Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E

Preliminary Examination

4E

ADDITIONAL MATHEMATICS

4049/2

Paper 2

30 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number in the spaces provided at the top of this page.
Write your answers in the spaces provided on the question paper.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use
90

This document consists of **18** printed pages including the cover page.

[Turn Over

Setter: Ms Elene Phang

Mathematical Formula

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \quad ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)....(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Given the curve $y = p(x+q)^2 + r$ is always negative. State two conditions for p and r. [2]

(b) **(i)** Determine the set of values of m for which the equation $x^2 + mx + 6 = -2x - m + 1$ has no real root. [3]

(ii) Hence, explain what can be deduced about the curve $y = x^2 + 2x + 6$ and the line $y = -2x - 1$. [2]

2 **(a)** Differentiate $4x^3 \ln x$ with respect to x. [2]

(b) Hence evaluate $\int_1^5 3x^2 \ln x \, dx$. [4]

(c) State the range of value of x for which $y = 4x^3 \ln x$ is an increasing function. [4]

3 **(a)** Find the independent term of $\left(x^3 + \dfrac{2}{x}\right)^8$. [3]

(b) Show that there is no constant term in the expansion of $\left(7x^4-4\right)\left(x^3+\dfrac{2}{x}\right)^8$. [3]

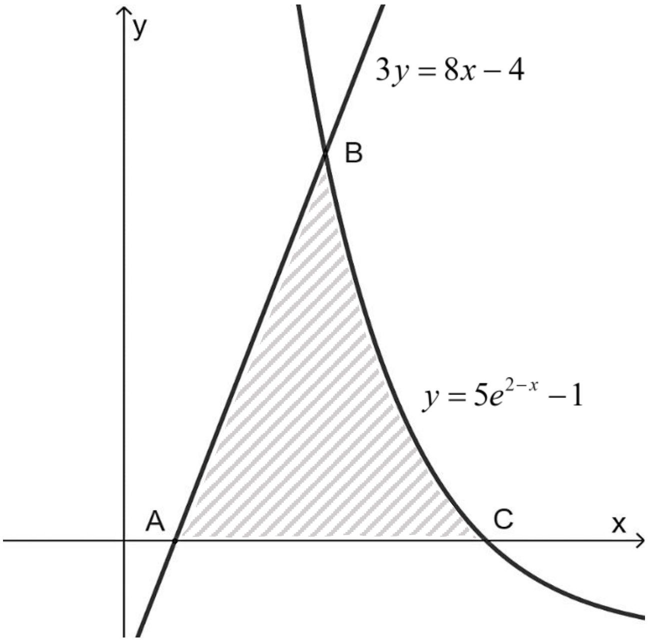
4 The expression $px^3+qx^2-29x-6$ where p and q are constants has a factor of $x+3$ and leaves a remainder of -24 when divided by $x-1$.

(a) Find the value of p and of q. [4]

(b) Using the values of p and q, solve the equation $px^3 + qx^2 - 29x - 6 = 0$. [3]

(c) Hence, solve $-6y^3 - 29y^2 + qy + p = 0$. [2]

5 The diagram shows part of the curve $y = 5e^{2-x} - 1$ which intersects the *x*-axis at *C* and the straight line $3y = 8x - 4$ at $B(2, 4)$.



(a) Show that the *x*-coordinate of *C* is 3.609. [2]

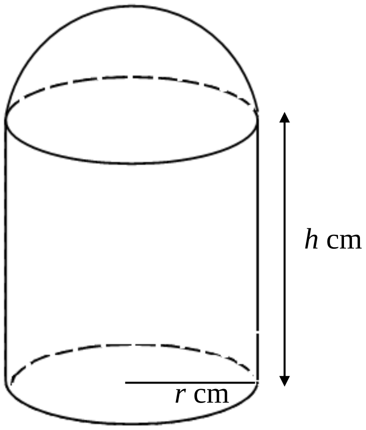
(b) Find the shaded area bounded by $3y = 8x - 4$, $y = 5e^{2-x} - 1$ and the x-axis. [6]

6 The equation of a curve is $y = \frac{2\sin x}{\cos x - 3}$.

(a) Express $\frac{dy}{dx}$ in the form $\frac{a + b\cos x}{(\cos x - 3)^2}$ where a and b are constants. [3]

(b) Given y is decreasing at 0.064 units per second, find the rate of change of x with respect to time when $x = \frac{\pi}{3}$. [3]

7 A water bottle in the shape of a hemisphere on a cylinder has a total external surface area of 900π cm^2.



(a) Express h in terms of r. [2]

(b) Show that the volume of the water bottle, V cm^3, is given by $V = 450\pi r - \frac{5}{6}\pi r^3$. [2]

(c) Find the value of r for which V has a stationary value and the stationary value of V. [4]

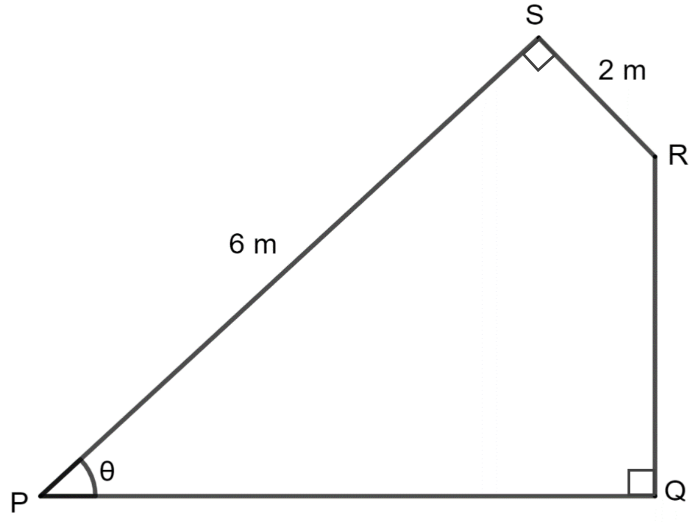
(d) Explain whether the volume of the water bottle can be further increased by changing the value of r in **(c)**. [2]

8 A cubic curve that passes through the x-axis at $x = 2$, has a coefficient of 1 for x^3 and $\frac{dy}{dx} = k(x-3)(x+1)$.

(a) Find the value of k and the equation of the curve. [4]

(b) The normal at P on the curve is parallel to the line $12y = x + 1$.
Find the x-coordinate of P. [3]

9 The diagram shows a solar panel on a roof top. It is given that $\angle PSR = \angle RQP = 90°$, SR = 2 m, SP = 6 m and $\angle SPQ = \theta$, where $0 < \theta < 90°$.



(a) **(i)** Show that the perimeter of the solar panel, P m, is given by $P = 8\sin\theta + 4\cos\theta + 8$. [3]

(ii) Express P in the form of $R\sin(\theta+\alpha)+8$, where $R>0$ and α is acute. [3]

(iii) Find the maximum perimeter and the corresponding value of θ. [2]

(b) (i) Express the area of the solar panel in terms of $a\sin 2\theta + b\sin^2\theta$, where a and b are integers. [2]

(ii) Hence, or otherwise, find the area of the solar panel when the perimeter is at its maximum. [2]

10 **(a)** Given that point $A\ (0,-3)$, $B\ (0,11)$, $C\ (6,5)$ lie on a circle. Show that the coordinates of the centre of the circle is $(-1,4)$. [4]

(b) Find the equation of the circle. [2]

(c) Point R and T are on the circle such that the tangents at these two points are parallel to the x-axis. Find the coordinates of R and T. [2]

11 (a) Show that $3\sin(A+B)+3\sin(A-B)=k\sin A\cos B$ and find the value of k. [3]

(b) Hence, find the exact value of $3\sin 105°+3\sin 15°$. [4]

End of Paper