NAN CHIAU HIGH SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS

MATHEMATICS　　　　　　　　　　　　　　　　　**4048/01**

Paper 1　　　　　　　　　　　　　　　　　**19 Aug 2021, Thursday**

　　　　　　　　　　　　　　　　　　　　　　2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [   ] at the end of each question or part question.
The total of the marks for this paper is 80.

This paper consists of **19** printed pages including the cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1   Calculate $6.2 \times 10^{-7} \times 2.7 \times 10^{-5}$, giving your answer in standard form.

Answer …..………..…..……….. [1]

**2**   Write the following numbers in order of size, starting with the smallest.

$$-\sqrt{9.87}, \qquad -\frac{22}{7}, \qquad -314.2\%, \qquad -3.143$$

Answer …..……….. , ………...…… , ……..……... , ………...…… [2]

3   Find the area of the smallest circle passing through the points $(2, 4)$ and $(5, 1)$. Express your answer in terms of π.

Answer ..…………..…… units^2 [2]

4

Insignificant reduction in food wastage from 2015 to 2019

Total Food Waste in Singapore (in kilotonnes)
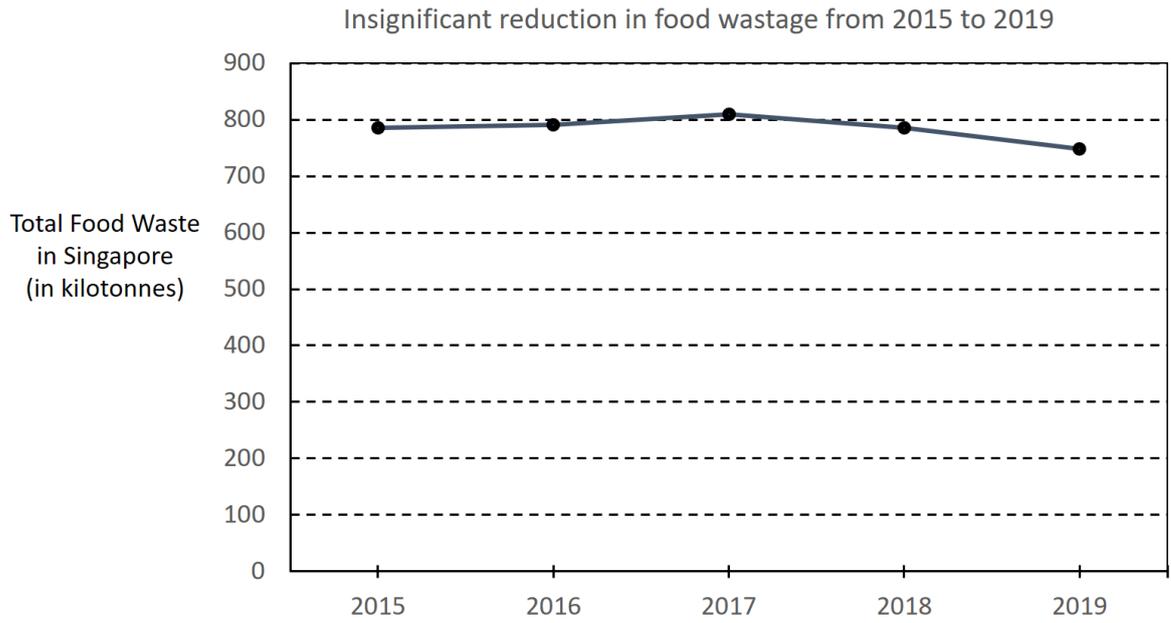
State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

...

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....

.... [2]

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5 Two consecutive positive integers are multiplied. The product when rounded to the nearest hundred is 202100. Find the sum of the two positive integers.

Answer .... [2]

6 The diagrams A, B, C, D, E and F show six graphs of different functions.

Diagram A	Diagram B
y O $\dfrac{1}{x}$	y O x

Diagram C	Diagram D
y O $\dfrac{1}{x}$	y O x

Diagram E	Diagram F
y O x	y O x
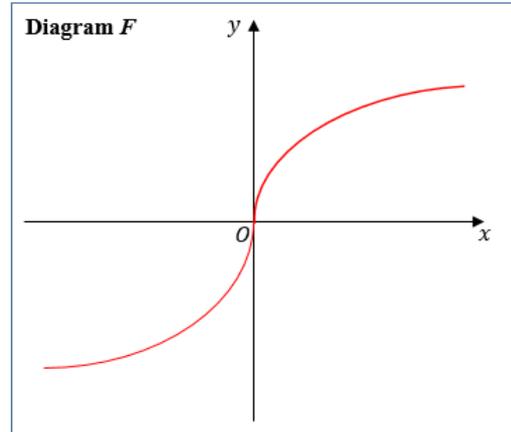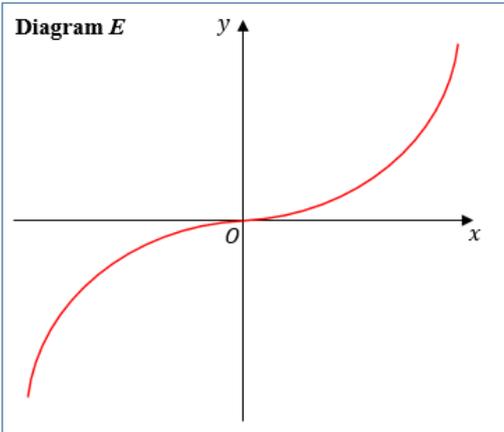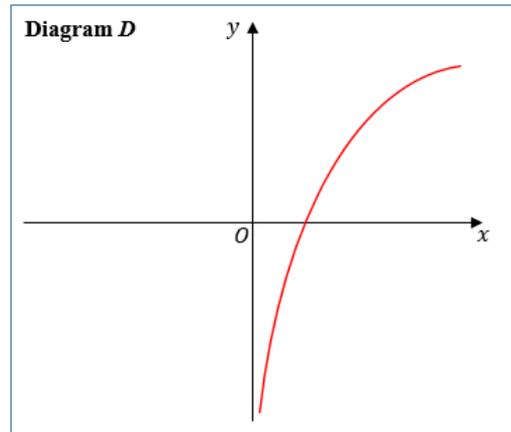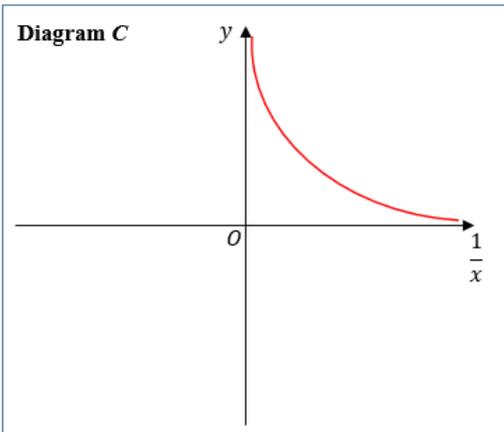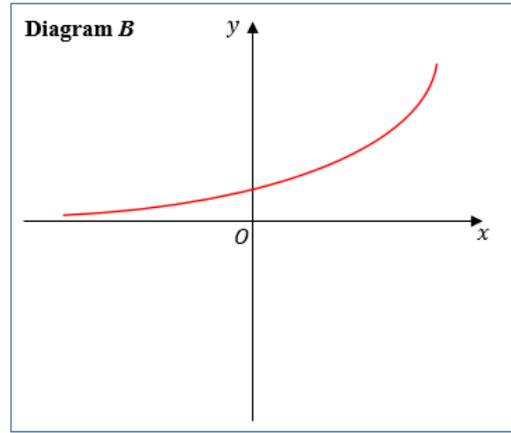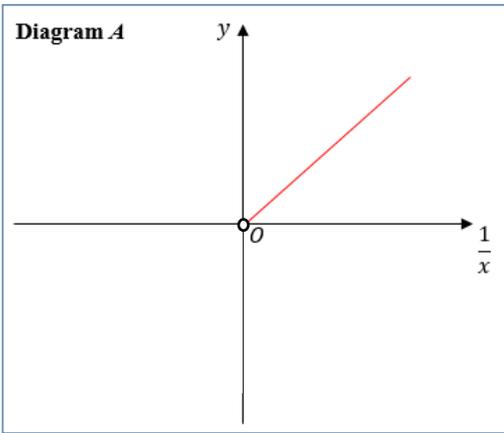
Select the diagram that best shows that

(a) y is inversely proportional to x,

Diagram ……………..… [1]

(b) $y = x^n$, where n is an odd integer greater than 1,

Diagram ……………..… [1]

(c) $y = n^x$, where n is an integer greater than 1.

Diagram ……………..… [1]

7   Naomi received a commission of $35 750 from the sale of a house. This sum of money was
divided among his expenditure, insurance premiums and savings in the ratio 5 : 4 : 6.
The savings is invested at a rate of 0.25% per annum compounded quarterly for a period of
2 years. Calculate the compound interest earned.

Answer $ ..……….…...………... [3]

8   In an experiment with 16 observations on x, the following results were obtained:

$$\sum x = 182 , \quad \sum x^2 = 3548$$

One observation with value 20 was detected to be wrongly recorded and was replaced by
its correct value 30. Find the corrected standard deviation.

Answer …..………….…..……….. [3]

9 The solution of the inequality $-1 < \dfrac{1-x}{2} \le k + 1$, where k is a constant, is represented on the number line below. Find the value of k.
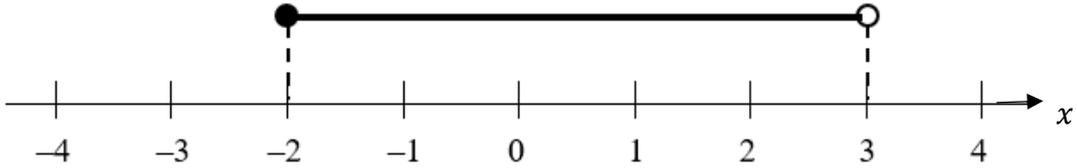
Answer $k =$ …….….…..……….. [3]

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10 **(a)** On the Venn diagram, shade the region which represents $(A \cup B') \cap (A' \cup B)$.

[1]
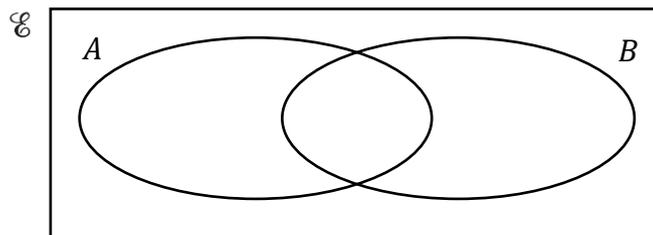
(b) $\mathcal{E} = \{(x, y): x + y \le 10\}$

$A = \{(0, 1), (1, 2), (1, 4), (1, 5)\}$

$B = \{(1, 0), (1, 2), (1, 3), (2, 1), (3, 7)\}$

Underline the correct statements from the list below.

$A \cap B = (1, 2)$ $\{1, 5\} \subset A$ $\emptyset \subset B$ $(2, 6) \in (A \cup B)'$ $A \subset B$

[2]

11   The   difference   of   the   two   roots   of   the   equation $3x^2 + 9x - k = 0$ is   13 units, where k is a constant. Find the value of k.

Answer $k = $ …………….....…. [3]

12   Find an expression, in terms of r, for the volume of the largest cube that can possibly fit inside a sphere of radius r cm.

Answer …..…………...…… cm^3 [3]

13 **(a)** The scale of a map is 5 cm : 1 km. A reservoir has an actual area of 0.7 km^2. Find the area, in square centimetres, of the reservoir on the map.

Answer ……….…..…… cm^2 [2]

(b)
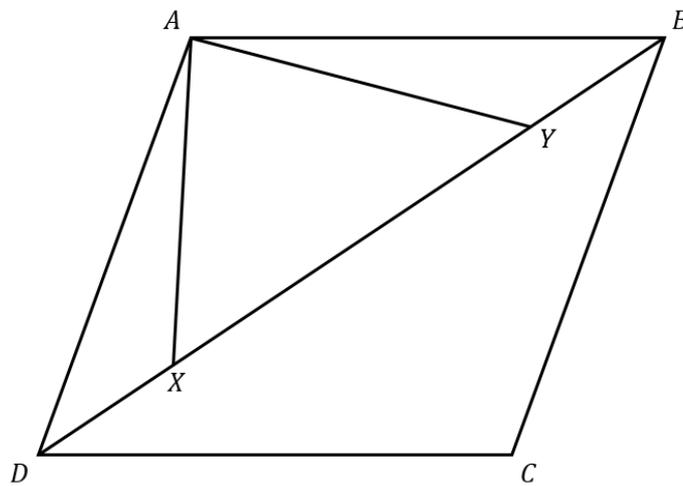
$ABCD$ is a rhombus. X and Y are points on BD such that $DX = YB$. Show that triangle ADX is congruent to triangle ABY. Give a reason for each statement you make.

Answer

[2]

14 A researcher conducted a Particle Filtration Efficiency (PFE) test on surgical masks and reusable masks which have been used for 24 hours. The box-and-whisker plots represent the distribution of PFE scores.
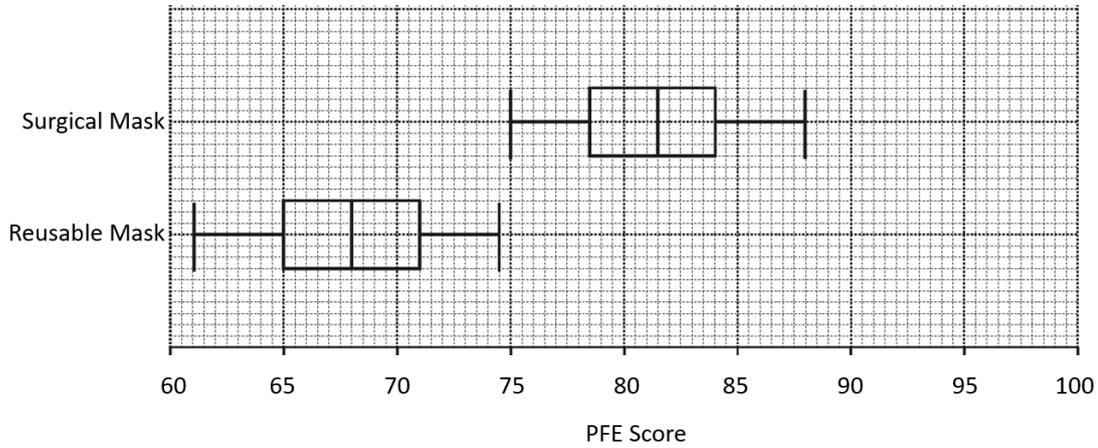
(a) Find the range of the PFE scores for reusable masks.

Answer …..………..…...……….. [1]

(b) Find the interquartile range of the PFE scores for surgical masks.

Answer …..………..…...……….. [1]

(c) Make two comparisons between the PFE scores for surgical masks and reusable masks.

1 ………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

2 ………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

……………………………………………………………………………… [2]

15 **(a)** Factorise completely $x^4 - y^4$.

Answer …………….……......……….. [2]

(b) Given that $x^2 - 2xy = 1 - y^2$, express y in terms of x.

Answer ………..……… or ……......……….. [3]

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16 A bird is sitting on the top, T, of a vertical pole 20 m above the ground and the angle of elevation of the bird at T from a point O on the ground is 45°. The bird takes one second to fly horizontally at a constant speed from T to a point B away from O, maintaining a constant height of 20 m above the ground throughout its flight. The angle of elevation of the bird at B from O is reduced to 30°. Find the speed of the bird in m/s.

Answer …..………………… m/s [3]

17 The population of a certain species of bacteria, N, is inversely proportional to the square root of the temperature, t °C, where $t > 0$. Find the percentage decrease in the temperature when the population is doubled.

Answer …..…………..…… % [4]

18 Leroy has written down four numbers. The mean of these numbers is $2x + y$, the median is $x + y$ and the mode is $x - y$, where x and y are constants and $x > y$.

Find the four numbers in terms of x and y.

Answer …….……... , …….……... , …….……... , …….……... [4]

19

Diagram 1   Diagram 2   Diagram 3   Diagram 4   Diagram 5
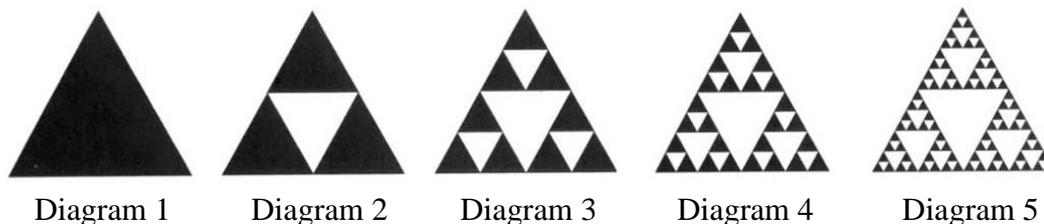
Diagram 1 shows an equilateral triangle with sides of length 1 unit. An inverted white equilateral triangle with sides of length $\frac{1}{2}$ unit is removed to form Diagram 2. Three inverted white equilateral triangles with sides of length $\frac{1}{4}$ unit is then removed from Diagram 2 to form Diagram 3. The process continues in this way.

(a) Find an expression, in terms of n, for the total number of black triangles in Diagram n.

Answer …..………..…..……….. [2]

(b) Taking the area of Diagram 1 to be a units^2, find an expression, in terms of a and n, for the total area of black triangles in Diagram n.

Answer …..………..…..… units^2 [2]

20 Figure 1 shows an open container in the form of an isosceles trapezoidal prism *ABCDEFGH* with square base *ABCD* of sides x cm, where x is a constant. The faces *ABFE* and *DCGH* are perpendicular to the base *ABCD*. The container is then tilted along *BC* such that the face *BCGF* and the base *ABCD* make an angle of $30°$ each with the horizontal surface, as shown in Figure 2. Water is then filled into this empty container at a constant rate of x^3 cm³/min. Assume that the thickness of the container is negligible in this question.
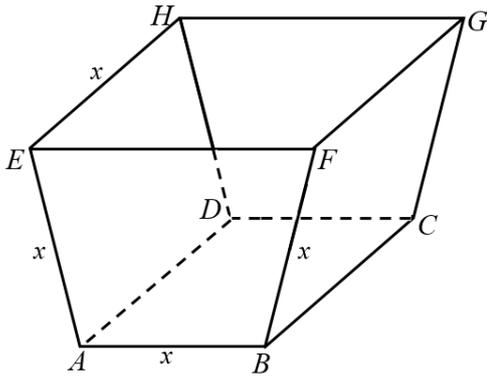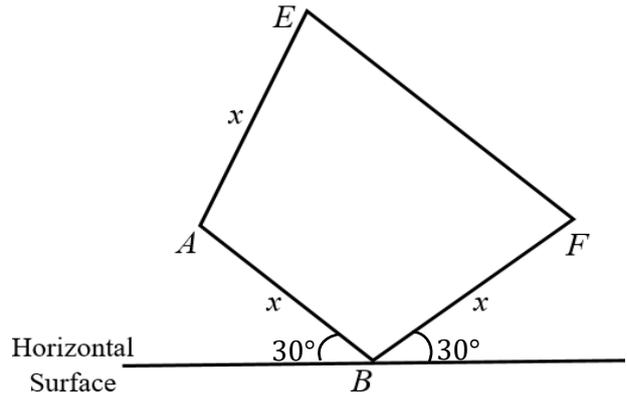
Figure 1   **Figure 2**

(a) Find an expression, in terms of x, for the height of the water level just before the water in the container overflows.

Answer …..……….……..… cm [1]

(b) Find the time it takes to first fill this container to the height found in part **(a)**, giving your answer to the nearest second.

Answer …..……….……..… seconds [3]

21 A closed conical ornament has a circular base of radius 10 cm and height 25 cm, and contains mercury to a depth of 15 cm as shown in the figure below. The ornament is then inverted. Find the new depth, h cm, of the mercury.

Mercury
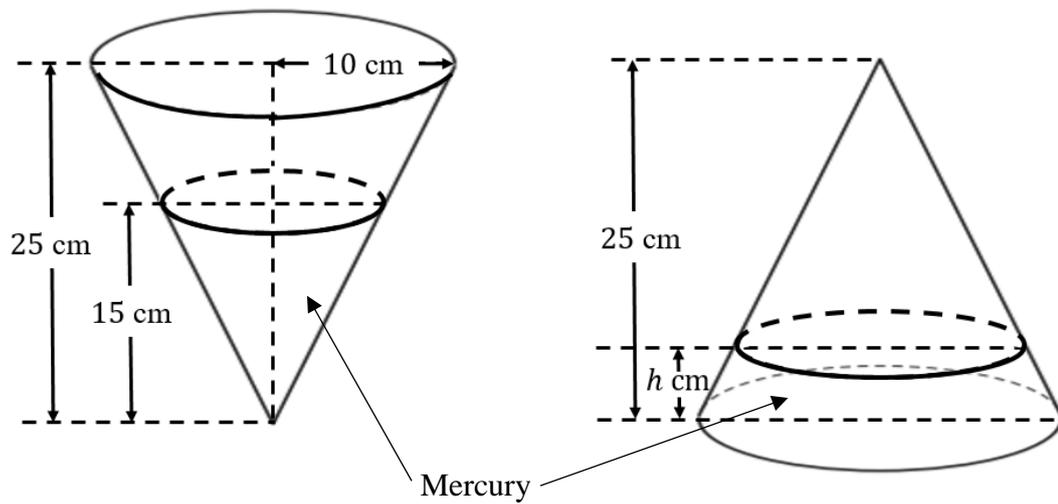
Answer $h =$ ……….…….……… cm [6]

22
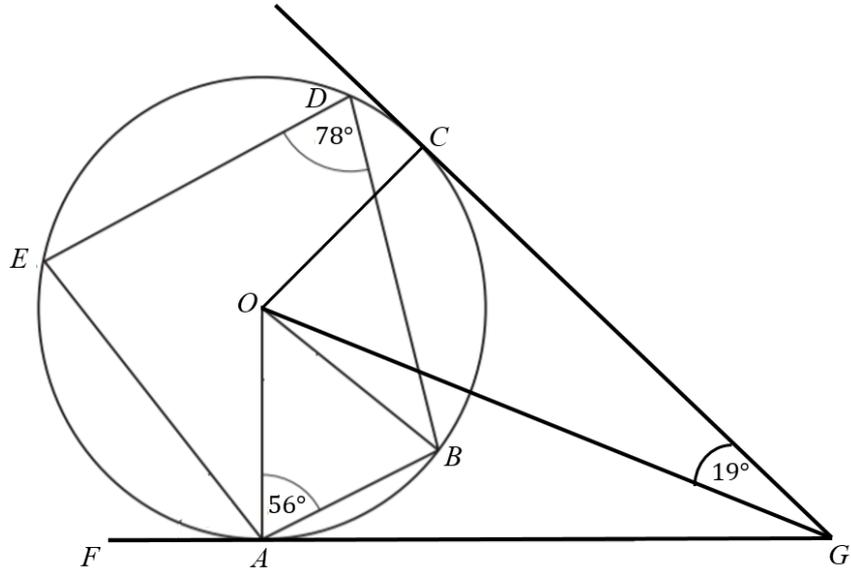
The diagram shows a circle, with centre *O*, passing through *A*, *B*, *C*, *D* and *E*. *CG* and *FG* are tangents to the circle at points *C* and *A* respectively. Angle $OAB = 56°$, angle $BDE = 78°$ and angle $OGC = 19°$.

Find, giving reasons for each answer, the value of

(a) angle EAF,

Answer …..………..…..……….. [3]

(b) angle BOG.

Answer …..………..…..……….. [3]

23 A game has two sets of eight icons. One icon from each set is randomly chosen to be displayed on each turn, starting with set 1. The icons for each set are as follows.

Set 1

Set 2

For example, if an [icon] icon is chosen from set 1 and an [icon] icon is chosen from

set 2, the display would be [icon] [icon] .

(a) Find the probability that, on one turn,

(i) [icon] [icon] is displayed,

Answer …..………..………….. [1]

(ii) at least one [icon] icon is displayed,

Answer …..………..………….. [2]

(b) The game is now programmed such that no two same icons will appear in the display.
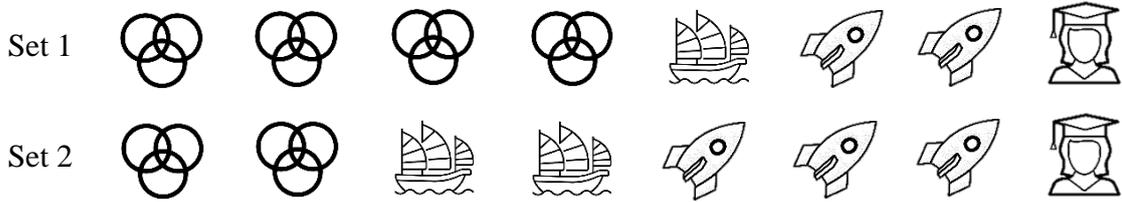
For example, if an icon is chosen in set 1, this icon will be removed from set 2 and an icon will be randomly chosen from the remaining six icons in set 2 as shown in the diagram below.

Set 2
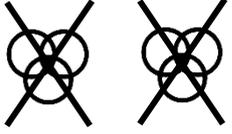
If now an icon is chosen in set 2, the display would be .

Draw a tree diagram to show the probabilities of all the possible outcomes.

[3]

End of Paper

Answer Key

Qn	Solutions
1	1.674×10^{-11}
2	-3.143 , $\quad -\frac{22}{7}$, $\quad -314.2\%$, $\quad -\sqrt{9.87}$
3	$4\frac{1}{2}\pi$
4	**Aspect/Feature (Any one):** - The scale chosen for the vertical axis is inappropriate, due to the units of the total food waste being in kilotonnes, OR - The spacing of the numbers / intervals on the vertical axis is/are more condensed, **How it may lead to Misinterpretation:** - thus resulting in the differences of the food wastage across the years to be less significant than it should be.
5	899
6	(a) Diagram A (b) Diagram E (c) Diagram B
7	71.66 (*nearest cents*)
8	10.4 (3 s.f.)
9	$k = 0.5$
10(a)	
10(b)	$\emptyset \subset B$ and $(2,6) \in (A \cup B)'$
11	$k = 120$
12	$\frac{8}{3\sqrt{3}}r^3 \quad cm^3$ Also Accept: $\frac{8\sqrt{3}}{9}r^3$ or $1.54r^3$ (3 s.f.)
13(a)	17.5 cm^2
13(b)	$\triangle ADX \equiv \triangle ABY$ (SAS)
14(a)	13.5
14(b)	5.5
14(c)	1. The PFE scores of reusable masks are (generally) lower as the median of 68 is lower than the median of 81.5 for the surgical masks. 2. The PFE scores of reusable masks are (slightly) less consistent as the IQR of 6 is higher than the IQR of 5.5 for the surgical masks.

15(a)	$(x - y)(x + y)(x^2 + y^2)$
15(b)	$y = x + 1 \ \ or \ \ y = x - 1$
16	14.6 m/s (3 s.f.)
17	75%
18	$x - y, \qquad x - y, \qquad x + 3y, \qquad 5x + 3y$
19(a)	3^{n-1} \qquad (b) $\left(\frac{3}{4}\right)^{n-1} a$
20(a)	$\frac{1}{2}x$
20(b)	26 seconds
21	$h = 1.95 \ cm \ (3 \ s.f.)$
22(a)	44°
22(b)	3°
23(a)(i)	$\frac{1}{8}$
23(a)(ii)	$\frac{5}{8}$
23(b)	