2021 Preliminary Examination
PU3 MATHEMATICS 9758/01
Solutions

Qn	Solution
1(i) [1]	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{4-x^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(4-x^2\right)^{-1}$ $=\frac{2x}{\left(4-x^2\right)^2}$
1(ii) [4]	$\int \frac{x^2}{\left(4-x^2\right)^2}\,\mathrm{d}x$ Let $u=\frac{1}{2}x$, $\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{2x}{\left(4-x^2\right)^2}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}$, $v=\frac{1}{4-x^2}$ $=\int\left(\frac{1}{2}x\right)\left[\frac{2x}{\left(4-x^2\right)^2}\right]\mathrm{d}x$ $=\left(\frac{1}{2}x\right)\left(\frac{1}{4-x^2}\right)-\int\left(\frac{1}{2}\right)\left(\frac{1}{4-x^2}\right)\mathrm{d}x$ $=\frac{x}{2\left(4-x^2\right)}-\frac{1}{2}\int\frac{1}{4-x^2}\,\mathrm{d}x$ $=\frac{x}{2\left(4-x^2\right)}-\frac{1}{2}\left(\frac{1}{2(2)}\right)\ln\left\lvert\frac{2+x}{2-x}\right\rvert+c$ $=\frac{x}{2\left(x^2+4\right)}-\frac{1}{8}\ln\left\lvert\frac{2+x}{2-x}\right\rvert+c$

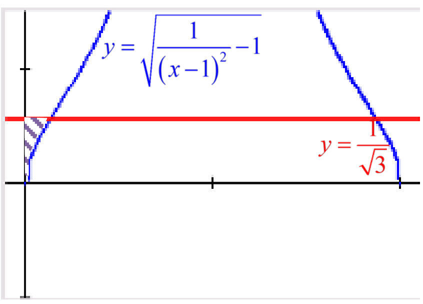
Qn	Solution
2(a) (i) [1]	$(0,b)\xrightarrow{\text{translate } b \text{ units in negaive } y \text{ direction}}(0,0)$ Note: $(a,0)\rightarrow(a,-b)$ (not required)
2(ii) [1]	$(a,0)\xrightarrow{\text{scale parallel to } x \text{ axis by scale factor } \frac{1}{a}}(1,0)$ $(0,b)\xrightarrow{\text{scale parallel to } x \text{ axis by scale factor } \frac{1}{a}}(0,b)$
2(b) (i) [2]	Range of g $=(-\infty,3)\cup(3,\infty)$ or $\mathbb{R}\setminus\{3\}$ Domain of h $=(-\infty,\infty)$ Since range of g $\subseteq$ domain of h $\Rightarrow$ hg exists
2(b) (ii) [3]	$\mathrm{hg}(x)=2-\left(3-\frac{1}{x-1}\right)=-1+\frac{1}{x-1}$

Qn	Solution
	<u>Method 1</u> $\left(\mathrm{hg}\right)^{-1}\left(3\right)=a$ $\mathrm{hg}\left(\left(\mathrm{hg}\right)^{-1}\left(3\right)\right)=\mathrm{hg}\left(a\right)$ $3=-1+\dfrac{1}{a-1}$ $\dfrac{1}{a-1}=4$ $a=\dfrac{5}{4}$ -- **<u>Method 2</u>** To find $\left(\mathrm{hg}\right)^{-1}$ Let $y=-1+\dfrac{1}{x-1}$ $y+1=\dfrac{1}{x-1}$ $x-1=\dfrac{1}{y+1}$ $x=\dfrac{1}{y+1}+1$ $\left(\mathrm{hg}\right)^{-1}\left(x\right)=\dfrac{1}{x+1}+1$ $\left(\mathrm{hg}\right)^{-1}\left(3\right)=\dfrac{1}{3+1}+1=\dfrac{5}{4}$

Qn	Solution
3(i) **[3]**	$y=ux^3\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=x^3\dfrac{\mathrm{d}u}{\mathrm{d}x}+3ux^2$ $x^2\left(x^3\dfrac{\mathrm{d}u}{\mathrm{d}x}+3ux^2\right)-3x\left(ux^3\right)+4=0$ $x^5\dfrac{\mathrm{d}u}{\mathrm{d}x}+3ux^4-3ux^4+4=0$ $x^5\dfrac{\mathrm{d}u}{\mathrm{d}x}+4=0$ $\dfrac{\mathrm{d}u}{\mathrm{d}x}=-\dfrac{4}{x^5}$
3(ii) **[4]**	$\dfrac{\mathrm{d}u}{\mathrm{d}x}=-\dfrac{4}{x^5}$

Qn	Solution
	Integrating with respect to x on both sides of the equation, $u=\int -\frac{4}{x^5}\,\mathrm{d}x$ $\quad =\int -4x^{-5}\,\mathrm{d}x$ $\quad = x^{-4}+c$ $y=ux^3 \Rightarrow u=\frac{y}{x^3}$ $\frac{y}{x^3}=x^{-4}+c$ $y=x^{-1}+cx^3$ When $x=1$, $y=3$, $c=2$ $\therefore y=x^{-1}+2x^3$

Qn	Solution
4(i) **[1]**	$\lvert\mathbf{a}\cdot\mathbf{c}\rvert$ is the length of projection of $\mathbf{c}$ onto $\mathbf{a}$ OR $\lvert\mathbf{a}\cdot\mathbf{c}\rvert$ is the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OA}$.
4(ii) **[3]**	$(2\mathbf{a}-\mathbf{b})\cdot(2\mathbf{a}-\mathbf{b}) = 4(\mathbf{a}\cdot\mathbf{a})-2(\mathbf{a}\cdot\mathbf{b})-2(\mathbf{b}\cdot\mathbf{a})+(\mathbf{b}\cdot\mathbf{b})$ $= 4\lvert\mathbf{a}\rvert^2-4(\mathbf{a}\cdot\mathbf{b})+\lvert\mathbf{b}\rvert^2 \quad (\because\ \mathbf{b}\cdot\mathbf{a}=\mathbf{a}\cdot\mathbf{b})$ $= 4(1)^2-4\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos 60^\circ+(2)^2$ $= 4-4(1)(2)\left(\frac{1}{2}\right)+4=4.$ $(2\mathbf{a}-\mathbf{b})\cdot(2\mathbf{a}-\mathbf{b}) = \lvert 2\mathbf{a}-\mathbf{b}\rvert^2=4 \Rightarrow \lvert 2\mathbf{a}-\mathbf{b}\rvert=2.$
4(iii) **[1]**	By Ratio Theorem, $\mathbf{c}=\frac{\mathbf{b}+2\mathbf{a}}{3}.$

4(iv) **[3]**	$\cos\angle AOC = \dfrac{\mathbf{a}\bullet\mathbf{c}}{\lvert\mathbf{a}\rvert\lvert\mathbf{c}\rvert}$ $= \dfrac{\mathbf{a}\bullet\left(\dfrac{\mathbf{b}+2\mathbf{a}}{3}\right)}{\lvert\mathbf{a}\rvert\lvert\mathbf{c}\rvert}$ [from **(iii)**] $= \dfrac{1}{3}\left(\dfrac{\mathbf{a}\bullet\mathbf{b}+2(1^2)}{\lvert\mathbf{c}\rvert}\right)$ [since $\mathbf{a}\bullet\mathbf{a} = \lvert\mathbf{a}\rvert^2 = 1^2$] $= \dfrac{1}{3}\left(\dfrac{\mathbf{a}\bullet\mathbf{b}+2}{\lvert\mathbf{c}\rvert}\right).$ $\cos\angle COB = \dfrac{\mathbf{b}\bullet\mathbf{c}}{\lvert\mathbf{b}\rvert\lvert\mathbf{c}\rvert}$ $= \dfrac{\mathbf{b}\bullet\left(\dfrac{\mathbf{b}+2\mathbf{a}}{3}\right)}{\lvert\mathbf{b}\rvert\lvert\mathbf{c}\rvert}$ [from **(iii)**] $= \dfrac{1}{3}\left(\dfrac{2^2+2(\mathbf{a}\bullet\mathbf{b})}{2\lvert\mathbf{c}\rvert}\right)$ [since $\mathbf{b}\bullet\mathbf{b} = \lvert\mathbf{b}\rvert^2 = 2^2$] $= \dfrac{1}{3}\left(\dfrac{2+\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{c}\rvert}\right) = \cos\angle AOC.$ Therefore line *OC* bisects the angle *AOB*.
Qn	**Solution**
5(i) **[4]**	$x = \tan\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \sec^2\theta$ $\displaystyle\int \frac{1}{\sqrt{x^2+1}}\,\mathrm{d}x = \int \frac{1}{\sqrt{\tan^2\theta+1}}\left(\sec^2\theta\right)\,\mathrm{d}\theta$ $\displaystyle = \int \frac{1}{\sec\theta}\left(\sec^2\theta\right)\,\mathrm{d}\theta$ $\displaystyle = \int \sec\theta\,\mathrm{d}\theta$ $= \ln\lvert\sec\theta+\tan\theta\rvert + c$ $= \ln\left\lvert\sqrt{x^2+1}+x\right\rvert + c$ Alternatively, $1+\tan^2\theta = \sec^2\theta \Rightarrow \sec\theta = \sqrt{1+x^2}.$
5(ii) **[4]**	$y = \sqrt{\dfrac{1}{(x-1)^2}-1}$ $y^2 = \dfrac{1}{(x-1)^2}-1$ $(x-1)^2 = \dfrac{1}{y^2+1}$ $x-1 = \pm\dfrac{1}{\sqrt{y^2+1}}$ 

$$x=1-\frac{1}{\sqrt{y^2+1}}\quad (\because x<1)$$

Volume of solid

$$=\pi\int_0^{\frac{1}{\sqrt{3}}}\left(1-\frac{1}{\sqrt{y^2+1}}\right)^2 \mathrm{d}y=\pi\int_0^{\frac{1}{\sqrt{3}}}1-\frac{2}{\sqrt{y^2+1}}+\frac{1}{y^2+1}\,\mathrm{d}y$$

$$=\pi\left[y-2\ln\left|\sqrt{y^2+1}+y\right|+\tan^{-1}y\right]_0^{\frac{1}{\sqrt{3}}}\quad \text{[from part (i)]}$$

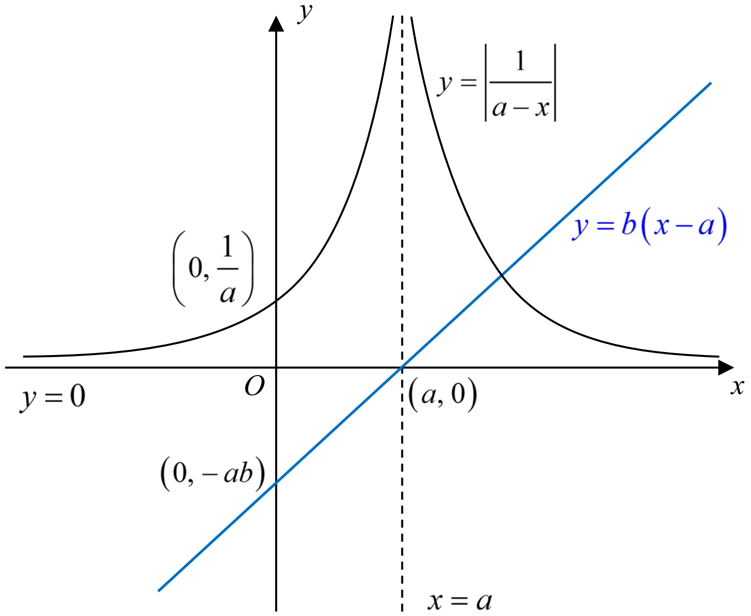
$$=\pi\left(\frac{1}{\sqrt{3}}-2\ln\left(\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1}+\frac{1}{\sqrt{3}}\right)+\tan^{-1}\frac{1}{\sqrt{3}}\right)-0$$

$$=\pi\left(\frac{1}{\sqrt{3}}-2\ln\left(\frac{2}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)+\frac{\pi}{6}\right)$$

$$=\pi\left(\frac{1}{\sqrt{3}}-2\ln\sqrt{3}+\frac{\pi}{6}\right)\text{ OR }\pi\left(\frac{1}{\sqrt{3}}-\ln 3+\frac{\pi}{6}\right)\text{ units}^3.$$

Qn	Solution

6(i)
[4]



6(ii)
[3]

<u>Method 1</u>: Simplifying the modulus using the graph

From the graph, the root of $\left|\frac{1}{a-x}\right|=b(x-a)$ is equal to the root of

$$-\left(\frac{1}{a-x}\right)=b(x-a)\Rightarrow\frac{1}{b}=(x-a)^2\Rightarrow x=a\pm\frac{1}{\sqrt{b}}.$$

Since $x>a$, $x=a+\frac{1}{\sqrt{b}}$.

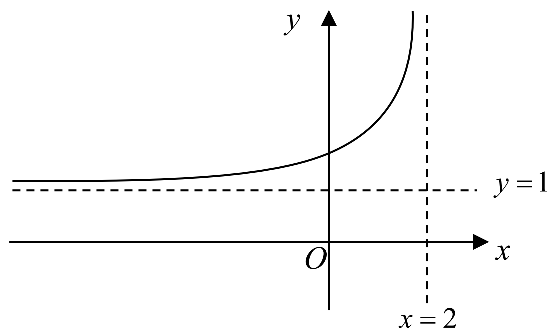
<u>Method 2</u>: Solve by squaring both sides

	$\left\lvert\frac{1}{a-x}\right\rvert=b(x-a)\Rightarrow\left(\frac{1}{a-x}\right)^2=b^2(x-a)^2$ $\Rightarrow\left(\frac{1}{b}\right)^2=(x-a)^2(a-x)^2=(x-a)^4$ $\Rightarrow\frac{1}{b}=(x-a)^2$ (since $b>0$) $\Rightarrow x=a\pm\frac{1}{\sqrt{b}}$. Since $x>a$, $x=a+\frac{1}{\sqrt{b}}$.
6(iii) **[2]**	From the sketch in part **(i)**, $x<a$ or $a<x<a+\frac{1}{\sqrt{b}}$ OR $x<a+\frac{1}{\sqrt{b}}$, $x\neq a$

Qn	Solution
7(i) **[2]**	$u_n=a^{n+1}-(n+1)^a$ $u_1=0\Rightarrow a^2-2^a=0\Rightarrow a^2=2^a$ … (*) **<u>Method 1</u>:** $u_3=a^4-4^a=(a^2)^2-4^a$ $=(2^a)^2-4^a$ [from (*)] $=(2^2)^a-4^a=4^a-4^a=0$. **<u>Method 2</u>:** $u_3=a^4-4^a=(a^2)^2-(2^2)^a$ $=(a^2)^2-(2^a)^2$ $=(a^2-2^a)(a^2+2^a)$ $=0$. [since $a^2-2^a=0$ from (*)]
7(ii) **[4]**	$u_n=2^{n+1}-(n+1)^2$ **<u>Method 1</u>**: Expansion

Qn	Solution
	$\sum_{r=1}^{n} u_r = \sum_{r=1}^{n}\left[2^{r+1}-(r+1)^2\right]$ $= \sum_{r=1}^{n} 2^{r+1} - \sum_{r=1}^{n}(r+1)^2$ $= \sum_{r=1}^{n} 2^{r+1} - \sum_{r=1}^{n}(r^2+2r+1)$ $= \sum_{r=1}^{n} 2^{r+1} - \left(\sum_{r=1}^{n} r^2 + 2\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1\right)$ $= \frac{4(2^n-1)}{2-1} - \left[\frac{n}{6}(n+1)(2n+1) + 2\left(\frac{n}{2}(n+1)\right) + n\right]$ $= 4(2^n-1) - \frac{n}{6}(n+1)(2n+1) - n(n+1) - n$ OR $4(2^n-1) - \frac{n}{6}(n+1)(2n+1) - n^2 - 2n$ OR $4(2^n-1) - \frac{n}{6}(n+1)(2n+1) - n(n+2)$ **<u>Method 2</u>**: Change of index $\sum_{r=1}^{n} u_r = \sum_{r=1}^{n}\left[2^{r+1}-(r+1)^2\right]$ $= \sum_{r=1}^{n} 2^{r+1} - \sum_{r=1}^{n}(r+1)^2$ $= \sum_{r=1}^{n} 2^{r} \times 2 - \sum_{r=2}^{n+1} r^2$ $= 2\sum_{r=1}^{n} 2^{r} - \left[\left(\sum_{r=1}^{n+1} r^2\right) - 1^2\right]$ $= 2\left[\frac{2(2^n-1)}{2-1}\right] - \left[\frac{n+1}{6}(n+1+1)(2(n+1)+1) - 1\right]$ $= 4(2^n-1) - \frac{1}{6}(n+1)(n+2)(2n+3) + 1$

Qn	Solution
7(iii) **[2]**	$\displaystyle\sum_{r=1}^{9}\left(v_{r+1}-v_r\right) = \cancel{v_2} - v_1$ $\qquad\qquad + \cancel{v_3} - \cancel{v_2}$ $\qquad\qquad + \cancel{\ldots}$ $\qquad\qquad + \cancel{v_9} - \cancel{v_8}$ $\qquad\qquad + v_{10} - \cancel{v_9}$ $\qquad\qquad = v_{10} - v_1$ $\displaystyle\sum_{r=1}^{10} u_r = 3587$. [from **(ii)** or from graphing calculator] Hence, $v_{10} - v_1 = 3587$ $\Rightarrow v_{10} = 3587 + v_1 = 3587 + u_1$ [since $v_1 = u_1$ (given)] $\Rightarrow v_{10} = 3587 + [2^{1+1} - (1+1)^2]$ (since $a = 2$) $\Rightarrow v_{10} = 3587 + 0 = 3587$.

Qn	Solution
8(i) **[2]**	
8(ii) **[3]**	$x = \cot t + 2 \qquad y = \sec t$ $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\operatorname{cosec}^2 t \qquad \dfrac{\mathrm{d}y}{\mathrm{d}t} = \sec t \tan t$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sec t \tan t}{-\operatorname{cosec}^2 t}$ $= \dfrac{\left(\dfrac{1}{\cos t}\right)\left(\dfrac{\sin t}{\cos t}\right)}{\left(-\dfrac{1}{\sin^2 t}\right)}$ $= -\dfrac{\sin^3 t}{\cos^2 t}$ Since $-\dfrac{\pi}{2} < t < 0$, $\sin^3 t < 0$, $\cos^2 t > 0$ Therefore $\dfrac{\mathrm{d}y}{\mathrm{d}x} > 0$, C is increasing
8(iii)	**<u>Method 1</u>**

Qn	Solution
[2]	Using GC, At $t=-\frac{\pi}{4}$, $x=1$, $y=1.4142$, $\frac{dy}{dx}=0.70711$, Gradient of normal $\frac{-1}{0.70711}=-1.4142$ Equation of normal $y-1.4142=-1.4142(x-1)$ $y=-1.4142x+2.8284$ $y=-1.41x+2.83$ (3s.f.) **Method 2** At $t=-\frac{\pi}{4}$, $x=1$, $y=\sec\frac{\pi}{4}=\sqrt{2}$, $\frac{dy}{dx}=-\frac{\sin^3\left(-\frac{\pi}{4}\right)}{\cos^2\left(-\frac{\pi}{4}\right)}=\frac{1}{\sqrt{2}}$, Gradient of normal $-\frac{1}{\left(\frac{1}{\sqrt{2}}\right)}=-\sqrt{2}$ Equation of normal $y-\sqrt{2}=-\sqrt{2}(x-1)$ $y=-\sqrt{2}x+2\sqrt{2}$
8(iv) [3]	Midpoint R $=\left(\frac{(\cot p+2)-2}{2},\frac{\sec p}{2}\right)$ $=\left(\frac{\cot p}{2},\frac{\sec p}{2}\right)$ $x=\frac{\cot p}{2}$ $\quad y=\frac{\sec p}{2}$ $\tan p=\frac{1}{2x}$ $\quad \sec p=2y$ **Method 1** Using trigonometric identity, $\tan^2 p+1=\sec^2 p$ $\left(\frac{1}{2x}\right)^2+1=(2y)^2$ $4y^2=\frac{1}{4x^2}+1$ --

Qn	Solution
	<u>Method 2</u> $\tan p = \dfrac{1}{2x}$ Using the right angle triangle, $\dfrac{1}{\left(\dfrac{2x}{\sqrt{4x^2+1}}\right)} = 2y$ □ $y = \dfrac{\sqrt{4x^2+1}}{4x}$ -- **<u>Method 3</u>** $\cos p = \dfrac{1}{2y}$ Using the right angle triangle, $\tan p = \dfrac{1}{2x}$ □ $\dfrac{1}{\sqrt{4y^2-1}} = 2x$ $\dfrac{1}{4y^2-1} = 4x^2$

Qn	Solution
9(a) (i) [4]	For $w = 1-\sqrt{3}\,\mathrm{i}$: $\lvert w\rvert = \lvert 1-\sqrt{3}\,\mathrm{i}\rvert = \sqrt{1^2+\left(\sqrt{3}\right)^2} = 2$ $\arg(w) = \arg\left(1-\sqrt{3}\,\mathrm{i}\right)$ $= -\tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right) = -\dfrac{1}{3}\pi.$ For $z = \sqrt{2}\left(\cos\dfrac{3}{4}\pi - \mathrm{i}\sin\dfrac{3}{4}\pi\right)$: $z = \sqrt{2}\left(\cos\dfrac{3}{4}\pi - \mathrm{i}\sin\dfrac{3}{4}\pi\right) = \sqrt{2}\left(\cos\left(-\dfrac{3}{4}\pi\right) + \mathrm{i}\sin\left(-\dfrac{3}{4}\pi\right)\right)$ Therefore $\lvert z\rvert = \sqrt{2}$ and $\arg(z) = -\dfrac{3}{4}\pi$. **<u>Method 1</u>** $\lvert w^2 z^*\rvert = \lvert w\rvert^2 \lvert z\rvert = 2^2\left(\sqrt{2}\right) = 4\sqrt{2}$

Qn	Solution
	$\arg\left(w^2 z^*\right) = \arg\left(w^2\right) + \arg\left(z^*\right)$ $= 2\arg(w) - \arg(z)$ $= 2\left(-\frac{1}{3}\pi\right) - \left(-\frac{3}{4}\pi\right) = \frac{1}{12}\pi.$ Therefore, $w^2 z^* = 4\sqrt{2}\left(\cos\frac{1}{12}\pi + \mathrm{i}\sin\frac{1}{12}\pi\right).$ **Method 2** $w = 2\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}},\ z = \sqrt{2}\mathrm{e}^{-\frac{3\pi}{4}\mathrm{i}} \Rightarrow z^* = \sqrt{2}\mathrm{e}^{\frac{3\pi}{4}\mathrm{i}}$ $w^2 z^* = \left(2\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}\right)^2 \left(\sqrt{2}\mathrm{e}^{\frac{3\pi}{4}\mathrm{i}}\right)$ $= 4\mathrm{e}^{-\frac{2\pi}{3}\mathrm{i}}\left(\sqrt{2}\mathrm{e}^{\frac{3\pi}{4}\mathrm{i}}\right)$ $= 4\sqrt{2}\ \mathrm{e}^{\frac{\pi}{12}\mathrm{i}}$ $w^2 z^* = 4\sqrt{2}\left(\cos\frac{1}{12}\pi + \mathrm{i}\sin\frac{1}{12}\pi\right)$
9(a) (ii) [2]	**Method 1** $w^n = 2^n\left[\cos\left(-\frac{n\pi}{3}\right) + \mathrm{i}\ \sin\left(-\frac{n\pi}{3}\right)\right]$ For $\operatorname{Re}\left(w^n\right) = 0$, $\operatorname{Re}\left(w^n\right) = 2^n \cos\left(-\frac{n\pi}{3}\right) = 0$ $\cos\left(-\frac{n\pi}{3}\right) = 0$ $-\frac{n\pi}{3} = \ldots,\ -\frac{3\pi}{2},\ -\frac{\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2},\ldots$ $-\frac{n\pi}{3} = \frac{(2k+1)\pi}{2}$, where $k \in \mathbb{Z}$ $n = -\frac{3(2k+1)}{2}$ Since $3(2k+1)$ is odd for all $k \in \mathbb{Z}$, $\frac{3(2k+1)}{2}$ is never an integer. Thus there is no integer value of n such that the real part of w^n is zero. -- **Method 2** For the real part of w^n to be zero, this means w^n is purely imaginary.

<table>
<tr><th>Qn</th><th>Solution</th></tr>
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$\arg\left(w^n\right) = ..., -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$

$= (2k+1)\frac{\pi}{2}$, where $k \in \mathbb{Z}$

Since $\arg\left(w^n\right) = n\arg(w) = -\frac{n}{3}\pi$,

$-\frac{n}{3}\pi = (2k+1)\frac{\pi}{2}$

$-\frac{n}{3} = \frac{2k+1}{2}$

$n = -\frac{3(2k+1)}{2}$

Since $3(2k+1)$ is odd for all $k \in \mathbb{Z}$, $\frac{3(2k+1)}{2}$ is never an integer. Thus there is no integer value of n such that the real part of w^n is zero.

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<tr><td>9(b)
(i)
[4]</td><td>

$3z^3 + 13z^2 + 20z + 14 = 0$

Since all coefficients of the equation are real and $-1+\mathrm{i}$ is a root, $-1-\mathrm{i}$ is another root.

A quadratic factor:

$\left[z-(-1+\mathrm{i})\right]\left[z-(-1-\mathrm{i})\right] = [z+1-\mathrm{i}][z+1+\mathrm{i}]$

$= \left[(z+1)-\mathrm{i}\right]\left[(z+1)+\mathrm{i}\right]$

$= (z+1)^2 - \mathrm{i}^2$

$= z^2 + 2z + 1 - (-1)$

$= z^2 + 2z + 2$

$\Rightarrow 3z^3 + 13z^2 + 20z + 14 = \left(z^2+2z+2\right)(az+b)$

<table>
<tr><th>Method 1</th><th>Method 2</th></tr>
<tr><td>Comparing z^3 terms:
$a = 3$

Comparing constant terms:
$2b = 14$
$\Rightarrow b = 7$</td><td>$$\begin{array}{r} 3z+7 \\ z^2+2z+2 \overline{\big)\, 3z^3+13z^2+20z+14} \\ \underline{-\left(3z^3+6z^2+6z\right)} \\ 7z^2+14z+14 \\ \underline{-\left(7z^2+14z+14\right)} \\ 0 \end{array}$$</td></tr>
</table>

We have, $3z^3 + 13z^2 + 20z + 14 = \left(z^2+2z+2\right)(3z+7)$. $3z+7=0 \Rightarrow z = -\frac{7}{3}$.

Therefore, the other roots are $-1-\mathrm{i}$ and $-\frac{7}{3}$.

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<tr><td>9(b)
(ii)
[2]</td><td>

Given $w^3 + 13w^2 + 60w + 126 = 0$

Divide 9 throughout:

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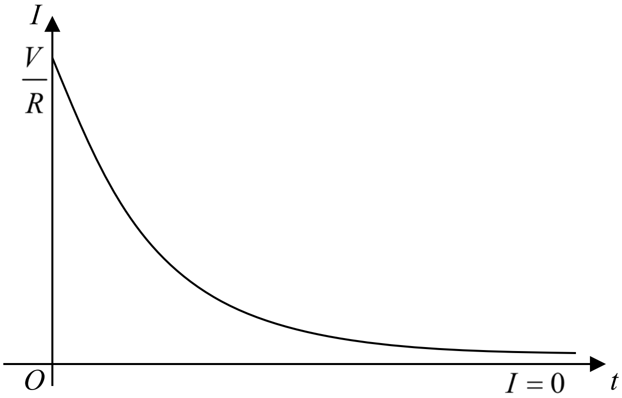
</table>

Qn	Solution
	$\frac{1}{9}w^3+\frac{13}{9}w^2+\frac{60}{9}w+\frac{126}{9}=0$ $\frac{1}{9}w^3+\frac{13}{9}w^2+\frac{20}{3}w+14=0$ $3\left(\frac{1}{3}w\right)^3+13\left(\frac{1}{3}w\right)^2+20\left(\frac{1}{3}w\right)+14=0$ So, z in $3z^3+13z^2+20z+14$ has been replaced with $\frac{1}{3}w$. $z=-1+\mathrm{i},\quad z=-1-\mathrm{i},\quad z=-\frac{7}{3}$ $\frac{1}{3}w=-1+\mathrm{i},\ \frac{1}{3}w=-1-\mathrm{i},\ \frac{1}{3}w=-\frac{7}{3}$ Therefore, $w=-3+3\mathrm{i},\ w=-3-3\mathrm{i},\ w=-7$.

Qn	Solution
10(i) **[2]**	To find maximum q, $\frac{\mathrm{d}q}{\mathrm{d}t}=0$, i.e. $I=0$. $0+\frac{q}{C}=V$ $q=VC$
10(ii) **[2]**	$RI+\frac{q}{C}=V$ Differentiate with respect to t, $R\frac{\mathrm{d}I}{\mathrm{d}t}+\frac{1}{C}\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)=\frac{\mathrm{d}V}{\mathrm{d}t}$ If V is a constant, i.e. $\frac{\mathrm{d}V}{\mathrm{d}t}=0$ $R\frac{\mathrm{d}I}{\mathrm{d}t}+\frac{1}{C}(I)=0$ $R\frac{\mathrm{d}I}{\mathrm{d}t}+\frac{I}{C}=0$, since $I=\frac{\mathrm{d}q}{\mathrm{d}t}$

Qn	Solution
10(iii) **[5]**	$R\dfrac{\mathrm{d}I}{\mathrm{d}t}+\dfrac{I}{C}=0$ $\dfrac{\mathrm{d}I}{\mathrm{d}t}=-\dfrac{I}{RC}$ $\displaystyle\int\frac{1}{I}\,\mathrm{d}I=\int-\frac{1}{RC}\,\mathrm{d}t$ $\ln\lvert I\rvert=-\dfrac{1}{RC}t+d$ $\lvert I\rvert=\mathrm{e}^{-\frac{1}{RC}t+d}=\mathrm{e}^{-\frac{1}{RC}t}\mathrm{e}^{+d}$ $I=A\mathrm{e}^{-\frac{1}{RC}t}$ When $t=0,\ I=\dfrac{V}{R}$ $A=\dfrac{V}{R}$ $\therefore I=\dfrac{V}{R}\mathrm{e}^{-\frac{1}{RC}t}$
10(iv) **[2]**	
10(v) **[1]**	As $t\to\infty$, $I\to 0$. In the long run, the current in the circuit tends to/approaches 0 amp.

Qn	Solution
11(a) **[2]**	Amount of money Ali paid at the end of 3 years $=\dfrac{36}{2}\left[2(200)+(36-1)(10)\right]$ $=13500$ Amount Ali owes the bank at the end of 3 years $=50000-13500$ $=36500$
11(b)(i)	At the end of 1 month, amount owed $=1.003(36500)-900$ $=35709.50$

Qn	Solution
[2]	At the end of 2 months, amount owed $=1.003(35709.50)-900$ $=34916.6285$ At the end of 3 months, amount owed $=1.003(34916.6285)-900$ $=34121.38$ $=34121$ (nearest dollar)
11 (b) (ii) [5]	(see below)

Month	Amount owed at the start of month	Amount owed at the end of month
1	$1.003(36500)$	$1.003(36500)-900$
2	$1.003[1.003(36500)-900]$ $=1.003^2(436500)-1.003(900)$	$1.003^2(36500)-1.003(900)$ -900
3	$1.003[1.003^2(36500)-1.003(900)-900]$ $=1.003^3(36500)-1.003^2(900)-1.003(900)$	$1.003^3(36500)-1.003^2(900)-1.003(900)$ -900
…	…	…
n		$1.003^n(36500)-1.003^{n-1}(900)$ $-1.003^{n-2}(900)-...-900$

On the last day of the nth month, Ali owed

$$1.003^n(36500)-1.003^{n-1}(900)-1.003^{n-2}(900)-...-900$$

$$=1.003^n(36500)-\left[900+1.003(900)...+1.003^{n-2}(900)+1.003^{n-1}(900)\right]$$

$$=1.003^n(36500)-900\left(1+1.003+...+1.003^{n-2}+1.003^{n-1}\right)$$

$$=1.003^n(36500)-900\left[\frac{1.003^n-1}{1.003-1}\right]$$

$$=1.003^n(36500)-300000\left(1.003^n-1\right)$$

When Ali pays off his study loan,

$$1.003^n(36500)-300000\left(1.003^n-1\right)\le 0$$

Using GC,

n	$1.003^n(36500)-300000\left(1.003^n-1\right)$
43	276.54
44	−622.6
45	−1524

Ali takes 44 months to pay off his study loan, i.e. he pays off his study loan in August 2027.

Qn	Solution
11(b) (iii) [2]	**Method 1:** Since Ali pays $\$(1.003\times 276.54)$ in the last month, total interest Ali paid $=43\times 900+(1.003\times 276.54)-36500$ $=2477.37$ (nearest cent)

Qn	Solution
	Method 2: Since Ali pays \$276.54 in the last month, total amount he paid $=13500+43\times 900+(1.003\times 276.54)$ $=52477.37$ Total amount of interest Ali paid $=52477.37-50000$ $=2477.37$ (nearest cent)
11(v) [3]	Let the amount that Ali pays per month upon graduation be \$$x$. $1.003^{36}(36500)-x\left[\dfrac{1.003^{36}-1}{1.003-1}\right]\le 0$ $40656.16902-37.956x\le 0$ $x\ge 1071.14$ Ali needs to pay \$1072 per month.