6\text{i}$ Comparing real part, $3^2+7^2+14=a \quad \Rightarrow a=72$ Let the other root be $x+y\text{i}$. $x^2+y^2+2\text{i}(x+y\text{i})=72+6\text{i}$ Comparing imaginary part, $2x=6 \quad \Rightarrow x=3$ Comparing real part, $3^2+y^2-2y=72$ $y^2-2y-63=0$ $(y+7)(y-9)=0$ $\therefore y=-7$ or $y=9$ Hence the other root is $3+9\text{i}$.
(b)	$w\text{i}+z=5 \quad \ldots(1)$ $w^2+(4\text{i}-1)z=-11+18\text{i} \quad \ldots(2)$ From [1], $z=5-w\text{i} \quad \ldots(3)$ Sub [3] into [2], $w^2+(4\text{i}-1)(5-w\text{i})=-11+18\text{i}$

	$\Rightarrow w^2+20\mathrm{i}+4w-5+\mathrm{i}w=-11+18\mathrm{i}$ $\Rightarrow w^2+(\mathrm{i}+4)w+(2\mathrm{i}+6)=0$ $\Rightarrow w=\dfrac{-(\mathrm{i}+4)\pm\sqrt{(\mathrm{i}+4)^2-4(1)(2\mathrm{i}+6)}}{2}$ $\Rightarrow w=\dfrac{-\mathrm{i}-4\pm\sqrt{\mathrm{i}^2+16+8\mathrm{i}-8\mathrm{i}-24}}{2}$ $\Rightarrow w=\dfrac{-\mathrm{i}-4\pm\sqrt{-9}}{2}$ $\Rightarrow w=\dfrac{-\mathrm{i}-4\pm 3\mathrm{i}}{2}$ $\Rightarrow w=\mathrm{i}-2$ or $-2\mathrm{i}-2$ Substitute w into [3]: $z=5-(\mathrm{i}-2)\mathrm{i}$ or $5-(-2\mathrm{i}-2)\mathrm{i}$ $\Rightarrow z=6+2\mathrm{i}$ or $3+2\mathrm{i}$ (b) (i) $\mathrm{i}z=\dfrac{(1+\mathrm{i})^6}{\left(-1-\mathrm{i}\sqrt{3}\right)^4}\Rightarrow \mathrm{i}(-1-\mathrm{i}\sqrt{3})^4 z=(1+\mathrm{i})^6$ $\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}(2\mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}})^4 z=(\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}})^6$ $z=\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}\left(\dfrac{1}{16}\mathrm{e}^{\mathrm{i}\frac{8\pi}{3}}\right)\left(8\mathrm{e}^{\mathrm{i}\frac{3\pi}{2}}\right)$ $z=\dfrac{1}{2}\mathrm{e}^{\mathrm{i}\frac{11\pi}{3}}=\dfrac{1}{2}\mathrm{e}^{-\mathrm{i}\frac{\pi}{3}}$

Q9	**Suggested Answers**
	(i) $l_1:\mathbf{r}=\begin{pmatrix}-1\\1\\2\end{pmatrix}+k\begin{pmatrix}-1\\1\\2\end{pmatrix}=\lambda\begin{pmatrix}-1\\1\\2\end{pmatrix},\ \lambda\in\mathbb{R}$ $l_1:\mathbf{r}$ $=\begin{pmatrix}-1\\1\\2\end{pmatrix}+k\begin{pmatrix}-1\\1\\2\end{pmatrix},\ k\in\mathbb{R}$ or AEF

	(ii) length of projection of **b** on l_1 = $\left|\begin{pmatrix}0\\2\\5\end{pmatrix}\cdot\frac{\begin{pmatrix}-1\\1\\2\end{pmatrix}}{\sqrt{(-1)^2+(1)^2+(2)^2}}\right|$  $= \frac{12}{\sqrt{6}} = 2\sqrt{6}$ $\lvert\mathbf{b}\rvert = \left\|\begin{pmatrix}0\\2\\5\end{pmatrix}\right\| = \sqrt{29}$ By Pythagoras' Theorem, d, Perpendicular distance from B to l_1 = $\sqrt{\lvert \mathrm{b}\rvert^2-\left(2\sqrt{6}\right)^2}$ $= \sqrt{29-4(6)} = \sqrt{5}$ Alternatively, Let C be foot of perpendicular from B to l_1, using length of projection of **b** on l_1 and O lies on l_1, $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$ $=2\sqrt{6}\,\hat{\underset{\sim}{a}}-\begin{pmatrix}0\\2\\5\end{pmatrix}$ $=2\sqrt{6}\times\frac{1}{\sqrt{6}}\begin{pmatrix}-1\\1\\2\end{pmatrix}-\begin{pmatrix}0\\2\\5\end{pmatrix}$ $=\begin{pmatrix}2\\0\\1\end{pmatrix}$ $\left\lvert\overrightarrow{BC}\right\rvert=\left\|\begin{pmatrix}2\\0\\1\end{pmatrix}\right\|=\sqrt{5}$ (iii) Since C lies on l_1, $\mathbf{c}=\lambda\begin{pmatrix}-1\\1\\2\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\overrightarrow{BC}=\lambda\begin{pmatrix}-1\\1\\2\end{pmatrix}-\begin{pmatrix}0\\2\\5\end{pmatrix}=\begin{pmatrix}-\lambda\\\lambda-2\\2\lambda-5\end{pmatrix}$

$$\left|\overrightarrow{BC}\right| = \left|\begin{pmatrix} -\lambda \\ \lambda-2 \\ 2\lambda-5 \end{pmatrix}\right| = \sqrt{5}$$

$$\sqrt{(-\lambda)^2 + (\lambda-2)^2 + (2\lambda-5)^2} = \sqrt{5}$$

$$6\lambda^2 - 24\lambda + 24 = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

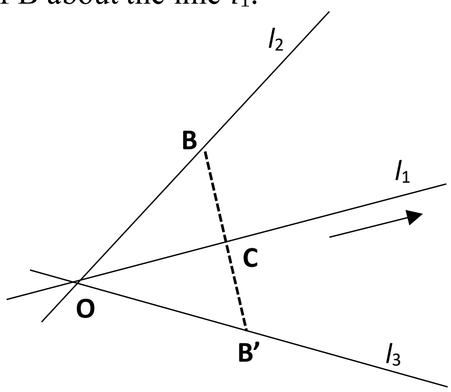
$$(\lambda-2)^2 = 0$$

$$\lambda = 2$$

$$\mathbf{c} = 2\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$$

(vi) $l_2 : \boldsymbol{r} = \mu\begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix},\ \mu \in \mathbb{R}$

Let B' be the point of reflection of B about the line l_1.
By mid-point theorem,



$$\overrightarrow{OC} = \frac{\overrightarrow{OB} + \overrightarrow{OB}'}{2}$$

$$\begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} + \overrightarrow{OB}'\right]$$

$$\overrightarrow{OB}' = 2\begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}$$

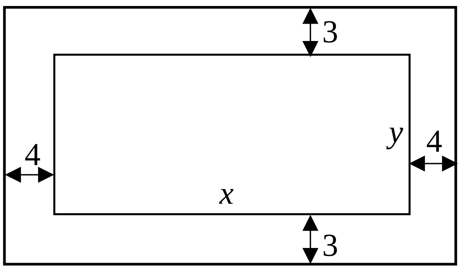
Since l_3 parallel to $\overrightarrow{OB'}$ and the origin O lies on l_3, $l_3 : \boldsymbol{r} = \gamma\begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix},\ \gamma \in \mathbb{R}$

So Cartesian equation of l_3 is : $-\dfrac{x}{4} = \dfrac{y}{2} = \dfrac{z}{3}$ or

$\dfrac{-x-4}{4} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$ or AEF

Q10	Suggested Answers
(i)	$u_{10}=1.5$ $ar^9=1.5$, $a=2$ $r^9=0.75$ $r=(0.75)^{\frac{1}{9}}\approx 0.96854$ $S_\infty=\dfrac{2}{1-(0.75)^{\frac{1}{9}}}\approx 63.574$ or $S_\infty=\dfrac{2}{1-0.96854}\approx 63.573$ Maximum amount to pay $<18\times S_\infty\approx \$1144.33$ or $\$1144.31$ Hence the cost will not exceed \$1200.
(ii)	$T=S_{10}=\dfrac{2\left(1-0.75^{\frac{10}{9}}\right)}{1-0.75^{\frac{1}{9}}}\approx 17.394\approx 17.4\text{m}$ or $T=S_{10}=\dfrac{2\left(1-0.96854^{10}\right)}{1-0.96854}\approx 17.394\approx 17.4\text{m}$
(iii)	**<u>Method 1 (consider U_1= 2):</u>** $\dfrac{10}{2}\left[2(2)+(10-1)d\right]=17.394$ $d\approx -0.0579\text{m}$
(iii)	**<u>Method 2 (consider U_{10} = 2):</u>** $\dfrac{10}{2}\left[2a+(10-1)d\right]=17.394$ $5\left[2a+9d\right]=17.394$-------(1) $a+9d=2$ ------------------(2) Subst. (2) into (1): $5(a+2)=17.394$ a = 1.4788 Hence, $\therefore d=\dfrac{2-1.4788}{9}\approx 0.0579\text{m}$
(iv)	**<u>Method 1:</u>** Original volume of paint = 5 litres Let u_n denotes the volume of paint in litres remaining after the nth refill. Before the 1st refill, the quantity of paint is reduced by a factor of $\dfrac{4}{5}$. $\therefore$ the volume of paint is reduced to $u_1=\dfrac{4}{5}\times 5$ $u_2=\dfrac{4}{5}\times u_1=\left(\dfrac{4}{5}\right)^2\times 5$

	$\therefore u_n = \left(\frac{4}{5}\right)^n \times 5 = 5\left(\frac{4}{5}\right)^n$
(iv)	**<u>Method 2:</u>** Let u_n denotes the volume of paint in litres remaining after the nth refill. $u_1 = 4$ Before the 2nd refill, the quantity of paint is reduced by a factor of $\frac{4}{5}$. $\therefore$ the volume of paint is reduced to $u_2 = \frac{4}{5} \times u_1 = \frac{4}{5} \times 4$ $u_3 = \frac{4}{5} \times u_2 = \left(\frac{4}{5}\right)^2 \times 4$ $\therefore u_n = 4\left(\frac{4}{5}\right)^{n-1}$
(v)	If the mixture is more than 80% turpentine, then it is less than 20% paint, ie 0.2 × 5 = 1 litre of paint. $u_n = 5\left(\frac{4}{5}\right)^n < 1$ or $u_n = 4\left(\frac{4}{5}\right)^{n-1} < 1$ Using GC, minimum $n = 8$ The minimum number of refills taken before the mixture is more than 80% turpentine is 8.

Q11	**Suggested Answers**
(i)	 $xy = 1200 \quad \Rightarrow \quad y = \frac{1200}{x}$ Let A be the total area of the canvas. To minimize $A = (x+8)(y+6)$ $= (x+8)\left(\frac{1200}{x} + 6\right)$ $= 1200 + 6x + \frac{9600}{x} + 48$ $= 6x + \frac{9600}{x} + 1248$ $\frac{dA}{dx} = 6 - \frac{9600}{x^2}$

$$\frac{\mathrm{d}A}{\mathrm{d}x}=0 \quad \Rightarrow \quad x^2=1600$$

$$x=40 \quad (\because\ x>0)$$

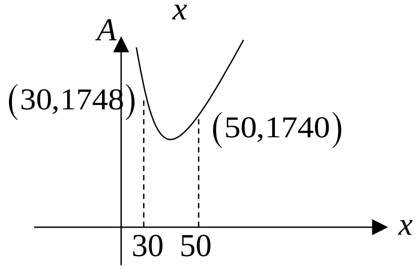
$$y=30$$

$$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=\frac{19200}{x^3}>0 \text{ since } x>0$$

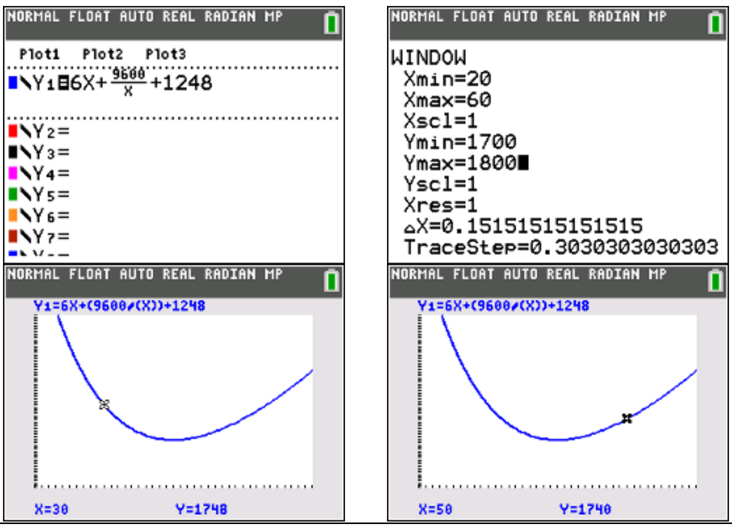
Hence A is minimum when $x=48$ cm and $y=36$ cm

(ii)

Graph of $A=6x+\dfrac{9600}{x}+1248$



From graph, largest possible area of the canvas is $1748\ \text{cm}^2$ when $x=30$ cm.



(iii)

$$A=\pi r^2$$

$$\frac{\mathrm{d}A}{\mathrm{d}t}=2\pi r\frac{\mathrm{d}r}{\mathrm{d}t} \quad \text{-------}(*)$$

When $t=0$, $A=\pi\left(\dfrac{2}{\sqrt{\pi}}\right)^2=4$

When $t=3$, $A=4+3\times 20=64$

Hence $64=\pi r^2 \quad \Rightarrow \quad r=\dfrac{8}{\sqrt{\pi}}$

	Sub $\frac{\mathrm{d}A}{\mathrm{d}t}=20$, $r=\frac{8}{\sqrt{\pi}}$ into (*): $20=2\pi\frac{8}{\sqrt{\pi}}\cdot\frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}r}{\mathrm{d}t}=0.705$ (3 s.f.) When $t=3$, rate of change of radius is 0.705 cm/min