1 The loudness, L dB, of a sound is given by $L = 10\ lg\left(\frac{I}{I_0}\right)$, where I is the sound intensity to be measured and I_0 is the minimum sound intensity that can be heard by a human.

(a) Given that the loudness of a thunder is 120 dB, find the ratio of the sound intensity of thunder to the minimum sound intensity that can be heard by a human. [2]

(b) Given that the minimum sound intensity that can be heard by a human occurs when L = 90 dB and $I = 10^{-3}$, find the loudness of a plane taking-off if its intensity is 100. [3]

2 The coordinates of three points A, B and C are $(-3\ ,1)$, $(6\ ,3)$ and $(1\ ,8)$ respectively. Given that $ABDC$ is a parallelogram, find

(a) the coordinates of D, [3]

(b) the area of the parallelogram $ABDC$. [2]

3 **(a)** Show that $4\sin^2 x - 2\cos^2 x$ can be written as $a + b\cos 2x$, where a and b are integers. [2]

(b) Hence sketch the graph of $y = 4\sin^2 x - 2\cos^2 x$ for $-90° \le x \le 270°$. [3]

4 **(a)** It is given that $(3x + k)$ is a factor of the polynomial

$81x^4 - 6x^3 - 9k^2x^2 - 11x - \frac{35}{9}$, show that $2k^3 + 33k - 35 = 0$. [2]

(b) Show that the equation $2k^3 + 33k - 35 = 0$ has only one real root for all real values of k. [4]

5 Integrate each of the following with respect to x.

(a) $\frac{1}{3}\tan^2\left(\frac{2}{5}x\right)$ [2]

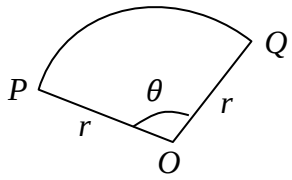
(b) $\frac{2-3x}{5-3x}$ [3]

6 **(a)** By squaring $\sin^2 x + \cos^2 x$, or otherwise, show that

$$\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\sin^2 2x.$$ [3]

(b) Hence, find the values of x for which $\sin^4 x + \cos^4 x = \frac{3}{5}$ for $\frac{3\pi}{2} \le x \le 2\pi$. [3]

7 A piece of wire with a fixed length, L cm, is bent to form the shape a sector of a circle, OPQ, centre O and radius r cm as shown in the diagram. The angle at the centre is θ radians.



(a) Show that the area, A cm^2, of the sector is given by $A = \frac{1}{2}rL - r^2$. [2]

(b)(i) Given that r can vary and A has a stationary value, find r in term of L and its corresponding value of θ. [3]

(b)(ii) Determine the nature of this stationary value. [1]

8 **(a)** The variables *x* and *y* increase in such a way that, when *x* is a particular value, the rate of increase of *y* with respect to time is half the rate of increase of *x* with respect to time. Given that $y = \frac{2\sqrt{6x+7}}{3}$, find this particular value of *x*. [3]

(b) The drug concentration, $C(t)$, in a person's bloodstream *t* hours after digesting a certain amount a drug, can be modelled by the function $C(t) = 0.845t - \frac{3t^3}{200}$.

Find the time interval when the drug concentration in the bloodstream is increasing. [4]

9 The height, *h* metres, of a stone from the ground after it has been thrown can be modelled by the equation $h = -ax^2 + x + \frac{2}{5}$, where *x* is the horizontal distance travelled by the stone in metres and *a* is a constant.

(a) Given that the maximum height attained by the stone is 2.4 metres, find the value of *a* and the corresponding horizontal distance, in metres, travelled by the stone. [5]

(b) Hence, solve the equation $-ax^2 + x + \frac{2}{5} = 0$ and explain the significant of the answer(s). [2]

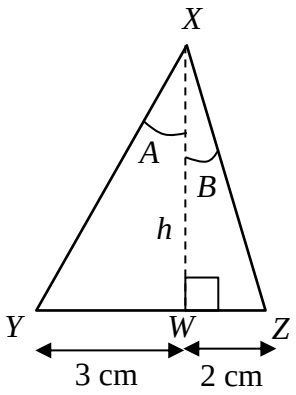
10 **(a)** Without the use of a calculator, express the principal value of $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ in radians as a multiple of π. [1]

(b) Without using a calculator, express $\cos\frac{7\pi}{12}$ in the form $\frac{\sqrt{a}-\sqrt{b}}{c}$, where *a*, *b* and *c* are integers. [2]

(c) Angles *A* and *B* both lie between $0°$ and $360°$. Given that $\sin A$ and $\cos B$ are both negative, explain whether you agree or disagree that $360° < A + B < 540°$. [2]

(d) The diagram shows a triangle XYZ of height h cm, YW = 3 cm, WZ = 2 cm and angle XWZ = 90°. Angles A and B are such that $A + B = 45°$, find the value of h.

[4]



11 **(a)** Given that $\int e^{-2x}\,\mathrm{f}(x)\,\mathrm{d}x = e^{-2x}\sin 4x + c$, find the value of $\mathrm{f}\left(\frac{\pi}{4}\right)$. [4]

(b) It is given that $\mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 4\sin\left(3x + \frac{\pi}{2}\right) + e^{2x}$. Given also that $\mathrm{f}'(0) = \frac{2}{3}$ and $\mathrm{f}(0) = \frac{65}{36}$, find the expression for $\mathrm{f}(x)$. [5]

12 The velocity, v m/s, of a particle moving in a straight line, t seconds after passing through a fixed point O, is given by $v = \frac{27}{2(3t+1)^2} - \frac{3t+1}{2}$.

(a) Find the initial acceleration of the particle. [2]

(b) Determine, with appropriate working, whether the velocity of the particle is increasing or decreasing. [2]

(c) Find the average speed of the particle during the first 6 seconds. [6]

13 **(a)(i)** The ninth term in the expansion of $\left(px - \frac{q}{x}\right)^n$, where p and q are constants, is independent of x. Find the value of n. [3]

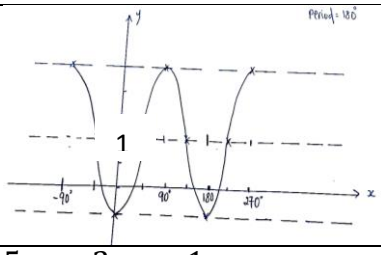
(a)(ii) Show that the ninth term is a positive number. [1]

(b)(i) Write down, and simplify, the first three terms in the expansion $\left(2 - \frac{x}{4}\right)^n$, where n is a positive integer greater than 2, in ascending power of x. [2]

(b)(ii) The first two non-zero terms in the expansion, in ascending power of x, of $(2 + x)\left(2 - \frac{x}{4}\right)^n$ are $a + bx^2$, where a and b are constants. Find the values of n, a and b. [4]

End of paper

Answer Keys for 2021 NCHS Prelim Exam Add Math Paper 1 (4049/01)

1a	$l: l_0 = 10^{12}: 1$	12a	$a = -82.5\, m/s^2$
1b	140 dB	12b	Since $\frac{dv}{dt} < 0, \therefore v \text{ is decreasing}$
2a	D (10, 10)	12c	5.07 m/s (3sf)
2b	55 units^2	13ai	16
3a	$1 - 3\cos 2x$	13bi	$2^n - n2^{n-3}x + n(n-1)2^{n-7}x^2 + \cdots.$
3b		13bii	n = 4 a = 32 b = - 5
5a	$\frac{5}{6}\tan\left(\frac{2}{5}x\right) - \frac{1}{3}x + c$		
5b	$x + \ln(5 - 3x) + c$		
6b	$x = 5.27, 5.73$		
7bi	$r = \frac{L}{4}; \theta = 2$		
7bii	A is a maximum value		
8a	1.5		
8b	$0 \le t < 4\frac{1}{3}$		
9a	$a = \frac{1}{8}$; x = 4		
9b	$x = 8.38 \quad or \quad x = -0.382$ Since distance > 0, $x = -0.382$ is rejected. 8.38m represents the **horizontal distance travelled by the stone when it hits the ground**.		
10a	$\frac{5\pi}{6}$		
10b	$\frac{\sqrt{2} - \sqrt{6}}{4}$		
10c	Disagree as $270° < A + B < 630°$ and not $360° < A + B < 540°$.		
10d	6		
11a	- 4		
11b	$f(x) = -\frac{4}{9}\sin\left(3x + \frac{\pi}{2}\right) + \frac{e^{2x}}{4} + \frac{1}{6}x + 2$		