Name:____________________________________ ( )	Class: Sec ________

90

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2021

Additional Mathematics **4049/02**

Paper 2 16 September 2021

Secondary 4 Express **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Question	Q1		Q2	Q3	Q4	Q5	Q6		Q7	Q8	Q9	Q10	
	i	ii					a	b				i	ii
Marks													

No of additional booklets/ writing paper used		**No of additional graph paper used**	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ⊗ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of ⊗ = $\frac{1}{2}bc \sin A$

Answer all the questions.

1 **(i)** Differentiate $x^2 \tan x$ with respect to x. **[2]**

(ii) Hence find $\int_0^{\frac{\pi}{4}} \left(2x^2 \sec^2 x + 4x \tan x + 4 \sin x\right) \mathrm{d}x$. **[4]**

2 **(i)** Prove that $\tan\left(x+135^\circ\right)+\tan\left(x-135^\circ\right)=\dfrac{4\tan x}{1-\tan^2 x}$ **[3]**

(ii) Hence solve the equation $\tan\left(x+135^\circ\right)+\tan\left(x-135^\circ\right)=6\tan x$ for $0^\circ \le \theta \le 180^\circ$ **[4]**

3 Given that $f(x) = 2x^3 - 3x^2 + 2x - 3$,

(i) show that $2x - 3$ is a factor of f(x) and hence factorise f(x) completely. **[3]**

(ii) Express $\dfrac{8x^2 - 11x + 5}{2x^3 - 3x^2 + 2x - 3}$ in partial fractions. **[5]**

4 **(i)** Solve the equation $\log_4 2y - \frac{\log_4(y-3)}{2} = 3\log_4 2$. **[4]**

(ii) Given that $\log_{27} z = \log_9 \sqrt{y}$ express *z* in terms of *y*. **[3]**

5 **(a)** Find the set of values of *k* for which the line $y = x + 2$ intersects the curve $y = 2x^2 - 3x + 10 + kx$ at exactly 2 points. **[3]**

(b) **(i)** Express $y = 2x^2 - 10x + c$ in the form $a(x-h)^2 + k$. **[2]**

(ii) If the minimum value of *y* is $-\frac{19}{2}$, find the value of *c* where *k* is in terms of c. **[2]**

6 **(a)** **(i)** It is given that $\int f(x)dx = \cos x + \frac{1}{2}\sin 2x + 4$.

Show that $f(x) = 1 - \sin x - 2\sin^2 x$. **[2]**

(ii) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = \frac{\pi}{6}$. **[6]**

6 Continued

(b) g(x) is such that $g''(x) = 4\cos 2x + 2$.

Given that $g'\left(\frac{\pi}{6}\right) = \sqrt{3}$, find $g'(x)$. **[4]**

7 Recorded values of the temperature, T °C, of a metal, t seconds after heating, are shown in the table below.

t	5	10	15	20	25	30
T	– 37	– 32	-25	-15	0	22

It is known that T and t are related by the equation $T = Ak^t - 50$, where A and k are constants.

(i) Draw the graph of $\lg(T+50)$ plotted against t, using a scale of 2 cm for 5 units

on the t axis and a scale of 4 cm for 0.5 unit on the $\lg(T+50)$ axis. **[3]**

7 Continued

(ii) Use your graph to estimate the value of each of the constants A and of k. **[5]**

(iii) State the initial temperature of the metal. **[1]**

(iv) Explain how the graph could be used to find the value of t when the temperature of the metal is 0 °C. **[1]**

8 The equation of a circle C is $x^2 + y^2 - 8x + 4y + 11 = 0$.

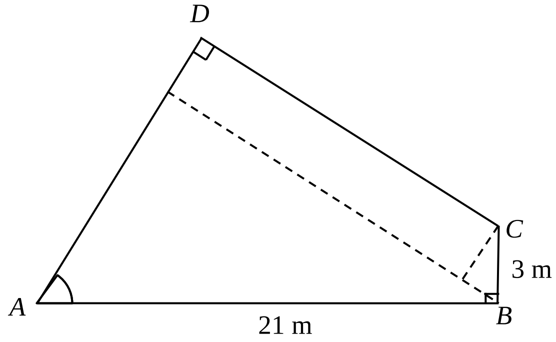
(i) Find the radius and the coordinates of the centre of the circle C. **[3]**

(ii) Find the values of k if the circle touches the line $y = k$. **[2]**

8 Continued

(iii) The line *l* with equation $y = -x + 5$ intersects the circle at the points *A* and *B*. Find the coordinates of *A* and *B* and hence find the shortest distance of the centre of the circle from *l*. **[7]**

9



The diagram shows a quadrilateral field *ABCD*, where *AB* = 21 m, *BC* = 3 m and angle *ABC* = angle *ADC* = 90°. Angle *BAD* = θ and can vary. The perimeter of the fencing around the quadrilateral field *ABCD* is *P* m.

(i) Show that $P = 24 + 24\sin\theta + 18\cos\theta$. **[4]**

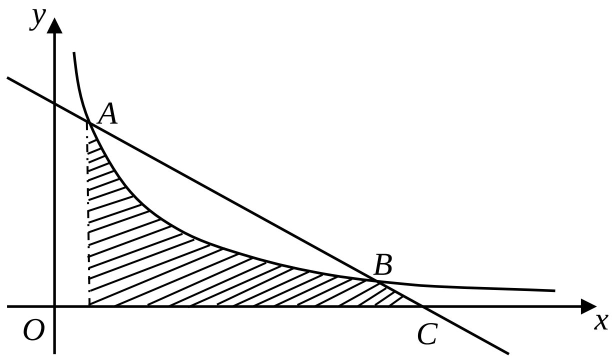
(ii) Express *P* in the form $R\sin(\theta + \alpha) + 24$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

9 **Continued**

(iii) Given that the total perimeter of the fencing is 53 m, find the value of θ. **[3]**

(iv) Explain why the total length of the fencing will never exceed a certain value and state this value. **[2]**

10



The diagram shows part of the curve $y = \dfrac{5}{2x-1}$. The line $9y = -10x + 55$ intersects the curve at points *A* and *B* and meets the *x*-axis at point *C*.

(i) Find the *x*-coordinates of points *A*, *B* and *C*. **[4]**

(ii) Find the area of the shaded region. **[5]**

END-OF-PAPER