Section A: Pure Mathematics [40 marks]

1 A curve C has parametric equations

$$x = t^3 - 12t, \quad y = t - 2 \quad \text{for } t \le 2.$$

(i) Sketch C, labelling the coordinates of any end points. [2]

A line l has equation $y = m(x+16)$, where m is positive. It is given that l intersects C at the points where $t = 2$ and $t = k$, where $k \le -4$.

(ii) Show that the area of the region bounded by C and l is $6k^2 - \dfrac{k^4}{4} - 16k + 12 + \dfrac{(k-2)^2}{2m}$. [5]

2 An Art teacher teaches her students to create patterns using squares of different sizes. One possible pattern is to begin with the first square with sides of length 2 mm. The first square is inscribed in the second square, where the corners of the first square coincide with the midpoints of the second square. She continues inscribing squares in this manner where the n^{th} square is inscribed in the $(n+1)^{\text{th}}$ square. **Figure 1** shows a piece of artwork after 4 squares are drawn.

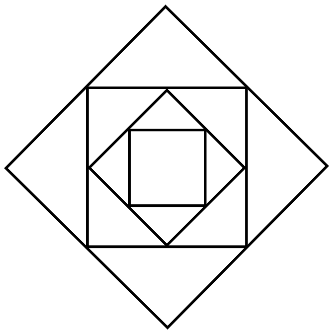

Figure 1

By using this pattern, Student A begins his artwork.

(i) Find, in terms of n, the length of the sides of the n^{th} square. [2]

(ii) A standard A4 paper measures 210 mm by 297 mm. Find the maximum number of complete squares that he can draw on the paper. [2]

Student B uses a giant drawing board and decides to make his artwork more eye-catching. He uses the same pattern and measurements as Student A, but he shades the 1^{st} square and also shades on any protruding areas covered by the 4^{th}, 7^{th} ,…,$(3N+1)^{\text{th}}$ squares, where N is a non-negative integer. A protruding area is defined by the region bounded by the newly drawn square and the square immediately preceding it. **Figure 2** shows a piece of artwork if he draws 4 squares.

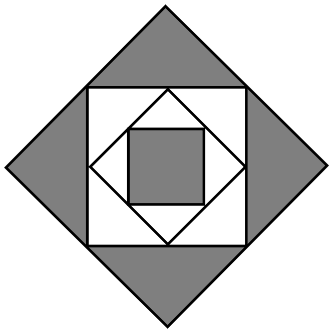

Figure 2

(iii) Find, in mm^2, the total shaded area as shown in **Figure 2**. [2]

(iv) Hence or otherwise, find the total shaded area if he draws 30 squares. Give your answer in m^2. [3]

3 Referred to the origin O, three distinct and non-collinear points A, B and C have position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively. Point L is the mid-point of BC. The position vector of a point P is given by $(1-k)\mathbf{a}+\frac{k}{2}(\mathbf{b}+\mathbf{c})$, where k is a non-zero constant and $k \neq 1$.

(i) Show that A, L and P are collinear. [3]

(ii) Show that $\frac{1}{2}\left|\overrightarrow{CP} \times \overrightarrow{CB}\right| = \frac{|1-k|}{2}\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}-\mathbf{a}\times\mathbf{c}\right|$. [3]

For the rest of the question, let $k = \frac{1}{2}$.

Let point Q be a point on the line passing through A and L. P and Q are distinct points and the areas of triangle CPB and triangle CQB are equal.

(iii) By considering part **(ii)**, find the position vector of Q in terms of $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$. [4]

(iv) Given that $\left|\overrightarrow{BC}\right| = 1$, interpret geometrically $\left|\overrightarrow{LP} \cdot \overrightarrow{BC}\right|$. [1]

4 **(a)** A quartic equation

$$\mathrm{i}z^4 + (-3-7\mathrm{i})z^3 + (21+17\mathrm{i})z^2 + (-51-15\mathrm{i})z + 45 = 0$$

has 4 distinct roots, z_1, z_2, z_3 and z_4 which are represented by points A, B, C and D respectively. It is given that $z_1 = -3\mathrm{i}$, $z_3 = 3$ and $\mathrm{Im}(z_4) > 0$.

(i) Find z_2 and z_4. [3]

(ii) Sketch the points A, B, C and D on an Argand diagram. [2]

(iii) Point E represents the complex number wz_3 such that $ABDE$ forms a parallelogram. Find w in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $0 \leq \theta < 2\pi$. [2]

(b) **Do not use a graphing calculator in answering this question.**

Express $\dfrac{(-4-4\mathrm{i})^5}{\left(-2\sqrt{3}+2\mathrm{i}\right)^7}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

(c) **Do not use a graphing calculator in answering this question.**

Given that $q = 1-\mathrm{i}$. Find the three smallest positive integers n for which $\left(\mathrm{i}q^n\right)^*$ is real and positive. [3]

Section B: Probability and Statistics [60 marks]

5 Twelve books, consisting of 5 identical Geography books, 4 identical Mathematics books and 3 identical Literature books, are arranged on a bookshelf that has a top rack and a bottom rack. Six books are chosen and arranged on each rack. Let

A be the event that **all** the Mathematics books are together,
B be the event that **all** the Literature books are on the same rack and separated.

(i) Find the number of ways to arrange the books if A and B occur. [2]

(ii) Find the number of ways to arrange the books if at least one Mathematics book is on the top rack. [4]

6 **(a)** For events F and G, it is given that $\mathrm{P}(F)=\frac{2}{5}$ and $\mathrm{P}(G)=\frac{2}{3}$. Find the greatest and least possible values of $\mathrm{P}(F'\cap G)$. [3]

(b) For events A, B and C, it is given that $\mathrm{P}(A)=\frac{3}{8}$, $\mathrm{P}(B)=\frac{2}{3}$, $\mathrm{P}(C)=\frac{5}{8}$, $\mathrm{P}(A\cap C)=\frac{1}{3}$ and $\mathrm{P}(A\cup B\cup C)=\frac{3}{4}$. It is also given that events A and B are independent, and that events B and C are independent.

(i) Find $\mathrm{P}(A'\cap B'|C')$. [3]

(ii) Find the exact value of $\mathrm{P}(A\cap B\cap C)$. [3]

7 The medical director of a hospital knows that the mean systolic blood pressure of patients who suffer from high blood pressure is 140 mmHg. He wishes to carry out a clinical trial to evaluate whether a new drug is effective in reducing the systolic blood pressure of patients who suffer from high blood pressure. The systolic blood pressure, x mmHg, of a random sample of 60 patients are summarized as follows.

$$\sum(x-140)=-37.6 \qquad \sum(x-140)^2=1012.17$$

(i) Calculate unbiased estimates of the population mean and variance of the systolic blood pressure of patients who suffer from high blood pressure [2]

(ii) Carry out the test, at 5% level of significance, for the medical director. You should state your hypotheses and define any symbols that you use. [5]

(iii) Upon closer inspection of the data of the sample of 60 patients, the director noted that the value of $\sum(x-140)$ is correct but the value of $\sum(x-140)^2$ should be larger instead. If a new test is carried out using this information at the 5% level of significance, explain whether the result of this test will differ from the result of the test in part **(ii)**. [3]

8 National Fruit Company owns a large tomato farm. The tomatoes produced are harvested and sold in boxes of 25. It is known that $100p\%$ of the tomatoes are rotten. For these boxes, the mean number of rotten tomatoes in a box is 1.

(i) Explain why the context above may not be well-modelled by a binomial distribution. [1]

Assume now that the context above is well-modelled by a binomial distribution.

(ii) State the value of p. [1]

(iii) Find the probability that a box chosen at random has less than 2 rotten tomato. [2]

(iv) A customer chose a box and inspected the contents individually. Find the probability that the twenty-first tomato is the fourth rotten tomato and no rotten tomatoes are found subsequently. [3]

Boxes that contains at least 24 tomatoes that are not rotten are deemed satisfactory.

(v) A customer first picks 3 boxes of tomatoes, of which at least 2 boxes are satisfactory. The customer then decides to buy another 5 boxes. Find the probability that exactly 6 of the 8 boxes are satisfactory. [4]

9 A shop sells two models of ovens produced by Factory A and Factory B. The lifespans of ovens produced by Factory A have the normal distribution with mean 13 years and standard deviation 6 months, while the lifespans of ovens produced by Factory B have the normal distribution with mean 15 years and standard deviation k months. The lifespan of any oven is independent of one another.

(i) Given that 90% of the ovens produced by Factory B exceeds a lifespan of 14 years. Show that $k = 9.3636$, correct to 5 significant figures. [3]

(ii) Find the probability that the lifespan of a randomly chosen oven produced by Factory B exceeds the lifespan of a randomly chosen oven produced by Factory A by less than 3 years. [2]

(iii) There is a probability of at least 0.4 that the lifespan of a randomly chosen oven produced by Factory A is within n years of 13 years. Find the least value of n, correct to 3 decimal places. [3]

(iv) Every oven produced by Factory A is wrapped in a box. A carton contains 20 of such boxes. If there are at least 3 ovens in a carton with lifespans of less than 12 years, the carton will be rejected. Find the probability that a carton is rejected. [3]

10 A circular card is divided into 3 sectors with values 0, 1, 2 and having angles $180°, (360p)°, (360q)°$ respectively where p and q are non-zero constants. The card has a pointer pivoted at its centre. After being set in motion, the pointer comes to rest randomly in one of the sectors.

In a game, a player gets to spin the pointer twice. The player's score is denoted by X. The player's score is

- the greater of the two values if the values shown on both spins are different.
- the sum of the two values obtained if the values shown on both spins are equal.

(i) Show that the probability that a player's score in a game is 2 is $0.5-p^2$. [3]

(ii) Find, in terms of p, the probability distribution of X. [2]

(iii) Given that $\mathrm{E}(X)=\dfrac{11}{9}$, find the exact value of $\mathrm{Var}(X)$. [4]

(iv) Find the probability that a player's mean score in 50 games is less than 1.5. [2]

A player plays 3 games. Let

A be the event that a player's total score in the 3 games is more than 5.

B be the event that a player's score is at least 2 in each of the 3 games,

(v) Without doing any calculation, explain why $\mathrm{P}(B)$ is less than $\mathrm{P}(A)$. [1]