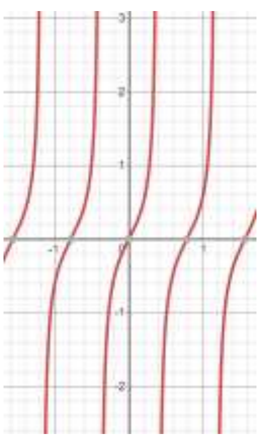
2021 PRELIMINARY EXAMINATION
SECONDARY 4E
AMATH PAPER 1 – MARK SCHEME

1	**(i)**	**EITHER** $y = 2\ln x - 3x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{x} - 3$ $\frac{2}{x} - 3 = 0$ $x = \frac{2}{3}$ **OR** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x\mathrm{e}^{-3x} - 3x^2\mathrm{e}^{-3x}}{x^2\mathrm{e}^{-3x}}$	M1- (also see OR) M1 A1
	(ii)	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{2}{x^2}$	M1- $-\frac{2}{x^2} + k$ (where k may not be 0) A1
	(iii)	When $x = \frac{2}{3}$, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -4.5 < 0$ Maximum point	B1
			TOTAL:6m
2	**(i)**	$V = \frac{1}{3}\pi(3x)^2(2x-1)$ $V = 3\pi x^2(2x-1)$ or $6\pi x^3 - 3\pi x^2$	M1 A1
	(ii)	$\frac{\mathrm{d}V}{\mathrm{d}x} = 3\pi\left[2x(2x-1) + x^2(2)\right]$ $= 3\pi\left[6x^2 - 2x\right]$ $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t}$ $100 = 3\pi\left[6(5)^2 - 2(5)\right] \cdot \frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.0758 \text{ cms}^{-1}$	M1- product rule or 1 of the 2 terms correct A1 M1 A1
			TOTAL: 6m

3	**(i)**	$2x^2+5x+c\geq 0$ $D=5^2-4(2)c\leq 0$ $c\geq \frac{25}{8}$	M1, M1 A1
	(ii)	$2x^2+5x+3=mx+1$ $2x^2+(5-m)x+2=0$ $D=(5-m)^2-4(2)(2)=0$ $m=1$ or $m=9$	M1 M1- D = 0 (Attempt to find D, but may be incorrect) A1
			TOTAL: 6m
4	**(a)(i)**	$\mathrm{f}^{(1)}(x)=xe^x+e^x$ or $(x+1)e^x$	M1- product and differentiate e^x A1
	(a)(ii)	$\mathrm{f}^{(2)}(x)=xe^x+2e^x$ or $(x+2)e^x$	B1- follow through their answer in (i)
	(b)	$\mathrm{f}^{(n)}(x)=xe^x+ne^x$ or $(x+n)e^x$	B1
	(c)	$\mathrm{f}^{(n)}(x)-\mathrm{f}^{(n-1)}(x)$ $=(x+n)e^x-(x+n-1)e^x$ $=e^x>0$	M1 A1
			TOTAL: 6m
5	**(i)**	$\cos(A+B)$ $=\cos A\cos B-\sin A\sin B$ $=\left(\frac{5}{13}\right)\left(\frac{7}{25}\right)-\left(-\frac{12}{13}\right)\left(-\frac{24}{25}\right)$ $=-\frac{253}{325}$	M1-formula plus substitution of some values seen M1- 2 of the 3 values are correct A1
	(ii)	$\tan(A+B)$ $=\frac{\tan A+\tan B}{1-\tan A\tan B}$ $=\frac{\left(-\frac{12}{5}\right)+\left(-\frac{24}{7}\right)}{1-\left(-\frac{12}{5}\right)\left(-\frac{24}{7}\right)}$ $=\frac{204}{253}$	M1 A1
	(iii)	$\tan(A+B)>0;\cos(A+B)<0$ Quadrant 3	M1 A1

			TOTAL: 7m
6	**(i)**	$\left(5-\sqrt{5}\right)^2-\frac{5}{2-\sqrt{5}}$ $=25-10\sqrt{5}+5-\frac{5\left(2+\sqrt{5}\right)}{4-5}$ $=30-10\sqrt{5}+10+5\sqrt{5}$ $=40-5\sqrt{5}$	M1- 30-10 $\sqrt{5}$ M1- rationalize M1- 10+5 $\sqrt{5}$ A1
	(ii)	$e^{x+1}\left(e^x-e^1\right)=0$ $e^x-e^1=0$ $e^x=e^1$ $x=1$	M1 M1 A1
			TOTAL: 7m
7	**(i)**	$a\tan\left(b\frac{\pi}{4}\right)=0$ $b\frac{\pi}{4}=\pi$ $b=4$ $a\tan\left(4\frac{\pi}{16}\right)=\frac{1}{2}$ $a=\frac{1}{2}$	B1 B1
	(ii)		B1- shape of tangent B1- number of cycles B1- asymptotes

	(iii)	$\frac{dy}{dx} = \frac{1}{2}(4)\sec^2 4x$ $\frac{dy}{dx} = 2\sec^2 4\left(\frac{\ }{4}\right)$ $= 2$	M1 A1
			TOTAL: 7m
8	**(a)(i)**	$N = \frac{10000}{2 + 4998e^0} = 2$	B1
	(a)(ii)	$N = \frac{10000}{2 + 4998e^{-1}}$ ≈ 5	M1 A1
	(a)(iii)	$\frac{10000}{2 + 4998e^{-0.2t}} \geq 2000$ $4000 + 9996000e^{-0.2t} \leq 10000$ $e^{-0.2t} \leq \frac{1}{1666}$ $-0.2t \leq \ln\left(\frac{1}{1666}\right)$ $t \geq 37.09$ 37 days have passed.	 M1 M1- apply logarithm A1
	(b)	When t is large, $N \approx \frac{10000}{2 + 4998(0)}$ $= 5000$ **Agree** with health expert as at most half the population would be infected.	 M1 A1
			TOTAL: 8m
9	**(a)**	Since C and E are midpoints of BD and DF respectively, by Mid-Point Theorem, CE is parallel to DF.	B1
	(b)(i)	Right angle in a semi-circle	B1

	(b)(ii)	$\frac{DF}{DE}=\frac{DB}{DC}$ $DF=\frac{DB\times DE}{DC}$ $DG^2=DF^2-FG^2$ $DG=\sqrt{DF^2-FG^2}$ $DG=\sqrt{(DF+FG)(DF-FG)}$ $DG=\sqrt{\left(\frac{DB\times DE}{DC}+FG\right)\left(\frac{DB\times DE}{DC}-FG\right)}$	B1- similar triangles B1- Pythagoras' theorem B1
	(c)	By Alternate Segment Theorem, $\angle HAD=\angle AGD$ $\angle AGD>\angle AFD$ $\angle HAD>\angle AFD$	B1 B1
			TOTAL: 7m

10	**(a)(i)**	Grad of $AC=\frac{1}{2}$ Grad of $BD=-2$ $y=-2x+c$ $14=-2(8)+c$ $c=30$ $y=-2x+30$ $0=-2h+30$ $h=15$ $D(15,0)$ Area $=\frac{1}{2}\begin{vmatrix}0 & 8 & 20 & 15 & 0\\5 & 14 & 15 & 0 & 5\end{vmatrix}$ $=175$ units^2	M1 M1 M1 A1 A1 M1 A1
	(a)(ii)	EITHER $M_{AC}=(10,10)$ $M_{BD}=(11.5,22)$ Since midpoints are not the same, *ABCD* is not a rhombus. OR $l_{AB}=\sqrt{64+81}=\sqrt{125}$ $l_{AD}=\sqrt{225+25}=\sqrt{250}$ Since the lengths are not the same, *ABCD* is not a rhombus.	M1- one correct A1 M1- one correct A1
	(b)	$M_{BD}=(10,10)$ $D(12,6)$	B1
			TOTAL: 10m
11	**(i)**	$T_{r+1}=\binom{6}{r}\left(-2x^2\right)^r$ $=\binom{6}{r}(-2)^r\left(x^{2r}\right)$ Since 2r is always even, non-constant terms have even powers of *x*.	M1 A1
	(ii)	$2\binom{6}{2}(-2)^2+3\binom{6}{1}(-2)$ $=84$	M1, M1 A1

	(iii)	$\left(2+3x^2\right)\left[6(-4x)(1-2x^2)^5\right]+6x(1-2x^2)^6$ $=6x(1-2x^2)^5\left[-4\left(2+3x^2\right)+1-2x^2\right]$ $=6x(1-2x^2)^5\left[-7-14x^2\right]$ $=-42x(1-2x^2)^5\left[1+2x^2\right]$	M1, M1 A1 (o.e)
	(iv)	Coeff of x^4 in $\left(2+3x^2\right)(1-2x^2)^6=84$ Differentiating Coeff of x^3 in $-42x(1-2x^2)^5\left[1+2x^2\right]=4(84)$ Coeff of x^3 in $x(1-2x^2)^5\left[1+2x^2\right]=\dfrac{4(84)}{-42}=-8$	M1 A1
			TOTAL: 10m
12	**(i)**	$t=0, v=2$	B1
	(ii)	$v=0$ $1+t=\sqrt{4t+9}$ $1+2t+t^2=4t+9$ $t^2-2t-8=0$ $(t-4)(t+2)=0$ $t=4$ or $t=-2$ (rej)	M1 M1- square both sides A1
	(iv)	$s=\int -1-t+\sqrt{4t+9}\,\mathrm{d}t$ $=-t-\dfrac{t^2}{2}+\dfrac{2}{3}(4t+9)^{3/2}+c$ $0=\dfrac{2}{3}(9)^{3/2}+c$ $c=-18$ $t=4, s=53.3$ m	M1- first 2 terms correct M1- 3rd term correct A1- value of *c* A1
	(iii)	$a=-1+\dfrac{1}{2}(4)(4t+9)^{-1/2}$ $t=2.\ a=-0.515\ \text{ms}^{-2}$	M1 A1
			TOTAL: 10m