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CANDIDATE NAME

CLASS

ADMISSION NUMBER

2021 Preliminary Examination
Pre-University 3

MATHEMATICS **9758/02**

Paper 2 **17 September 2021**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your admission number, name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Qn No.	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	*	Total
Score													
Max Score	**4**	**6**	**8**	**10**	**12**	**7**	**7**	**9**	**11**	**12**	**14**		**100**

This document consists of **7** printed pages.

Section A: Pure Mathematics [40 marks]

1 Given that θ is a sufficiently small angle, show that

$$\frac{1}{\sin 2\theta + \cos\theta} \approx 1 + a\theta + b\theta^2,$$

where a and b are constants to be determined. [4]

2 It is given that y and x are related by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 - 2y + 5}{y - 2}.$$

Given that that $y = 1$ when $x = 0$, find the particular solution for the above differential equation. [6]

3 It is given that $\mathrm{f}(x) = \ln(2 + 2\sin x)$.

(i) Show that $\mathrm{f}''(x) = \dfrac{k}{1 + \sin x}$, where k is a constant to be found. [3]

(ii) Hence find the Maclaurin series for $\mathrm{f}(x)$, up to and including the term in x^3. [4]

(iii) Use the series in part **(ii)** to approximate the value of $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$. [1]

4 The plane p passes through the points with coordinates $(-k,\ 2,\ 5)$, $(0,\ 2,\ -1)$ and $\left(-\frac{1}{2},\ 3,\ -1\right)$, and the line l has equation $\frac{x+2}{-3}=y-2=\frac{z-4}{k}$, where k is a constant.

(i) Show that the cartesian equation of the plane is $6x+3y+kz=6-k$. [2]

(ii) Show that the l cannot be perpendicular to p. [2]

For the rest of this question, let $k=-2$.

(iii) Given that l meets p at point N, find the coordinates of N. [3]

(iv) Another plane π is parallel to the plane p. Given that the distance between p and π is 11 units, find the possible points of intersection between l and π. [3]

5 Parameterisation is the process of finding parametric equations of a curve. The position of a point that moves on a curve in two-dimensional space is determined by the time needed to reach the point when starting from a fixed origin.

For example, if (x, y) are the coordinates of the point, the movements of the x-coordinate and y-coordinate of the point are described by a pair of parametric equation, $x=\mathrm{f}(t)$, $y=\mathrm{g}(t)$ where t is a parameter and denotes the time. For example, to parametrise the equation of the curve $y=x^2+1$, let $x=t$, then $y=t^2+1$.

A curve D has parametric equations

$$x=t+\frac{1}{t}+4, \qquad y=t-\frac{1}{t}+1, \qquad t\le -1.$$

A curve E has equation

$$y=x-2-\frac{1}{x-3}, \qquad x\le 2.$$

(i) Show that curve D and E intersect only once at $t=-1$ and hence find the coordinates of the point of intersection. [3]

(ii) Sketch the graph of curve D, indicating clearly the point of intersection found in part **(i)**. [2]

(iii) Using $x = t + 3$, parameterise the equation of E. Sketch the graph of E on the same diagram in part **(ii)**. [3]

Given that D intersects the y-axis at $t = -\sqrt{3} - 2$.

(iv) Find the area of the finite region bounded by D, E and the y-axis, giving your answer correct to four decimal places. [4]

Section B: Probability and Statistics [60 marks]

6 The events A and B are such that $\mathrm{P}(A) = 0.6$, $\mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A \cap B') = 0.55$.

(i) Find the probability that B occurs. [1]

(ii) Find the probability that neither A nor B occurs. [1]

A third event C is such that B and C are independent and $\mathrm{P}(C) = 0.6$.

(iii) Find $\mathrm{P}(B' \cap C)$. [2]

(iv) Hence find the range of values of $\mathrm{P}(A \cap B' \cap C)$. [3]

7 A company produces packets of almond flour with each packet weighing μ_0 grams. In a quality control inspection, the production manager wishes to check if the mean mass of almond flour per packet is overstated.

(i) Explain why the production manager should take a sample of at least 30 packets of almond flour, and state how these packets should be chosen. [2]

The production manager takes a suitable sample of 40 packets of almond flour and finds that the mean mass of almond flour per packet is 248.5 grams and its standard deviation is 4.3 grams.

(ii) Given that the production manager concludes that the mean mass of almond flour per packet is not overstated at the 1% level of significance, find the range of values of μ_0, giving your answer to 2 decimal places. [4]

(iii) Explain what is meant by "at the 1% level of significance" in the context of the question. [1]

8 A doctor prescribes a specific medication to 12 of his patients who developed a particular type of allergy in his clinic. Past records show that $100p\%$ of adults with such allergy report symptomatic relief after consuming the medication. The number of patients who reported symptomatic relief after consuming the medication is assumed to follow a binomial distribution.

(i) Write down in terms of p, the probability that 10 patients reported symptomatic relief after consuming the medication. [1]

(ii) It is given that the modal number of patients who reported symptomatic relief after consuming the medication is 10. Use this information to find exactly the possible range of values of p. [4]

Suppose now $p = 0.8$.

In another clinic, 15 patients who developed the allergy were also prescribed with the same medication.

(iii) Find the probability that all patients reported symptomatic relief after consuming the medication. [1]

In order to have a sensing of the effectiveness of the medication, the doctor in this clinic checks the number of patients who reported symptomatic relief after consuming the medication. He takes 3 random samples of 15 patients each who had been prescribed with the medication because of the allergy.

(iv) Find the probability that one of the samples has at least 8 patients who reported symptomatic relief after consuming the medication, and the other two samples each has all patients who reported symptomatic relief after consuming the medication. [3]

9 Two notes are drawn, at random and without replacement, from a bag containing n \$1 notes, two \$2 notes and one \$5 note, where $n \geq 2$. It is assumed that all the notes are identical in size. The random variable M is the absolute difference in the amount of money, in dollars, between two notes drawn.

(i) Determine the probability distribution of M, simplifying the probabilities as far as possible. [5]

(ii) Find $\mathrm{E}(M)$ and show that $\mathrm{Var}(M) = \dfrac{36[\mathrm{f}(n)]}{(n+3)^2(n+2)^2}$, where $\mathrm{f}(n)$ is a cubic polynomial to be determined. [5]

(iii) Given that $\text{Var}(M) = \frac{77}{36}$, find the value of n. [1]

10 **(i)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A) and (B) below. In each case, your diagram should include 6 points in the first quadrant, approximately equally spaced with respect to x. The letters a, b, c and d represent constants.

(A) $y = ax^2 + b$, where a is negative and b is positive,
(B) $y = c\ln x + d$, where c is negative and d is positive. [2]

A pot-in-pot cooler is an affordable electricity-free evaporative cooling device used to maintain a low temperature inside an inner compartment. In an experiment, water is filled in a particular type of pot-in-pot cooler and the temperature of water is measured by a thermocouple inside the cooler at different time interval. The following table gives details of the temperature of water measured over a period of time.

Time taken from the start of experiment (t hours)	0.25	0.5	0.75	1	1.5	2	3
Temperature of water (W °C)	19.5	18.4	17.1	16.6	16.1	15.7	15.4

(ii) Draw a scatter diagram for these values. Use your diagram to explain whether a linear model is appropriate to model these values. [2]

(iii) Find, correct to 4 decimal places, the product moment correlation coefficient between

(a) t and W,
(b) t^2 and W,
(c) $\ln t$ and W. [3]

(iv) Using your answers to part **(i)**, **(ii)** and **(iii)**, explain which of

$$W = at + b,\ W = ct^2 + d \text{ or } W = e\ln t + f,$$

where a, b, c, d, e and f represent constants, is the most appropriate model.
Find the equation of a suitable regression line in this case. [3]

(v) Use your equation to estimate the time taken to reduce the temperature of water in the pot-in-pot cooler to 16 °C and explain whether your estimate is reliable. [2]

11 **In this question, you should assume that *L* and *S* follow independent normal distributions. You should also state clearly the mean and variance of all distributions you use.**

An oil company supplies engine oil in cans of two capacities, large and small.

The amount, L millilitres, of oil in a large can is normally distributed, where $L \sim \mathrm{N}(5000,\ \sigma^2)$. The amount, S millilitres, of oil in a small can is normally distributed, where $S \sim \mathrm{N}(1000,\ 25)$.

(i) Find $\mathrm{P}\left(5000-\sigma < L < 5000+2\sigma\right)$. [2]

(ii) The probability that a randomly chosen large can has more than 4990 millilitres of oil is 0.943. Find σ^2. [2]

Use $\sigma^2 = 40$ for the rest of the question.

(iii) Find the probability that the amount of oil in a randomly chosen small can is at most 1002 millilitres. [1]

(iv) Twenty small cans of oil are randomly chosen. Find the probability that fewer than ten cans have the amount of oil in the can to be at most 1002 millilitres. [2]

(v) Find the probability that the amount of oil in a randomly chosen large can exceeds five times the amount of oil in a randomly chosen small can by more than 30 millilitres. [3]

The oil company fills the large cans with Type *A* lubricating oil and small cans with Type *B* lubricating oil, and sells the Type *A* industrial lubricating oil at \$0.13 per millilitres and Type *B* industrial lubricating oil at \$0.05 per millilitres. The oil company supplies 6 large cans of oil and 2 small cans of oil to a manufacturing firm.

(vi) Find the probability that the total cost incurred for the manufacturing firm is at least \$3995. [4]

End of Paper