

Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887



PRELIMINARY EXAMINATION 2021 Secondary 4

Friday
4 August 2021

MATHEMATICS Paper 2

4048/02
2 h 30 min

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

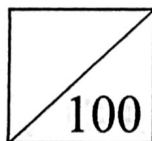
The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 100.



This question paper consists of 21 printed pages and 3 blank pages.

2

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

1 (a) Simplify $\frac{25m^2 - 4}{4 - 4m - 15m^2}$.

Answer [3]

(b) Given that $r = 3 - \sqrt{\frac{p^2}{4} - q}$,

(i) evaluate r when $p = -8$ and $q = 7$,

Answer [1]

(ii) express p in terms of q and r .

Answer [3]

4

(c) The value of a 2-digit number can be written as $10x + y$ where x is the tens digit of the number and y is its units digit.

(i) When the digits x and y are reversed, the value is increased by 27.
Show that $y - x = 3$. [1]

(ii) The sum of the original number and the number with reverse digits is 121.
Show that $x + y = 11$. [1]

(iii) Hence, find the original number.

Answer [3]

- 2 Bag A has 4 green counters, 6 red counters and 2 yellow counters.

- (a) Alice takes a counter at random from Bag A, notes its colour and replaces it in the bag. She repeated this process n times. Write down an expression, in terms of n , for the probability that she takes only green counter.

Answer [1]

- (b) Bill takes a counter at random from Bag A, notes its colour and replaces it back. He then takes a second counter at random from Bag A.

Calculate the probability that Bill takes only one yellow counter.

Answer [2]

Bag B has 3 green counters and 3 red counters.

- (c) Charles takes a counter at random from Bag A and places it into Bag B. He then takes a counter at random from Bag B and places it into Bag A. Find the probability that after these two moves, Bag A has 4 green counters, 6 red counters and 2 yellow counters.

Answer [3]

- 3 (a) The table shows the height, h cm, of 60 plants.

Height (h cm)	$10 < h \leq 20$	$20 < h \leq 30$	$30 < h \leq 40$	$40 < h \leq 50$
Frequency	7	14	27	12

- (i) Calculate an estimate of the standard deviation of the height of the plants.

Answer cm [1]

- (ii) Two plants are chosen at random. Find the probability that both plants have height greater than 30 cm.

Answer [2]

- (b) A toothpaste supplier delivers tubes of toothpaste to 3 different shops A, B and C. The number of tubes of toothpaste supplied per delivery to each shop, the sizes and sale prices of the tubes, together with the number of deliveries made to each shop over a 6-month period are shown in the table below.

Size of tube		Number of tubes of toothpaste per delivery			Number of deliveries over 6 months
		small	medium	large	
Name of shop	A	68	85	105	4
	B	75	—	62	7
	C	—	130	98	5
Sale price per tube		\$2.50	\$4.20	\$6.00	

It is given that $T = \begin{pmatrix} 68 & 85 & 105 \\ 75 & 0 & 62 \\ 0 & 130 & 98 \end{pmatrix}$.

- (i) Write down the matrix P such that TP gives the total number of tubes of toothpaste delivered to each shop per delivery.

Answer $P =$ [1]

- (ii) (a) Write down a 1×3 matrix D to represent the number of deliveries over 6 months to each shop.

Answer $D = \dots\dots\dots$ [1]

- (ii) (b) Evaluate DT .

Answer $DT = \dots\dots\dots$ [1]

- (c) State what the elements of DT represent.

Answer

.....

 [1]

- (iii) Q is a matrix that represents the total amount of money that would be obtained from each shop from the sale of all the tubes of toothpaste per delivery. By matrix multiplication, find Q .

Answer $Q = \dots\dots\dots$ [2]

- 4 (a) Alan drove a distance of x km and his car used 7.5 litres of fuel. Write down an expression, in terms of x , for the rate of fuel used by his car in litres per 100 km.

Answer litres per 100 km [1]

- (b) Ben drove a distance of $(x + 30)$ km and his car used 8 litres of fuel. Write down an expression, in terms of x , for the rate of fuel used by his car in litres per 100 km.

Answer litres per 100 km [1]

- (c) The rate of fuel used by Alan's car is 1.5 litres per 100 kilometre more than the rate of fuel used by Ben's car. Form an equation to represent this information and show that it can be simplified to $3x^2 + 190x - 45000 = 0$.

Answer [3]

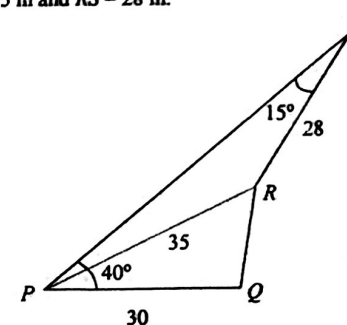
- (d) Solve the equation $3x^2 + 190x - 45000 = 0$, giving both answers correct to two decimal places.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- (e) Each litre of fuel costs \$2.42.
Calculate the amount Alan paid for the petrol if he drove 120 km.

Answer \$ $\dots\dots\dots$ [2]

- 5 The diagram shows four points P , Q , R and S on level ground.
 Q is due east of P .
Angle $SPQ = 40^\circ$ and angle $PSR = 15^\circ$.
 $PQ = 30$ m, $PR = 35$ m and $RS = 28$ m.



- (a) Calculate

- (i) the bearing of R from P ,

Answer $\dots\dots\dots^\circ$ [3]

- (ii) QR ,

Answer $\dots\dots\dots$ m [2]

- (iii) the area of triangle PRS .

Answer m^2 [2]

A vertical mast stands at R and the angle of elevation of the top of the mast from P is 18° .

- (b) (i) Calculate the height of the mast.

Answer m [2]

- (ii) A bird lands on the top of the mast.
Find its largest angle of depression of a point along PQ produced.

Answer $^\circ$ [3]

- 6 (a) It is given that $\overrightarrow{PQ} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ and the position vector of P is $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Find

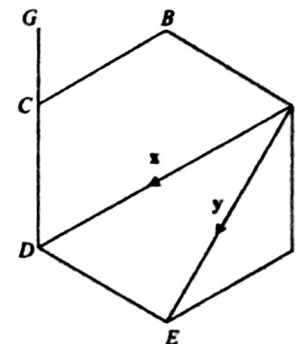
(i) $|\overrightarrow{PQ}|$,

Answer units [1]

- (ii) the coordinates of Q .

Answer (.....,) [2]

(b)



$ABCDEF$ is a regular hexagon.

G is a point on DC extended such that $2DG = 3DC$.

$\overline{AD} = x$ and $\overline{AE} = y$.

- (i) Express, as simply as possible, in terms of x and/or y ,

(a) \overline{AB} ,

Answer [1]

(b) \overline{AF} ,

Answer [1]

(c) \overline{BG} ,

Answer [2]

- (ii) M is the mid-point of AF .
 H is a point on BE where $4HE = 3BE$.

- (a) Find
- \overline{HG}
- .

Answer [2]

- (b) Using vectors, determine whether
- H
- lies on
- MG
- . [2]

- 7 Figure I shows a piece of wood is in the form of a right circular cone with a base diameter of 18 cm.

- (a) The curved surface area of the cone is $135\pi \text{ cm}^2$.
 Find the height of the cone.

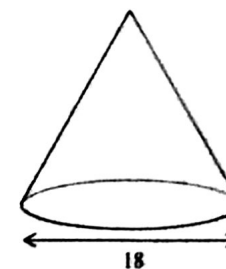


Figure I

Answer cm [3]

The cone in Figure I is cut into two portions by a plane parallel to its base.
 The upper portion is a cone of base diameter 6 cm and the lower portion, as shown in Figure II, is a frustum of height x cm.

- (b) Find the value of
- x
- .

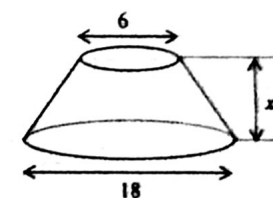


Figure II

Answer $x =$ [2]

- (c) A right cylindrical hole of diameter 6 cm is drilled through the frustum as shown in Figure III.

The wood weighs 810 kg per cubic metres.

Find the mass, in grams, of the solid which remains in the frustum.

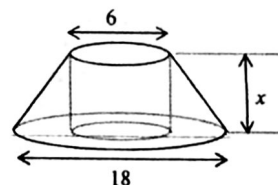


Figure III

Answer g [3]

- 8 The table shows some values for $y = \frac{3}{10}x^3 - x + 1$ for $-3 \leq x \leq 3$.

x	-3	-2	-1	0	1	2	3
y	p	0.6	1.7	1	0.3	1.4	6.1

- (a) Find the value of p .

Answer $p = \dots\dots\dots$ [1]

- (b) Using a scale of 2 cm to 1 unit on both axes, draw a horizontal axis for $-3 \leq x \leq 3$ and a vertical axis for $-4.5 \leq y \leq 6.5$ on the grid opposite.
On your axes, plot the points given in the table and join them with a smooth curve. [3]
- (c) By drawing a tangent, find the gradient of the curve at $x = 2$.

Answer [2]

- (d) The equation $\frac{3}{10}x^3 - x = k - 1$, where k is a positive integer, has exactly one solution for $-3 \leq x \leq 3$. Write down an inequality for k .

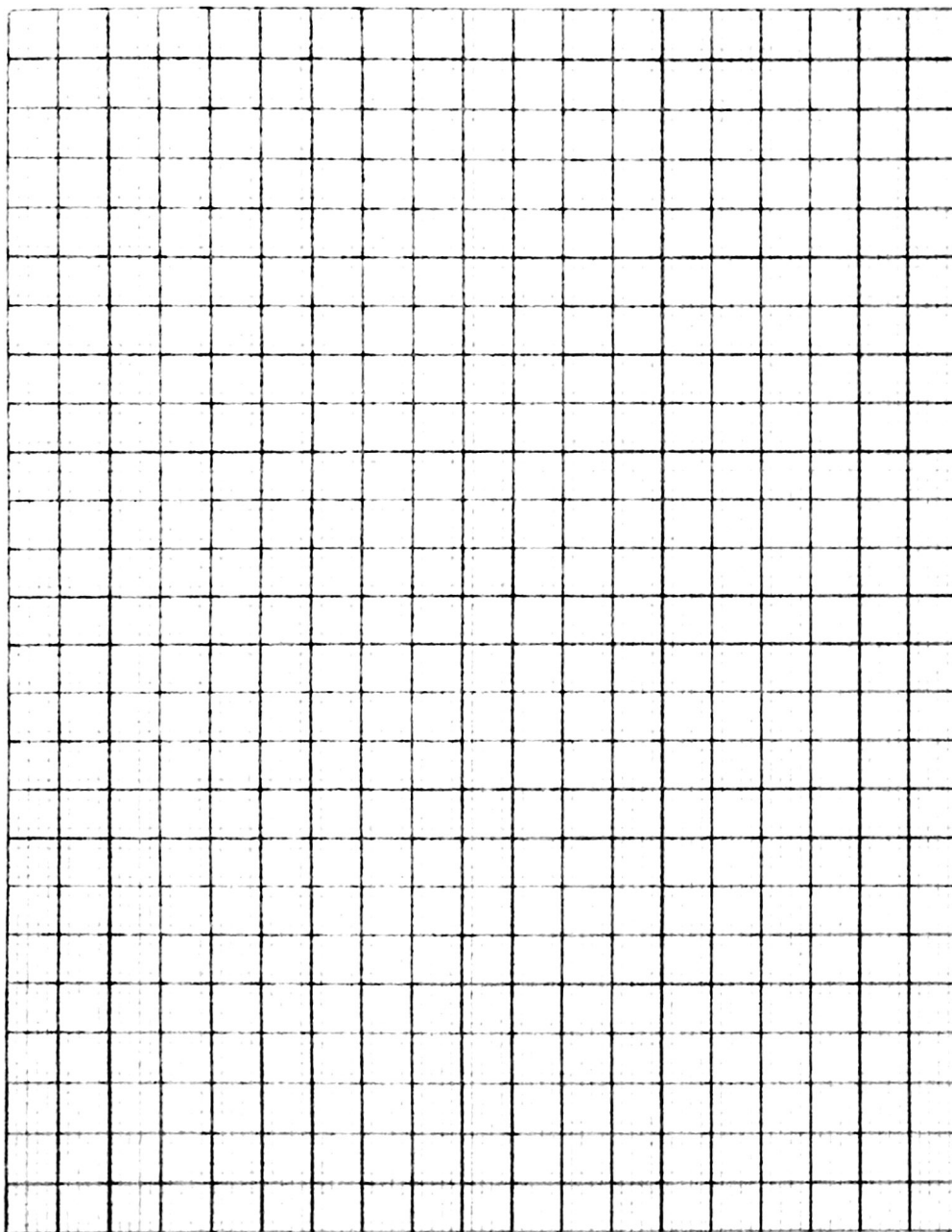
Answer [1]

- (e) (i) The line $y = ax + b$ can be drawn on the same grid to solve the equation $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$. Find the value of a and the value of b .

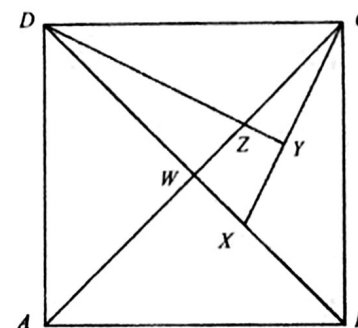
Answer $a = \dots\dots\dots$ and $b = \dots\dots\dots$ [2]

- (ii) Draw the line $y = ax + b$, for $-3 \leq x \leq 3$, to solve the equation $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]



- 9 The diagram shows a square $ABCD$.
The diagonals of the square intersect at W and X is a point on the diagonal BD .
 Y lies on CX such that DY is perpendicular to CX and DY meets AC at Z .



- (a) Show that triangles DWZ and CYZ are similar.

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[2]

- (b) Write down another pair of triangles that are similar but not congruent.

Answer Δ and Δ [1]

- (c) Every month, John randomly selects an 'Employee of the Month' from all his employees. Mr Tan comments that the chance of randomly selecting a male supervisor as the employee of the month is most unlikely.
Do you agree with Mr Tan's comment? Justify your answer. [2]

- (d) John states that the difference between the salary of the lowest paid employee and the mean salary of all the employees as a percentage of the mean salary of all the employees is less than 25%.
Verify, showing all calculations, whether his statement is valid. [5]

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1	(a)	$\frac{5m+2}{2+3m}$	
	(b)	(i) 0	
		(ii) $p = \pm 2\sqrt{(3-r)^2 + q}$	
	(c)	(iii) 47	
2	(a)	$\left(\frac{1}{3}\right)^{-}$	
	(b)	$\frac{5}{18}$	
	(c)	$\frac{1}{2}$	
3	(a)	(i) 9.10 (3sf)	
	(ii)	247	
		590	
	(b)	(i) $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$	
	(ii)	(a) $\begin{pmatrix} 4 & 7 & 5 \end{pmatrix}$	
	(b)	$\begin{pmatrix} 797 & 990 & 1344 \end{pmatrix}$	
	(c)	It represents the total number of tubes of toothpaste of each size delivered to all the shops over 6 months.	
	(iii)	$\begin{pmatrix} 1157 \\ 559.5 \\ 1134 \end{pmatrix}$	
4	(a)	750	
	(b)	$\frac{800}{x}$	
		$x+30$	
	(d)	94.84, -158.17	
	(c)	\$22.97	
5	(a)	(i) 061.9°	
	(ii)	16.5 m	
	(iii)	222 m ²	
	(b)	(i) 11.4 m	
	(ii)	shortest distance = $35\sin 28.0502^\circ$ = 16.4585 m angle of depression = $\tan^{-1} \frac{11.3721}{16.4585}$ = 34.6° (1dp)	

6	(a)	(i) 7.81	
	(ii)	$(-3, 2)$	
	(b)	(i) $-y + x$	
		(b) $y - \frac{1}{2}x$	
		(c) $\frac{3}{4}x - \frac{1}{2}y$	
	(ii)	(a) $x - y$	
	(b)	Since $\overline{MG} = 2\overline{HG}$ and G is a common point, H lies on BG .	
7	(a)	12 cm	
	(b)	8	
	(c)	611 g	
8	(a)	-4.1	
	(c)	2.6	
	(d)	$2 \leq k \leq 6$ or $1 < k < 7$	
	(e)	(i) $a = \frac{1}{2}$ and $b = 2$	
		(ii) -1.75, -0.75, 2.5	
9	(b)	$\triangle CXW$ and $\triangle DXY$	
	(d)	(i) $\frac{2}{5}$	
		(ii) 90 cm ²	
10	(a)	\$44815.61	
	(b)	\$137.28	
	(c)	I agree with Mr Tan's claim.	
	(d)	22.4% < 25%, John's statement is valid.	