1 **(a)** Given the solution set of $3x^2 < hx + k$ is $-2\frac{1}{3} < x < 2\frac{1}{2}$, where h and k are constants. Find the value of h and of k. [3]

(b) Find the range of values of m for which the curve $y = x^2 + (m+2)x + 1 - m$ does not intersect the line $y + x = 1$. [3]

2 The equation of a curve is $y = e^{3x-5x^2+ln2x}$, where $x > 0$.

(a) Obtain an expression for $\frac{dy}{dx}$. [2]

(b) Find the coordinates of the stationary point of the curve and leave your answer in exact form. [3]

(c) Determine the nature of the stationary point of the curve. [2]

3 Express $\frac{8x^3-5x+3}{x(4x^2-9)}$ in partial fractions. Hence find $\int \frac{8x^3-5x+3}{2x(4x^2-9)}\,dx$. [8]

4 Solve the following equations.

(a) $\log_3\sqrt{3x^2-3} = 2 + \dfrac{1}{\log_{(3x-3)}9}$. [4]

(b) $2^{4x+3} + 14(4^x) = 15$. [4]

5 **(a)** Show that $\frac{1+\sec 2x}{\tan 2x} = \cot x$. [4]

(b) Given that $y = \ln[\sin(nx)]$ where n is a constant, find an expression for $\frac{dy}{dx}$. [2]

(c) Using the results from parts (**a**) and (**b**), find $\int \frac{1+\sec 8x}{2\tan 8x} - 1\,dx$. [3]

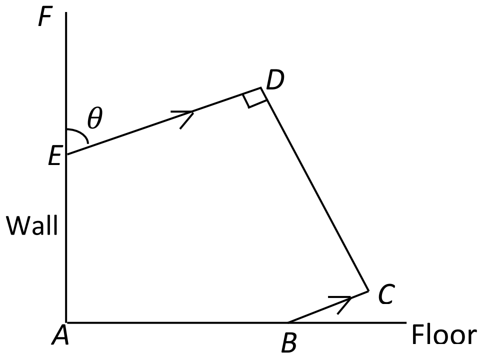
6 A circle with centre C touches the y-axis and intersects x-axis at $x = -2$ and $x = -8$. Point C lies above the x-axis.

(a) Find the equation of the circle. [4]

(b) If the point $(-3, k)$ lies inside the circle, find the range of values of k. [3]

(c) Find the length of the chord such that the coordinates of the mid-point of the chord is $(-3, 5)$. [3]

7 The side view of a sculpture *ABCDE* lying on the horizontal floor *AB* and leaning against a vertical wall *AEF* is shown below. Given *DE* = 17 m, *CD* = 20 m, *BC* = 2 m, angle $EDC = \frac{\pi}{2}$ radians, angle *DEF* = θ and *ED* is parallel to *BC*.



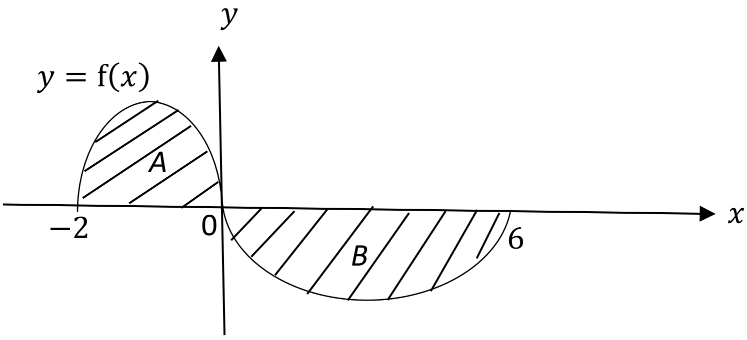
(a) Show that the length AB is $15\sin\theta + 20\cos\theta$. [2]

(b) Express AB in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(c) Find the values of θ for which $AB = 23$ m. [3]

(d) Write down the maximum value of AB and find the corresponding value of θ. [3]

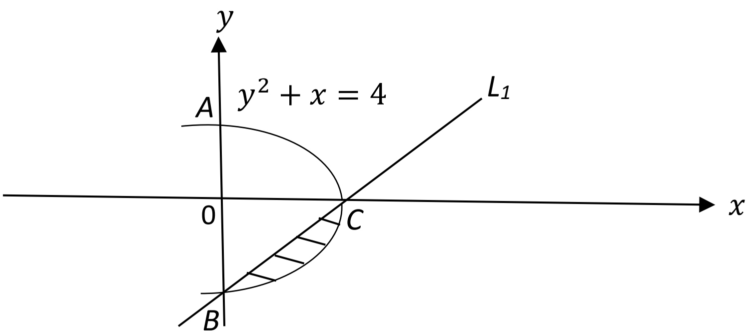
8 **(a)** The diagram shows part of the curve $y = \mathrm{f}(x)$. Shaded area *A* and *B* are bounded by the curve and *x*-axis. Given $\int_{-2}^{0} \mathrm{f}(x)\mathrm{d}x = h$ and $\int_{0}^{6} \mathrm{f}(x)\mathrm{d}x = q$, where h and q are constants, find an expression in terms of h and q for each of the following. Hence interpret how each expression is related to the area *A* and *B*.



(i) $\int_{-2}^{6} 5\,\mathrm{f}(x)\mathrm{d}x$ [3]

(ii) $\int_{6}^{0} \mathrm{f}(x)\,\mathrm{d}x - \int_{0}^{-2} \mathrm{f}(x)\,\mathrm{d}x$ [3]

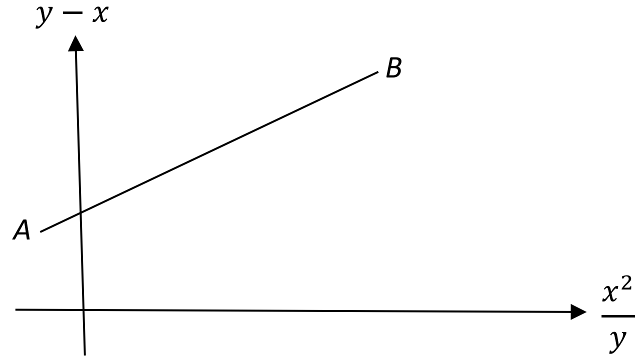
(b) The diagram shows the curve $y^2 + x = 4$ intersecting the line L_1 at point *B* and *C*. Both points *A* and *B* lie on the *y*-axis while point *C* lies on the *x*-axis. Find the area of the shaded region bounded by the line and curve. [6]



9 **(a)** Express h and k in terms of x given that $(6^{x-3})(24^{3x+1})(27^{3-2x}) = (a^h)(b^k)$, where *a* and *b* are prime numbers and *a* < *b*. [4]

(b) Hence find the value of x given further that $(6^{x-3})(24^{3x+1})(27^{3-2x}) = (ab)^q$, where *q* is a rational number. [2]

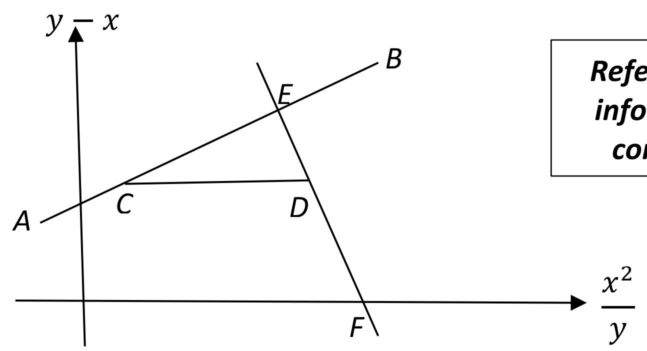
10 (a)



The diagram shows part of a straight line graph AB drawn to represent the equation $y^2 = hx^2 + y(k+x)$, where h and k are constants. Given that the line passes through $\left(\frac{1}{2}, 3\right)$ and $(2, 5)$, find the value of h and of k. [4]

(b) If the diagram from part **(a)** is being drawn to scale on a graph paper, find the equation of a suitable straight line which should be added to the graph in order to find the value of *x* for which $h + 2 + \frac{ky}{x^2} = \frac{5y}{x^2}$. [2]

(c)



Refer to part (a) information for continuation

EF is a normal to the line *AB* at $E(6, p)$, intersects the *x*-axis at *F* where *p* is a constant. A horizontal line meets *AB* at *C* and *EF* at *D*. *ACEB* and *EDF* are straight lines.

(i) Find the value of acute angle *ECD*. [2]

(ii) Find the value of *p*. [2]

(iii) Find the coordinates of *F*. [3]

End of Paper

Answers

1a) $h = \frac{1}{2}, \quad k = 17.5$

1b) $-9 < m < -1$

2a) $\frac{dy}{dx} = 2xe^{3x-5x^2}\left(3 - 10x + \frac{1}{x}\right)$ OR $2e^{3x-5x^2}(-10x^2 + 3x + 1)$

2b) $\left(\frac{1}{2}, e^{\frac{1}{4}}\right)$

2c) $\left(\frac{1}{2}, e^{\frac{1}{4}}\right)$ is a maximum stationary point.

3) $2 - \frac{1}{3x} + \frac{5}{2(2x-3)} - \frac{11}{6(2x+3)}$, $\quad x - \frac{1}{6}lnx + \frac{5}{8}ln(2x-3) - \frac{11}{24}ln(2x+3) + c$

4a) $x = 80$

4b) $x = -0.208$

5b) $\frac{dy}{dx} = n\cot(nx)$

5c) $\frac{1}{8}lnsin(4x) - x + C$

6a) $radius = 5, \quad (x+5)^2 + (y-4)^2 = 25$

6b) $-0.583 < k < 8.58$

6c) Length of cord = $4\sqrt{5}$ = 8.94

7b) $AB = 25\cos(\theta - 0.644)$

7c) $\theta = 1.05 \quad or \quad 0.241$

7d) $max\ AB = 25, \ \theta = 0.644$

8ai) $5(h+q)$, 5 times of (area A – area B)

8aii) $-q + h$, Area A + area B

8b) $\frac{4}{3}$

9a) $h = 10x \quad and \quad k = 7 - 2x$

9b) $x = \frac{7}{12}$

10a) $h = \frac{4}{3} \quad and \quad k = \frac{7}{3}$

10b) $y - x = -2\frac{x^2}{y} + 5$

10ci) ∠ECD = 53.1 or 0.927 radian

10cii) $p = 10\frac{1}{3}$

10ciii) $F\left(19\frac{7}{9}, 0\right)$