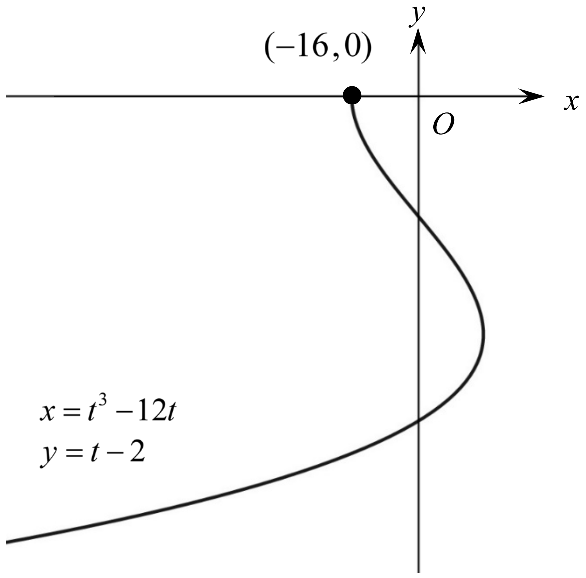
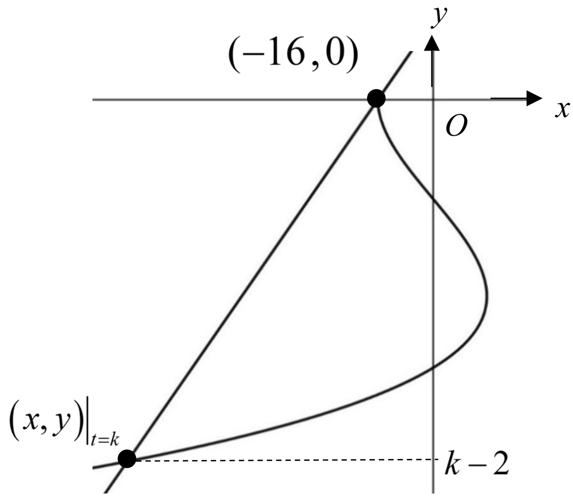
<table>
<tr><th>Q1</th><th>Suggested Solutions</th></tr>
<tr><td>(i)</td><td>
</td></tr>
<tr><td>(ii)</td><td>

$$y=m\left(x+16\right)\Rightarrow x=\frac{y}{m}-16$$

$$A=\int_{k-2}^{0}x\,\mathrm{d}y-\int_{k-2}^{0}\left(\frac{y}{m}-16\right)\mathrm{d}y$$

$$=\int_{k}^{2}\left(t^3-12t\right)\left(1\,\mathrm{d}t\right)-\left[\frac{y^2}{2m}-16y\right]_{k-2}^{0}$$

$$=\left[\frac{t^4}{4}-6t^2\right]_{k}^{2}-\left[0-\left(\frac{\left(k-2\right)^2}{2m}-16(k-2)\right)\right]$$

$$=\left[\left(\frac{16}{4}-24\right)-\left(\frac{k^4}{4}-6k^2\right)\right]+\left[\frac{\left(k-2\right)^2}{2m}-16(k-2)\right]$$

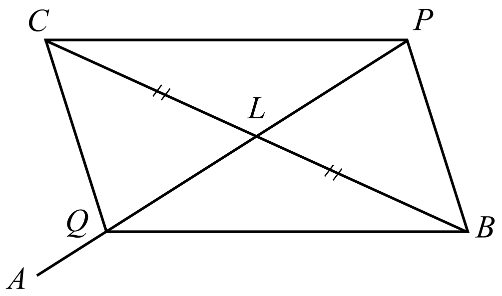
$$=6k^2-\frac{k^4}{4}-16k+12+\frac{\left(k-2\right)^2}{2m}$$</td></tr>
</table>

Q2	**Suggested Solutions**
(i)	<table><tr><th>n</th><th>Length of n^{th} square</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>$\left(2^{\frac{1}{2}}\right)2 = 2^{\frac{3}{2}}$</td></tr><tr><td>3</td><td>$\left(2^{\frac{2}{2}}\right)2 = 2^{\frac{4}{2}}$</td></tr><tr><td>: .</td><td>: .</td></tr><tr><td>n</td><td>$2^{\frac{n+1}{2}}$</td></tr></table> The length of the n^{th} square is $2^{\frac{n+1}{2}}$ mm.
(ii)	$2^{\frac{n+1}{2}} < 210$ $\frac{n+1}{2} < \frac{\ln(210)}{\ln 2}$ $n < 14.428$ Hence maximum number of square is 14.
(iii)	From part **(i)**, length of the n^{th} square is $2^{\frac{n+1}{2}}$. Therefore, area of the n^{th} square $= \left(2^{\frac{n+1}{2}}\right)^2 = 2^{n+1}$. Area of the 1st square $= 2^2$ Area of the 4th square – Area of the 3rd square $= 2^5 - 2^4$ $= 2^4(2-1)$ $= 2^4$ Total shaded area in Figure 2 $= 2^2 + 2^4 = 20$
(iv)	<table><tr><th>n</th><th>Area of n^{th} square</th><th>Protruding area of n^{th} square</th></tr><tr><td>1</td><td>2^2</td><td>2^2</td></tr><tr><td>2</td><td>2^3</td><td>$2^3 - 2^2 = 2^2(2-1) = 2^2$</td></tr><tr><td>3</td><td>2^4</td><td>$2^4 - 2^3 = 2^3(2-1) = 2^3$</td></tr><tr><td>4</td><td>2^5</td><td>2^4</td></tr><tr><td>: .</td><td>: .</td><td>: .</td></tr><tr><td>7</td><td>2^8</td><td>2^7</td></tr><tr><td>: .</td><td>: .</td><td>: .</td></tr><tr><td>28</td><td>2^{29}</td><td>2^{28}</td></tr></table> He will only shade up to the 28th square if he draws 30 squares.

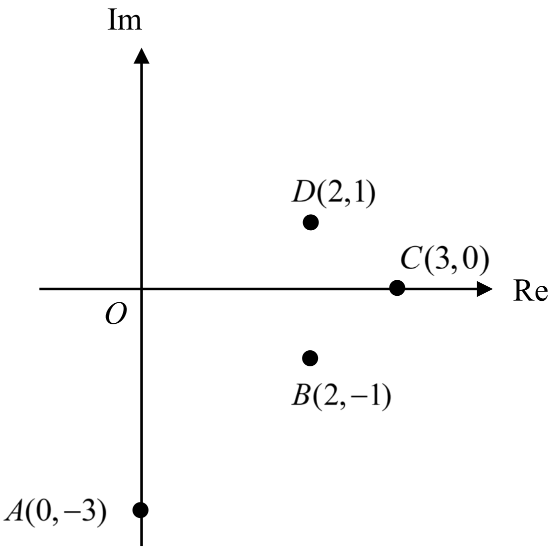
Q2	**Suggested Solutions**
	Total shaded area $= 2^2 + 2^4 + 2^7 + \ldots + 2^{28}$ $= 4 + \dfrac{2^4(2^{3(9)} - 1)}{\left(2^3 - 1\right)}$ $= 306{,}783{,}380 \text{ mm}^2$ $= 307 \text{ m}^2$ (3 s.f.).

Q3	**Suggested Solutions**
(i)	$\overrightarrow{AL}$ $=\overrightarrow{OL}-\overrightarrow{OA}$ $=\dfrac{\mathbf{b}+\mathbf{c}}{2}-\mathbf{a}$ $\overrightarrow{AP}$ $=\overrightarrow{OP}-\overrightarrow{OA}$ $=(1-k)\mathbf{a}+k\dfrac{\mathbf{b}+\mathbf{c}}{2}-\mathbf{a}$ $=k\left(\dfrac{\mathbf{b}+\mathbf{c}}{2}-\mathbf{a}\right)$ $=k\overrightarrow{AL}$ OR $\overrightarrow{PL}$ $=\overrightarrow{OL}-\overrightarrow{OP}$ $=\dfrac{\mathbf{b}+\mathbf{c}}{2}-(1-k)\mathbf{a}-k\dfrac{\mathbf{b}+\mathbf{c}}{2}$ $=(1-k)\left(\dfrac{\mathbf{b}+\mathbf{c}}{2}-\mathbf{a}\right)$ $=(1-k)\overrightarrow{AL}$ Since $\overrightarrow{AP}$ is **parallel** to $\overrightarrow{AL}$ with a **common point** L, A, L and P are collinear.
(ii)	$\dfrac{1}{2}\left\|\overrightarrow{CP}\times\overrightarrow{CB}\right\|$ $=\dfrac{1}{2}\left\|\left(\overrightarrow{OP}-\overrightarrow{OC}\right)\times\left(\overrightarrow{OB}-\overrightarrow{OC}\right)\right\|$ $=\dfrac{1}{2}\left\|\left((1-k)\mathbf{a}+k\dfrac{(\mathbf{b}+\mathbf{c})}{2}-\mathbf{c}\right)\times(\mathbf{b}-\mathbf{c})\right\|$ $=\dfrac{1}{2}\left\|(1-k)\mathbf{a}\times(\mathbf{b}-\mathbf{c})+\dfrac{k}{2}(\mathbf{b}+\mathbf{c})\times(\mathbf{b}-\mathbf{c})-\mathbf{c}\times(\mathbf{b}-\mathbf{c})\right\|$ $=\dfrac{1}{2}\left\|\begin{array}{l}(1-k)\mathbf{a}\times(\mathbf{b}-\mathbf{c})\\+\dfrac{k}{2}(\mathbf{b}\times\mathbf{b}-\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{b}-\mathbf{c}\times\mathbf{c})\\-\mathbf{c}\times\mathbf{b}+\mathbf{c}\times\mathbf{c}\end{array}\right\|$

Q3	Suggested Solutions
	$=\frac{1}{2}\left\lvert\begin{matrix}(1-k)\mathbf{a}\times(\mathbf{b}-\mathbf{c}) \\ +\frac{k}{2}(\mathbf{0}-\mathbf{b}\times\mathbf{c}-\mathbf{b}\times\mathbf{c}-\mathbf{0}) \\ +\mathbf{b}\times\mathbf{c}+\mathbf{0}\end{matrix}\right\rvert$ $=\frac{1}{2}\lvert(1-k)\mathbf{a}\times(\mathbf{b}-\mathbf{c})+(1-k)(\mathbf{b}\times\mathbf{c})\rvert$ $=\frac{\lvert 1-k\rvert}{2}\lvert\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}-\mathbf{a}\times\mathbf{c}\rvert$
(iii)	$k=\frac{1}{2}$ $\overrightarrow{OP}=(1-k)\mathbf{a}+\frac{k}{2}(\mathbf{b}+\mathbf{c})=\frac{1}{2}\mathbf{a}+\frac{1}{4}(\mathbf{b}+\mathbf{c})$ The points *A, L, P* and *Q* are collinear. Let *l* be the line passing through points *A* and *L*. $l:\mathbf{r}=(1-k)\mathbf{a}+\frac{k}{2}(\mathbf{b}+\mathbf{c})$ where $k\in\mathbb{R}, k\neq 1$ Since *P* and *Q* lie on the line passing through *A* and *L*, hence $\overrightarrow{OQ}=(1-\lambda)\mathbf{a}+\frac{\lambda}{2}(\mathbf{b}+\mathbf{c})$ for some real constant λ. Hence, $\overrightarrow{OQ}$ has the same form as $\overrightarrow{OP}$. This implies that area of triangle *CQB* $=\frac{\lvert 1-\lambda\rvert}{2}\lvert\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}-\mathbf{a}\times\mathbf{c}\rvert$ by **(ii)**. Area of triangle *CQB* = Area of triangle *CPB* $\frac{\lvert 1-\lambda\rvert}{2}\lvert\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}-\mathbf{a}\times\mathbf{c}\rvert=\frac{\lvert 1-0.5\rvert}{2}\lvert\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}-\mathbf{a}\times\mathbf{c}\rvert$ $\frac{\lvert 1-\lambda\rvert}{2}=\frac{\lvert 1-0.5\rvert}{2}$ $\lvert 1-\lambda\rvert=0.5$ $1-\lambda=0.5$ or -0.5 $\lambda=0.5$ (value of k that gives point P) or 1.5. $\overrightarrow{OQ}=(1-1.5)\mathbf{a}+\frac{1.5}{2}(\mathbf{b}+\mathbf{c})$ $=-0.5\mathbf{a}+0.75(\mathbf{b}+\mathbf{c})$

Q3	Suggested Solutions
	Alternative Method (Geometrical)  $CPQB$ is a parallelogram. $k=\dfrac{1}{2}$ $\overrightarrow{OP}=(1-k)\mathbf{a}+\dfrac{k}{2}(\mathbf{b}+\mathbf{c})=\dfrac{1}{2}\mathbf{a}+\dfrac{1}{4}(\mathbf{b}+\mathbf{c})$ $\overrightarrow{QC}=\overrightarrow{BP}$ $\overrightarrow{OC}-\overrightarrow{OQ}=\overrightarrow{OP}-\overrightarrow{OB}$ $\overrightarrow{OQ}=\overrightarrow{OB}+\overrightarrow{OC}-\overrightarrow{OP}$ $=\mathbf{b}+\mathbf{c}-\left[\dfrac{1}{2}\mathbf{a}+\dfrac{1}{4}(\mathbf{b}+\mathbf{c})\right]$ $=-0.5\mathbf{a}+0.75(\mathbf{b}+\mathbf{c})$
(iv)	$\left\lvert\overrightarrow{LP}\cdot\overrightarrow{BC}\right\rvert$ is the length of projection of $\overrightarrow{LP}$ onto $\overrightarrow{BC}$.

Note that the formula for length of projection of is $\lvert h\cdot\hat{a}\rvert$ where $\hat{a}$ is an unit vector. Hence this works because $\left\lvert\overrightarrow{BC}\right\rvert=1$.

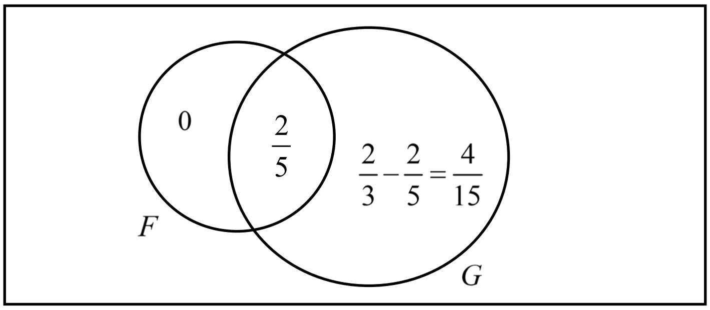
Q4	Suggested Solutions
(a) **(i)**	$\mathrm{i}z^4+(-3-7\mathrm{i})z^3+(21+17\mathrm{i})z^2+(-51-15\mathrm{i})z+45=0$ $(z-3)(z+3\mathrm{i})(\mathrm{i}z^2+az+b)=0$ $(z^2+(3\mathrm{i}-3)z-9\mathrm{i})(\mathrm{i}z^2+az+b)=0$ Note that the coefficients *a* and *b* may not be real. The conjugate root theorem also do not apply in this case because the equation is not one with all coefficients real. By comparing coefficient of constant term: $(-9\mathrm{i})(b)=45 \Rightarrow b=5\mathrm{i}$ *z* term: $-9a\mathrm{i}+b(3\mathrm{i}-3)=-51-15\mathrm{i}$ $-9a\mathrm{i}-15-15\mathrm{i}=-51-15\mathrm{i}$ $-9a\mathrm{i}=-36$ $a=\dfrac{4}{\mathrm{i}}$ $a=-4\mathrm{i}$ $(z^2+(3\mathrm{i}-3)z-9\mathrm{i})(\mathrm{i}z^2-4\mathrm{i}z+5\mathrm{i})=0$ $(z^2+(3\mathrm{i}-3)z-9\mathrm{i})(\mathrm{i})(z^2-4z+5)=0$ Solving $(z^2-4z+5)=0$ by GC, $z_2=2-\mathrm{i}$ and $z_4=2+\mathrm{i}$ since $\mathrm{Im}(z_4)>0$.
(ii)	
(iii)	From the Argand diagram, for *ABDE* to form a parallelogram, $E(0,-1)$. Therefore, $wz_3=-\mathrm{i}$. $wz_3=-\mathrm{i}$ $3w=\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}$ $w=\frac{1}{3}\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}$

Q4	Suggested Solutions
	$w=\frac{1}{3}\mathrm{e}^{\mathrm{i}\left(\frac{3\pi}{2}\right)}$
(b)	$\left(-4-4\mathrm{i}\right)^5=\left[\sqrt{4^2+4^2}\right]^5\mathrm{e}^{\mathrm{i}\left(-\frac{3\pi}{4}\right)(5)}$ $=\left(\sqrt{32}\right)^5\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ $\left(-2\sqrt{3}+2\mathrm{i}\right)^7=\left(\sqrt{12+4}\right)^7\mathrm{e}^{\mathrm{i}\left(\frac{5\pi}{6}\right)(7)}$ $=4^7\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{6}\right)}$ $\frac{\left(-4-4\mathrm{i}\right)^5}{\left(-2\sqrt{3}+2\mathrm{i}\right)^7}=\frac{\left(\sqrt{32}\right)^5\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}}{4^7\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{6}\right)}}$ $=\frac{1}{2\sqrt{2}}\mathrm{e}^{\mathrm{i}\frac{5\pi}{12}}$
(c)	$\arg\left(\mathrm{i}q^n\right)^*=-\arg\left(\mathrm{i}q^n\right)$ $=-\left[\arg\left(\mathrm{i}\right)+n\arg\left(q\right)\right]$ $=-\left(\frac{\pi}{2}-\frac{n\pi}{4}\right)$ $\arg\left(\mathrm{i}q^n\right)^*=2k\pi,\ k\in\mathbb{Z}$ Real and positive implies that $-\left(\frac{\pi}{2}-\frac{n\pi}{4}\right)=2k\pi$. $\frac{n}{4}-\frac{1}{2}=2k$ $n=8k+2$ The three smallest positive integers are 2, 10 and 18.

Q5	**Suggested Solutions**
(i)	No of ways $= 2\times 3\times {}^4C_3 = 24$
(ii)	<u>**Using complement method**</u> Total number of ways to arrange the 12 books $= \dfrac{12!}{3!4!5!} = 27720$ 3 cases if there is no Mathematics book on the top rack. Case 1: 4 Mathematics and 2 Literature on the bottom rack. No of ways $= \left(\dfrac{6!}{4!2!}\right)\left(\dfrac{6!}{5!}\right) = 90$ Case 2: 4 Mathematics and 2 Geography books on the bottom rack. No of ways $= \left(\dfrac{6!}{4!2!}\right)\left(\dfrac{6!}{3!3!}\right) = 300$ Case 3: 4 Mathematics and 1 Geography and 1 Literature book on the bottom rack. No of ways $= \left(\dfrac{6!}{4!}\right)\left(\dfrac{6!}{4!2!}\right) = 450$ Total no of ways $= 27720 - 90 - 300 - 450 = 26880$

Q6	Suggested Solutions

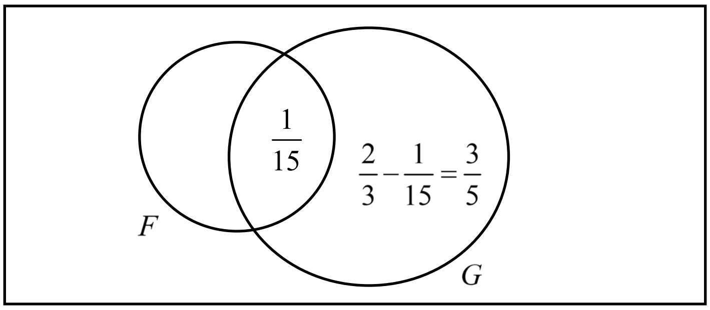
(a) Consider $F \subseteq G$:



Least $\mathrm{P}(F'\cap G)$ is $\frac{4}{15}$.

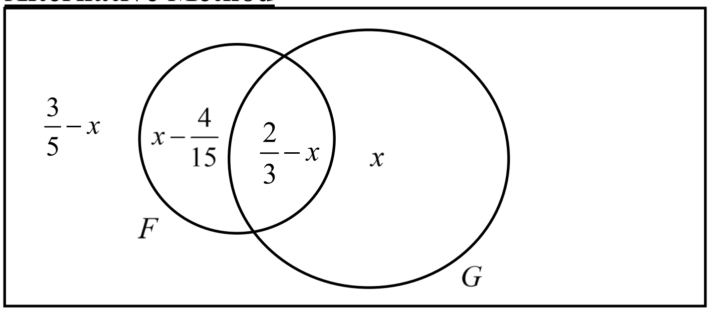
$\frac{2}{3}+\frac{2}{5}=\frac{16}{15}>1$

$\therefore \mathrm{P}(F\cap G)=\frac{16}{15}-1=\frac{1}{15}$



Greatest $\mathrm{P}(F'\cap G)$ is $\frac{3}{5}$.

Alternative Method



Let $\mathrm{P}(F'\cap G)=x$.

$\mathrm{P}(F\cap G)=\mathrm{P}(G)-\mathrm{P}(F'\cap G)=\frac{2}{3}-x$.

$\mathrm{P}(F\cap G')=\mathrm{P}(F)-\mathrm{P}(F\cap G)=\frac{2}{5}-\left(\frac{2}{3}-x\right)=x-\frac{4}{15}$.

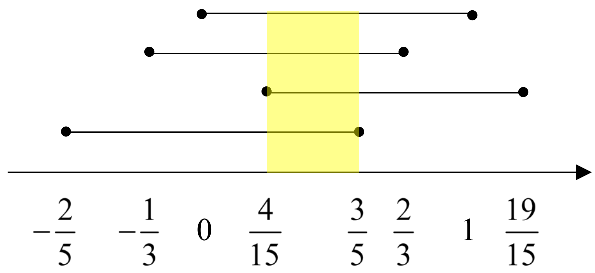
$\mathrm{P}(F'\cap G')=1-\mathrm{P}(F\cup G)=1-\left(x-\frac{4}{15}+\frac{2}{3}\right)=\frac{3}{5}-x$.

Therefore, $0\le x\le 1$ and

Q6	Suggested Solutions

$0 \le \frac{2}{3} - x \le 1$ and $0 \le x - \frac{4}{15} \le 1$ and $0 \le \frac{3}{5} - x \le 1$.

This is equivalent to $0 \le x \le 1$ and
$-\frac{1}{3} \le x \le \frac{2}{3}$ and $\frac{4}{15} \le x \le \frac{19}{15}$ and $-\frac{2}{5} \le x \le \frac{3}{5}$.



$\frac{4}{15} \le x \le \frac{3}{5}$

Greatest $\mathrm{P}(F' \cap G)$ is $\frac{3}{5}$. Least $\mathrm{P}(F' \cap G)$ is $\frac{4}{15}$.

(b)(i)

$$\begin{aligned} \mathrm{P}(A' \cap B' | C') &= \frac{\mathrm{P}(A' \cap B' \cap C')}{\mathrm{P}(C')} \\ &= \frac{1 - \mathrm{P}(A \cup B \cup C)}{1 - \mathrm{P}(C)} \\ &= \frac{1 - \frac{3}{4}}{1 - \frac{5}{8}} \\ &= \frac{2}{3} \end{aligned}$$

(b)(ii)

Since events A and B are independent,
$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B) = \frac{1}{4}$.
Since events B and C are independent,
$\mathrm{P}(B \cap C) = \mathrm{P}(B)\mathrm{P}(C) = \frac{5}{12}$.

$$\begin{aligned} \mathrm{P}(A \cup B \cup C) = {} & \mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C) \\ & - \mathrm{P}(A \cap B) - \mathrm{P}(A \cap C) - \mathrm{P}(B \cap C) \\ & + \mathrm{P}(A \cap B \cap C) \end{aligned}$$

$$\frac{3}{4} = \frac{3}{8} + \frac{2}{3} + \frac{5}{8} - \frac{1}{4} - \frac{1}{3} - \frac{5}{12} + \mathrm{P}(A \cap B \cap C)$$

$$\mathrm{P}(A \cap B \cap C) = \frac{1}{12}$$

Q7	**Suggested Solutions**
(i)	$\bar{x}=\dfrac{-37.6}{60}+140=139.3733\approx 139\ (3\text{ s.f.})$ $s^2=\dfrac{1}{59}\left[1012.17-\dfrac{(-37.60)^2}{60}\right]$ $=16.7560565$ $=16.8\ (3\text{ s.f.})$
(ii)	Let μ be the **population mean** systolic blood pressure of patients who suffer from high blood pressure. $\mathrm{H_0}:\mu=140$ vs $\mathrm{H_1}:\mu<140$ Level of significance: 5% (lower tailed). Under $\mathrm{H_0}$, $\overline{X}$ is approximately normal by Central Limit Theorem since $n=60(\geq 30)$ is large. Hence, $Z=\dfrac{\overline{X}-140}{\sqrt{S^2/60}}\sim \mathrm{N}(0,1)$ approximately. **Method 1 : Using *p*-value** By GC, p-value $=0.11783>0.05$ **Method 2: Using critical region and test statistic, z** Critical region: $z<-1.64485$ $z=\dfrac{139.3733-140}{\sqrt{16.7560565/60}}$ $=-1.18590>-1.6449$ Do not reject $\mathrm{H_0}$. We conclude that there is insufficient evidence at 5% level of significance to claim that the new drug is effective in reducing the systolic blood pressure of patients suffering from high blood pressure.
(iii)	Since $\sum(x-140)^2$ is larger, the value of s^2 increases. The new observed test statistic value gets smaller (closer to zero) and thus less negative than the original test statistic value. Therefore, new observed z-value > original observed z-value > z_{critical}. The new test statistic value remains to be outside the critical region (new p-value gets larger which exceeds the critical region). So the result of this test will not differ from the result of the test in part **(ii)**.
(iii)	**Alternative Method** Since $\sum(x-140)^2>1012.17$,

Q7	**Suggested Solutions**
	then $s^2 = \frac{1}{59}\left[\sum(x-140)^2 - \frac{(-37.60)^2}{60}\right] > 16.75601$ Test statistic: $Z = \frac{\overline{X}-140}{\sqrt{s^2/60}} = \sqrt{\frac{60}{s^2}}\left(\overline{X}-140\right)$ Therefore, new $z = \sqrt{\frac{60}{s^2}}(139.3733-140) > -1.6448$ Do not reject H_0. Hence, the result of the new test remains unchanged from the result of the test carried out in part **(ii)**. Detailed manipulation: $\frac{1}{s^2} < \frac{1}{16.75601}$ $\sqrt{\frac{60}{s^2}} < \sqrt{\frac{60}{16.75601}}$ $\sqrt{\frac{60}{s^2}}(139.3733-140) > \sqrt{\frac{60}{16.75601}}(139.3733-140) > -1.6449$ Therefore, new $z = \sqrt{\frac{60}{s^2}}(139.3733-140) > -1.6448$

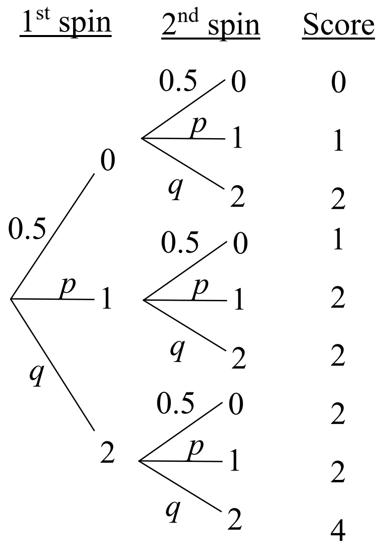
Q8	Suggested Solutions
(i)	The **probability** that a tomato is rotten **may not be the same** for all tomatoes because the tomatoes from the farm may be subjected to different treatment, thereby affecting the quality of the tomatoes. OR Whether a tomato is rotten may not be **independent** of whether another tomato is rotten because a rotten tomato may **affect** the quality of other tomatoes from the same plant.
(ii)	$25(p)=1$ $p=\frac{1}{25}=0.04$
(iii)	Let X be the number of rotten tomatoes out of 25. $X \sim \mathrm{B}(25, 0.04)$ $\mathrm{P}(X<2)=\mathrm{P}(X\le 1)$ $=0.735810$ $=0.736(3\text{ s.f.})$
(iv)	Let Y be the number of rotten tomatoes out of 20. $Y \sim \mathrm{B}(20, 0.04)$ Required probability $=\mathrm{P}(Y=3)(0.04)(0.96)^4$ $=(0.0364499)(0.04)(0.96)^4$ $=0.00124$ (3 s.f.)
(v)	Let Q be the number of satisfactory boxes out of 3. $Q\sim \mathrm{B}(3, 0.73581)$ Let R be the number of satisfactory boxes out of 5 $R\sim \mathrm{B}(5, 0.73581)$ $\mathrm{P}(\text{exactly 6 boxes satisfactory} \mid Q\ge 2)$ $=\frac{\mathrm{P}(\text{exactly 6 boxes satisfactory and } Q\ge 2)}{\mathrm{P}(Q\ge 2)}$ $=\frac{\mathrm{P}(Q=2)\mathrm{P}(R=4)+\mathrm{P}(Q=3)\mathrm{P}(R=3)}{\mathrm{P}(Q\ge 2)}$ $=\frac{\mathrm{P}(Q=2)\mathrm{P}(R=4)+\mathrm{P}(Q=3)\mathrm{P}(R=3)}{1-\mathrm{P}(Q\le 1)}$ $=0.335\,(3\text{ s.f.})$

Q9	**Suggested Solutions**
(i)	Let B denote the lifespan (in **years**) of an oven produced by Factory B. $B \sim \mathrm{N}\left(15, \left(\frac{k}{12}\right)^2\right)$ $\mathrm{P}(B > 14) = 0.9$ $\mathrm{P}\left(Z > \frac{14-15}{\left(\frac{k}{12}\right)}\right) = 0.9$ $-\frac{12}{k} \approx -1.281551567$ $k \approx 9.3636497$ $= 9.3636$ (5 s.f.) <u>OR</u> Let B denote the lifespan (in **months**) of an oven produced by Factory B. $B \sim \mathrm{N}(180, k^2)$ $\mathrm{P}(B > 14 \times 12) = 0.9$ $\mathrm{P}\left(Z > \frac{168-180}{k}\right) = 0.9$ $-\frac{12}{k} \approx -1.281551567$ $k \approx 9.3636497$ $= 9.3636$ (5 s.f.)
(ii)	$\mathrm{E}(B - A) = 15 - 13 = 2$ $\mathrm{Var}(B - A) = \left(\frac{9.3636}{12}\right)^2 + 0.5^2 \approx 0.85886809$ $B - A \sim \mathrm{N}(2, 0.85887)$ $\mathrm{P}(0 < B - A < 3) = 0.844$ (3 s.f.)
(iii)	Let A denote the lifespan (in **years**) of an oven produced by Factory A. $A \sim \mathrm{N}(13, 0.5^2)$

Q9	Suggested Solutions
	$\mathrm{P}(13-n \le A \le 13+n) \ge 0.4$ $\mathrm{P}(A \le 13-n) \le 0.3$ Let $\mathrm{P}(A \le a) = 0.3$ From GC, $a \approx 12.7377997$ $13-n \le 12.7378$ $n \ge 0.2622$ Least $n = 0.263$ (3 d.p.)
(iv)	$\mathrm{P}(A < 12) \approx 0.022750062$ Let X denote the number of oven, out of 20, with lifespan less than 12 years. $X \sim \mathrm{B}(20, 0.022750)$ $\mathrm{P}(X \ge 3) = 1 - \mathrm{P}(X \le 2)$ $\approx 1 - 0.9899538$. $= 0.0100$ (3 s.f.)

Q10	Suggested Solutions

(i)



$$\begin{aligned}\mathrm{P}(X=2)&=0.5q+p^2+pq+0.5q+pq\\&=q+2pq+p^2\\&=(0.5-p)+2p(0.5-p)+p^2 \quad \because 0.5+p+q=1\\&=0.5-p^2\end{aligned}$$

(ii)

$\mathrm{P}(X=0)=(0.5)^2=0.25$

$\mathrm{P}(X=1)=0.5p+p(0.5)=p$

$\mathrm{P}(X=2)=0.5-p^2$ from part **(i)**

$\mathrm{P}(X=4)=q^2=(0.5-p)^2$

x	0	1	2	4
$\mathrm{P}(X=x)$	0.25	p	$0.5-p^2$	$(0.5-p)^2$

(iii)

$$\mathrm{E}(X)=\frac{11}{9}$$

$$0+p+2\left(0.5-p^2\right)+4(0.5-p)^2=\frac{11}{9}$$

$$p+1-2p^2+4\left(0.25-p+p^2\right)=\frac{11}{9}$$

$$2-3p-2p^2=\frac{11}{9}$$

$$18p^2-27p+7=0$$

$$(3p-1)(6p-7)=0$$

$$p=\frac{1}{3} \text{ or } p=\frac{7}{6} \text{ (Rej } \because 0\le p\le 1)$$

Q10	Suggested Solutions

x	0	1	2	4
$\mathrm{P}(X=x)$	0.25	$\frac{1}{3}$	$\frac{7}{18}$	$\frac{1}{36}$

$$\mathrm{E}\left(X^2\right)=0+\frac{1}{3}+2^2\left(\frac{7}{18}\right)+4^2\left(\frac{1}{36}\right)$$

$$=\frac{7}{3}$$

$$\mathrm{Var}(X)=\mathrm{E}\left(X^2\right)-\left[\mathrm{E}(X)\right]^2$$

$$=\frac{7}{3}-\left(\frac{11}{9}\right)^2$$

$$=\frac{68}{81}$$

(iv)

Since $n=50$ is large, $\bar{X}\sim \mathrm{N}\left(\frac{11}{9},\ \frac{68}{81(50)}\right)$ approximately by Central Limit Theorem.

$\mathrm{P}\left(\bar{X}<1.5\right)=0.984$

(v)

B is a proper **subset** of A.

(If a player scores at least 2 in each of the three games, then a player's total score in the three games will be at least 6 which is more than 5. Therefore, all the possible outcomes of event B are also outcomes of event A.)

Furthermore, there are outcomes in event A that are not outomes of event B. For example, a player can score a combination of 0, 2, and 4 in each of the 3 games. The total score is 6 which is more than 5, but the player did not score at least 2 in each of the 3 games.

Therefore, $\mathrm{P}(B)$ is less than $\mathrm{P}(A)$.