Class	Index Number

Name : ________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2021
Secondary 4

Wednesday 18 August 2021

ADDITIONAL MATHEMATICS
Paper 2

4049/02
2 h 15 min

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

	90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots \binom{n}{r} a^{n-r} b^r + \cdots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

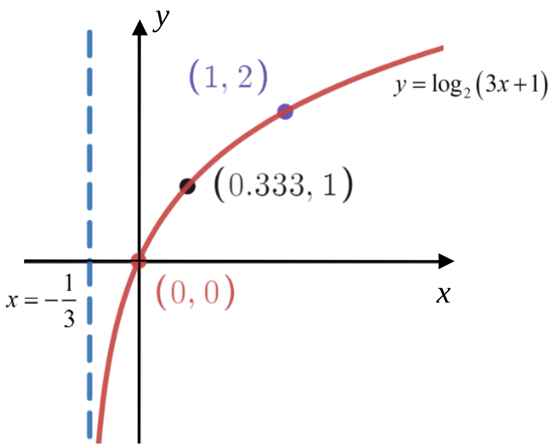
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Sketch the graph of of $y=\log_2(3x+1)$ and label the point where $x=\frac{1}{3}$. Explain why $x>-\frac{1}{3}$. [3]



Asymptote – **B1**

Shape and point $(\frac{1}{3},1)$ -**B1**

$$y=\log_2(3x+1)$$
$$2^y=3x+1$$
$$2^y>0 \quad \textbf{A1}$$
$$3x+1>0$$
$$x>-\frac{1}{3}$$

(b) Solve $\log_2(3x+1)+\frac{1}{2}\log_{\sqrt{2}}(3x-1)=1$. [4]

$$\log_2(3x+1)+\frac{1}{2}\log_{\sqrt{2}}(3x-1)=1$$
$$\log_2(3x+1)+\frac{1}{2}\left(\frac{\log_2(3x-1)}{\log_2 2^{\frac{1}{2}}}\right)=1 \quad \textbf{M1} \text{ – change base}$$
$$\log_2(3x+1)+\left(\frac{1}{2}\times 2\right)\log_2(3x-1)=1$$
$$\log_2(3x+1)(3x-1)=1$$
$$(3x+1)(3x-1)=2 \quad \textbf{M1}$$
$$9x^2-1=2$$
$$x^2=\frac{1}{3}$$

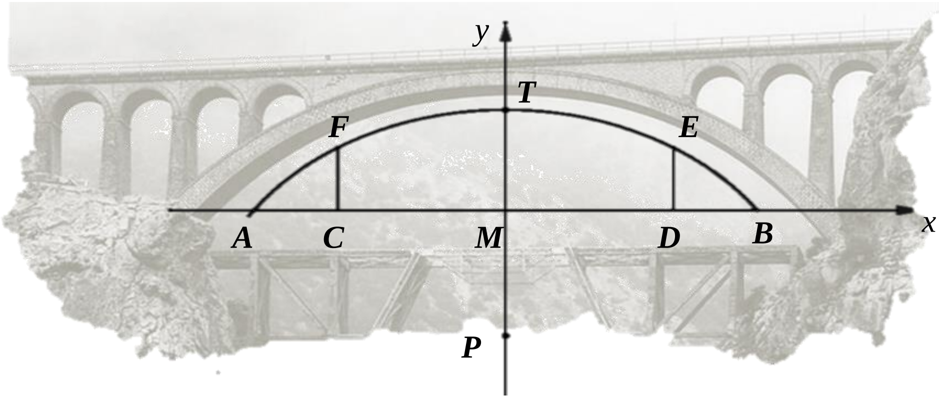
$x=\sqrt{\frac{1}{3}}$ **A1**

$x=0.192$

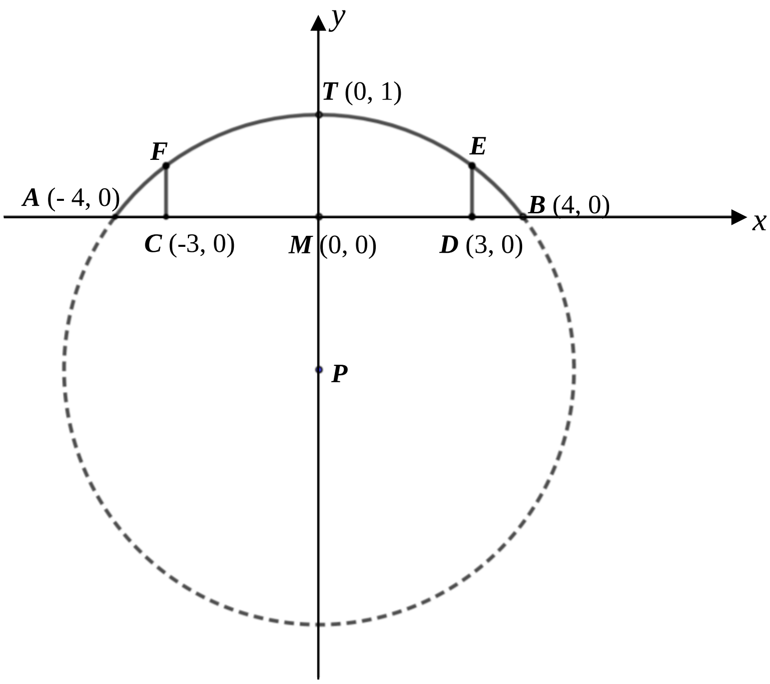
$x=-\sqrt{\frac{1}{3}}$ **A1**

(rejected)

2



The diagram above shows an arch bridge. The arc *AFTEB* is part of a circle with centre *P*, and *AB* is a chord 8 m. Point *T* is 1 m vertically above point *M* which is the midpoint of *AB*. *CF* and *DE* are perpendicular to chord *AB*. The arc *AFTEB* can be modelled onto the coordinate plane below, where *C* and *D* are (-3, 0) and (3, 0) respectively.



(i) Show that *P* is $(0,-7.5)$. [2]

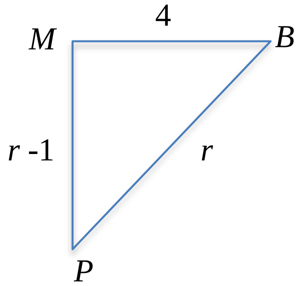
$$r^2=(r-1)^2+4^2$$

$$r^2=r^2-2r+1+16$$

M1

$$2r=17$$

$$r=8.5$$



A1

$\therefore P(0,-7.5)$

2 **(ii)** Find the equation of the circle. [1]

Centre $P(0,-7.5)$ and radius 8.5

Equation of circle is

$$x^2+(y+7.5)^2=8.5^2$$

$$x^2+\left(y+\frac{15}{2}\right)^2=\frac{289}{4} \quad \textbf{B1}$$

$$4x^2+4\left(y+\frac{15}{2}\right)^2=289$$

(iii) Calculate the height of the pillar *CF*. [2]

At $x = 3$,

$$3^2+(y+7.5)^2=\frac{289}{4} \quad \textbf{M1}$$

$$(y+7.5)^2=\frac{253}{4}$$

$$y+7.5=\pm\sqrt{\frac{253}{4}}$$

$$y=0.453 \text{ or } y=-15.5$$

$\therefore$ height of the pillar = 0.453 m **A1**

(iv) Find the equation of tangent at point *B*. [3]

PB is perpendicular to the tangent at point *B*.

Gradient *PB* = $\frac{7.5}{4}=\frac{15}{8}$ **M1**

Gradient of tangent at point *B* = $-\frac{8}{15}$ **M1**

Equation of tangent at *B* is

$$y=-\frac{8}{15}(x-4)$$

$$y=-\frac{8}{15}x+\frac{32}{15} \quad \textbf{A1}$$

$$15y=-8x+32$$

3 (a) Solve $\sqrt{4x+12}-\sqrt{x+3}=2$. [3]

$$\sqrt{4x+12}-\sqrt{x+3}=2$$
$$\sqrt{4(x+3)}-\sqrt{x+3}=2$$
M1 $$2\sqrt{x+3}-\sqrt{x+3}=2$$
$$\sqrt{x+3}=2 \quad \textbf{M1}$$
$$x+3=4$$
$$x=1 \quad \textbf{A1}$$

$$\sqrt{4x+12}=2+\sqrt{x+3} \quad \textbf{M1}$$
$$4x+12=4+4\sqrt{x+3}+(x+3)$$
$$3x+5=4\sqrt{x+3}$$
$$9x^2+30x+25=16(x+3)$$
$$9x^2+14x-23=0$$
$$x=1 \text{ or } x=-\frac{23}{9}(NA)$$
A1 – must show both

(b) The volume of right circular cone is 4π cm^3. The radius of its base is $(1+\sqrt{2})$cm. Find, without the use of a calculator, the height of the cone in the form $(a+b\sqrt{2})$cm, where a and b are integers. [3]

$$\text{Vol of cone}=\frac{1}{3}\pi r^2 h$$
$$4\pi=\frac{1}{3}\pi\left(1+\sqrt{2}\right)^2 h \quad \textbf{M1}$$
$$12=\left(1+2\sqrt{2}+2\right)h$$
$$h=\frac{12}{\left(3+2\sqrt{2}\right)}\times\frac{\left(3-2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)} \quad \textbf{M1}$$
$$=\frac{12\left(3-2\sqrt{2}\right)}{9-8}$$
$$=12\left(3-2\sqrt{2}\right)$$
$$=36-24\sqrt{2} \quad \textbf{A1}$$

4 A particle moving in a straight line, passes through a fixed point *O* with a velocity of 4 m/s. Its acceleration *t* seconds after passing through *O*, is $(6t-8)\,\mathrm{m\,s^{-2}}$. Find

(i) the minimum velocity of the particle, [4]

$a=(6t-8)$

$v=\int 6t-8\;dt$

$=3t^2-8t+c$, c is an arbitrary constant **M1**

At $t=0$, $v=4$ $\therefore c=4$

$v=3t^2-8t+4$ **M1**

Min velocity when $a=0$, $t=\frac{8}{6}=\frac{4}{3}$ **M1**

At $t=\frac{4}{3}, \frac{d^2v}{dt^2}=6>0$

$\therefore$ minimum velocity, $v=3\left(\frac{4}{3}\right)^2-8\left(\frac{4}{3}\right)+4=-\frac{4}{3}$ m/s **A1**

(ii) the time when the particle first comes to an instantaneous rest, [2]

$v=0$

$3t^2-8t+4=0$ **M1**

$(3t-2)(t-2)=0$

$t=\frac{2}{3}$ or $t=2$

$\therefore$ it first comes to rest at $t=\frac{2}{3}$ s **A1**

(iii) the distance travelled in the 2$^{\text{nd}}$ second. [3]

$s=\int_1^2 3t^2-8t+4dt$ **M1**

$=\left[t^3-4t^2+4t\right]_1^2$ **M1**

$=\left[(8-16+8)-(1-4+4)\right]$

$=1$ **A1**

or

$s=t^3-4t^2+4t+c$ **M1**

$t=0$, $s=0$ $\therefore c=0$

at $t=1$, $s=1-4+4=1$

at $t=2$, $s=8-16+8=0$ **M1**

$\therefore$ distance travelled = 1m **A1**

5 **(i)** Prove that $\operatorname{cosec} 2x + \cot 2x = \cot x$. [3]

$$\operatorname{cosec} 2x + \cot 2x = \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$$

$$= \frac{1 + \cos 2x}{\sin 2x} \quad \textbf{M1}$$

$$= \frac{2\cos^2 x}{2\sin x \cos x} \quad \textbf{M1}$$

$$= \frac{\cos x}{\sin x} \quad \textbf{A1}$$

$$= \cot x$$

(ii) Hence, deduce the value of $\cot 15°$ in surd form. [2]

$$\cot 15° = \operatorname{cosec} 30° + \cot 30°$$

$$= \frac{1}{\sin 30°} + \frac{1}{\tan 30°} \quad \textbf{M1}$$

$$= \frac{1}{1/2} + \frac{1}{1/\sqrt{3}}$$

$$= 2 + \sqrt{3} \quad \textbf{A1}$$

(iii) Using part **(i)**, solve $\operatorname{cosec} 2x + \cot 2x = 6 - 5\tan x$ for $0° \le x \le 360°$ [4]

$$\operatorname{cosec} 2x + \cot 2x = 6 - 5\tan x$$

$$\cot x = 6 - 5\tan x$$

$$\frac{1}{\tan x} = 6 - 5\tan x \quad \textbf{M1}$$

$$5\tan^2 x - 6\tan x + 1 = 0$$

$$(5\tan x - 1)(\tan x - 1) = 0 \quad \textbf{M1}$$

$$\tan x = \frac{1}{5} \quad \text{or} \quad \tan x = 1$$

$$\alpha = 11.30993° \qquad x = \underbrace{45°, 225°}_{\textbf{A1}}$$

$$x = \underbrace{11.3°, 191.3°}_{\textbf{A1}}$$

6 **(a)** It is given that $g(x)=2e^{x}-3\sqrt{e^{x}}$. Solve the equation $g(x)+1=0$. [4]

$2e^{x}-3\sqrt{e^{x}}+1=0$

Let $p=e^{\frac{1}{2}x}$,

$2p^{2}-3p+1=0$ **M1**

$(2p-1)(p-1)=0$

$p=\frac{1}{2}$

$e^{\frac{1}{2}x}=\frac{1}{2}$

$\frac{1}{2}x=\ln\left(\frac{1}{2}\right)$

$x=-1.39$ **A1**

$p=1$ **M1**

$e^{\frac{1}{2}x}=1$

$\frac{1}{2}x=0$

$x=0$ **A1**

(b) The function f (x) is such that $f'(x)=3e^{x}+e^{-2x}$.

(i) Given that $f(0)=4$, find an expression for f (x). [3]

$f'(x)=3e^{x}+e^{-2x}$

$f(x)=\int 3e^{x}+e^{-2x}dx$

$=3e^{x}-\frac{e^{-2x}}{2}+c$ **M1**

$f(0)=4$

$4=3e^{0}-\frac{1}{2}e^{0}+c$ **M1**

$4=3-\frac{1}{2}+c$

$c=\frac{3}{2}$

$\therefore f(x)=3e^{x}-\frac{e^{-2x}}{2}+\frac{3}{2}$ **A1**

(ii) Show that $\int_{-\ln 2}^{0} f'(x)dx=k$, where k is a constant to be determined. [3]

$$\int_{-\ln 2}^{0} f'(x)\,dx = k$$

M1 $$\left[3e^{x}-\frac{1}{2}e^{-2x}\right]_{-\ln 2}^{0}=k$$

$$k=\left[\left(3e^{0}-\frac{1}{2}e^{0}\right)-\left(3e^{-\ln 2}-\frac{1}{2}e^{2\ln 2}\right)\right]$$ **M1**

$$=2\frac{1}{2}-\left(\frac{3}{2}-\frac{1}{2}\times 4\right)$$

$$=3$$ **A1**

7 **(a)** **(i)** An equation of a curve is $y=x^4+2x^3$. Find the coordinates of the stationary points and determine the nature of the stationary points. [5]

$$y=x^4+2x^3$$

$$\frac{dy}{dx}=4x^3+6x^2$$ **M1**

$$\frac{dy}{dx}=0$$

$$2x^2(2x+3)=0$$

$$x=0 \quad \text{or} \quad x=-\frac{3}{2}$$

	$x<0$	$x=0$	$x>0$
$\frac{dy}{dx}$	+ve	0	+ve

M1

(0, 0) is a point of inflexion **A1**

$$\frac{d^2y}{dx^2}=12x^2+12x$$ **M1**

At $x=-\frac{3}{2}$, $\frac{d^2y}{dx^2}=9>0$

At $x=-\frac{3}{2}$, $y=-\frac{27}{16}$

$\left(-\frac{3}{2},-\frac{27}{16}\right)$ is a minimum point **A1**

(ii) Explain whether y is increasing or decreasing for $-1.5<x<0$. [2]

$$\frac{dy}{dx}=4x^3+6x^2=2x^2(2x+3)$$

$x^2>0, -\frac{3}{2}<x<0$ **M1**

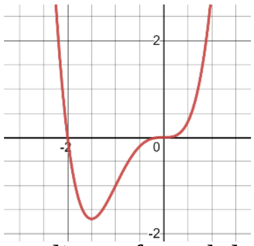
$(2x+3)>0$

$\therefore \frac{dy}{dx}>0$ **A1**

y is an increasing function for $-1.5<x<0$.

OR

	$x=-1.5$	$x>-1.5$	$x<0$	$x\ =0$
$\frac{dy}{dx}$	0	+ve	+ve	0



Describe clearly that gradient of graph of *x*> -1.5 is positive and the gradient of graph less than zero is also positive, hence *y* is an increasing function for $-1.5<x<0$.

7 **(b)** Two variables *x* and *y* are related by the equation $8y=\left(\frac{x}{2}-1\right)^4$. Given that both *x* and *y* vary with time, find the value of *x* at the instant when the rate of change of *y* is twice the rate of change of *x*. [3]

$$y=\frac{1}{8}\left(\frac{x}{2}-1\right)^4$$

$$\frac{dy}{dx}=\frac{1}{8}\times 4\left(\frac{x}{2}-1\right)^3\left(\frac{1}{2}\right)$$

$$=\frac{1}{4}\left(\frac{x}{2}-1\right)^3$$ **M1**

$$\frac{dy}{dt}=2\left(\frac{dx}{dt}\right)$$

$$2\left(\frac{dx}{dt}\right)=\frac{1}{4}\left(\frac{x}{2}-1\right)^3\left(\frac{dx}{dt}\right)$$

$$2=\frac{1}{4}\left(\frac{x}{2}-1\right)^3$$ **M1**

$$8=\left(\frac{x}{2}-1\right)^3$$

$$\frac{x}{2}-1=2$$

$$\frac{x}{2}=3$$

$$x=6$$ **A1**

8 The population P, in millions, of a country was recorded on January of the various years and the results are shown in the table below.

Year	2005	2010	2015	2020
P	12.95	14.67	17.52	22.11

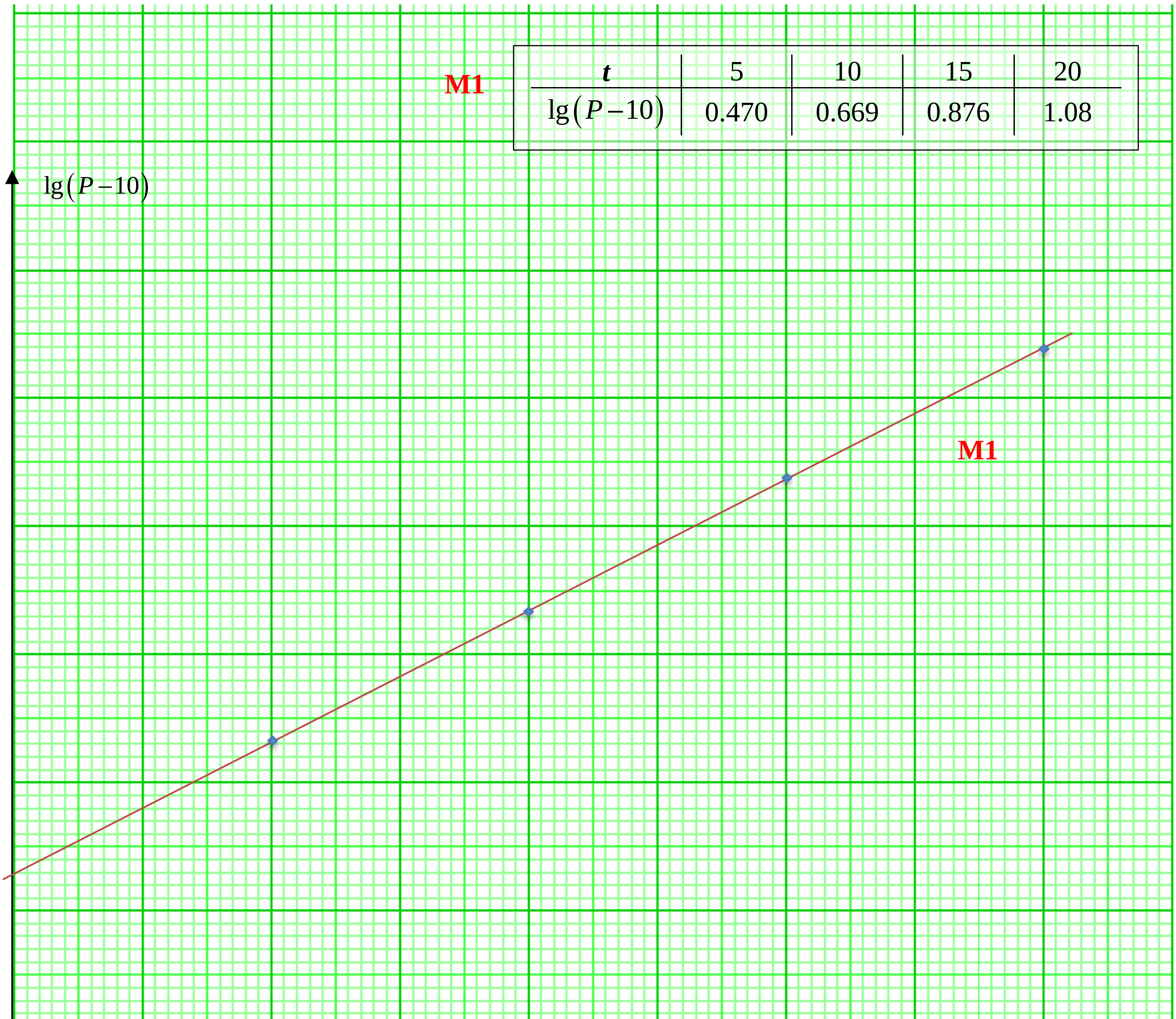
Given that $P=10+ab^t$, where t is the time measured in years from 2000 and a and b are constants.

$$\lg(P-10)=\lg a+\lg b$$

(i) Draw the graph of $\lg(P-10)$ plotted against t, for $0\le t\le 25$. [3]

M1

t	5	10	15	20
$\lg(P-10)$	0.470	0.669	0.876	1.08



t

Axes -M1

8 **(ii)** Use the graph to estimate the values of a and b. [4]

$$P = 10 + ab^t$$
$$P - 10 = ab^t$$
$$\lg(P-10) = \lg a + t\lg b$$

$\lg b$ is the gradient

$$\lg b = \frac{1.08-0.27}{20} \quad \textbf{M1}$$
$$= 0.0405$$
$$b = 1.09$$

$$[0.0405 \le \lg b \le 0.0415]$$
$$1.09 \le b \le 1.1 \quad \textbf{A1}$$

lg a is the vertical intercept

$$\lg a = 0.27$$
$$a = 1.86 \quad \textbf{A1}$$
$$[0.25 \le \lg a \le 0.27]$$
$$1.78 \le a \le 1.86$$

(iii) Explain how the graph could be used to find the year in which the population will reach 33.6 millions. [2]

To reach population of 33.6 millions, take $\lg(33.6-10) = \lg 23.6$
$= 1.37$

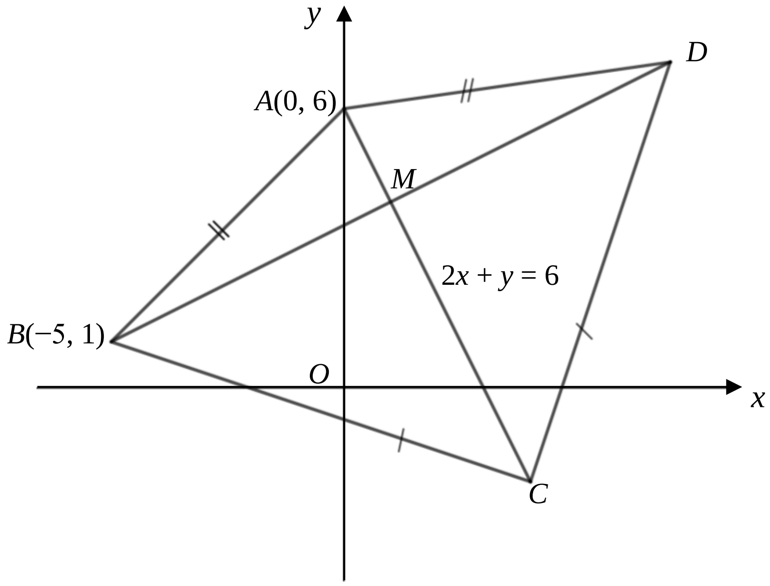
1. Draw a horizontal line $\lg(P-10) = 1.37$ **M1**

2. The point of intersection with the linear graph will give you value t.

A1

3. The year is $2000 + t$

9 Solution to this question by accurate drawing will not be accepted.



The diagram shows a kite *ABCD* with *AB* = *AD* and *CB* = *CD*. The diagonals intersect at *M*. It is given that the coordinates of *A* and *B* are $(0,\ 6)$ and $(-5, 1)$ respectively and the equation of *AC* is $2x + y = 6$.

Find

(i) the equation of *BD*. [2]

Gradient of *BD* = $\frac{1}{2}$ **M1**

Equation of *BD* is

$$y - 1 = \frac{1}{2}(x + 5)$$

$$y = \frac{1}{2}x + \frac{7}{2} \quad \textbf{A1}$$

$$2y = x + 7$$

(ii) the coordinates of *M* and of *D*. [4]

$$2x+y=6\cdots\cdots(1)$$
$$y=\frac{1}{2}x+\frac{7}{2}\cdots\cdots(2)$$
$$6-2x=\frac{1}{2}x+\frac{7}{2}$$
$$\frac{5}{2}x=\frac{12-7}{2} \quad \textbf{M1}$$
$$x=1$$
$$y=4$$

$M\ (1, 4)$ **A1**

Midpoint BD = M

$$\left(\frac{-5+x}{2}, \frac{1+y}{2}\right)=(1,\ 4) \quad \textbf{M1}$$

$$\frac{-5+x}{2}=1 \qquad \frac{1+y}{2}=4$$
$$-5+x=2 \qquad 1+y=8$$
$$x=7 \qquad y=7$$

$D\ (7, 7)$ **A1**

9 Given further that the area of the triangle ABD is $\frac{1}{3}$ of the area of the triangle CBD,

(iii) find the coordinates of C, [2]

By similar Δs,

$$\frac{1}{x}=\frac{1}{4} \quad \textbf{M1}$$
$$x=4$$

Or

$$\sqrt{x^2+(2x)^2}=4\sqrt{1^2+2^2}$$
$$\sqrt{5x^2}=4\sqrt{5} \quad \textbf{M1}$$
$$5x^2=80$$
$$x=4$$

A1 $C=(4, -2)$

(iv) find the area of the kite $ABCD$. [2]

Area of kite $ABCD$ =

$$\frac{1}{2}\begin{vmatrix} -5 & 4 & 7 & 0 & -5 \\ 1 & -2 & 7 & 6 & 1 \end{vmatrix}=\frac{1}{2}\left[(10+28+42)-(4-14-30)\right]$$

M1

$$=\frac{1}{2}[120]$$
$$=60 \text{ units}^2 \quad \textbf{A1}$$

10 **(i)** Solve the equation $4\cos 2A = 3 - 2\sin A$ for $0 \le A \le 2\pi$. [4]

$$4\cos 2A = 3 - 2\sin A$$

$$4\left(1-2\sin^2 A\right) = 3 - 2\sin A \quad \textbf{M1}$$

$$4 - 8\sin^2 A = 3 - 2\sin A$$

$$8\sin^2 A - 2\sin A - 1 = 0$$

$$\left(4\sin A + 1\right)\left(2\sin A - 1\right) = 0 \quad \textbf{M1}$$

$\sin A = -\dfrac{1}{4}$

basic$\angle$, $\alpha = 0.25268$

$\angle A = \pi + \alpha,\ 2\pi - \alpha$

$= 3.39, 6.03$ **A1**

$\sin A = \dfrac{1}{2}$

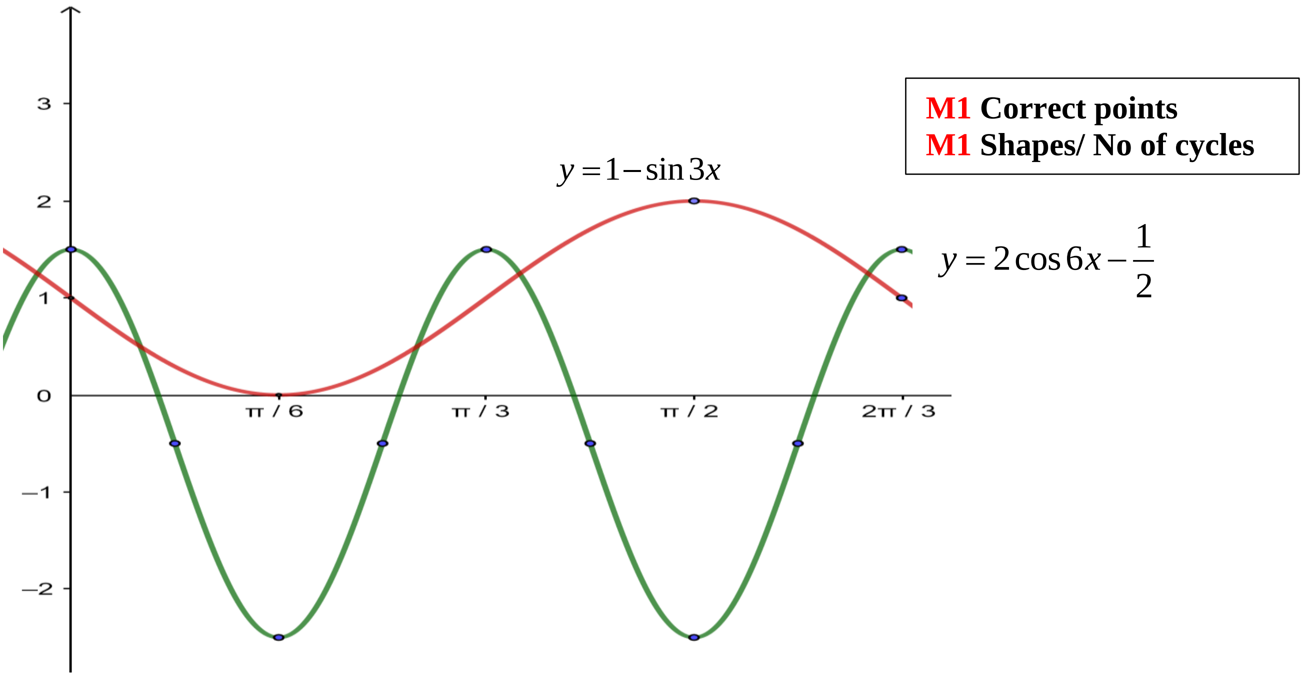
$A = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ **A1**

(ii) It is given that $f(x) = 2\cos 6x - \dfrac{1}{2}$ and $g(x) = 1 - \sin 3x$. State the period of f(x) and g(x), in terms of π. [2]

Period of f(x) = $\frac{2\pi}{6}=\frac{\pi}{3}$ **B1**

Period of g(x) = $\frac{2\pi}{3}$ **B1**

10 **(iii)** Sketch, on the same axes, the graphs of for $y=\mathrm{f}(x)$ and $y=\mathrm{g}(x)$ for $0\le x\le\frac{2\pi}{3}$. [4]



(iv) Explain how the solutions of the equations in part **(i)** could be used to find the x-coordinates of the points of intersection of the graphs of **(ii).** [2]

$$2\cos 6x - \frac{1}{2} = 1 - \sin 3x$$
$$4\cos 6x - 1 = 2 - 2\sin 3x$$
M1
$$4\cos 6x = 3 - 2\sin 3x$$
$$\text{Let } 3x = A$$
$$4\cos 2A = 3 - 2\sin A$$

The solutions in part **(i)** divided by 3 will be the *x* coordinates of the points of intersection of the graphs of **(ii)** **A1**

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