Name:________________________________ ( )	Class: Sec ________

90

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2021

Additional Mathematics 4049/01

Paper 1 31 August 2021

Secondary 4 Express **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Question	Q1	Q2	Q3	Q4	Q5	Q6	Q7		Q8	Q9	Q10	Q11		Q12
							i,ii	iii				i,ii	iii,iv	
Marks														

No of additional booklets/ writing paper used		No of additional graph paper used	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Area of $\triangle = \frac{1}{2}bc\sin A$

Answer all the questions.

1 The curve $y = \ln\left(x^2 e^{-3x}\right)$ has a stationary point.

(i) Find the x-coordinate of the stationary point. **[3]**

(ii) Find $\dfrac{d^2y}{dx^2}$. **[2]**

(iii) Determine the nature of the stationary point. **[1]**

2 Water is poured into an inverted cone. The radius of its circular base is $3x$ cm and its vertical height is $2x \quad 1$ cm.

(i) Express the volume of the cone in terms of x. **[2]**

(ii) If the water is poured into the container at a constant rate of $100\ \text{cm}^3\text{s}^{-1}$, calculate the rate of change of x when $x = 5$. **[4]**

3 The equation of the curve is given by $y = 2x^2 + 5x + c$, where c is a constant.

(i) Find the range of values of c such that $y \geq 0$. **[3]**

(ii) Given that $c = 3$, find the values of m such that $y = mx + 1$ is a tangent to the curve. **[3]**

4 It is given that $y = \mathrm{f}(x) = xe^x$ and that $\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n} = \mathrm{f}^{(n)}(x)$.

(a) Find, in terms of *x*,

(i) $\mathrm{f}^{(1)}(x)$ **[2]**

(ii) $\mathrm{f}^{(2)}(x)$ **[1]**

(b) Hence, write down, in terms of *n* and *x*, $\mathrm{f}^{(n)}(x)$. **[1]**

(c) Show that $\mathrm{f}^{(n)}(x) - \mathrm{f}^{(n-1)}(x)$ is always positive, for all values of *n*. **[2]**

5 **Do not use a calculator for this whole question.**

It is given that *A* and *B* are angles in the same quadrant such that

$\cos A = \frac{5}{13}$ and $\tan B = -\frac{24}{7}$.

(a) Find the values of

(i) $\cos(A+B)$, **[3]**

(ii) $\tan(A+B)$. **[2]**

(b) Use your answers in **(a)** to deduce which quadrant the angle $(A+B)$ must lie in. Explain your answer. **[2]**

6 **(i)** Express $\left(5-\sqrt{5}\right)^2-\frac{5}{2-\sqrt{5}}$, in the form of $p+q\sqrt{5}$, where *p* and *q* are constants to be found. **[4]**

(ii) Solve $e^{2x+1}-e^{x+2}=0$. **[3]**

7 It is given that $f(x) = a\tan(bx)$ where *a* and *b* are positive integers and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

(i) The graph $y = f(x)$ passes through $\left(\frac{\pi}{4}, 0\right)$ and $\left(\frac{\pi}{16}, \frac{1}{2}\right)$.

Show that the smallest possible value of *b* is 4 and find the value of *a*. **[2]**

(ii) Sketch the graph $y = f(x)$. **[3]**

(iii) Find the exact value of the gradient of the tangent at $x = \frac{\pi}{4}$. **[2]**

8 There is a virus outbreak in a small town of 10000 residents. The spread of the virus through the town is modelled by $N = \dfrac{10000}{2 + 4998\mathrm{e}^{-0.2t}}$, where N is the number of residents infected after t days. An emergency will be declared if 20% of the residents have been infected.

(a) Find the

(i) number of residents who contracted the virus at the start of the outbreak, **[1]**

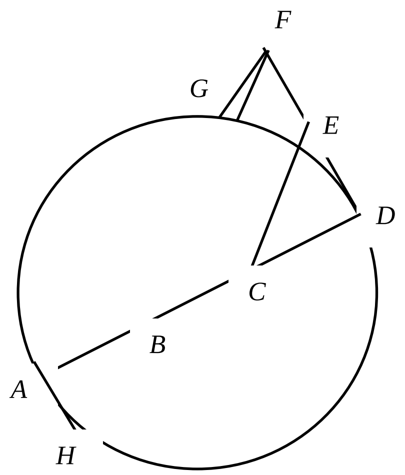
(ii) number of residents infected after 5 days, rounding off your answer to the nearest integer, **[2]**

(iii) number of days passed before an emergency is declared. **[3]**

8 Continued

(b) A health expert claims that the virus will not infect the entire population.
Do you agree with the health expert? Explain your answer. **[2]**

9



In the diagram above, $AB = BC = CD$ and E is a midpoint of DF which is a tangent to the circle. The line HA is another tangent to the circle.

(a) Show that CE is parallel to BF. **[1]**

(b) If AD is the diameter of the circle,

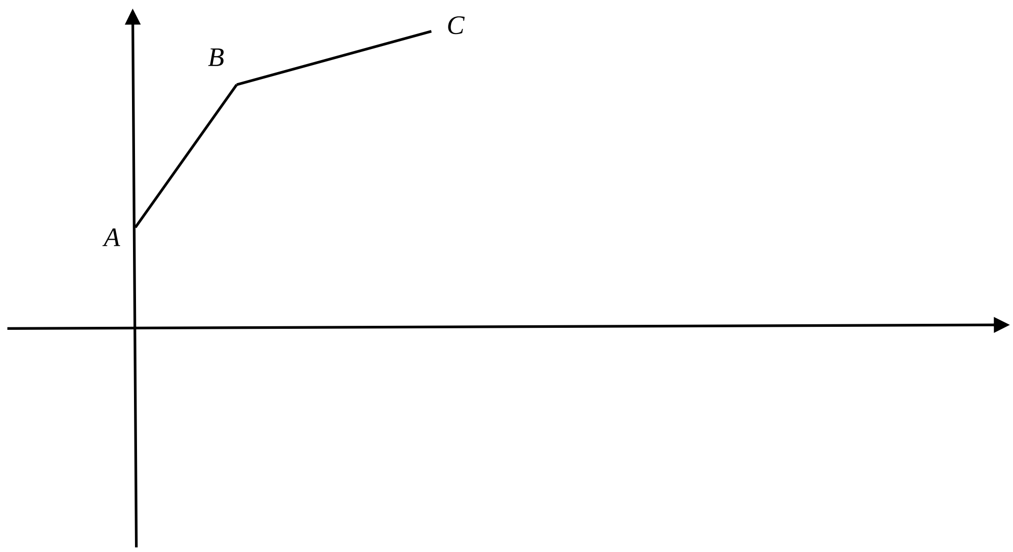
(i) explain why $\angle AGD = 90°$; **[1]**

(ii) prove that $DG = \sqrt{\left(\frac{BD \times DE}{CD} + FG\right)\left(\frac{BD \times DE}{CD} - FG\right)}$. **[3]**

9 **Continued**

(c) If AD is **not** the diameter of the circle, compare the angles HAD and AFD.
Explain your answer. **[2]**

10 The diagram shows three vertices of a **kite** *ABCD* where *A*(0, 5), *B*(8, 14) and *C*(20,15). *AB* = *BC*. The last vertex of the kite, *D*, has coordinates *(h, k)* where *h* and *k* are integers.



(a) When $k = 0$,

(i) find the area of the kite, and **[7]**

10 **Continued**

(a) **(ii)** show that *ABCD* is **not** a rhombus. **[2]**

(b) Find the coordinates of *D* if *ABCD* **is** a rhombus. **[1]**

11 **(i)** Explain why all the non-constant terms in the expansion of $\left(1-2x^2\right)^6$ have even powers of *x*. **[2]**

(ii) Find the coefficient of x^4 in the expansion of $\left(2+3x^2\right)\left(1-2x^2\right)^6$. **[3]**

11 **Continued**

(iii) Differentiate, with respect to x, $\left(2+3x^2\right)\left(1-2x^2\right)^6$. **[3]**

(iv) Using your answers in **(ii)** and **(iii)**, find the coefficient of x^3 in the expansion of $x\left(1+2x^2\right)\left(1-2x^2\right)^5$. **[2]**

12 A cyclist starts from a point *A* and travels in a straight line until he comes to a rest at a point *B*. During the motion, his velocity, $v\ \text{ms}^{-1}$, is given by $v=-1-t+\sqrt{4t+9}$, where t is the time in seconds after leaving *A*.

Find the

(i) initial velocity, **[1]**

(ii) time taken to travel from *A* to *B*, **[3]**

12 Continued

(iii) distance *AB*, **[4]**

(iv) acceleration when $t = 2$. **[2]**

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