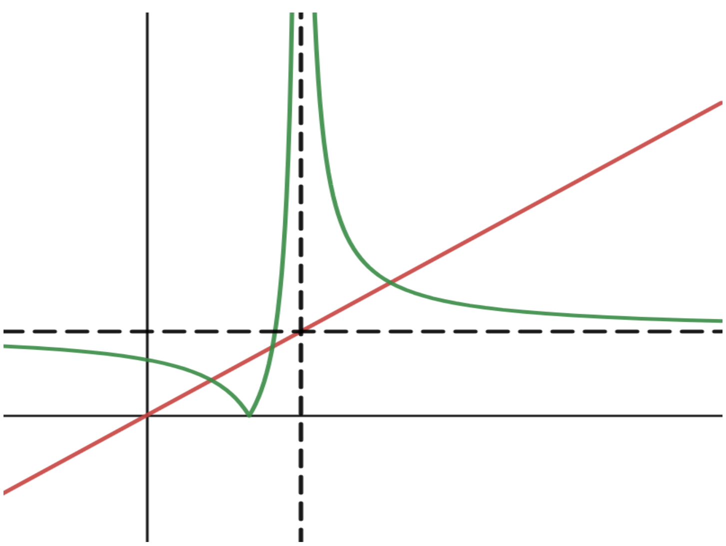
2021 NYJC JC2 Prelim 9758/1 Solutions

Qn	
1(a)	$(2\mathbf{r}-\mathbf{p})\times\mathbf{p}=\mathbf{0}$ means $2\mathbf{r}-\mathbf{p}$ is parallel to $\mathbf{p}$ $2\mathbf{r}-\mathbf{p}=k\mathbf{p}$ $2\mathbf{r}=(k+1)\mathbf{p}$ $\mathbf{r}=\frac{k+1}{2}\mathbf{p}$ $\mathbf{r}$ represents the set of position vectors of points that lies on the line through the origin and parallel to **p.**
1(b)	Area $\triangle PQR$ $=\frac{1}{2}\lvert\overrightarrow{QR}\times\overrightarrow{QP}\rvert$ $=\frac{1}{2}\lvert\lambda\mathbf{p}\times(\mathbf{p}-\mathbf{q})\rvert$ $=\frac{1}{2}\lvert\lambda(\mathbf{p}\times\mathbf{q})\rvert$ $=\frac{-\lambda}{2}\lvert(\mathbf{p}\times\mathbf{q})\rvert$ $k=-\lambda/2$
2(i)	

2(ii)	The reflected part of $y=\left\|\dfrac{ax-3a+2}{3-x}\right\|$ is $-\dfrac{ax-3a+2}{3-x}$. Solving $-\dfrac{ax-3a+2}{3-x}=\dfrac{a}{3}x$: $ax-3a+2=\dfrac{a}{3}x^2-ax$ $ax^2-6ax+9a-6=0$ $x=\dfrac{6a\pm\sqrt{(-6a)^2-4(a)(9a-6)}}{2a}=\dfrac{6a\pm\sqrt{24a}}{2a}=3\pm\sqrt{\dfrac{6}{a}}$ $x^2-6x+9-\dfrac{6}{a}=0$ $(x-3)^2=\dfrac{6}{a}$ $x=3\pm\sqrt{\dfrac{6}{a}}$ Solving $\dfrac{ax-3a+2}{3-x}=\dfrac{a}{3}x$: $\dfrac{a}{3}x^2-3a+2=0$ $x^2=\dfrac{9a-6}{a}$ $x=\sqrt{9-\dfrac{6}{a}}$ (Reject $x=-\sqrt{9-\dfrac{6}{a}}$) $\therefore\left\|\dfrac{ax-3a+5}{3-x}\right\|>\dfrac{a}{3}x$ $\Rightarrow x<3-\sqrt{\dfrac{6}{a}}$ or $\sqrt{9-\dfrac{6}{a}}<x<3$ or $3<x<3+\sqrt{\dfrac{6}{a}}$
3(i)	$\dfrac{1}{r!(r+2)}=\dfrac{r+1}{(r+2)!}=\dfrac{r+2-1}{(r+2)!}=\dfrac{1}{(r+1)!}-\dfrac{1}{(r+2)!}$ Alternative: $\dfrac{1}{r!(r+2)}=\dfrac{r+1}{(r+2)!}$ $\dfrac{A}{(r+2)!}+\dfrac{B}{(r+1)!}=\dfrac{A+B(r+2)}{(r+2)!}$ $\therefore r+1=A+B(r+2)$ By comparing coeff: $A=-1,\ B=1$

	$\displaystyle\sum_{r=1}^{n}\frac{1}{r!(r+2)}=\sum_{r=1}^{n}\left[\frac{1}{(r+1)!}-\frac{1}{(r+2)!}\right]$ $\displaystyle=\frac{1}{2!}-\cancel{\frac{1}{3!}}$ $\displaystyle+\cancel{\frac{1}{3!}}-\cancel{\frac{1}{4!}}$ $\vdots$ $\displaystyle+\cancel{\frac{1}{n!}}-\cancel{\frac{1}{(n+1)!}}$ $\displaystyle+\cancel{\frac{1}{(n+1)!}}-\frac{1}{(n+2)!}$ $\displaystyle=\frac{1}{2}-\frac{1}{(n+2)!}$
3(ii)	$\displaystyle\sum_{r=1}^{\infty}\frac{1+\left(\frac{1}{3}\right)^{r}r!(r+2)}{r!(r+2)}=\sum_{r=1}^{\infty}\frac{1}{r!(r+2)}+\sum_{r=1}^{\infty}\left(\frac{1}{3}\right)^{r}$ As $n\to\infty$, $\displaystyle\frac{1}{(n+2)!}\to\infty$, thus $\displaystyle\sum_{r=1}^{\infty}\frac{1}{r!(r+2)}$ converges. Since $\left\|\frac{1}{3}\right\|<1$, thus $\displaystyle\sum_{r=1}^{\infty}\left(\frac{1}{3}\right)^{r}$ converges. Therefore, $\displaystyle\sum_{r=1}^{\infty}\frac{1+\left(\frac{1}{3}\right)^{r}r!(r+2)}{r!(r+2)}$ converges. $\displaystyle\sum_{r=1}^{\infty}\frac{1+\left(\frac{1}{3}\right)^{r}r!(r+2)}{r!(r+2)}=\frac{1}{2}+\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}+\frac{1}{2}=1$
4(i)	Take an arbitrary point (x,y) on C. Distance between (x,y) and $(1,0)$, $s=\sqrt{(x-1)^{2}+y^{2}}$ $s^{2}=(x-1)^{2}+y^{2}$
4(ii)	Differentiating w.r.t. x gives $\displaystyle 2s\frac{\mathrm{d}s}{\mathrm{d}x}=2(x-1)+2y\frac{\mathrm{d}y}{\mathrm{d}x}$. When s takes a minimum value, $\displaystyle\frac{\mathrm{d}s}{\mathrm{d}x}=0$. Thus

	$2(x-1)+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ $x-1+\left(\dfrac{x}{1-x}\right)\left[\dfrac{1-x-x(-1)}{(1-x)^2}\right]=0$ $\dfrac{x}{(1-x)^3}=1-x$ $x=(1-x)^4$ $x^4-4x^3+6x^2-5x+1=0$ By GC (Poly Root Finder), $x\approx 0.2755$ (to 4 d.p.) $y=\dfrac{0.2755}{1-0.2755}\approx 0.3803$ (to 4 d.p.) So the point on C having a minimum distance from the point $(1,0)$ is $(0.2755, 0.3803)$.
4(iii)	The minimum distance $=\sqrt{(0.2755-1)^2+0.3803^2}$ ≈ 0.818
5(a)	(i) $(a,0)\to(\frac{a+3}{2},0)$; $(0,b)\to(\frac{3}{2},b)$ $(\frac{a+3}{2},0)$ is the only point. Asymptote: $y=k$ (ii) $(a,0)\to(0,a)$ $(0,b)\to(b,0)$ Asymptote: $x=k$
5(b) (i)	$y=\ln\left(\dfrac{e^x+5}{e^x-1}\right)\Rightarrow e^y=\dfrac{e^x+5}{e^x-1}\Rightarrow(e^y-1)e^x=e^y+5$ $\Rightarrow e^x=\dfrac{e^y+5}{e^y-1}\Rightarrow x=\ln\left(\dfrac{e^y+5}{e^y-1}\right)$ $\therefore g^{-1}(x)=\ln\left(\dfrac{e^x+5}{e^x-1}\right)$ $D_{g^{-1}}=R_g=(0,\infty)$

<table>
<tr><td>(b)
(ii)</td><td>
Note: g is a self-inverse function
$\mathrm{g}^{2021}\mathrm{h}(x)=\ln 2$
$\mathrm{gh}(x)=\ln 2$
<table>
<tr><td><u>Method 1:</u>
$\mathrm{h}(x)=\mathrm{g}^{-1}(\ln 2)=\ln 7$
$1+\sqrt{9-(x-2)^2}=\ln 7$</td>
<td><u>Method 2(not recommended):</u>
$\mathrm{gh}(x)=\ln 2$
$\ln\left(\dfrac{\mathrm{e}^{1+\sqrt{9-(x-2)^2}}+5}{\mathrm{e}^{1+\sqrt{9-(x-2)^2}}-1}\right)=\ln 2$
$\dfrac{\mathrm{e}^{1+\sqrt{9-(x-2)^2}}+5}{\mathrm{e}^{1+\sqrt{9-(x-2)^2}}-1}=2$
$\mathrm{e}^{1+\sqrt{9-(x-2)^2}}=7$
$1+\sqrt{9-(x-2)^2}=\ln 7$</td></tr>
</table>
$9-(x-2)^2=(\ln 7-1)^2$
$(x-2)^2=9-(\ln 7-1)^2=(2+\ln 7)(4-\ln 7)$
$x=2\pm\sqrt{(2+\ln 7)(4-\ln 7)}$
</td></tr>
<tr><td>6(a)</td><td>
$\displaystyle\int_{-3}^{-1}\frac{|x+2|}{x^2+4x+5}\mathrm{d}x=-\int_{-3}^{-2}\frac{x+2}{x^2+4x+5}\mathrm{d}x+\int_{-2}^{-1}\frac{x+2}{x^2+4x+5}\mathrm{d}x$
$\displaystyle=-\frac{1}{2}\int_{-3}^{-2}\frac{2x+4}{x^2+4x+5}\mathrm{d}x+\frac{1}{2}\int_{-2}^{-1}\frac{2x+4}{x^2+4x+5}\mathrm{d}x$
$\displaystyle=\frac{1}{2}\left(-\Big[\ln\left|x^2+4x+5\right|\Big]_{-3}^{-2}+\Big[\ln\left|x^2+4x+5\right|\Big]_{-2}^{-1}\right)$
$\displaystyle=\frac{1}{2}[(-\ln 1+\ln 2)+(\ln 2-\ln 1)]=\ln 2$
</td></tr>
<tr><td>6(b)
(i)</td><td>
$\displaystyle\int\frac{\sin(\ln x)}{x}\mathrm{d}x=\int\frac{1}{x}\sin(\ln x)\mathrm{d}x=-\cos(\ln x)+c$
</td></tr>
<tr><td>6(b)
(ii)</td><td>
$\displaystyle\int x\sin(\ln x)\mathrm{d}x=\int x\left(\frac{x}{x}\right)\sin(\ln x)\mathrm{d}x$
$\displaystyle=\int x^2\left(\frac{1}{x}\right)\sin(\ln x)\mathrm{d}x$
$\displaystyle=x^2(-\cos(\ln x))-\int 2x(-\cos(\ln x)\mathrm{d}x$
$\displaystyle=-x^2\cos(\ln x)+2\int x(\frac{x}{x})\cos(\ln x)\mathrm{d}x$
$\displaystyle=-x^2\cos(\ln x)+2\left[x^2\sin(\ln x)-\int 2x\sin(\ln x)\mathrm{d}x\right]$
$\displaystyle 5\int x\sin(\ln x)\mathrm{d}x=-x^2\cos(\ln x)+2x^2\sin(\ln x)$
$\displaystyle\int x\sin(\ln x)\mathrm{d}x=\frac{1}{5}(-x^2\cos(\ln x)+2x^2\sin(\ln x))+c$
</td></tr>
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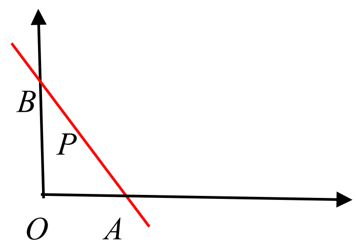
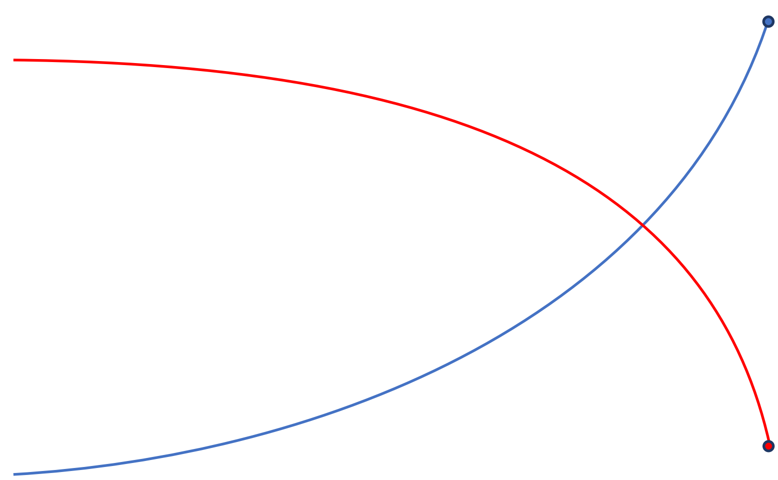
7(i)	$y=\ln(1+\mathrm{e}^x)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^x}{1+\mathrm{e}^x}$ $(1+\mathrm{e}^x)\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^x$ $(1+\mathrm{e}^x)\frac{\mathrm{d}y}{\mathrm{d}x}-\mathrm{e}^x=0$ (shown)
7(ii)	$(1+\mathrm{e}^x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^x\frac{\mathrm{d}y}{\mathrm{d}x}-\mathrm{e}^x=0$ $(1+\mathrm{e}^x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\mathrm{e}^x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^x\frac{\mathrm{d}y}{\mathrm{d}x}-\mathrm{e}^x=0$ $(1+\mathrm{e}^x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+2\mathrm{e}^x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^x\frac{\mathrm{d}y}{\mathrm{d}x}-\mathrm{e}^x=0$ When $x=0$, $y=\ln 2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^0}{1+\mathrm{e}^0}=\frac{1}{2}$ $(1+\mathrm{e}^0)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^0(\frac{1}{2})-\mathrm{e}^0=0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{4}$ By Maclaurin's series, $y=\ln 2+\frac{1}{2}x+\frac{\frac{1}{4}}{2!}x^2+\cdots$ $y=\ln 2+\frac{1}{2}x+\frac{1}{8}x^2+\cdots$ Since $y=\ln(1+\mathrm{e}^x),\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^x}{1+\mathrm{e}^x}$ $\frac{\mathrm{e}^x}{1+\mathrm{e}^x}=\frac{1}{2}+\frac{1}{4}x+...$
7(iii)	Using MF26, $\frac{\mathrm{e}^x}{1+\mathrm{e}^x}=\mathrm{e}^x(1+\mathrm{e}^x)^{-1}$ $=(1+x+\cdots)(1+1+x+\cdots)^{-1}$ $\approx(1+x)(2^{-1}(1+\frac{x}{2})^{-1})$ $=\frac{1}{2}(1+x)(1+\frac{x}{2}+\cdots)$ $=\frac{1}{2}(1+x+\frac{x}{2}+\cdots)$ $=\frac{1}{2}+\frac{1}{4}x+...$ OR

	<table><tr><td>$$\begin{aligned}\frac{e^x}{1+e^x} &= \frac{1}{1+e^{-x}} = \left(1+e^{-x}\right)^{-1} \\ &= (1+1-x+\cdots)^{-1} \\ &= \frac{1}{2}\left(1-\frac{x}{2}\right)^{-1} \\ &= \frac{1}{2}\left(1+\frac{x}{2}+\cdots\right) \\ &= \frac{1}{2}+\frac{1}{4}x+\cdots\end{aligned}$$</td><td>$$\begin{aligned}\frac{e^x}{1+e^x} &= 1-\frac{1}{1+e^x} = 1-(1+e^x)^{-1} \\ &= 1-(1+1+x+\cdots)^{-1} \\ &= 1-\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1} \\ &= 1-\frac{1}{2}\left(1-\frac{x}{2}+\cdots\right) \\ &= \frac{1}{2}+\frac{1}{4}x+\cdots\end{aligned}$$</td></tr></table>
8(i)	 Let the tangent to C at the point $P\left(a\sin^5 t, a\cos^5 t\right)$ cut the x and y-axes at A and B respectively. $$\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \frac{-5a\sin t\cos^4 t}{5a\cos t\sin^4 t} = -\cot^3 t.$$ Equation of tangent at P is $$y - a\cos^5 t = -\cot^3 t\left(x - a\sin^5 t\right).$$ At A, $y = 0$. Substituting into equation of tangent gives $$\begin{aligned} x &= a\sin^5 t + \frac{a\cos^5 t}{\cot^3 t} \\ &= a\sin^5 t + a\sin^3 t\cos^2 t \\ &= a\sin^3 t\left(\sin^2 t + \cos^2 t\right) \\ &= a\sin^3 t\end{aligned}$$ At B, $x = 0$. Substituting into equation of tangent gives $$\begin{aligned} y &= a\cos^5 t + a\sin^5 t\cot^3 t \\ &= a\cos^5 t + a\sin^2 t\cos^3 t \\ &= a\cos^3 t\left(\cos^2 t + \sin^2 t\right) \\ &= a\cos^3 t\end{aligned}$$ $$\begin{aligned} OA^{\frac{2}{3}} + OB^{\frac{2}{3}} &= \left(a\sin^3 t\right)^{\frac{2}{3}} + \left(a\cos^3 t\right)^{\frac{2}{3}} \\ &= a^{\frac{2}{3}}\left(\sin^2 t + \cos^2 t\right) = a^{\frac{2}{3}}\end{aligned}$$

8(ii)	Midpoint of AB is $M\left(\frac{a\sin^3 t}{2},\frac{a\cos^3 t}{2}\right)$. Set $x=\frac{a\sin^3 t}{2}$ and $y=\frac{a\cos^3 t}{2}$ $\Rightarrow a\sin^3 t=2x$ and $a\cos^3 t=2y$ $\Rightarrow \left(a\sin^3 t\right)^{\frac{2}{3}}=(2x)^{\frac{2}{3}}$ and $\left(a\cos^3 t\right)^{\frac{2}{3}}=(2y)^{\frac{2}{3}}$ $\Rightarrow a^{\frac{2}{3}}\sin^2 t=(2x)^{\frac{2}{3}}$ and $a^{\frac{2}{3}}\cos^2 t=(2y)^{\frac{2}{3}}$ Adding, $2^{\frac{2}{3}}\left(x^{\frac{2}{3}}+y^{\frac{2}{3}}\right)=a^{\frac{2}{3}}\left(\sin^2 t+\cos^2 t\right)$ $\Rightarrow x^{\frac{2}{3}}+y^{\frac{2}{3}}=\left(\frac{a}{2}\right)^{\frac{2}{3}}$
8(iii)	The curve C is symmetrical about the x and y axes. Area enclosed by C $=4\int_0^a y\ \mathrm{d}x$ $=4\int_0^{\frac{\pi}{2}} a\cos^5 t\left(5a\sin^4 t\cos t\right)\ \mathrm{d}t$ $=20a^2\int_0^{\frac{\pi}{2}}\sin^4 t\cos^6 t\ \mathrm{d}t$ $\approx 0.37a^2$
9(i)	$2z-1=\|w\|\Rightarrow z=\frac{\|w\|+1}{2}\in\mathbb{R}$, $2z^3-5z^2+2z\in\mathbb{R}$. Thus for $a\in\mathbb{R}$, $2z^3-5z^2+2z+(a+3)\mathrm{i}=0$ $\Rightarrow 2z^3-5z^2+2z=0$ and $a+3=0$ $\Rightarrow z(2z-1)(z-2)=0$ and $a=-3$ $z(2z-1)(z-2)=0\Rightarrow z=0,\frac{1}{2},2$. If $z=0$, then $\|w\|=2z-1=-1<0$ which is impossible. If $z=\frac{1}{2}$, then $\|w\|=0\Rightarrow w=0$ which contradicts $w\neq 0$. Hence $z=2$.
9(ii)	$\|w\|=2(2)-1=3$. $w=3\left(\cos\frac{\pi}{3}+\mathrm{i}\sin\frac{\pi}{3}\right)=\frac{3}{2}+\frac{3\sqrt{3}}{2}\mathrm{i}$.

9(iii)	$\left\lvert w^n \right\rvert > 20212021$ $\lvert w \rvert^n > 20212021$ $3^n > 20212021$ $n > \dfrac{\lg 20212021}{\lg 3} \approx 15.3$ So least n is 16.
9(iv)	For w^k to be a positive real number, $\arg w^k = 2m\pi,\ m = 0, 1, 2, \ldots$ $\Rightarrow k \arg w = 2m\pi,\ m = 0, 1, 2, \ldots$ $\Rightarrow \dfrac{k\pi}{3} = 0, 2\pi, 4\pi, \ldots$ $\Rightarrow k = 0, 6, 12, \ldots$ So least positive integer k is 6.
10(i)	$\cos\theta = \dfrac{\begin{pmatrix}0\\0\\1\end{pmatrix} \cdot \begin{pmatrix}1\\4\\6\end{pmatrix}}{\sqrt{1^2+4^2+6^2}} = \dfrac{6}{\sqrt{53}}$ $\theta = 34.5^o$
10(ii)	$\mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}0\\0\\1\end{pmatrix} + s\begin{pmatrix}1\\4\\6\end{pmatrix}$ $\mathbf{n} = \begin{pmatrix}0\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\4\\6\end{pmatrix} = \begin{pmatrix}-4\\1\\0\end{pmatrix}$ $\mathbf{r} \cdot \begin{pmatrix}-4\\1\\0\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}-4\\1\\0\end{pmatrix} = -2$ $-4x + y = -2$
10(iii)	$\begin{pmatrix}2\\6\\12\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}0\\0\\1\end{pmatrix} + s\begin{pmatrix}1\\4\\6\end{pmatrix}$ $2 = 1 + s \Rightarrow s = 1$ $12 = 3 + t + 6(1) \Rightarrow t = 3$

10(iv)	$l:\mathbf{r}=\begin{pmatrix}3\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\0\end{pmatrix},\lambda\in\mathbb{R}$ $\overrightarrow{OF}=\begin{pmatrix}3\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\0\end{pmatrix}$ Let the final position vector of balloon be *X*. $\overrightarrow{XF}\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}=0$ $\left(\begin{pmatrix}3\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}-\begin{pmatrix}2\\6\\12\end{pmatrix}\right)\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}=0$ $2-4+\lambda(4+1)=0$ $\lambda=\frac{2}{5}$ $\overrightarrow{OF}=\begin{pmatrix}3\\2\\1\end{pmatrix}+\frac{2}{5}\begin{pmatrix}2\\1\\0\end{pmatrix}=\frac{1}{5}\begin{pmatrix}19\\12\\5\end{pmatrix}$ $XF=\sqrt{(1\frac{4}{5})^2+(-3\frac{3}{5})^2+(-11)^2}=11.7$
11(i)	 *O* *y* *x* 74 1 21

11(ii)	Using GC, the coordinates of the equilibrium point is (16.3, 49.7).
11(iii)	Let q_e be the quantity at equilibrium point. Thus $\text{C.S.} = \int_0^{q_e} D(q)\,\mathrm{d}q - p_e q_e$ $= \int_0^{q_e} \left(75 - 1.22^q\right)\mathrm{d}q - p_e q_e$ $= \left[75q - \dfrac{1.22^q}{\ln(1.22)}\right]_0^{q_e} - p_e q_e$ $= 289.40$ $\text{P.S.} = p_e q_e - \int_0^{q_e} S(q)\,\mathrm{d}q$ $= p_e q_e - \int_0^{q_e} \left(2\left(1.22^q\right) - 1\right)\mathrm{d}q$ $= p_e q_e - \left[\dfrac{2\left(1.22^q\right)}{\ln(1.22)} - q\right]_0^{q_e}$ $= 578.80$
11(iv)	$75 - 1.22^{q_e} = 2(1.22^{q_e}) - 1$ $\Rightarrow 3(1.22)^{q_e} = 76$ Let q_e^* be the quantity at the new equilibrium point. Thus $75 - 1.22^{q_e^*} = 2(1.22^{q_e^*-a}) - 1 = 2(1.22^{-a})(1.22^{q_e^*}) - 1$ $\Rightarrow \left(1 + 2(1.22^{-a})\right)(1.22)^{q_e^*} = 76$ Dividing, we have $(1.22)^{q_e^* - q_e} = \dfrac{3}{1 + 2(1.22^{-a})}$ Also, we have $p_e = 2(1.22)^{q_e} - 1$ $p_e^* = S(q_e^* - a) = 2(1.22)^{q_e^*-a} - 1 = 2(1.22)^{-a}(1.22)^{q_e^*} - 1$ Thus $\dfrac{p_e^* + 1}{p_e + 1} = \dfrac{2(1.22)^{-a}(1.22)^{q_e^*}}{2(1.22)^{q_e}}$ $= (1.22)^{-a}(1.22)^{q_e^* - q_e}$ $= \dfrac{3(1.22)^{-a}}{1 + 2(1.22^{-a})}$ $= \dfrac{3}{(1.22)^a + 2}$

11(v)	$p = 75 - 1.22^q$ $\Rightarrow q = \dfrac{\ln(75-p)}{\ln(1.22)}$ Using (iv), $p_e^* = \dfrac{3(p_e+1)}{(1.22)^4+2} - 1 = 35.0588$ Using GC, $Increase = \displaystyle\int_{p_e^*}^{p_e} \frac{\ln(75-p)}{\ln(1.22)}\,\mathrm{d}p$ $= 255$ (to nearest whole number)