



**JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)**

CANDIDATE NAME

CLASS

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INDEX NUMBER

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MATHEMATICS

4048/01

Paper 1

20 Aug 2021

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 80.

For Examiner's Use
80

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 By rounding each number to 1 significant figure, estimate the value of $\frac{1\,988 \times 6.32}{342.5 - 142.5}$.
Show your working clearly.

Answer [2]

- 2 Triangle ABC is a right-angled triangle.

Given that two of its lengths are 11 cm and 5 cm, find two possible lengths for the third side of the triangle.

Answer cm or cm [2]

- 3 The frequency table below shows the number of books read by a group of students in a month.

Number of books	1	2	3	4	5	6
Number of students	9	4	x	8	6	5

- (a) Write down the least possible value of x if the mode is 3.

Answer $x = \dots\dots\dots$ [1]

- (b) Find the largest value of x if the median is 4.

Answer $x = \dots\dots\dots$ [1]

- 4 (a) Express 3 780 as the product of its prime factors in index notation.

Answer $3\,780 = \dots\dots\dots$ [1]

- (b) The lowest common multiple of 3 780 and integer k is 7 560.
The highest common factor of 3 780 and integer k is 60.

Find the smallest possible integer value of k .

Answer $k = \dots\dots\dots$ [2]

5 Make x the subject of the given formula $\frac{3}{y} = \sqrt{2x^2 - 1}$.

Answer $x = \dots\dots\dots$ [3]

- 6 Solve the pair of given equations simultaneously.

$$\begin{aligned}2x - 3y &= 13, \\3x - 12y &= 42.\end{aligned}$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [3]

7 The speed of gamma rays in air is approximately 2.983×10^{10} cm/s.

(a) Express this speed in metres per second. Give your answer in standard form.

Answer m/s [1]

(b) Calculate the time taken, in microseconds, for gamma rays to travel 10 000 metres.
[micro = 10^{-6}]

Answer μ s [2]

8 An apartment is selling at \$750 000.

There is a scheme where the buyer is required to pay a downpayment of 15%, the remaining of which is a loan at a simple interest rate of 4.2% per annum to be repaid in monthly instalments over 20 years.

Calculate the amount of each monthly instalment.

Answer \$ [3]

9 For a regular polygon with n sides, the ratio of the size of an interior angle to its exterior angle is 7 : 2.

(a) Find the size of the interior angle.

Answer° [1]

(b) Find the value of n .

Answer $n =$ [2]

10 A forest reserve of area 225 km^2 is represented by an area of 36 cm^2 on a map.

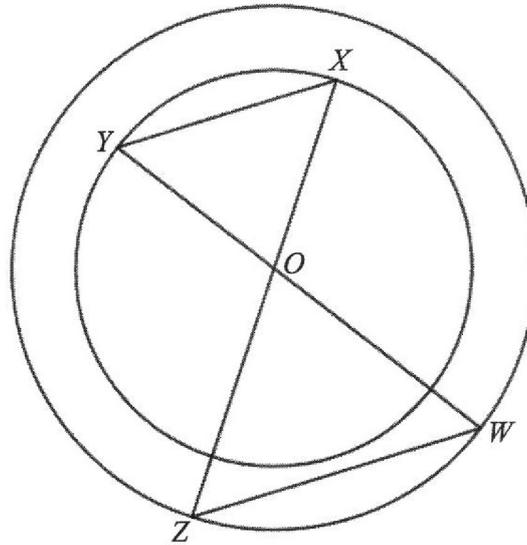
(a) Given that the scale of the map is $1 : n$, find the value of n .

Answer $n = \dots\dots\dots$ [2]

(b) Find the actual distance, in kilometres, of a trail represented by 2.1 cm on the map.

Answer $\dots\dots\dots \text{ km}$ [1]

- 11 In the diagram, O is the centre of two concentric circles.
 X and Y lie on the circumference of the smaller circle, while W and Z lie on the circumference of the larger circle.
 WY and XZ intersect at O .



With clear reasoning, prove that triangle WOX and triangle ZOY are congruent.

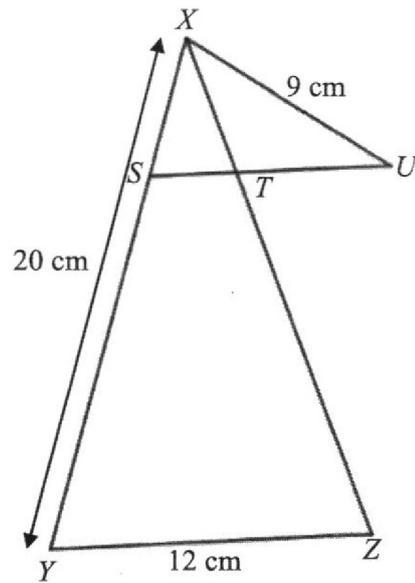
Answer

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..... [3]

12 In the diagram, triangle XYZ is similar to triangle UXS .



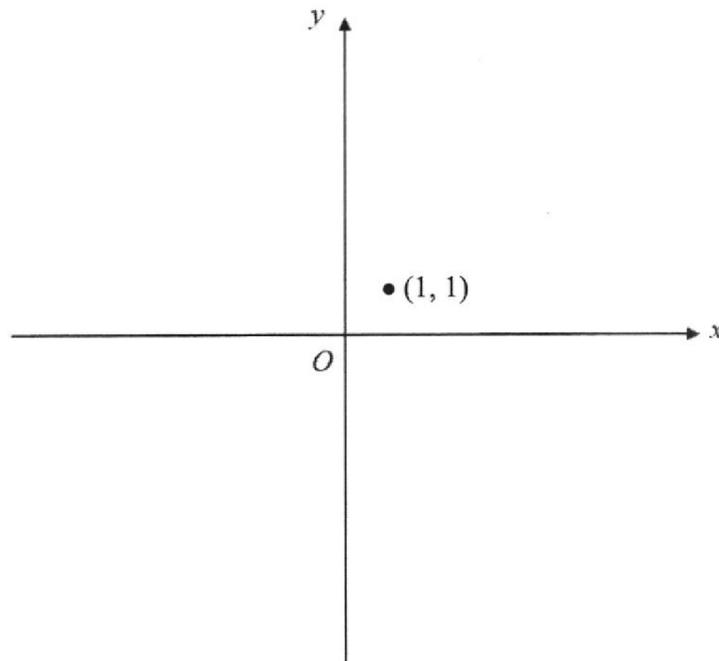
Given that $US = UX = 9$ cm, $XY = XZ = 20$ cm and $YZ = 12$ cm, find the length of SY .

Answer cm [3]

- 13 (a) Sketch the graph of $y = \frac{2}{x^2}$ on the axes below.
The point (1, 1) is shown in the diagram.

Answer

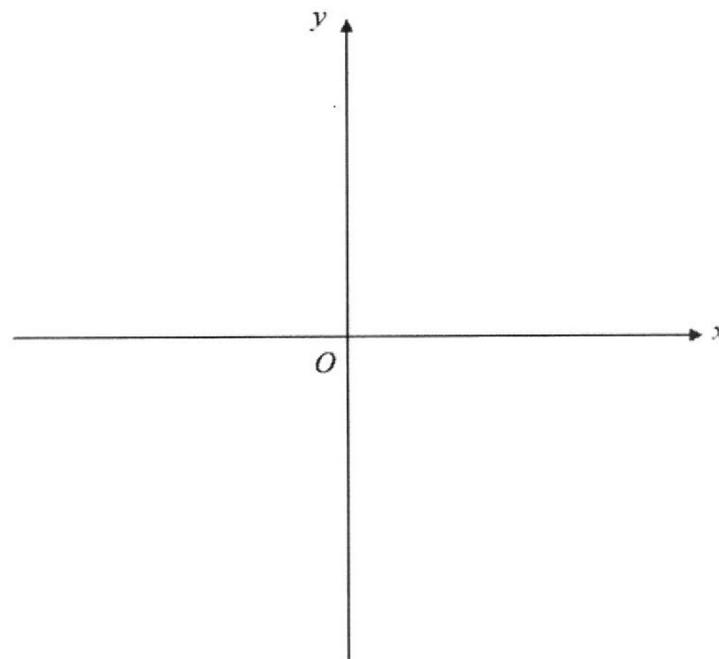
[2]



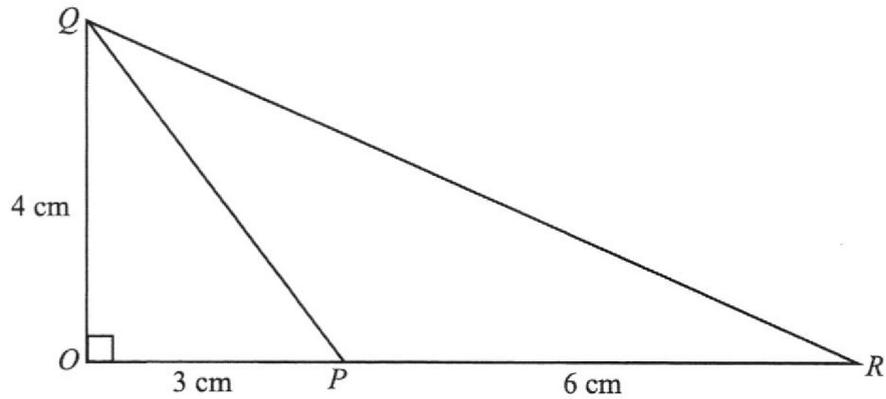
- (b) Sketch the graph of $y = 5^x$ on the axes below.
Indicate clearly where the graph intersects with the axes on your sketch.

Answer

[1]



- 14 The diagram shows a right-angled triangle OQR , where $OQ = 4$ cm and P is a point on OR such that $OP = 3$ cm and $PR = 6$ cm.



Express as a fraction in its simplest form,

(a) $\cos \angle QPR$,

Answer [1]

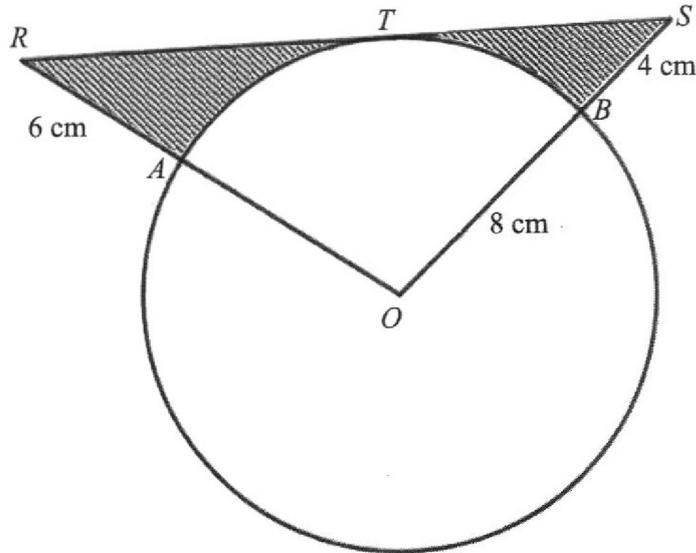
(b) $\tan \angle PRQ$,

Answer [1]

(c) $\frac{\sin \angle PQR}{\sin \angle PRQ}$.

Answer [2]

- 15 The diagram shows a circle of radius 8 cm and centre O .
 The tangent to the circle at T meets OA produced at R and OB produced at S .
 The area of minor sector OAB is 57.5 cm^2 , $AR = 6 \text{ cm}$ and $BS = 4 \text{ cm}$.



- (a) Show that angle AOB is 1.797 radians.

Answer

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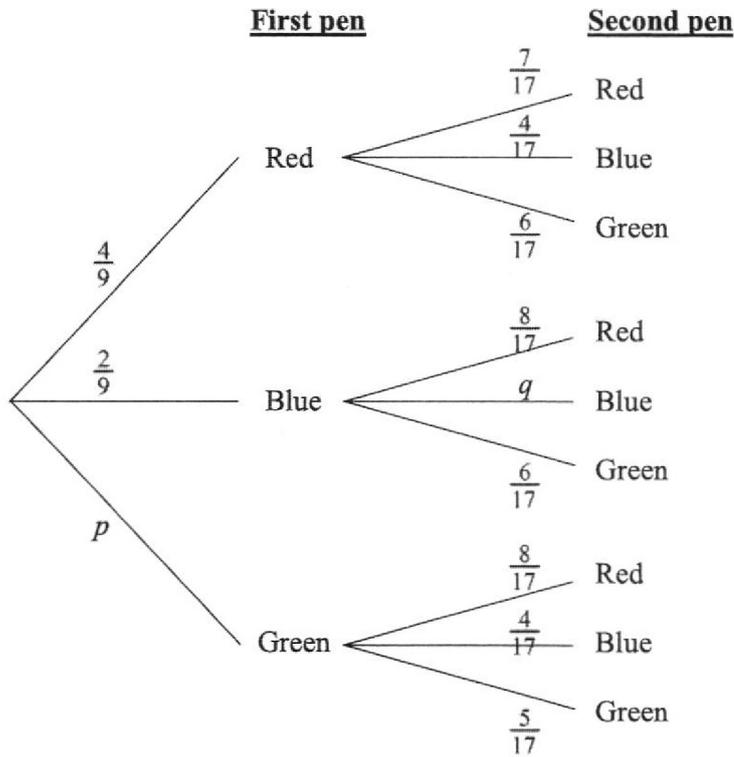
.....

..... [2]

- (b) Calculate the shaded area.

Answer cm^2 [2]

- 16 Jane had a box filled with 8 red pens, 4 blue pens and 6 green pens. She took out two pens at random, one after the other, without replacement.



- (a) Write down the value of p and of q as a fraction in its simplest form.

Answer $p = \dots\dots\dots$

$q = \dots\dots\dots$ [1]

- (b) Calculate the probability that

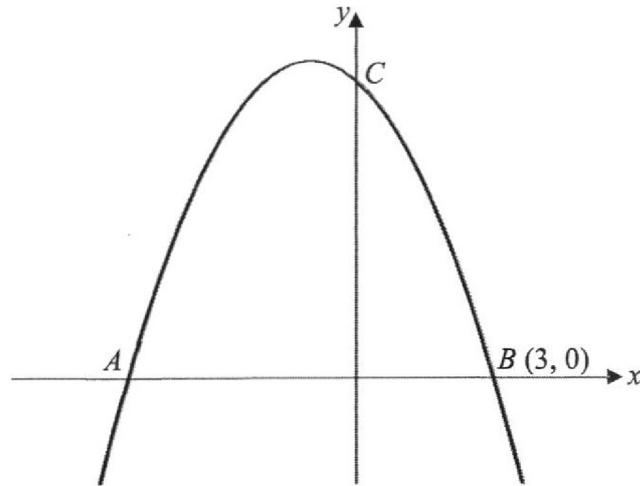
- (i) one pen is blue and the other is green,

Answer $\dots\dots\dots$ [1]

- (ii) both pens are of different colours.

Answer $\dots\dots\dots$ [2]

- 17 The diagram below shows the sketch of the graph $y = -x^2 + px + q$.
 The graph cuts the x -axis at points A and $B(3, 0)$.
 The graph also cuts the y -axis at point C .
 The equation of the line of symmetry of the graph is $x = -1$.



- (a) Write down the coordinates of the point A .

Answer $A(\dots\dots\dots, \dots\dots\dots)$ [1]

- (b) Find the value of p and of q .

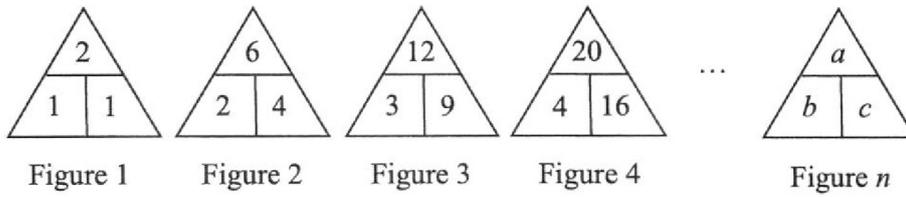
Answer $p = \dots\dots\dots$

$q = \dots\dots\dots$ [3]

- (c) Write down the coordinates of the point C .

Answer $C(\dots\dots\dots, \dots\dots\dots)$ [1]

18 Study the pattern below.



(a) Write down the values of a , b and c in Figure 11.

Answer $a = \dots\dots\dots$
 $b = \dots\dots\dots$
 $c = \dots\dots\dots$ [2]

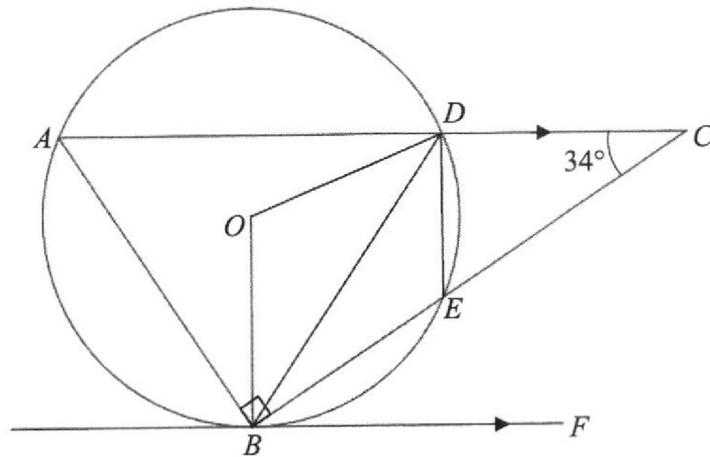
(b) Write down an expression, in terms of n , for a in Figure n .

Answer $a = \dots\dots\dots$ [1]

(c) Explain why the value of a is even for all integer values of n .

Answer $\dots\dots\dots$
 $\dots\dots\dots$
 $\dots\dots\dots$
 $\dots\dots\dots$ [2]

19



The points A, B, E and D lie on the circle with centre O .
 BEC is a straight line and BF is a tangent to the circle at point B .
 It is given that AC is parallel to BF , angle $ABC = 90^\circ$ and angle $ACB = 34^\circ$.

Stating reasons clearly, calculate

(a) angle OBE ,

Answer $^\circ$ [2]

(b) angle DEB ,

Answer $^\circ$ [2]

(c) angle BOD .

Answer $^\circ$ [1]

- 20 The diagram shows the map of a garden in the shape of a quadrilateral $ABCD$. The scale of the map is 1 cm to 10 m.

(a) Using a pair of compasses and ruler, construct

(i) the perpendicular bisector of AB , and [1]

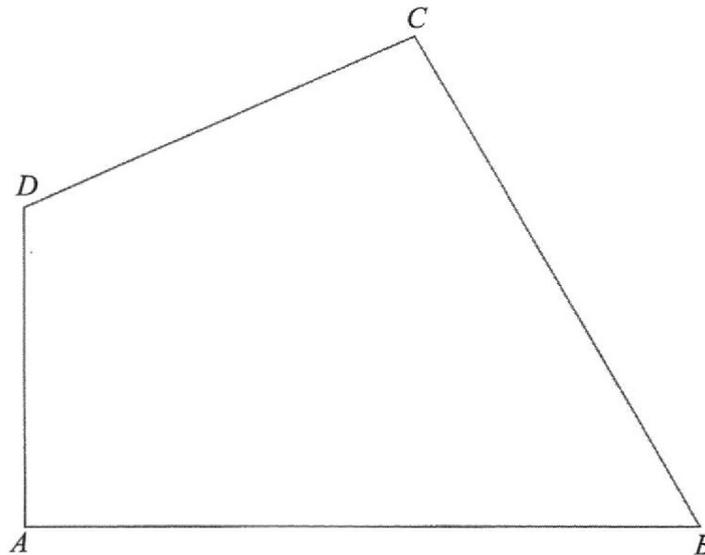
(ii) the angle bisector of angle ABC . [1]

(b) A statue S is located inside the garden $ABCD$ such that it is 55 m from D and equidistant from A and B .

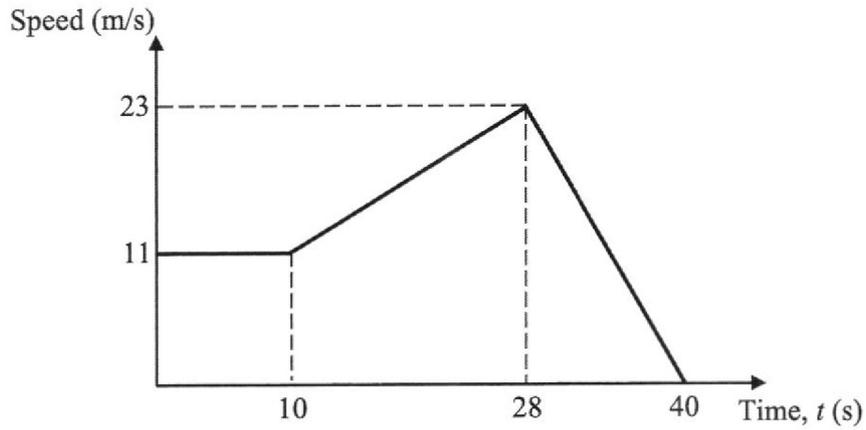
Label the exact position of S . [2]

(c) A pine tree T is located inside the garden $ABCD$ such that it is equidistant from AB and BC , and equidistant from A and B .

Label the exact position of T . [1]



- 21 The diagram shows the speed-time graph of an object over a period of 40 seconds.



- (a) Calculate the deceleration of the object at $t = 35$.

Answer m/s^2 [1]

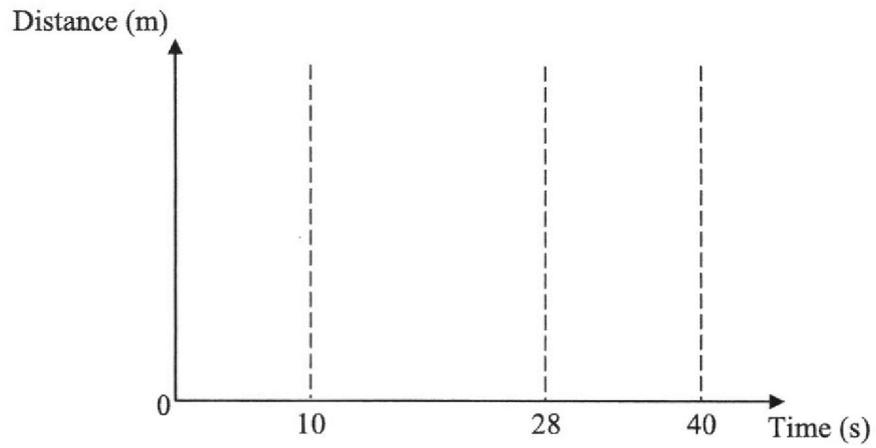
- (b) Find the speed of the object at $t = 18$.

Answer m/s [2]

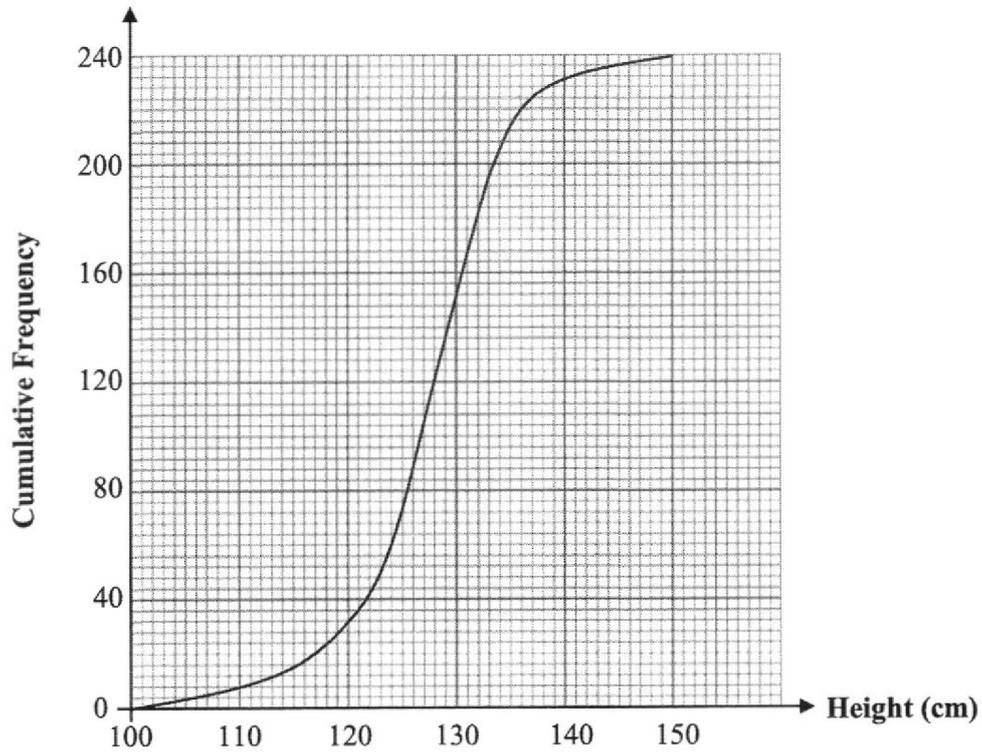
- (c) On the axes in the answer space below, sketch the distance-time graph of the object and indicate clearly, on the vertical axis, the distance travelled at $t = 10$, $t = 28$ and $t = 40$.

Answer

[3]



- 22 The cumulative frequency graph shows the distribution of heights of a sample of 240 children from ABC Primary School.



- (a) Find the interquartile range.

Answer cm [2]

- (b) It is given that 15% of the students are taller than h cm.

Find the value of h .

Answer $h =$ [2]

- (c) The height of 240 children from XYZ Primary School have a larger median and a smaller standard deviation.

Describe how their cumulative frequency curve will differ from ABC Primary School.

Answer

.....

.....

..... [2]

End of Paper

Answer Key:

1. 60

2. 12.1, 9.80

3. (a) 10 (b) 5

4. (a) $2^2 \times 3^3 \times 5 \times 7$ (b) 120

5. $x = \pm \sqrt{\frac{y^3 + 9}{18}}$

6. $x = 2, y = -3$

7. (a) 2.983×10^8 (b) 33.5

8. \$4 887.50

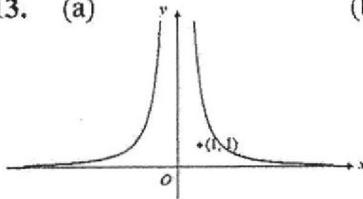
9. (a) 140 (b) 9

10. (a) 250 000 (b) 5.25

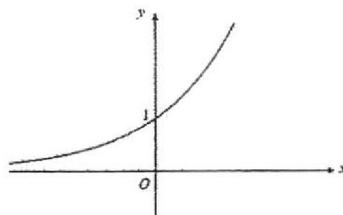
11. $OW = OZ$ (radii of larger circle); $OX = OY$ (radii of smaller circle);
 $\angle WOX = \angle ZOY$ (vert. opp. angles) so $\triangle WOX \cong \triangle ZOY$ (SAS)

12. 14.6

13. (a)



(b)



14. (a) $-\frac{3}{5}$ (b) $\frac{4}{9}$ (c) $\frac{6}{5}$

15. (a) $0.5(8^2)\theta = 57.5$; $\theta = \frac{57.5(2)}{8^2} = 1.796875 = 1.797$ radians (b) 24.4

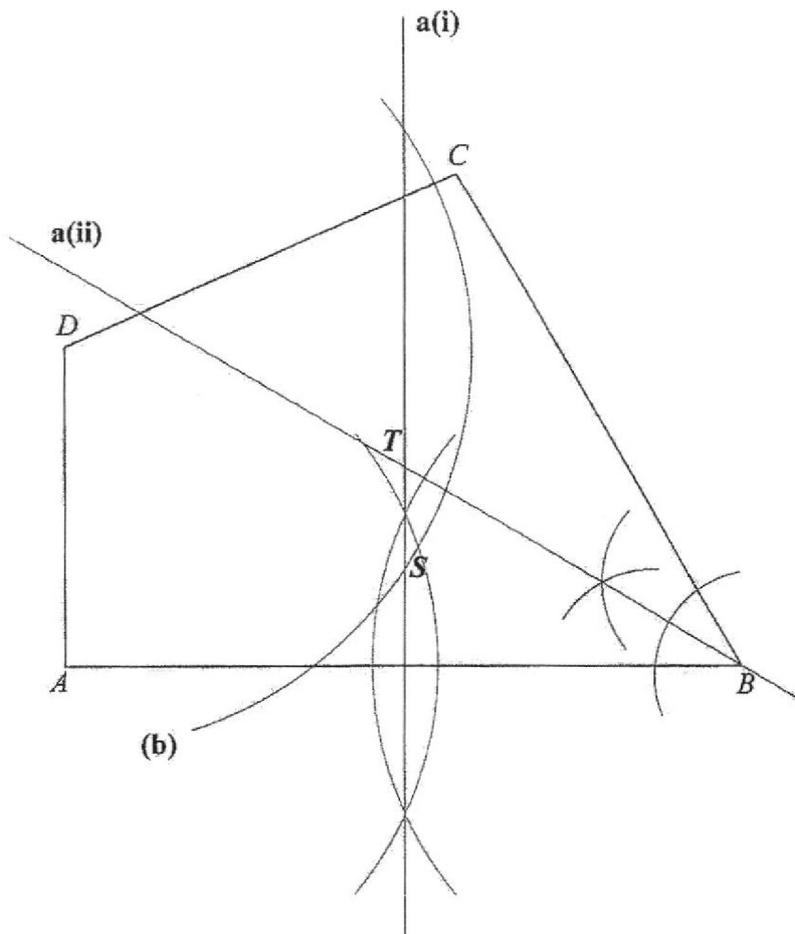
16. (a) $p = \frac{1}{3}, q = \frac{3}{17}$ (b)(i) $\frac{8}{51}$ (ii) $\frac{104}{153}$

17. (a) $(-5, 0)$ (b) $p = -2, q = 15$ (c) $(0, 15)$

18. (a) $a = 132, b = 11, c = 121$ (b) $n(n+1)$
(c) When n is odd, $(n+1)$ is even, product of $n(n+1)$ is even.
When n is even, $(n+1)$ is odd, product of $n(n+1)$ is still even.
So for all values of n , the value of a is even. (shown)

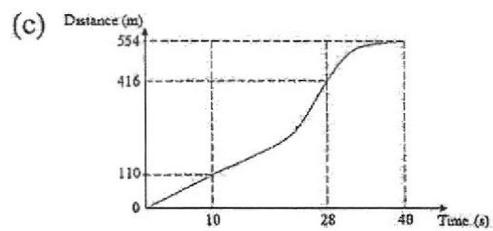
19. (a) 56 (b) 124 (c) 112

20.



21. (a) 1.92

(b) 16.3



22. (a) 8

(b) 133

(c) The curve will be shifted to the right and it will be steeper.