2021 PRELIMINARY EXAMINATION
SECONDARY 4E
AMATH PAPER 2 – MARK SCHEME

1	**(i)**	$\frac{d}{dx}\left(x^2 \tan x\right)$ $= x^2 \frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}\left(x^2\right)$ $= x^2 \sec^2 x + 2x \tan x$	 M1 A1
	(ii)	$\int_0^{\frac{\pi}{4}} \left(2x^2 \sec^2 x + 4x \tan x + 4\sin x\right) dx$ $= 2\int_0^{\frac{\pi}{4}} \left(x^2 \sec^2 x + 2x \tan x + 2\sin x\right) dx$ $= 2\left[x^2 \tan x - 2\cos x\right]_0^{\frac{\pi}{4}}$ $= 2\left\{\left(\frac{\pi}{4}\right)^2 \tan\frac{\pi}{4} - 2\cos\frac{\pi}{4} - \left(\frac{\pi}{4}\right)^2 \tan 0 + 2\cos 0\right\}$ $= 2.40527 \approx 2.41$	 M1 M1 M1 A1
			TOTAL:6m
2	**(i)**	$= \frac{\tan x + \tan 135^\circ}{1 - \tan x \tan 135^\circ} + \frac{\tan x - \tan 135^\circ}{1 + \tan x \tan 135^\circ}$ $= \frac{\tan x - 1}{1 + \tan x} + \frac{\tan x + 1}{1 - \tan x}$ $= \frac{-1 - \tan^2 x + 2\tan x + 1 + 2\tan x + \tan^2 x}{1 - \tan^2 x}$ $= \frac{4\tan x}{1 - \tan^2 x}$ (proven)	M1 M1 A1
	(ii)	$\tan\left(x + 135^\circ\right) + \tan\left(x - 135^\circ\right) = 6\tan x$ For $0^\circ < x < 360^\circ$ $\frac{4\tan x}{1 - \tan^2 x} = 6\tan x$ $4\tan x = 6\tan x(1 - \tan^2 x)$ $2\tan x[2 - (3 - 3\tan^2 x)] = 0$ $2\tan x(-1 + 3\tan^2 x) = 0$ $\therefore$ 2tan x = 0 or $\tan^2 x = \frac{1}{3}$	 M1 M1 M1

		$x = 0^\circ, 30^\circ, 150^\circ, 180^\circ, 210^\circ, 330^\circ$ Since $0^\circ \le x \le 180^\circ$, so $x = 0^\circ, 30^\circ, 150^\circ, 180^\circ$	A1
			TOTAL: 7m
3	**(i)**	Let $x = \frac{3}{2}$, $2\left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)^2 + 2\left(\frac{3}{2}\right) - 3 = 0$ by Factor Theorem, $(2x-3)$ is a factor of $2x^3 - 3x^2 + 2x - 3$ (shown) $2x^3 - 3x^2 + 2x - 3 = (2x-3)(x^2 + bx + 1)$ By comparing coefficient of x $b = 0$ $\therefore 2x^3 - 3x^2 + 2x - 3 = (2x-3)(x^2+1)$	M1 M1 – by comparing coefficient/Long division A1
	(ii)	$\frac{8x^2 - 11x + 5}{2x^3 - 3x^2 + 2x - 3} = \frac{A}{(2x-3)} + \frac{Bx + C}{(x^2+1)}$ $8x^2 - 11x + 5 = A(x^2+1) + (Bx + C)(2x-3)$ when $x = \frac{3}{2}$ $8\left(\frac{3}{2}\right)^2 - 11\left(\frac{3}{2}\right) + 5 = A\left[\left(\frac{3}{2}\right)^2 + 1\right]$ $\frac{13}{2} = \frac{13}{4}A$ $A = 2$ when $x = 0$ $5 = 2(1) + (c)(-3)$ $c - -1$ when $x = 1$ $2 = 2(2) - B + 1$ $B - 3$	M1 M1 M2 – find A, B and C M1 – only find 2 A1

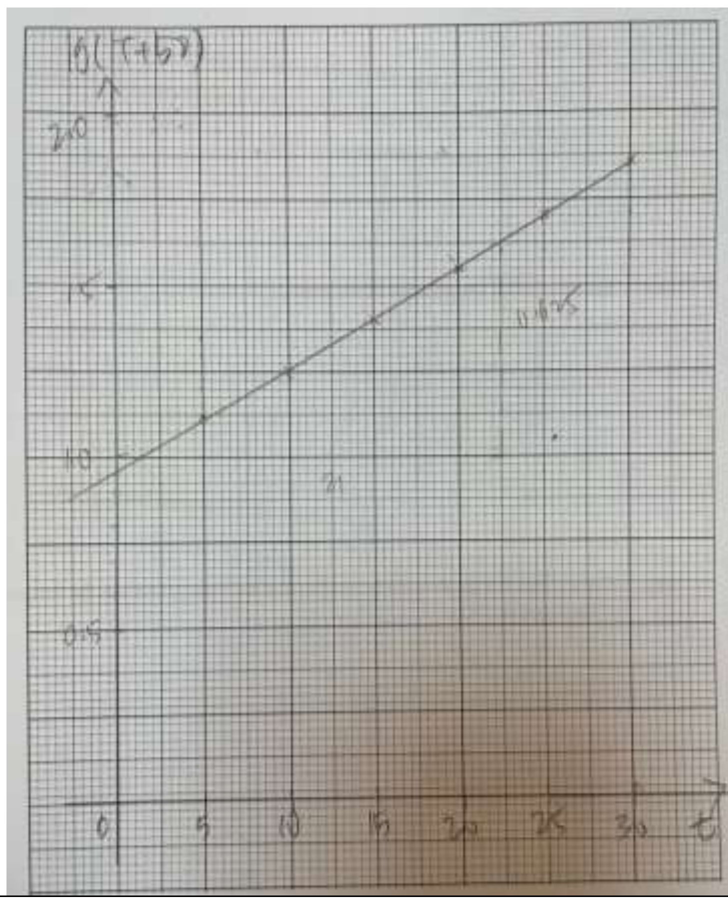
		$\frac{8x^2-11x+5}{2x^3-3x^2+2x-3}=\frac{2}{(2x-3)}+\frac{3x-1}{(x^2+1)}$	
			TOTAL: 8m
4	**(i)**	$\log_4 2y-\frac{\log_4(y-3)}{2}=3\log_4 2$ $\log_4 2y-\log_4(y-3)^{\frac{1}{2}}=\log_4 2^3$ $\log_4\frac{2y}{\sqrt{(y-3)}}=\log_4 8$ $\frac{2y}{\sqrt{(y-3)}}=8$ $2y=8\sqrt{(y-3)}$ $4y^2-64y+192=0$ $y^2-16y+48=0$ $(y-4)(y-12)=0$ $y=4\quad or\quad y=12$	M1 – power law M1 – quotient law M1 A1
	(ii)	$\log_{27} z=\log_9\sqrt{y}$ $\frac{\log_3 z}{\log_3 27}=\frac{\log_3\sqrt{y}}{\log_3 9}$ $\frac{\log_3 z}{\log_3 3^3}=\frac{\log_3\sqrt{y}}{\log_3 3^2}$ $\frac{\log_3 z}{3\log_3 3}=\frac{\log_3\sqrt{y}}{2\log_3 3}$ $\frac{\log_3 z}{3}=\frac{\log_3\sqrt{y}}{2}$ $\log_3 z=\frac{3}{2}\log_3 y^{\frac{1}{2}}$ $\log_3 z=\log_3 y^{\frac{1}{2}\times\frac{3}{2}}$ $z=y^{\frac{3}{4}}$	M1 – change of base law M1 A1
			TOTAL: 7m
5	**(a)**	$2x^2+kx-3x+10=x+2$ $2x^2+kx-4x+8=0$ $2x^2+(k-4)x+8=0$	M1

		For 2 points, Discriminant > 0 $(k-4)^2-4(2)(8)>0$ $k^2-8k-48>0$ $(k-12)(k+4)>0$ $k<-4$ or $k>12$	M1 A1
	(bi)	$y=2x^2-10x+c$ $y=2\left[x^2-5x+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2\right]+c$ $y=2\left(x-\frac{5}{2}\right)^2+\frac{2c-25}{2}$	M1 A1
	(bii)	$\frac{2c-25}{2}=-\frac{19}{2}$ $2c=-19+25$ $c=3$	M1 A1
			TOTAL: 8m
6	**(ai)**	$f(x)=\frac{d}{dx}\left(\cos x+\frac{1}{2}\sin 2x+4\right)$ $=-\sin x+\frac{1}{2}(2)\cos 2x$ $=\cos 2x-\sin x$ $=1-\sin x-2\sin^2 x$	M1 A1
	(aii)	$f(x)=\cos 2x-\sin x$ $f'(x)=-2\sin 2x-\cos x$ When $x=\frac{\pi}{6}$,	M1 M1

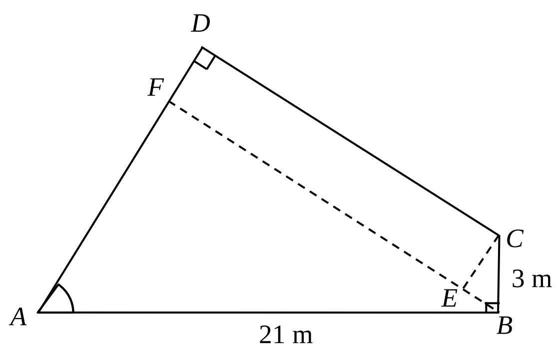
		$f'(\frac{\pi}{6}) = -2\sin 2\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{6}\right)$ $= -2\sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{6}\right)$ $= -2\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2}$ $= -\frac{3\sqrt{3}}{2}$ Gradient of tangent = $-\frac{3\sqrt{3}}{2}$ Gradient of normal $= \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$ $f(\frac{\pi}{6}) = \cos 2\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)$ $= \cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right)$ $= \frac{1}{2} - \frac{1}{2}$ $= 0$ Point $\left(\frac{\pi}{6}, 0\right)$ Equation of normal $y = \frac{2\sqrt{3}}{9}x + c$ $0 = \frac{2\sqrt{3}}{9}\left(\frac{\pi}{6}\right) + c$ $c = -\frac{\sqrt{3}\pi}{27}$ $y = \frac{2\sqrt{3}}{9}x - \frac{\sqrt{3}\pi}{27}$ or $27y = 6\sqrt{3}x - \sqrt{3}\pi$ or $y = \frac{2}{3\sqrt{3}}x - \frac{\pi}{9\sqrt{3}}$	M1 M1 M1 A1(accept non-rationalised surd answer)
	(b)	Given $g''(x) = 4\cos 2x + 2$	

		$g'(x)=\int(4\cos 2x+2)\,dx$ $=\dfrac{4\sin 2x}{2}+2x+c$ $=2\sin 2x+2x+c$ $g'\left(\dfrac{\pi}{6}\right)=2\sin 2\left(\dfrac{\pi}{6}\right)+2\left(\dfrac{\pi}{6}\right)+c$ $\sqrt{3}=2\sin\left(\dfrac{\pi}{3}\right)+\dfrac{\pi}{3}+c$ $=2\left(\dfrac{\sqrt{3}}{2}\right)+\dfrac{\pi}{3}+c$ $c=-\dfrac{\pi}{3}$ $g'(x)=2\sin 2x+2x-\dfrac{\pi}{3}$	M1 M1 M1 A1
			TOTAL: 12m
7	**(i)**		Table of values – B1 Plot all points – B1 Straight line drawn – B1

t	5	10	15	20	25	30
T	1.114	1.255	1.398	1.544	1.7	1.857



	(ii)	$T = Ak^t - 50$ $\lg(T+50) = \lg Ak^t$ $\lg(T+50) = \lg A + t\lg k$ Gradient, $\lg k = \frac{0.9}{30}$ $= 0.03\ (\pm 0.005)$ $k = 10^{0.03}$ $= 1.07151$ $= 1.07$ (3 s.f.) $\lg(T+50)$ -intercept, $\lg A = 0.95(\pm 0.05)$ $A = 10^{0.95}$ $= 8.9125$ $= 8.91$ (3 s.f.)	M1 M1 M1 M1 M1
	(iii)	From (ii), When $t = 0$, $\lg(T+50) = 0.95(\pm 0.05)$ $T + 50 = 8.9125$ $T = -41.0875$ $= -41.1°C$ (1 d.p.)	B1
	(iv)	When T = 0, $\lg(T+50)$ = 1.6989 = 1.7 (1 d.p.) Read off from the graph where $\lg(T+50)$ = 1.7, and the value of t can be obtained	B1
			TOTAL: 10m
8	**(i)**	$x^2 + y^2 - 8x + 4y + 11 = 0$ $x^2 - 8x + 16 + y^2 + 4y + 4 = -11 + 16 + 4$ $(x-4)^2 + (y+2)^2 = 3^2$ Centre of the circle is $(4, -2)$ and Radius = 3 units	M1 A1 A1
	(ii)	$k = -2 + 3 = 1 \quad or$ $k = -2 - 3 = -5$	A1 A1

	(iii)	$(5-y)^2+y^2-8(5-y)+4y+11=0$ $25-10y+y^2+y^2-40+8y+4y+11=0$ $2y^2+2y-4=0$ $y^2+y-2=0$ $y=1$ or $y=-2$ Sub $y=1 \quad \therefore x=4 \quad \Rightarrow A(4,1)$ Sub $y=-2 \quad \therefore x=5+2=7 \quad \Rightarrow B(7,-2)$ Midpoint $=\left(\frac{4+7}{2},\frac{1-2}{2}\right)=\left(\frac{11}{2},\frac{-1}{2}\right)$ Perpendicular distance from C to line $=\sqrt{\left(4-\frac{11}{2}\right)^2+\left(-2+\frac{1}{2}\right)^2}$ $=\sqrt{4.5}$ or 2.12 units	M1 M1 M1 M1 M1 M1 A1
			TOTAL: 12m
9	**(i)**	Add in *EF*, parallel to *CD*  $\cos\theta=\frac{AF}{21}\Rightarrow AF=21\cos\theta$ $\angle CBE=90°-(90°-\theta)=\theta$ $\sin\theta=\frac{CE}{3}\Rightarrow CE=3\sin\theta$ $DF=CE=3\sin\theta$ $AD=3\sin\theta+21\cos\theta$ $\sin\theta=\frac{BF}{21}\Rightarrow BF=21\sin\theta$	 M1 M1

		$\cos\theta = \frac{BE}{3} \Rightarrow BE = 3\cos\theta$ $DC = FE = 21\sin\theta - 3\cos\theta$ $P = AB + BC + DC + DF + AF$ $= 21 + 3 + (21\sin\theta - 3\cos\theta) + 3\sin\theta + 21\cos\theta$ $P = 24 + 24\sin\theta + 18\cos\theta.$ (shown)	M1 A1
	(ii)	Let $24\sin\theta + 18\cos\theta = R\sin(\theta + \alpha)$ $R = \sqrt{24^2 + 18^2} = 30$ $\tan\alpha = \frac{18}{24}$ $\alpha = 36.86989° = 36.9°$ $(to\ 1\ dp)$ $P = 30\sin(\theta + 36.9°) + 24$	M1 M1 A1
	(iii)	$P = 30\sin(\theta + 36.86989°) + 24 = 53$ $\sin(\theta + 36.86989°) = \frac{29}{30}$ $\theta + 36.86989° = 75.1649°$ $\theta = 38.295°$ $\theta = 38.3°$	M1 M1 A1
	(iv)	$P = 30\sin(\theta + 36.86989°) + 24$ Max of $\sin(\theta + 36.86989°) = 1$ Max L = 30(1) + 24 = 54 m The total length of fencing needed will not exceed 54 m.	M1 A1
			TOTAL: 12m
10	**(i)**	$\frac{5}{2x-1} = \frac{-10x+55}{9}$ $45 = (2x-1)(-10x+55)$ $20x^2 - 120x + 100 = 0$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $\therefore\ x = 1$ or $x = 5$	M1 M1 M1

		When $y=0$, $-10x+55=0$ $x=\frac{11}{2}$ or 5.5 ∴ x-coordinate of A is 1 ∴ x-coordinate of B is 5 ∴ x-coordinate of C is 5.5	A1 – for matching the x values to the right point.
	(ii)	Area of shaded region $=\int_1^5 \frac{5}{2x-1}\,dx+\frac{1}{2}\times 0.5\times\frac{5}{9}$ $=5\left[\frac{\ln(2x-1)}{2}\right]_1^5+\frac{5}{36}$ $=\frac{5}{2}\left[\ln 9-\ln 1\right]+\frac{5}{36}$ $=2.8854$ (3sf) $= 2.89$ (3sf)	M1, M1 M1 M1 A1
			TOTAL: 9m