Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E Preliminary Examination 4E

ADDITIONAL MATHEMATICS 4049/2

Paper 2

30 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

MARK SCHEME

1a)	$p < 0$ B1 $r < 0$ B1
b) (i)	$x^2 + (m+2)x + 5 + m = 0$ $b^2 - 4ac < 0$ $(m+2)^2 - 4(1)(5+m) < 0$ M1 $m^2 + 4m + 4 - 20 - 4m < 0$ $m^2 - 16 < 0$ $(m-4)(m+4) < 0$ M1 $-4 < m < 4$ A1
(ii)	$m = 2,$ and $-4 < 2 < 4$ M1 The curve and the line do not intersect. A1
2a)	$\frac{d}{dx} 4x^3 \ln x = 12x^2 \ln x + \frac{4x^3}{x}$ $= 12x^2 \ln x + 4x^2$ A2
b)	$\int_1^5 12x^2 \ln x \, dx = \left[4x^3 \ln x\right]_1^5 - \int_1^5 4x^2 \, dx$ M1 $\int_1^5 12x^2 \ln x \, dx = \left[4x^3 \ln x - \frac{4}{3}x^3\right]_1^5$ M1 for integrating $4x^2$ $\int_1^5 3x^2 \ln x \, dx = \left[x^3 \ln x - \frac{1}{3}x^3\right]_1^5$ M1 divide by 4 on both sides $= 160$ A1
c)	$12x^2 \ln x + 4x^2 > 0$ M1 $4x^2 (3\ln x + 1) > 0$ M1 for factorising since $4x^2 > 0$ and $x \neq 0,$ $3\ln x + 1 > 0$ M1 $x > e^{-\frac{1}{3}}$ or $x > 0.717$ A1
3a)	$(x^3 + 2x^{-1})^8$ $\binom{8}{r} (x^3)^{8-r} (2^{-1})^r$ $x^{24-3r} \times x^{-r} = x^0$ M1 $24 - 4r = 0$ $r = 6$ M1 $\binom{8}{6} (x^3)^2 (2x^{-1})^6 = 1792$ A1

b)	$24-4r=-4$ M1 $r=7$ $\binom{8}{7}\left(x^3\right)^1\left(2x^{-1}\right)^7$ M1 $=1024x^{-4}$ $7x^4\left(1024x^{-4}\right)-4\left(1792\right)$ M1 $=0$
4a)	Let f (x) be $px^3+qx^2-29x-6$. $\mathrm{f}\left(-3\right)=0$ $p\left(-3\right)^3+q\left(-3\right)^2-29\left(-3\right)-6=0$ M1 $q=3p-9$ $\mathrm{f}\left(1\right)=-24$ $p\left(1\right)^3+q\left(1\right)^2-29\left(1\right)-6=-24$ M1 $p+q=11$ $p=5$ A1 $q=6$ A1
b)	$5x^3+6x^2-29x-6=0$ $\left(x+3\right)\left(5x^2-9x-2\right)=0$ M1 $\left(x+3\right)\left(5x+1\right)\left(x-2\right)=0$ M1 $x=-3,\quad x=-\frac{1}{5},\quad x=2$ A1
c)	$-6y^3-29y^2+6y+5=0$ $\div y^3$ $-6-\frac{29}{y}+\frac{6}{y^2}+\frac{5}{y^3}=0$ $\frac{1}{y}=x$ M1 $y=-\frac{1}{3},\quad y=-5,\quad y=\frac{1}{2}$ A1
5a)	$5e^{2-x}-1=0$ M1 $x=2-\ln\left(\frac{1}{5}\right)$ M1 $x=3.609$

b)	$y=0, \quad x=\frac{1}{2}$ M1 for finding x-coordinate of A $\frac{1}{2}\times\frac{3}{2}\times 4$ M1 $=3 \quad \text{units}^2$ $\int_2^{3.609} 5e^{2-x}-1 \quad dx$ M1 $=\left[-5e^{2-x}-x\right]_2^{3.609}$ M1 $=2.390562 \quad \text{units}^2$ $3+2.390562$ M1 $=5.39 \quad \text{units}^2$ A1
6a)	$\frac{dy}{dx}=\frac{2\cos x(\cos x-3)-2\sin x(-\sin x)}{(\cos x-3)^2}$ M1 $=\frac{2\cos^2 x-6\cos x+2\sin^2 x}{(\cos x-3)^2}$ M1 $=\frac{2-6\cos x}{(\cos x-3)^2}$ A1
b)	$\frac{dy}{dx}=\frac{2-6\cos\left(\frac{\pi}{3}\right)}{\left(\cos\frac{\pi}{3}-3\right)^2}$ M1 $=-\frac{4}{25}$ $-0.064=-\frac{4}{25}\times\frac{dx}{dt}$ M1 $\frac{dx}{dt}=\frac{2}{5}$ units/sec A1
7a)	$2\pi r^2+\pi r^2+2\pi rh=900$ M1 $3\pi r^2+2\pi rh=900$ $h=\frac{900-3r^2}{2r}$ A1

b)	$V = \pi r^2\left(\frac{900-3r^2}{2r}\right)+\frac{2}{3}\pi r^3$ M1 $= \frac{900\pi r^2}{2} - \frac{3\pi r^3}{2r} + \frac{2}{3}\pi r^3$ $= 450\pi r - \frac{5}{6}\pi r^3$ M1
c)	$\frac{dV}{dr} = 450\pi - \frac{5\pi r^2}{2}$ M1 $450\pi - \frac{5\pi r^2}{2} = 0$ M1 $r^2 = 180$ $r = 13.416 \quad (\text{reject } -13.416)$ $= 13.4$ A1 $V = 12600 \text{ cm}^3$ A1
d)	$\frac{d^2V}{dr^2} = -5\pi r$ M1 $= -211$ Volume is <u>maximum</u> at $r = 13.4$ Therefore, <u>NO</u>, volume **cannot** increase further. A1
8a	$y = \int k\left(x^2-2x-3\right)dx$ $= k\left(\frac{x^3}{3}-x^2-3x\right)+c$ M1 $k = 3$ A1 $y = x^3-3x^2-9x+c$ $0 = (2)^3-3(2)^2-9(2)+c$ $M1$ $c = 22$ $y = x^3-3x^2-9x+22$ A1
b)	$\text{Gradient of normal} = \frac{1}{12}$ $3\left(x^2-2x-3\right) = -12$ M1 $x^2-2x-3 = -4$ $x^2-2x+1 = 0$ M1 $(x-1)^2 = 0$ $x = 1$ A1

9a (i)	$PQ = 6\cos\theta + 2\sin\theta$ M1 $RQ = 6\sin\theta - 2\cos\theta$ M1 $P = 6\cos\theta + 2\sin\theta + 6\sin\theta - 2\cos\theta + 6 + 2$ M1 $= 8\sin\theta + 4\cos\theta + 8$
(ii)	$R = \sqrt{8^2 + 4^2}$ M1 $= \sqrt{80}$ $= 8.9442$ $\tan\alpha = \frac{4}{8}$ M1 $\alpha = 26.565°$ $P = \sqrt{80}\sin(\theta + 26.6°) + 8$ A1
(iii)	Max $P = 16.9$ m A1 $(\theta + 26.6565°) = 90°$ $\theta = 63.4°$ A1
b) (i)	Area of solar panel $= \frac{1}{2} \times 6\cos\theta \times 6\sin\theta + \frac{1}{2} \times 2\cos\theta \times 2\sin\theta + 2\sin\theta(6\sin\theta - 2\cos\theta)$ M1 $= 18\sin\theta\cos\theta + 2\sin\theta\cos\theta + 12\sin^2\theta - 4\sin\theta\cos\theta$ $= 16\sin\theta\cos\theta + 12\sin^2\theta$ $= 8\sin 2\theta + 12\sin^2\theta$ A1
(ii)	Area of solar panel $= 8\sin 2(63.43439°) + 12\sin^2(63.43439°)$ M1 $= 16\ \text{m}^2$ A1
10a)	$y = \frac{11-3}{2}$ $y = 4$ M1 Let center of circle be $(p, 4)$ $(p-0)^2 + (4--3)^2 = (p-6)^2 + (4-5)^2$ M1 $p^2 + 7^2 = p^2 - 12p + 36 + 1$ M1 $12 = -12p$ $p = -1$ M1 $(-1, 4)$
b)	$r = \sqrt{(6+1)^2 + (5-4)^2}$ M1 $= \sqrt{50}$ units $(x+1)^2 + (y-4)^2 = 50$ A1
c)	$(-1,\ 4 + 5\sqrt{2})$ and $(-1,\ 4 - 5\sqrt{2})$ A1 each

11a)	$3\sin(A+B)+3\sin(A-B)$ $=3\sin A\cos B+3\sin B\cos A+3\sin A\cos B-3\sin B\cos A$ M1 $=6\sin A\cos B$ A1 $k=6$ A1
b)	$3\sin(60°+45°)+3\sin(60°-45°)$ $=6\sin 60°\cos 45°$ M1 for each angle $=6\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}$ M1 $=\frac{3\sqrt{6}}{2}$ A1