



**JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)**

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS

4048/02

Paper 2

24 August 2021

2 hours and 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all your work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's use
100

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1 (a) Solve the inequality $-\frac{3x+4}{2} < \frac{5-x}{7}$.

Answer [2]

(b) Simplify $\left(\frac{64a^6}{b^{-15}}\right)^{-\frac{1}{3}}$, expressing your answer in positive index notation.

Answer [2]

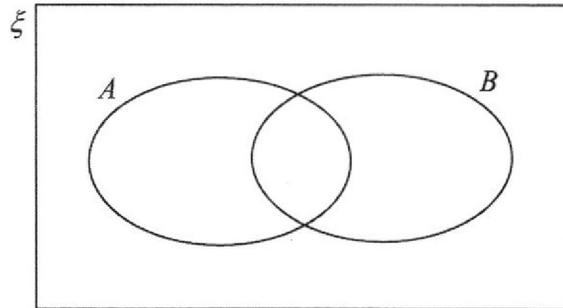
(c) Express $\frac{2}{2x-3} + \frac{7}{2x^2+x-6}$ as a single fraction in its simplest form.

Answer [3]

(d) Simplify $\frac{4u^2 - v^2}{6uv - 3v^2}$.

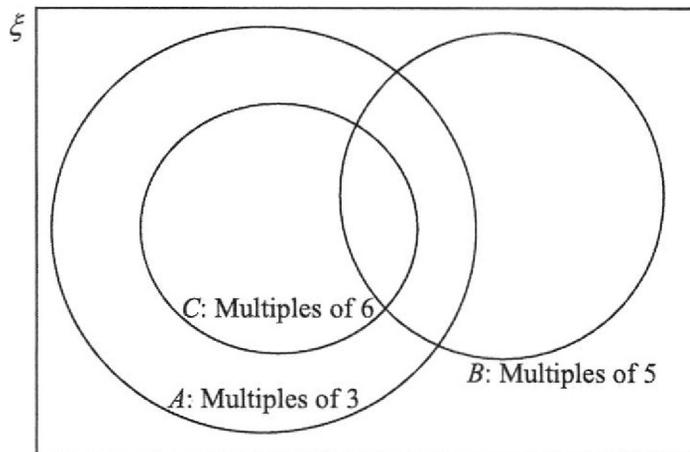
Answer [2]

- 2 (a) In the Venn diagram below, shade the region $A' \cap B$. [1]



- (b) (i) Place the five numbers in their correct positions in the Venn diagram below.

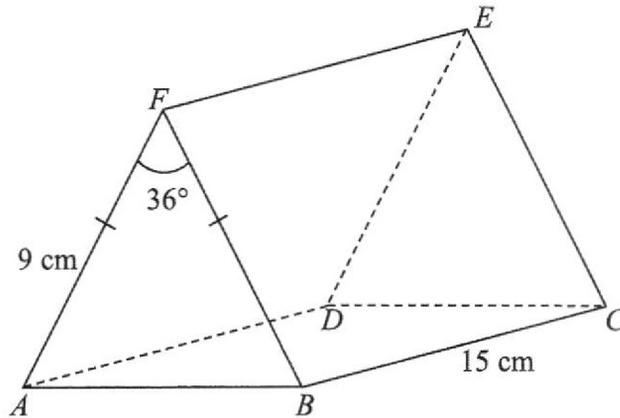
35, 45, 51, 60 and 76 [2]



- (ii) Use \emptyset , \in , \notin , \subset or $\not\subset$ to complete the statement: $51 \text{ ____ } B$.

Answer [1]

- 3 The diagram shows a solid prism $ABCDEF$ whose cross section is an isosceles triangle. It is given that $AF = 9$ cm, $BC = 15$ cm and angle $AFB = 36^\circ$.



- (a) Calculate the volume of the prism $ABCDEF$.

Answer cm^3 [3]

- (b) The prism is then melted and moulded into smaller spheres of radius 0.8 cm. Johnny claims that there will be 166 spheres formed.

State, showing your calculations clearly, if Johnny's claim is correct.

Answer
 [3]

- 4 Estate E and Estate W are 35 km apart.
Jason cycles at an average speed of x km/h from Estate E to Estate W .
On his return journey, he cycles 4 km/h faster.

(a) Write an expression, in terms of x , for the number of hours

(i) Jason takes to cycle from Estate E to Estate W ,

Answer h [1]

(ii) Jason takes to cycle from Estate W to Estate E .

Answer h [1]

(b) Given that Jason completes the return journey 20 minutes faster, form an equation in x , and show that it reduces to $x^2 + 4x - 420 = 0$. [3]

(c) Solve the equation $x^2 + 4x - 420 = 0$.

Answer $x =$ or [3]

(d) Write down the time taken, in hours, for Jason to cycle from Estate E to Estate W .

Answer h [1]

- 5 (a) Given that $\begin{pmatrix} 9 & 2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2y \end{pmatrix} = \begin{pmatrix} x \\ 2x-7 \end{pmatrix}$, find the value of x and of y .

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [2]

- (b) Robert loves to eat Beef-burgers and Chicken-burgers.
The matrix, \mathbf{T} , shows the total number of Beef-burgers and Chicken-burgers he consumed in a particular week.

$$\mathbf{T} = \begin{matrix} & \begin{matrix} \text{Beef} & \text{Chicken} \end{matrix} \\ \begin{pmatrix} 5 & 3 \\ 1 & 2 \end{pmatrix} & \begin{matrix} \text{Monday to Friday} \\ \text{Saturday and Sunday} \end{matrix} \end{matrix}$$

- (i) A Beef-burger has 300 calories and a Chicken-burger has 270 calories.

Represent these calories in a 2×1 column matrix \mathbf{C} .

$$\text{Answer } \mathbf{C} = \begin{pmatrix} & \\ & \end{pmatrix} [1]$$

- (ii) (a) Evaluate the matrix $\mathbf{N} = \mathbf{TC}$.

$$\text{Answer } \mathbf{N} = \begin{pmatrix} & \\ & \end{pmatrix} [2]$$

- (b) State what the elements in \mathbf{N} represent.

Answer $\dots\dots\dots$
 $\dots\dots\dots$ [1]

- (iii) (a) Evaluate the matrix $\mathbf{A} = \frac{1}{7}(1 \ 1)\mathbf{N}$.

$$\text{Answer } \mathbf{A} = \left(\begin{array}{c} \\ \end{array} \right) [1]$$

- (b) Explain what the matrix \mathbf{A} represents.

Answer

..... [1]

- (iv) Robert hopes to reduce his total calories intake by 50% on weekdays and maintain his total calories intake on weekend.

Using matrix multiplication, calculate the total amount of calories he should consume in the following week.

Answer [2]

- 6 The variables x and y are connected by the equation $y = x^3 - 3x + 1$.
Some corresponding values of x and y are given in the table below.

x	-3	-2	-1	0	1	2	3
y	-17	k	3	1	-1	3	19

- (a) Find the value of k .

Answer $k = \dots\dots\dots$ [1]

- (b) In the grid on page 11, draw the graph of $y = x^3 - 3x + 1$ for $-3 \leq x \leq 3$. [2]

- (c) Use your graph to solve $x^3 - 3x + 1 = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ or $\dots\dots\dots$ [2]

- (d) By drawing a tangent, find the gradient of the curve at $x = -1.5$.

Answer $\dots\dots\dots$ [2]

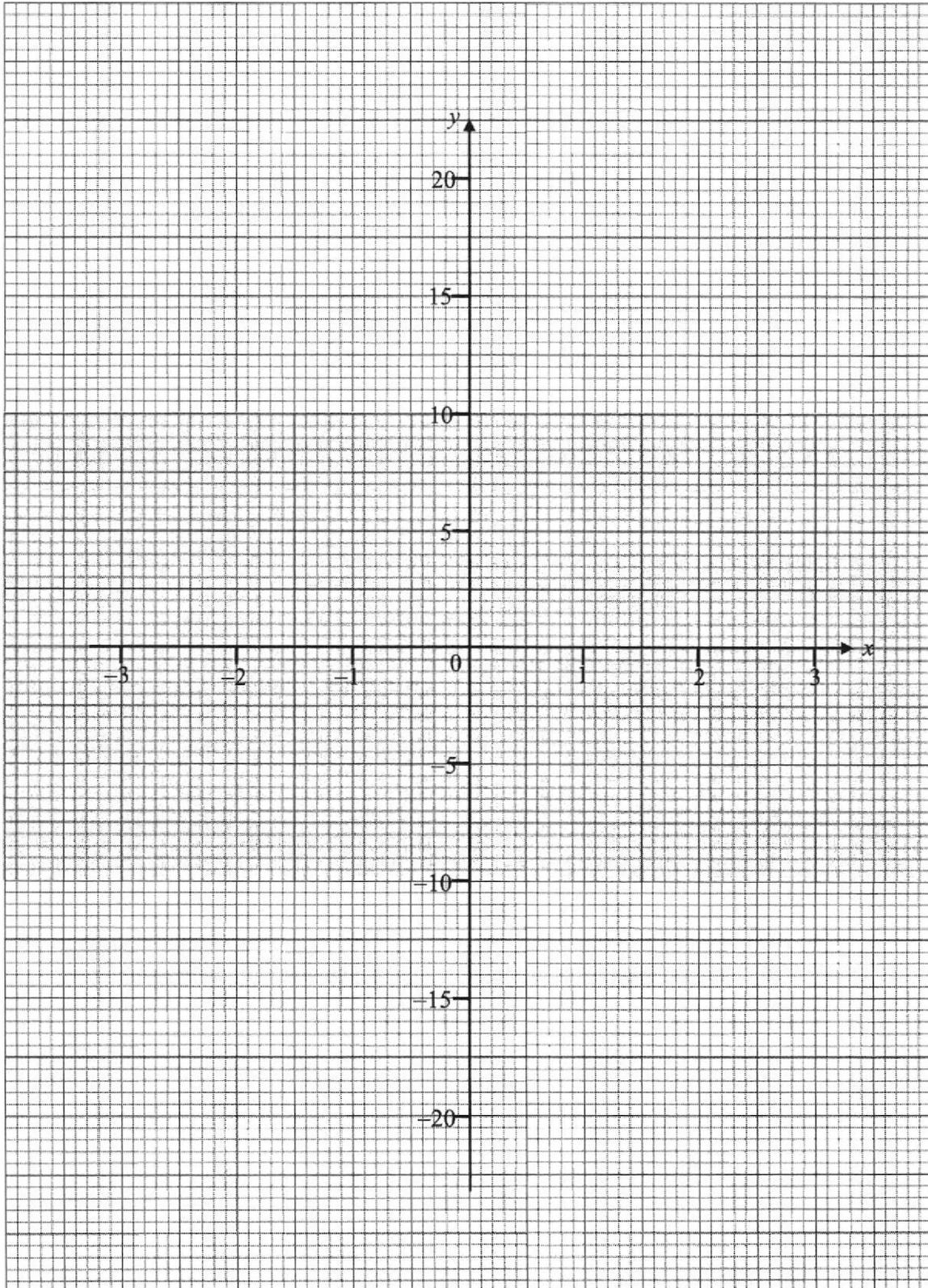
- (e) (i) In the same grid, draw the graph of $y = 3x - 1$. [1]

- (ii) The line $y = 3x - 1$ and the curve $y = x^3 - 3x + 1$ can be used to solve the equation $x^3 = ax + b$.

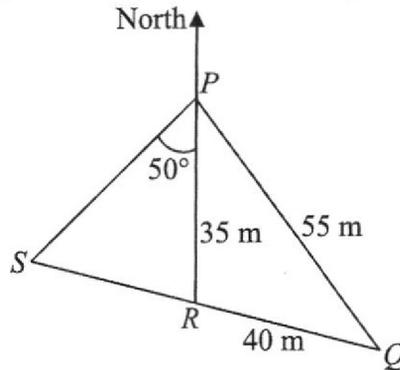
Find the value of a and of b .

Answer $a = \dots\dots\dots$

$b = \dots\dots\dots$ [2]



- 7 P , Q , R and S are points on a triangular garden.
 SRQ is a straight line.
 It is given that $PQ = 55$ m, $RQ = 40$ m, $PR = 35$ m and angle $SPR = 50^\circ$.



(a) Find

- (i) the bearing of S from P ,

Answer $^\circ$ [1]

- (ii) angle PRQ ,

Answer $^\circ$ [3]

- (iii) the area of triangle PQR .

Answer m^2 [2]

- (b) A gardener wants to make a shortest pathway from R to PQ .

Calculate the length of the pathway.

Answer m [2]

- (c) A tree of height 4 m was planted at R .

Find the greatest angle of elevation of the top of the tree when viewed from a point along PQ .

Answer[°] [2]

- 8 (a) The stem and leaf diagram below shows the marks obtained by 30 students in a Science test marked out of 80 marks.

1	8
2	
3	
4	2 2 3 4 5 6 7 8
5	0 1 1 2 4 8 9
6	3 3 3 3 3 6 7 7
7	0 3 5 5 6 6

Key: 2 | 3 means 23 marks

- (i) Find the median mark.

Answer [1]

- (ii) Explain why the mean may not be an appropriate average to be used to summarise the marks obtained by these students.

Answer

..... [1]

- (iii) A distinction is awarded to the top 30% of the students.

Find the minimum mark a student needs to score in order to achieve distinction.

Answer [1]

- (iv) Find the standard deviation of the marks.

Answer [1]

- (v) The standard deviation of the marks obtained by a second group of students who took the same test was 9.85.

Use this information to comment on the marks obtained by the two groups of students.

Answer

..... [1]

- (b) At the cashier counter of a shop, a customer may pay for his purchase by NETS (electronic payment), credit card or cash.

The probability that a customer makes payment by NETS is $\frac{2}{3}$ and the probability that payment is made by credit card is $\frac{1}{4}$.

- (i) Find the probability that a customer pays by cash.

Answer [1]

- (ii) Two customers are paying for their purchases.

Find the probability that

- (a) both customers pay by credit card,

Answer [1]

- (b) no cash is used when they make their payments.

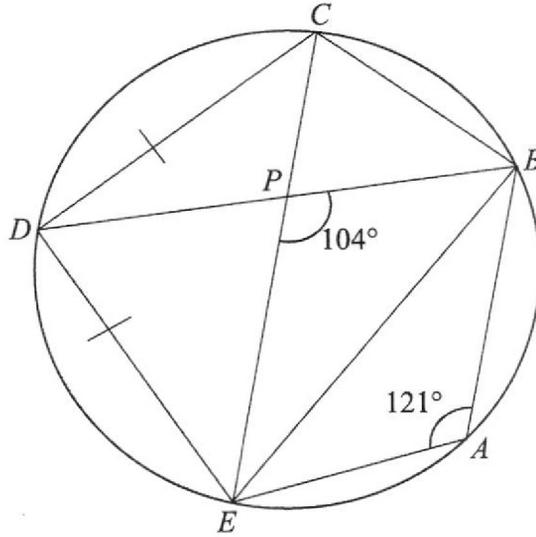
Answer [1]

- (iii) Three customers are paying for their purchases.

Find the probability that the first cash transaction occurs with the third customer and not before.

Answer [2]

- 9 In the diagram, not drawn to scale, A, B, C, D and E are points on the circle. CPE and BPD are straight lines. It is given that $DC = DE$, angle $EPB = 104^\circ$ and angle $BAE = 121^\circ$.



- (a) Find angle BDE .

Answer° [1]

- (b) Hence show that angle $DEC = 45^\circ$.

[1]

- (c) Prove that CE is the diameter of the circle.

Answer
 [2]

- (d) Hence find angle CDB .

Answer° [1]

- (e) Find angle BEP .

Answer° [1]

- (f) Explain why P cannot be the centre of the circle.

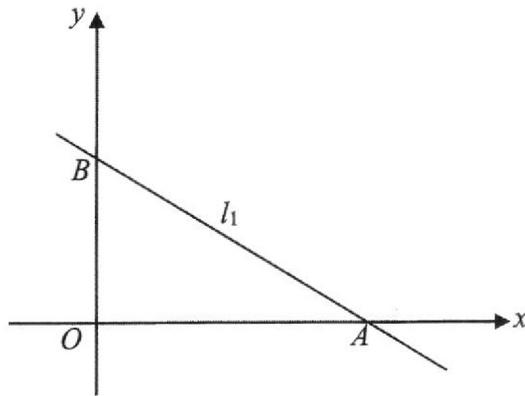
Answer
 [2]

- (g) It is further given that $EP = 9$ cm.

Find the length of DP .

Answer cm [2]

- 10 The equation of the line l_1 is $x + 4y = 20$.
The line cuts the x -axis at point A and the y -axis at point B .



- (a) Find the length of AB .

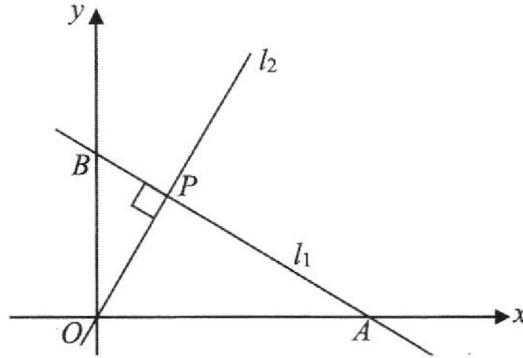
Answer units [3]

A point C lies on the line l_1 such that it is equidistant from the coordinate axes.

- (b) Show that the coordinates of point C is $(4, 4)$. [2]

Answer

Another line l_2 , which is perpendicular to l_1 , passes through the origin O and intersects l_1 at P .



- (c) The product (gradient of AB) \times (gradient of OP) = -1 .

Use this information to find the equation of the line OP .

Answer [3]

- (d) Hence find the coordinates of P .

Answer (.....,) [2]

- (e) State a pair of similar triangles from the diagram.

Answer and [1]

- 11 Rebecca always travels by taxi to work.
To get a taxi, she has two options.
She can either flag down a taxi at the roadside, or book one via a mobile app.
The fare calculation for each option is as shown below.

Option 1: Flag down a taxi at the roadside.

Flag Down charge (First 1 km or less)	\$3.20
1) Every 450 m thereafter or less up to 10 km 2) Every 350 m thereafter or less after 10 km 3) Every 45 seconds of waiting* time or less	\$0.22
Surcharges	
Monday – Friday (Except Public Holidays): 06 00 to 09 29 Monday – Sunday & Public Holidays: 18 00 to 23 59	25% of metered fare
Midnight to Before 05 59	50% of metered fare
* <i>Waiting time refers to time during journey when vehicle is not moving due to traffic light or road congestions.</i>	

[resources: www.cdgtaxi.com.sg/ride-with-us/fares/ (modified)]

Option 2: Book a taxi via mobile app.

Fares are charged based on both the total distance travelled and the total time taken for the journey.

Base fare	\$2.80
Per Kilometre fare (distance)	\$0.50
Per Minute fare (time)	\$0.15
Booking fee	
Current Booking fee	\$3.30
Advance Booking fee (at least half an hour in advance)	\$8.00
# For every confirmed ride, minimum fare is \$6.00	

[resources: www.cdgtaxi.com.sg/ride-with-us/fares/ (modified)]

For both options:

Rebecca works only on weekdays (Monday to Friday).

Her workplace is 17 km from home.

Her journey to work will take 25 minutes inclusive of 6 minutes of 'waiting' time.

In order to reach her workplace on time, she needs to get into a taxi by 07 30.

For option 2 only:

As a regular user of the taxi services, Rebecca holds a Loyalty card that will entitle her to a 20% discount for fare exceeding \$10.00, if she chooses Option 2.

- (a) How much does it cost Rebecca to travel by taxi to work on Monday if she chooses Option 1?

Answer \$ [4]

- (b) Rebecca is required to work on all weekdays in August 2021, including 9 August National Day, which is a Public Holiday.

Suggest the **minimum** budget that Rebecca needs to set aside for transport to work in the month of August 2021.

Show your workings clearly.

[7]

Answer

Work schedule for August 2021

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

End of Paper

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Answers to Paper 2:

1 (a) $x > -2$, (b) $\frac{1}{4a^2b^5}$, (c) $\frac{2x+11}{(2x-3)(x+2)}$, (d) $\frac{2u+v}{3v}$

3 (a) 357 cm^3 , (b) claim is correct

4 (a)(i) $\frac{35}{x}$ h, (ii) $\frac{35}{x+4}$ h, (c) 18.6 or -22.6 , (d) 1.88 hours

5 (a) $x = 5$ and $y = 1$, (b)(i) $\begin{pmatrix} 300 \\ 270 \end{pmatrix}$, (ii)(a) $\begin{pmatrix} 2310 \\ 840 \end{pmatrix}$, (ii)(b) amount of calories consumed for

'Monday to Friday' and 'Saturday and Sunday' respectively, (iii)(a) (450), (iii)(b) **average amount** of calories consumed per day in that week, (iv) 1995 calories

6 (a) $k = -1$, (c) $x \approx -1.9$ or 0.4 or 1.5 , (d) 3.375, (e)(ii) $a = 6$ and $b = -2$

7 (a)(i) 230° , (ii) 94.1° , (iii) 698 m^2 , (b) 25.4 m, (c) 9.0°

8 (a)(i) 58.5 marks, (ii) because there was an outlier of 18 marks, (iii) 66 marks, (iv) 13.2 marks, (v) marks obtained by the second group of students was more consistent because their SD was smaller

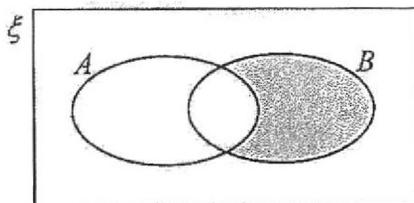
8 (b)(i) $\frac{1}{12}$, (ii)(a) $\frac{1}{16}$, (ii)(b) $\frac{121}{144}$, (iii) $\frac{121}{1728}$

9 (a) 59° , (b) 45° , (d) 31° , (e) 31° , (g) 7.42 cm,

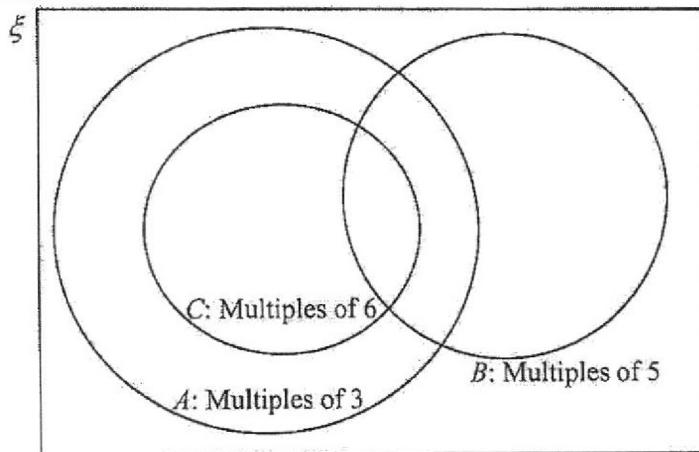
10 (a) 20.6 units, (c) $y = 4x$, (d) $P = (1\frac{3}{17}, 4\frac{12}{17})$, (e) $\angle AOB$ and $\angle APO$ (or $\angle BOA$ and $\angle BPO$, or $\angle BPO$ and $\angle OPA$)

11 (a) \$17.20, (b) \$323 [accept \$323 – \$330]

2 (a)



(b)(i)

(ii) $51 \notin B$